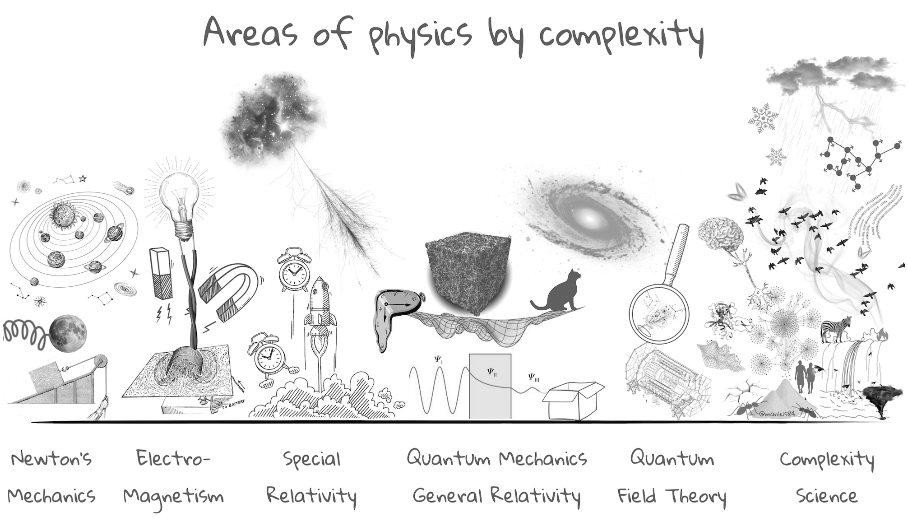
UNIVERSITÀ DEGLI STUDI DI PADOVA



Tasks # 34 and #42

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Final Report

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1 | Task 34: Game Theory on networks

Task leader(s): *Edoardo D'Amore*

1.1 | Ultimatum Game

The Ultimatum Game [5] is a game where two players, an *offerer* and a *respondent*, bargain in order to divide a fixed reward (we will consider it as 1) between them. Each player i is characterized by two parameters:

- the **offer ratio** p_i, which corresponds to the fraction of the total reward that the offerer proposes to the respondent;
- the **acceptance threshold** q_i, which is the minimum fraction the respondent is willing to accept.

We have considered three ways to initialize these parameters:

- **empathetic** (EMP): $q_i = p_i$;
- **pragmatic** (PRG): $q_i = 1 - p_i$;
- **random** (RND): p_i and q_i are independent.

In our setting each player will correspond to a node in a network. Each node will interact with all of its neighbors each round and play two games with any of them: one where it acts as the offerer and one where it acts as the respondent. After a round is finished, the **payoffs** are computed for each player:

$$\Pi_i = \sum_{j \in \mathcal{N}_i} (\Delta\Pi_{ij}^O + \Delta\Pi_{ij}^R) \tag{1.1}$$

where $\mathcal{N}_i$ is the set of neighbors of node i, $\Delta\Pi_{ij}^O$ and $\Delta\Pi_{ij}^R$ are the payoffs resulting from the interactions between node i and node j, where i acted as the offerer and the respondent respectively. Afterwards each player will update its parameters based on its **update rule** [1]:

- REP: a node $j \in \mathcal{N}_i$ is selected randomly. If $\Pi_j > \Pi_i$ then node i will copy node j's strategy with probability:

$$P_{ij} = \frac{\Pi_j - \Pi_i}{2\max\{k_i, k_j\}}$$

- UI: the node $j = \underset{j \in \mathcal{N}_i}{\operatorname{argmax}}\, \Pi_j$ is selected. If $\Pi_j > \Pi_i$ then node i copies its strategy.

- MOR: node i copies the strategy of node $j \in \mathcal{N}_i$ with probability:

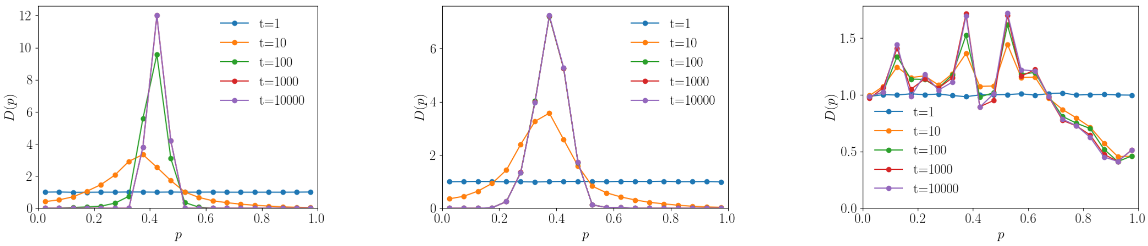
$$P_{ij} = \frac{\Pi_j}{\sum_{l \in \mathcal{N}_i} \Pi_l}$$

Along with p_j and q_j, the update rule is also adopted.

1.2 Simulation results

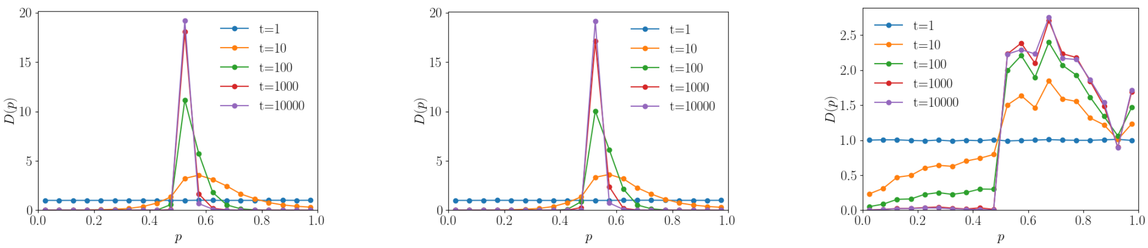
Four network topologies where considered: Erdos-Reyni (ER), Barabasi-Albert (BA), Stochastic-block-model (SBM) and the real-world network of Western States Power Grids of the United States (RW) (see appendix A.2 for RW). All of these networks have $N = 5000$ nodes.

EMP and PRG players From figures 1.1 and 1.2 we can observe how homogeneous networks (ER and SBM) tend to have a single clear peak for $D(p)$. BA networks, on the other hand, show a much more varied landscape of $D(p)$, and it can be seen that this variability is mainly present in higher degree nodes ($k \gtrsim 10$), as shown in figure 1.3.



(a) ER (b) SBM (c) BA

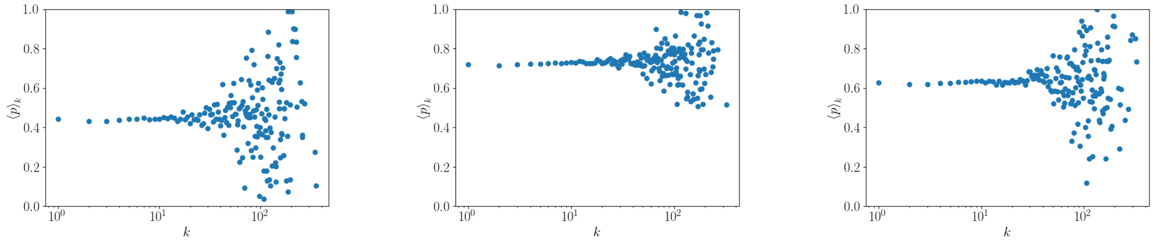
Figure 1.1: Distribution of p for EMP players.



(a) ER (b) SBM (c) BA

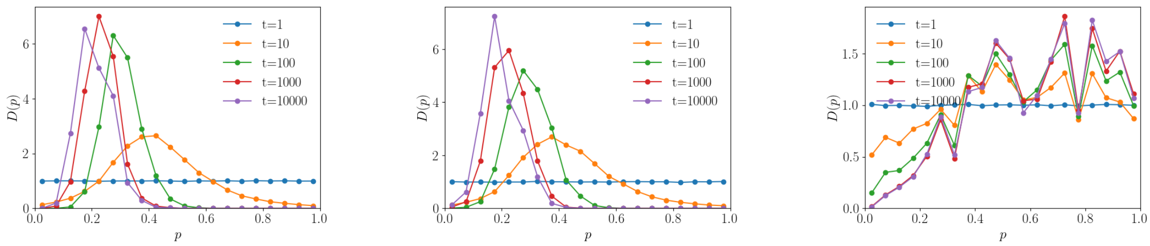
Figure 1.2: Distribution of p for PRG players.

RND players Once again ER and SBM networks show similar $D(p)$ and $D(q)$ (figures 1.4 and 1.5). BA networks are much more scattered across the (p, q) strategy space with respect to homogeneous networks.



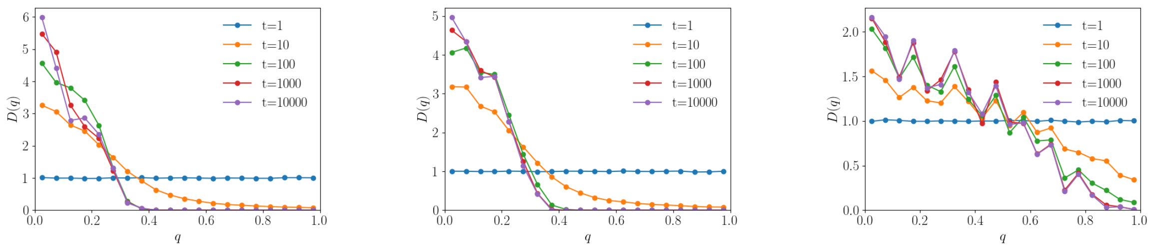
(a) EMP (b) PRG (c) RND

Figure 1.3: Average value of p w.r.t. the degree in BA networks.



(a) ER (b) SBM (c) BA

Figure 1.4: Distribution of p for RND players.



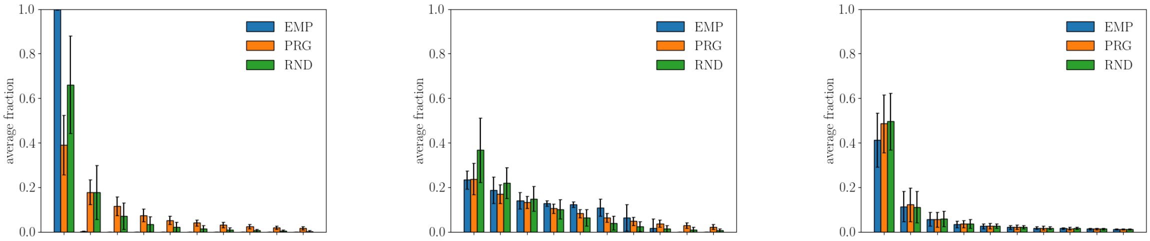
(a) ER (b) SBM (c) BA

Figure 1.5: Distribution of q for RND players.

Strategy frequency Finally we can observe a difference in behaviour between ER and SBM nets (figure 1.6). While ER nets tend favor just one strategy, especially for EMP and RND players, SBM nets clearly allow much more strategies to coexist in similar freqencies. This could be explained given the modular structure of the SBM networks. BA networks, as seen before, have most of their small degree nodes follow the same strategy.

1.3 Conclusions

Simulation results show that the evolution of game strategies greatly depends on the network's structure. Players in homogeneos network tend to converge to a smaller set of strategies with respect to players on scale-free networks, where instead high-degree nodes show a much greater range of values for (p, q). Additionally, as can be seen in appendix A.1, ER and SBM networks tend to greatly favor the REP update rule, while



(a) ER (b) SBM (c) BA

Figure 1.6: Frequency of the 10 most popular strategies.

BA ones allow for more competition, particularly between REP and UI.

2 | Task 42: Public transport in UK

Task leader(s): *Edoardo D'Amore*

2.1 | Dataset

The objective of this task is to divide by city the dataset provided by [2].

The tables considered are:

- `nodes`: contains nodes corresponding to the stops in public transport.
- `edges`: contains the connections between the nodes.
- `stops`: contains geographical information about the nodes.
- `cities`: contains all cities in the United Kingdom with a population of at least 50 thousands people [3].

2.2 | Data processing

The first step has been the sanitization of the tables, which consisted mainly in reformatting specific columns in order to correctly cast them to numerical values.

In order to collect all nodes related to a city, the following pipeline has been followed:

1. join the `nodes` and the `stops` tables on the `ATCOCode` attribute. If one the `cities` is present in either the `NatGazLocality`, the `ParentLocality` or the `GrandParentLocality` attributes, then it is assigned to the node.
2. nodes that are within a 0.5Km radius from eachother are assigned to the same group. Groups are divided based on how many different cities their nodes are assigned to:
   - 0: the group is discarded;
   - 1: the group is classified as "safe";
   - 2 or more: the group is classified as "unsafe".
3. nodes in the "safe" groups are all assigned to the same city.
4. nodes in the "unsafe" groups are assigned to the same city of the node they are closest to.

To obtain the edges it is sufficient to filter the `edges` table by considering only the links between two nodes previously retrieved.

For each city, two files are produced: `nodes.csv` and `edges.csv`. Note that, for some cities, it was not possible to correctly retrieve any node by following this procedure.

2.3 Basic analysis

In this section we will consider the networks of three cities in the United Kingdom: London, Birmingham and Liverpool.

Degree distribution From figure 2.1 it is possible to observe that the three networks show similar degree distribution.

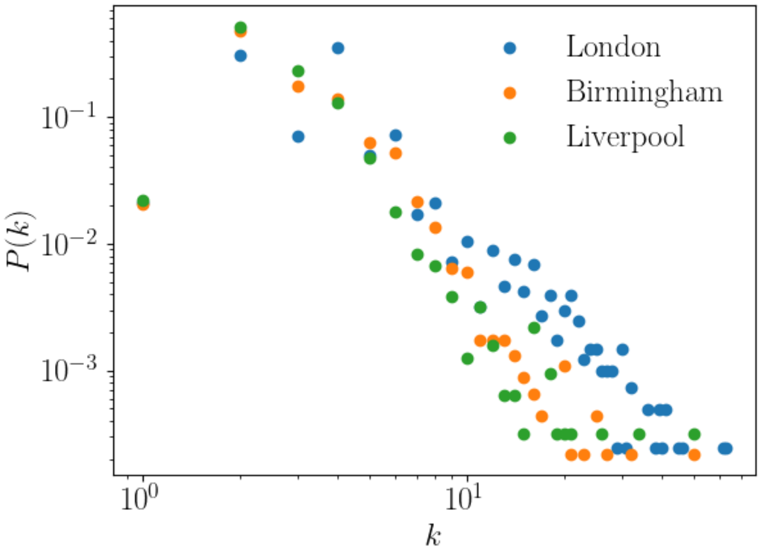


Figure 2.1: Degree distributions.

Assortativity coefficients Even though these are directed networks, most of the links are reciprocated, resulting in similar values for the assortativity coefficients (table 2.1).

City	$r^{(\text{out,out})}$	$r^{(\text{in,out})}$	$r^{(\text{out,in})}$	$r^{(\text{in,in})}$
London	0.28	0.27	0.28	0.27
Birmingham	0.28	0.30	0.29	0.29
Liverpool	0.26	0.28	0.26	0.25

Table 2.1: Assortativity coefficients.

Centrality measures Figures 2.2 and 2.3 show the closeness and betweenness centrality measures respectively, for each node in the three networks.

Nearest-neighbors' average degree In figure 2.4 we can observe how the results are quite analougous for the three cities, with London having on average higher values.

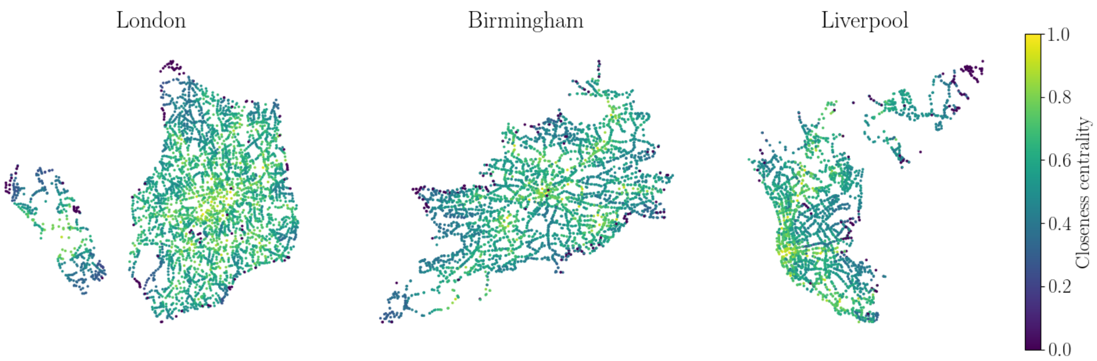


Figure 2.2: Closeness centrality.

Figure 2.3: Betweenness centrality.

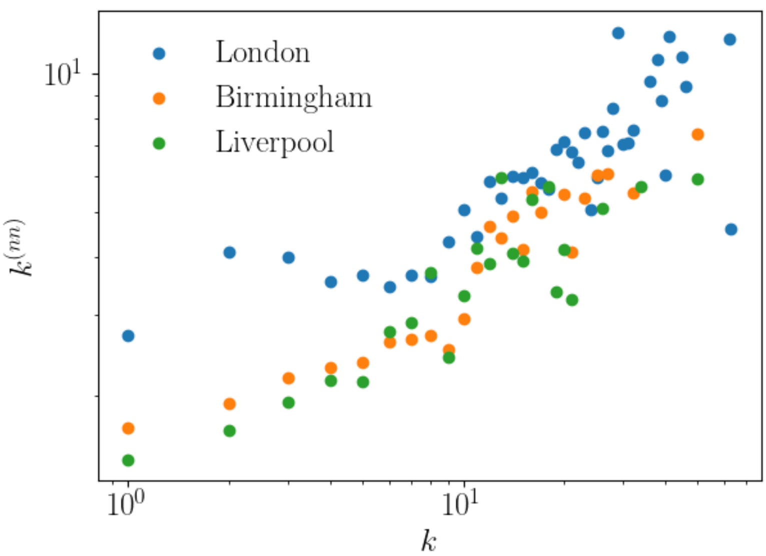


Figure 2.4: Nearest-neighbors' average degree.

3 | Bibliography

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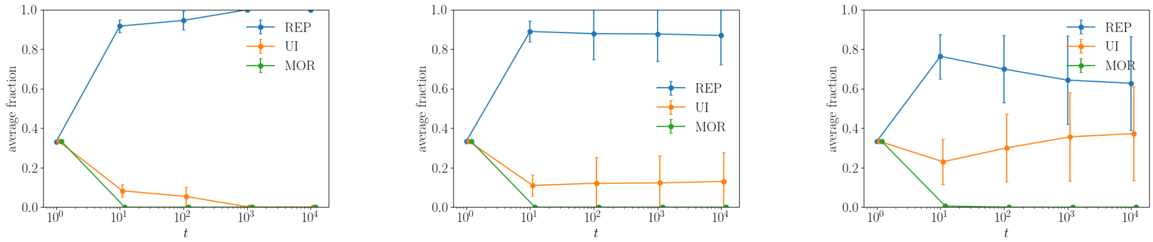
A | Task 34

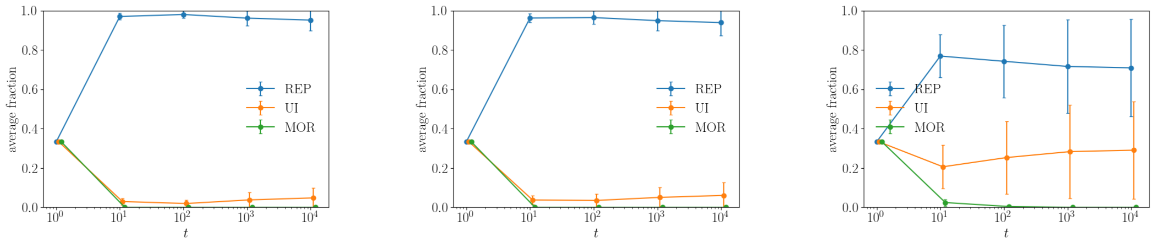
A.1 | Additional results

Update rule distributions Figures A.1, A.2 and A.3 show how homogeneous networks tend to overwhelmingly prefer the REP update rule against the other two. Barabasi-Albert networks however show more variability and allow also the UI update rule to be competitive, while still favoring the REP one.



(a) ER (b) SBM (c) BA

Figure A.1: Update rule distribution over time for EMP players.

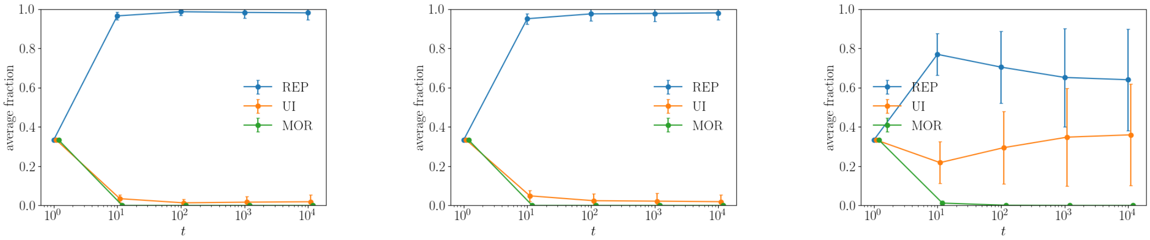


(a) ER (b) SBM (c) BA

Figure A.2: Update rule distribution over time for PRG players.

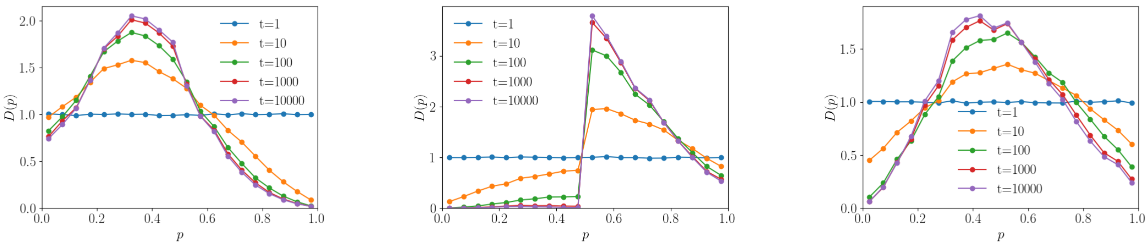
A.2 | Simulation on a real-world network

The Ultimatum Game has been run also on the network of the Western States Power Grid of the United States [6] [4]. Here are reported the figures relative to $D(p)$ (figure A.4) and the update rule distribution over time (figure A.5).



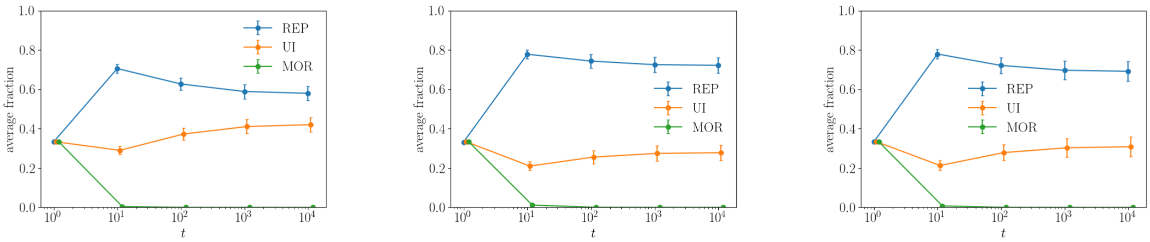
(a) ER (b) SBM (c) BA

Figure A.3: Update rule distribution over time for RND players.



(a) EMP (b) PRG (c) RND

Figure A.4: $D(p)$ in the real-world network.



(a) EMP (b) PRG (c) RND

Figure A.5: Update rule distribution over time in the real-world network.