UNIVERSITÀ DI PISA

Master of Science in Computer Engineering

Project Report
The Christmas shop

Performance Evaluation of Computer Systems and Networks

R. Xefraj, E. Casapieri, A. Di Donato

Academic Year 2019/2020

Contents

Chapter 1

Introduction

1.1 Problem description

Consider a large store with a number of tills equal to n. Each cashier serves customers at the checkout, with a certain checkout rate μ_{c}, according to a FIFO policy. The customers may wish to wrap their purchase with a certain probability. There are two possible approaches, either to convert a number of k tills to wrapping tills or to have the wrapping done directly in the same tills where the checkout takes place. The wrapping will be processed with a certain wrap rate μ_{w} and the tills will adopt a FIFO policy in this case too.

1.2 Objectives

The main objective of this project is to select the best policy between the two possible, taking into account the specific needs of the store. In particular:

- The total number of tills.
- The interrarrival rate.
- The check-out rate.
- The gift-wrap rate.
- The probability a customer will request to gift-wrap the purchased goods.

1.3 Performance indexes

To fulfilled the objectives, the following performance indexes will be taken into consideration:

- The queuing and response time for customers who want to checkout only and for those who also want to wrap the purchased goods.
- Fairness of the response time for the checkout only costumers and checkout and wrap costumers.

1.4 Scenarios

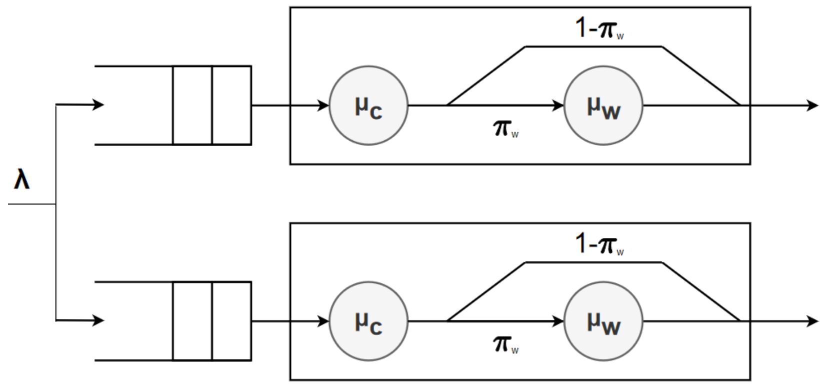
These are the scenarios that have been taken into consideration:

- Uniform distribution of interarrival, checkout and gift-wrap times;
- Exponential distribution of interarrival, checkout and gift-wrap times;

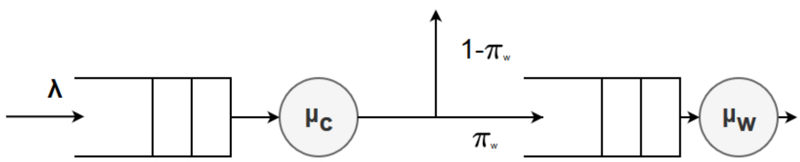
Chapter 2

Model

2.1 Description



(a) Model with Policy 1



(b) Model with Policy 2

Figure 2.1: Policies

Figure 2.1 shows the models of the two possible policies where:

- λ is the average interarrival rate for customers;
- μ_{c} is the average service rate of the checkout till at one till;
- μ_{w} is the average service rate of the wrap till;
- π_{w} is the probability that a customer requests to wrap a purchase;

2.2 Assumptions

During the system modeling the following assumptions were made:

- There is no fixed length on the queues, so there is no maximum number of customers in each of them.
- All queues, as previously mentioned, use FIFO policy.
- There are no differences in term of service between tills in the same scenario analysis (Every checkout till has the same checkout rate and every wrapping till has the same wrapping rate).
- A costumer cannot change his position in the queue.
- A costumer cannot change queue.

- A customer decides whether or not to wrap his purchase before the checkout; this, regardless of the queue in each tills.
- A customer in the queue cannot leave the queue until it has been served.
- The costumers selects the tills by chance.

2.3 Factors

The system, both in policy 1 and policy 2, is affected by the following factors:

- checkout rate μ_c
- wrapping rate μ_w
- wrapping probability π_w
- interarrival rate of customers λ

2.4 Exponential scenario - Theoretical analysis

To have realistic results we selected the checkout rate and wrapping rate in a range to match the real rates of cashiers. Since we must grant stability in our simulations the only factor we have the power to control is the inter-arrival rate. We theoretically analyzed both polices for exponential distribution.

2.4.1 Policy 1

The Policy 1 is composed by N M/Cox/1 queue for each till the stability condition is:

$$\rho = \frac{\lambda}{n_{tills}} (\sum_{j=1}^{m} \frac{\prod_{k=1}^{j-1} p_k}{\mu_j}) < 1$$

$$\lambda < \frac{1}{\sum_{j=1}^{m} \frac{\prod_{k=1}^{j-1} p_k}{\mu_j}} n_{tills}$$

2.4.2 Policy 2

Taken the system following the *Policy 2* and given the total system input λ:
since Jackson's Theorem hypothesis holds, if:

$$\forall i: \ \rho_i = \frac{\lambda_i}{\mu_i} < 1$$

ours OJN admits a product form:

$$\rho_{\underline{n}} = \rho(n_1, ..., n_m) = \prod_{i=1}^{m} p_i(n_i)$$

Let's compute the interarrival rates and utilizations for checkout tills and wrap tills starting from the total interarrival rate:

$$\lambda_c = \frac{\lambda}{n_{checkout}}, \quad \rho_c = \frac{\lambda}{n_{checkout} \cdot \mu_c}$$

$$\lambda_w = \frac{\lambda \cdot \pi_w}{n_{wrap}}, \quad \rho_w = \frac{\lambda \cdot \pi_w}{n_{wrap} \cdot \mu_w}$$

Now we can compute the stability condition of each till:

$$\begin{cases} \dfrac{\lambda}{n_{\text{checkout}}} \cdot \dfrac{1}{\mu_{\text{c}}} < 1 \;\} \text{Stability condition checkout M/M/1} \\ \\ \lambda \cdot \dfrac{\pi_{\text{w}}}{n_{\text{wrap}}} \cdot \dfrac{1}{\mu_{\text{w}}} < 1 \end{cases}$$

$$\begin{cases} \lambda < n_{\text{checkout}} \cdot \mu_{\text{c}} \\ \\ \lambda < \dfrac{n_{\text{wrap}} \cdot \mu_{\text{w}}}{\pi_{\text{w}}} \end{cases}$$

So our system is stable iff every till in our system is stable, and this condition is true if the total interarrival rate of our system is:

$$\lambda < min(\mu_{\text{c}} \cdot n_{\text{checkout}}, \frac{n_{\text{wrap}} \cdot \mu_{\text{wrap}}}{\pi_{\text{wrap}}})$$

Since every till of the system is an M/M/1 we have:

$$p_{\text{i}}(n_{\text{i}}) = (1 - \rho_{\text{i}}) \cdot \rho_{\text{i}}^{n_{\text{i}}} \quad \forall n_{\text{i}}$$

$$p_{\underline{\text{n}}} = \prod_{i=1}^{m} (1 - \rho_{\text{i}}) \cdot \rho_{\text{i}}^{n_{\text{i}}}$$

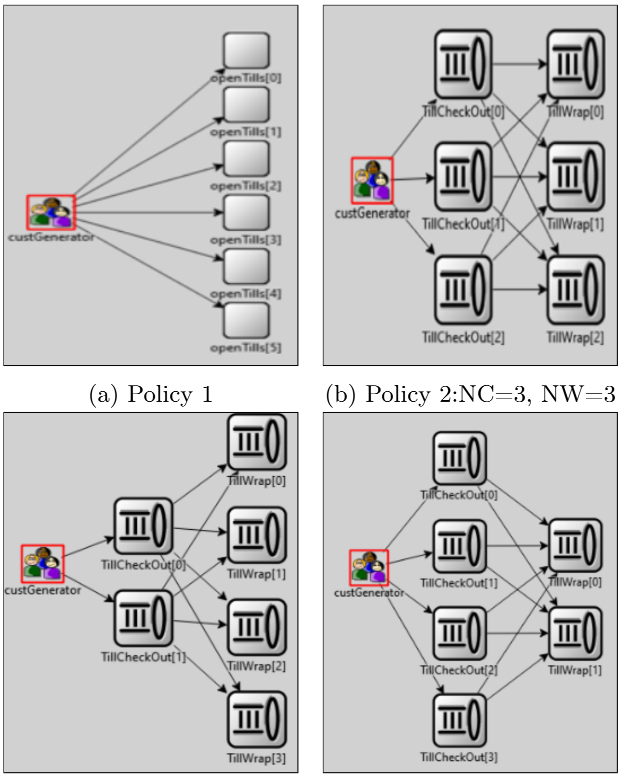
$$E[N] = \sum_{i=1}^{m} E[N_{\text{i}}] = \sum_{i=1}^{m} \frac{\rho_{\text{i}}}{1 - \rho_{\text{i}}}$$

$$E[R] = \frac{E[N]}{\lambda} \quad \text{(Little's Law)}$$

Chapter 3

Implementation

3.1 Code overview



(a) Policy 1 (b) Policy 2:NC=3, NW=3

(c) Policy 2:NC=2, NW=4 (d) Policy 2:NC=4, NW=2

Figure 3.1: Diagram of the Omnet++ network

The Omnet++ network diagram changes depending on the policy being considered. Specifically, within policy 1 there are 2 types of node:

- **Customer Generator (custGenerator)**: generates the customers using a configurable distribution.
- **Cashier (openTills)**: it takes care of the checkout of customer orders and also does the wrapping if required. The type of request made by each customer is determined in the *custGenerator* node.

Within policy 2, the node in charge of customer generation is still present; unlike policy 1, in this case a cashier can take care of either checkout or wrapping:

- **Cashier in charge of checkout (TillCheckout)**: it takes care of the chekout once the client has been generated. After the checkout is complete, if required (the choice is made within the customer generation node), the customer is directed to the wrapping till.
- **Cashier in charge of checkout (TillWrap)**: only customers who have requested wrapping are received here. Once finished within this node, customers leave the store.

3.2 Model Validation

In order to verify that the implementation exactly reflects our model, we carried out the following tests.

Test 1: Constant interarrival rate, checkout rate and wrapping rate, wrapping probability equal to 1 and 0

The object of this test is to check the correctness of the costumer's flow in the system. Requirements: all costumers should wrap the purchased goods, no queue should occur in our system, a fixed number of costumer should be served by the end of the simulation, each wrapping till has served the same number of costumers.

Test 2: Constant interarrival rate, checkout rate and wrapping rate, changing wrapping probability

The objective of this test is to check the correctness of the probabilistic distribution of the costumer in the wrapping tills. Still no queue should occur in the tills.

Test 3: Exponential interarrival rate, checkout rate and wrapping rate changing wrapping probability and till full utilization

The objective of this test is to try the correctness of the theoretical formula we calculated for the interarrival rate. We impose the utilization of each till to 1 (by imposing the interarrival rate), and expect the waiting and response time to never reach stability.

Test 4: Exponential interarrival rate, checkout rate and wrapping rate changing wrapping probability not full utilization

The objective of this test is to try the correctness of the theoretical formula we calculated for the interarrival rate. We impose the utilization of each till (by imposing the interarrival rate), and expect the waiting and response time reach stability after some time.

Chapter 4

Experiments

For the experimental analysis the following information have been considered:

- **Response/Waiting time Checkout** is measured for the customers that require only the checkout. For policy 1 the response time is taken from the tills when this service is done and the customer exit the shop whereas the waiting time is taken when that type of customer exit the queue. In policy 2 the response time is taken only from the checkout tills (first layer of tills) whereas the waiting time only when the customer exit the first queue.
- **Response/Waiting time Checkout and Wrap** is measured for the customers that require the checkout and wrap service. In the policy 1 the response time is taken from the tills when both services are done whereas the waiting time when that type of customer exit the queue. In policy 2 the response time is taken only from the wrapping tills (second layer of tills) and is the total time the costumer was in the shop. Whereas the waiting time is the time spent in both the queues (checkout and warp queue).
- **Cumulative response time Checkout and Wrap** is not a pure signal but has been obtained by merging the samples of the response checkout + response checkout and wrap (or waiting checkout + waiting checkout and wrap).

4.1 Warm-up and simulation time

In order to define the required warm-up period, we simulated for 10 times (with different seeds) our worst case scenarios for each policy:

Policy 1 - Exponential - 6 Tills	$\lambda = 1/25$, $\mu_{\mathrm{c}} = 1/30$, $\mu_{\mathrm{w}} = 1/40$, $\pi_{\mathrm{w}} = 80\%$
Policy 2 - Exponential - [2,3,4] Checkout Tills, [4,3,2] Wrap Tills	$\lambda = 1/25$, $\mu_{\mathrm{c}} = 1/30$, $\mu_{\mathrm{w}} = 1/40$, $\pi_{\mathrm{w}} = 80\%$
Policy 1 - Uniform - 6 Tills	$\lambda = 1/20$, $\mu_{\mathrm{c}} = 1/30$, $\mu_{\mathrm{w}} = 1/40$, $\pi_{\mathrm{w}} = 80\%$
Policy 2 - Uniform - [2,3,4] Checkout Tills, [4,3,2] Wrap Tills	$\lambda = 1/20$, $\mu_{\mathrm{c}} = 1/30$, $\mu_{\mathrm{w}} = 1/40$, $\pi_{\mathrm{w}} = 80\%$

From experimental results, the inter-arrival for the uniform distribution has been chosen smaller than the one in the exponential scenario since otherwise the utilization of each till was too small to infer significant results.
For each Uniform distribution, given the mean of the distribution [$\delta = \lambda$, μ_{c}, μ_{w}], the interval [a,b] has been chosen as following [a=$1/10\delta$, b=$19/10\delta$].
We took in account that the policy 1 and 2 could have different warm-up times, so we took the maximum warm-up time from all the scenarios. The warm-up has been estimated for the exponential distribution and uniform distribution of the RVs.
By comparing all the plots of the mean response times, the warm-up period has been selected 60000s for both distributions. After the estimation of the warm-up time new simulations has been executed in order to estimate the sample mean and variance without the warm-up interval. By using the obtained values it has been estimated that simulation time must be 200000s in order to have a 10% relative accuracy with 95% confidence.

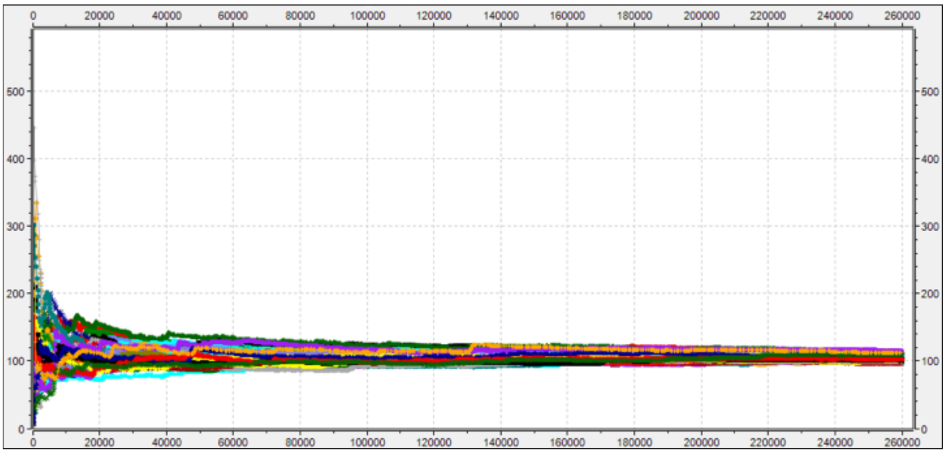


Figure 4.1: NTills= 6, $\lambda = 1/25$, $\mu_c = 1/30$, $\mu_w = 1/40$, $\pi_w = 80\%$, exponential distribution

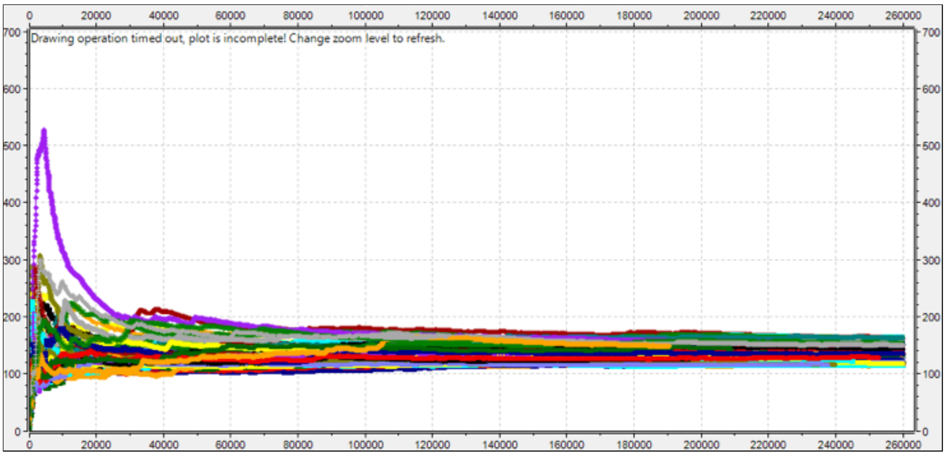


Figure 4.2: NC = [2,3,4], NW = [4,3,2], $\lambda = 1/25$, $\mu_c = 1/30$, $\mu_w = 1/40$, $\pi_w = 80\%$, exponential distribution

4.2 $2^k r$ Factorial Analysis

To see which are the most influencing factor of our simulations it has been performed a $2^k r$ (r = 10) factorial analysis for both policies of the cumulative response time. First policy factors:

- A = initialInterArrivalRate - Exponential: [1/25, 1/30] - Uniform: [1/20, 1/25]
- B = checkoutRate [1/25, 1/30]
- C = wrappingRate [1/35, 1/40]
- D = wrappingProbability [0.6, 0.8]

By plotting the QQ plot of the residuals the normal hypothesis has been tested, and the constant standard deviation hypothesis has been verified by plotting the residuals vs the avg predicted response time. The most influencing factor on the response time is the wrapping probability with a 42,72% impact in the exponential scenario and 44.15% in the uniform scenario. The second most influencing factor is the checkout rate with a 23.06% impact in the exponential scenario and 21.50% in the uniform scenario. As we expected the checkout rate is an important factor since both type of customer (checkout only/ checkout and wrap) have to request that service. The influnce of the Interarrival is 10.66% for Exponential distribution and 16.96% for the uniform one.
Since we must take care also of how many tills must be converted in gift-wrap points, in the second policy this must be taken in account as a factor.
Second policy factors:

- A = checkout only tills [4, 3] (over 6 total tills)
- B = initialInterArrivalRate Exponential: [1/25, 1/30] - Uniform: [1/20, 1/25]
- C = checkoutRate [1/25, 1/30]
- D = wrappingRate [1/35, 1/40]
- E = wrappingProbability [0.6, 0.8]

Also with this policy we have as a dominating factor the wrapping probability with a 48,69% impact in the exponential scenario and 40.27% in the uniform one.
The number of checkout only tills (A) has an influence of 8.18% in the exponential scenario and 9.42% in the Uniform. The number of checkout only tills has a strong interplay with the wrapping probability factor (Exp: 3.78%; Uniform: 9.42%).
We also analyzed the Policy 2 in the exponential scenario by fixing the number of checkout tills.

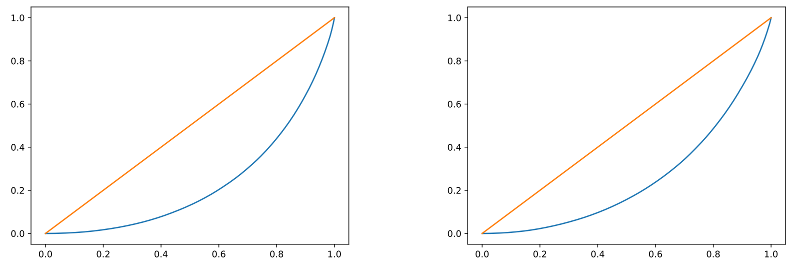
- For the configuration NC=2 NW=4 - The checkout rate has the strongest impact (46.33%)
- For the configuration NC=3 NW=3 - The result is very similar to the Policy 1
- For the configuration NC=4 NW=2 - The wrapping probability has the strongest impact (61.25%)

4.3 Fairness - Lorenz curve

In our system the desired behaviour (like in any other system) is that customers that want the same kind of service will have more or less the same response time; this is highly improbable since the two kind of customer (checkout-only and checkout+wrap) will both have to checkout. Worst-case scenarios were considered for the fairness analysis. To calculate the fairness the Gini coefficient has been used (ratio of the area between the line of perfect equality and the observed Lorenz curve to the area between the line of perfect equality and the line of perfect inequality: 0 Indicates perfect equality; 1 Indicates perfect inequality).

Checkout fairness We expect the policy 2 to be fairer than the policy 1, since in the latter a customer which want only to checkout will also has to wait all the customers ahead of him that want also to gift-wrap their purchased goods.
The Gini coefficients are the following:

Policy	**Exponential**	**Uniform**
Policy 1 NTills = 6	0.540	0.444
Policy 2 NC = 4	0.489	0.313
Policy 2 NC = 3	0.495	0.335
Policy 2 NC = 2	0.502	0.397

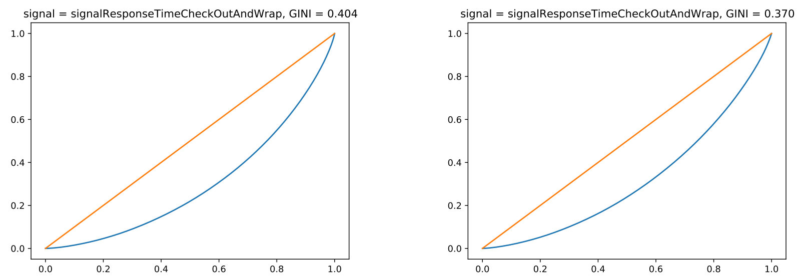


(a) NTills = 6, policy 1 (b) NC = 3, NW = 3, policy 2

Figure 4.3: Checkout fairness exponential distribution

Checkout and Wrap fairness We expect very similar results for both policies. The Gini coefficients are the following:

Policy	Exponential	Uniform
Policy 1 NTills = 6	0.408	0.301
Policy 2 NC = 4 NW = 2	0.406	0.319
Policy 2 NC = 3 NW = 3	0.378	0.247
Policy 2 NC = 2 NW = 4	0.380	0.260



(a) NTills = 6, policy 1 (b) NC = 3, NW = 3, policy 2

Figure 4.4: Checkout and Wrap fairness exponential distribution

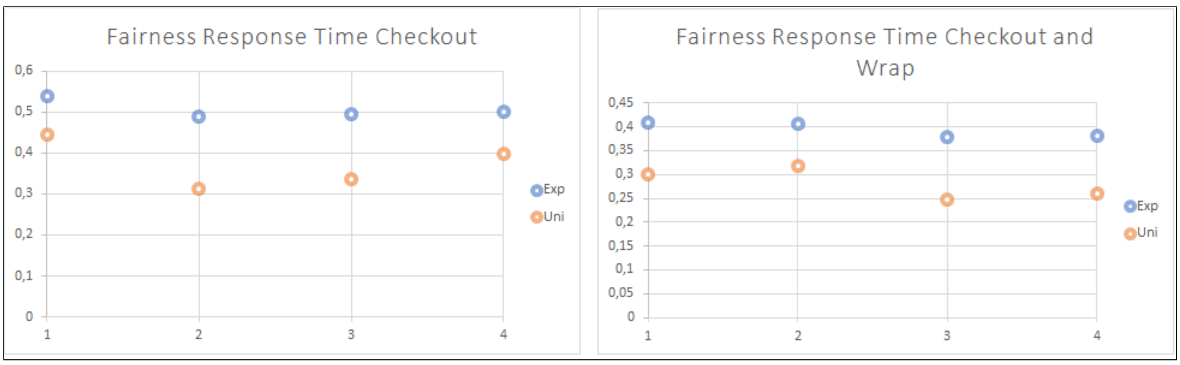


Figure 4.5: 1: N=6, 2: NC=4 NW=2, 3: NC=3 NW=3, 4: NC=2 NW=4

As we expected the policy 2 is fairer than policy 1 for the checkout only costumers. We can also notice that, even if the uniform distribution has a bigger mean inter arrival than the exponential one, is always more fair (due to the less variance of the distribution).

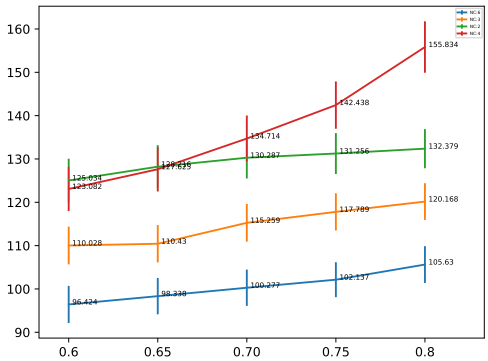
4.4 Changing Factors

4.4.1 Changing Wrapping Probability

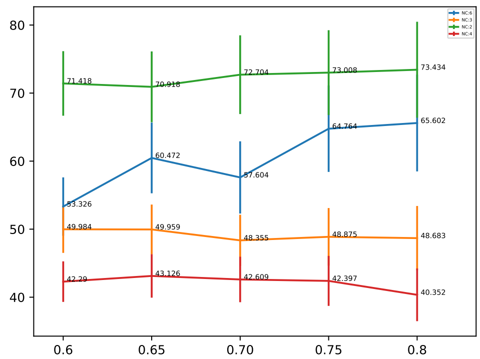
In this section we want to check if there is a winner policy by changing the wrapping probability, and in case the policy two is the winner how many tills must be converted to achieve that result.

4.4.1.1 Exponential

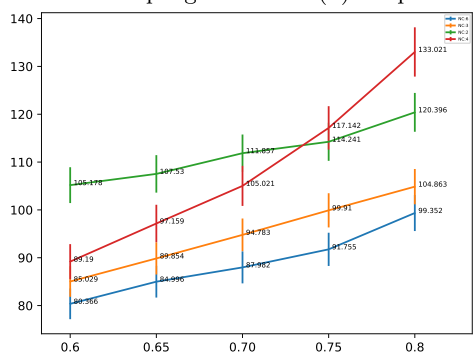
In order to obtain the below results the most meaning-full scenario has been considered: Number of tills = 6, $\lambda = 1/25$, $\mu_{checkout} = 1/30$, $\mu_{wrap} = 1/40$, CI = 0.95. In particular, the parameters listed above were fixed and the wrapping probability was varied. Below there are the graphs comparing policy 1 with policy 2 (each colour corresponds to a different distribution of tills: e.g. NC = 3 corresponds to having NW = 3 out of a total of 6 tills) for 3 different signals; on the x-axis we have the wrapping probability and on the y-axis the mean response time.



(a) ResponseTimeCheckoutAndWrap signal



(b) ResponseTimeCheckout signal



(c) CumulativeResponseTime signal

Figure 4.6: Number of tills = 6, $\lambda = 1/25$, μ_c=1/30, μ_w=1/40, CI = 0.95

General conclusions:

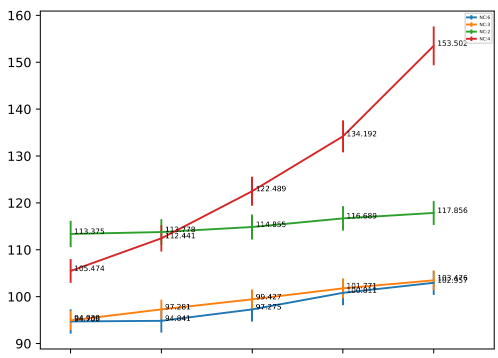
- On average, the mean response time increases as the number of customers requesting their purchases to be wrapped increases.
- For the policy 2 the mean response time of checkout is left almost unchanged by the increasing wrapping probability. This is due to the fact that the same number of customers are served in the checkout till indipendently of the number of customers that request the wrap service;
- The mean response time of the checkout+wrap service is highly influenced by the wrapping probability factor, since, in particular on the policy 2, the number of customers in the queue of the wrapping tills increase.

It is also important to notice that the policy 2 scenario with 4 checkout tills and 2 wrapping tills (red line in the plots) is not able to manage quite well the increasing number of customer that require to gift-wrap the purchased goods. In particular we can see that this scenario worsens significantly as the demand for wrapping increases; this is not the case in the opposite scenario with 4 wrapping tills and two checkout tills (green line). A comparison of policies shows:

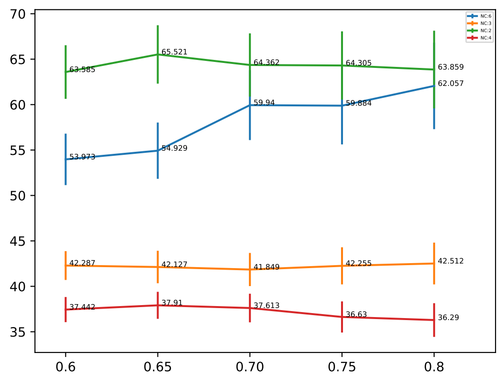
- For checkout-only costumers, policy 1 is not the most suitable; this is because they will most likely have to wait many checkout-and-wrap costumers to wrap their goods.
- For the checkout-and-wrap costumers the policy 1 is the best since once the checkout is finished they have no more to join a queue and have at their disposal all the available tills to wrap the purchased goods.
- Analyzing the cumulative response time is not possible to infer a winner from policy 1 and policy 2 with nc=3 and nw=3 with the CI=0.95, but we can conclude that the policy 1 gives better response times than policy 2 with nc=2 nw=4 and nc=4 nw=2.

4.4.1.2 Uniform

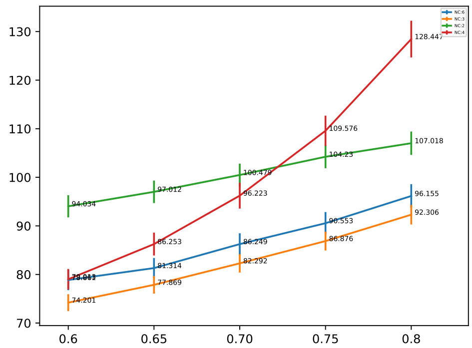
In order to obtain the below results the most meaning-full scenario has been considered: Number of tills = 6, Inter-arrival= Uniform(2s, 38s) Checkout = Uniform(3s,57s), Wrap = Uniform(4s, 76s), CI = 0.95. In particular, the parameters listed above were fixed and the wrapping probability was varied.



(a) ResponseTimeCheckoutAndWrap signal



(b) ResponseTimeCheckout signal



(c) CumulativeResponseTime signal

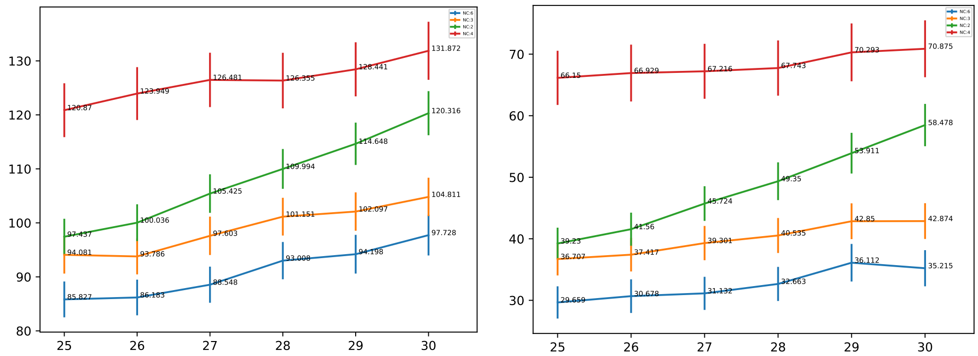
Figure 4.7: Number of tills = 6, Inter-arrival=Uniform(2s, 38s) Checkout=Uniform(3s,57s), Wrap=Uniform(4s, 76s), CI = 0.95

At first sight the exponential and uniform result are quite similar despite the Policy 2 NC=NW=3 being below the Policy 1. By only watching the cumulative response is not easy to tell why this happens. By analyzing the response time of the checkout only costumer and checkout+wrap costumer we can notice that the pattern followed are the same as the exponential: with regard to checkout only, the time saved is greater in Policy 1 compared to Policy 2 when comparing Uniform with Exponential, on the other hand for checkout and wrap customers the time difference is minimal compared to the Exponential scenario. So in the cumulative response the extra time spent by checkout only costumers is not recovered by checkout and wrap customers. The two scenarios are probably not perfectly comparable because of the substantial difference in the parameters used in them; the fact that the trend in each case is the same is a good result.

Since we have overlapping result, we can not assert a winner with CI equal to 0.95 but we can say which is not the best configuration. We can see that increasing the wrapping probability the mean response time increases. We can say with 0.95 Confidence that the scenario 2 with NC=3 and NW=3 is better than the scenario NC=4 NW=2, and the scenario where NC=2 and NW=4.

4.4.2 Changing the checkout rate

4.4.2.1 Exponential



(a) CumulativeResponseTime signal (b) CumulativeWaitingTime signal

Figure 4.8: Number of tills = 6, λ = 1/25, π_{w} = 0.8, μ_w=1/40, CI = 0.95

- It is evident from the plots that on average the policy 2 with NC = 4 and NW = 2 (red line) is the worst. This is due to the contribution of the time values of the customers requiring also the wrapping service, that in this scenario are the 80% of the total. Because of that, the high values of the response times are not so much caused by the variation of the checkout time, remaining quite stable, but from the fact that much more customers queue up to the wrapping tills that are just two. This also explains why the green line (policy 2 with NC = 2 and NW = 4) remains on average better than the red one.
- The policy 1 on average tends to have the best behavior. This is mainly due to the fact the it is the winner considering only the customer requiring the complete service.

4.4.2.2 Uniform

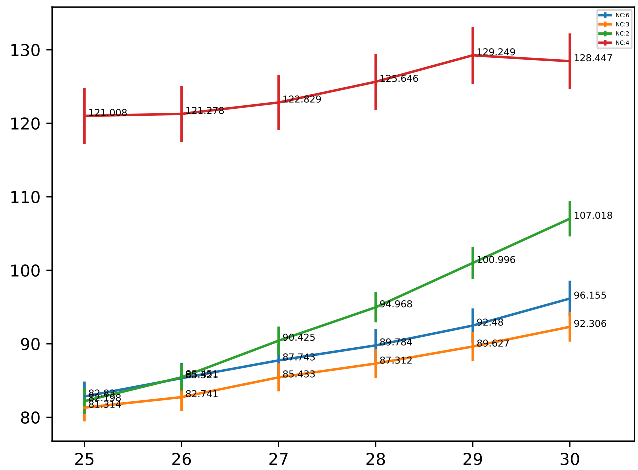
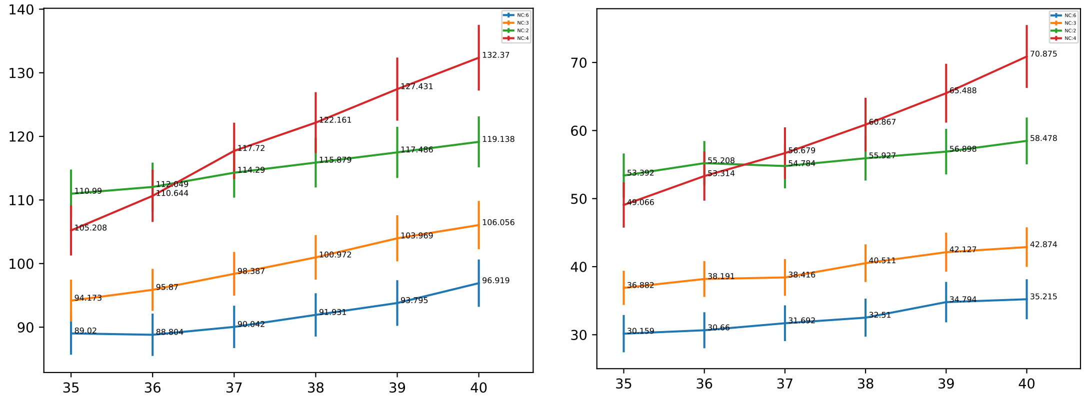


Figure 4.9: Number of tills = 6, λ = 1/20, π_{w} = 0.8, μ_w=1/40, CI = 0.95

The things said for the exponential scenario still holds.

4.4.3 Changing the wrapping rate

4.4.3.1 Exponential



(a) CumulativeResponseTime signal

(b) CumulativeWaitingTime signal

Figure 4.10: Number of tills = 6, λ = 1/25, π_{w} = 0.8, μ_c=1/30, CI = 0.95

- On average the policy 2 with NC = 4 and NW = 2 is more influenced than the others by the decreasing of the wrapping rate. This can be explained by the fact that many customers in this scenario require the complete service, so even a little decrease in the wrapping rate can lengthen the queues at the wrapping tills.
- The policy 1 also here tends to have the best behavior due to the best performance for what concerns the customers requiring the complete service. It should also be noted that the other configurations of policy 2 also have an almost stable behaviour.

4.4.3.2 Uniform

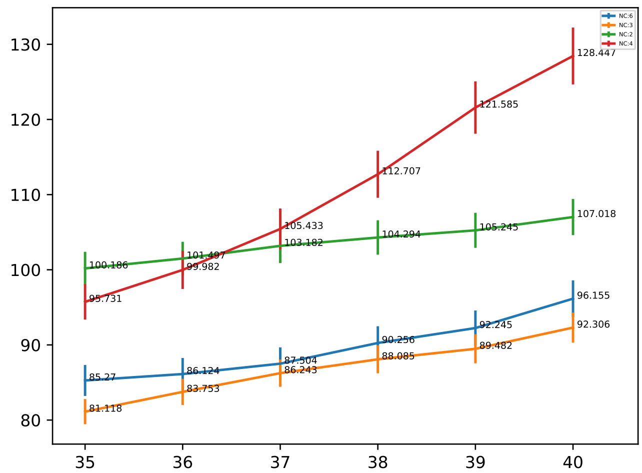


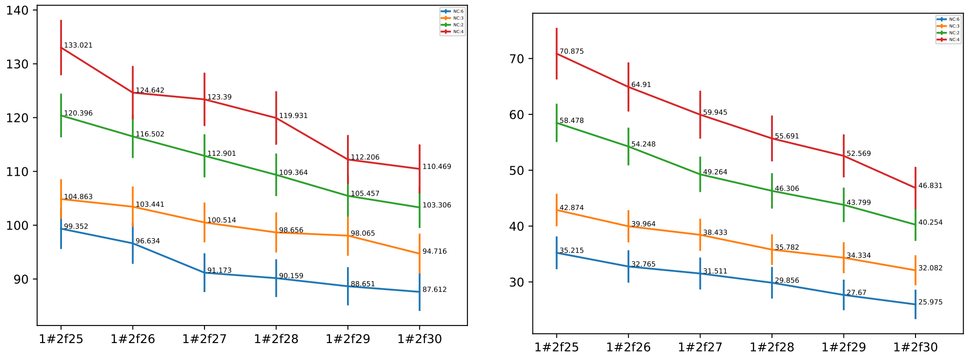
Figure 4.11: Number of tills = 6, λ = 1/20, π_{w} = 0.8, μ_c=1/30, CI = 0.95

The things said for the exponential scenario still holds.

4.4.4 Changing the interarrival rate

4.4.4.1 Exponential

As the inter-arrival rate decreased, it was observed, that within the various configurations the response time decreased. As in the other cases, policy 1 tends to behave better here due to the contribution of the high number of wrapping requests.



(a) CumulativeResponseTime signal (b) CumulativeWaitingTime signal

Figure 4.12: Number of tills = 6, $\pi_{\mathrm{w}} = 0.8$, μ_c=1/30, μ_w=1/40, CI = 0.95

4.4.4.2 Uniform

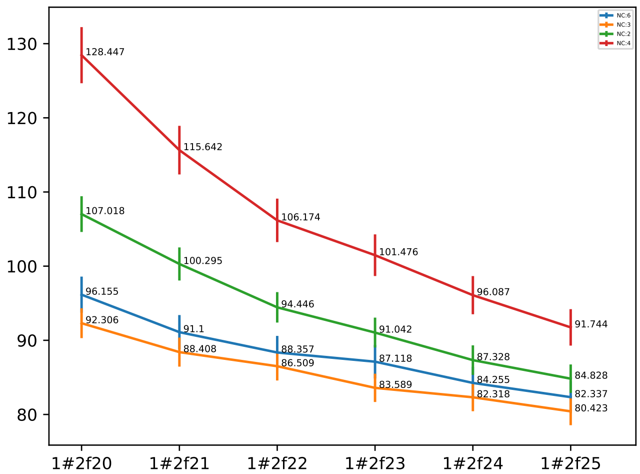


Figure 4.13: Number of tills = 6, $\pi_{\mathrm{w}} = 0.8$, μ_c=1/30, μ_w=1/40, CI = 0.95

4.5 A bigger shop: 12 Tills Analysis

In this section we analyzed the result for a bigger shop (12 total Tills instead of 6) to check if for a bigger shop a different solution is more suitable.

4.5.1 Description

We used the same model as for the 6 tills shop. Obviously the theoretical analysis still hold. The checkout rate, the wrapping rate are the same since usually the cashiers serve the costumer with a fixed rate. The wrapping probability is the same since the costumer doesn't decide to wrap or not in function of the shop dimensions. The only factor that has been changed is the interval rate, since we assumed that a bigger shop is able to manager more customers. By using the formulas found during the analysis we choose a suitable range for the interarrival.
We estimated the warm-up time by simulating 10 different seeds for each scenario (same procedure done for the 6Tills scenario). The warm-up time is 60000s.

4.5.2 $2^k r$ Factorial Analysis And Fairness Results

$2^k r$ factorial analysis: The main objective of this analysis was to check if, in the scenario 2, in a bigger shop the number of tills to be converted becomes a more influencing factor or not.
Scenario 2 Factors:

- **A = checkout only tills [4,6]**
- **B = initialInterArrivalRate [1/12.5, 1/15]**
- C = checkoutRate [1/25, 1/30]
- D = wrappingRate [1/35, 1/40]
- E = wrappingProbability [0.6, 0.8]

The factors has the same influences of the 6 Tills scenarios. This result is not surprising because if we see the scenarios as a whole of M/Cox/1 Tills for Policy 1 and M/M/1 Tills for Policy 2 with have the same inter arrival rate at each till (since all the factor stays the same).

Fairness: no significant difference has been noticed from the 6 Tills scenario.
Changing factors: No significant difference has been noticed from the 6 Tills scenario.

4.6 Conclusions

We have seen that there are two different winner considering the two different probability distributions.

- **Exponential:** For the exponential scenario the best configuration is the one where every till perform both the checkout and wrapping service (Policy 1). We can also notice that the Policy 2 with NC = NW = N/2 gives very close mean response times.
- **Uniform:** For the uniform scenario is more difficult to assert a winner between Policy 1 and Policy 2 (NC = NW = N/2) since most of the time the mean results overlap each other, so we can not tell which of the two is the clear best with CI=95%.

General conclusions: In general, balanced configurations, with the same number of checkout tills and wrapping tills, are the ones in which the mean response time is the lowest; there is no advantage in favouring the number of tills of one type over those of the other type. In particular we noticed that for the Policy 2 with less checkout tills than wrapping tills the checkout becomes the bottle neck since every wrap customer has to perform also the checkout service. When we have more checkout tills than wrapping till, when the probability that a customer will ask to wrap the purchased goods increases, the bottleneck is on the wrapping tills (as we can see in Figure 4.6 (a)).