# R Notebook

## Implementation

In this section we present some practical illustrations of the algorithms previously discussed. We start by considering a random walk plus noise model, i.e. the most basic form of a linear Gaussian state-space model.

$$y_t|x_t \sim N(x_t, \sigma^2) \tag{1}$$
$$x_t|x_{t-1} \sim N(x_{t-1}, \tau^2) \tag{2}$$
$$x_0 \sim N(m_0, C_0) \tag{3}$$

As we already know, in this case the filtering distribution can be computed in closed form solutions using the Kalman filter. However, this toy example is useful to understand the logic of the algorithms and compare the performances of different filtering strategies.

**Kalman Filter**
The Kalman Filter for this model can be easily implemented. Starting from the filtering distribution at period $t-1$, $x_{t-1}|y_{1:t-1} \sim N(m_{t-1}, C_{t-1})$, we compute:

- the one-step-ahead predictive distribution at time $t-1$

$$\begin{aligned} x_t|y_{1:t-1} &\sim N(a_t, R_t) \\ a_t &= m_{t-1} \\ R_t &= C_{t-1} + \tau^2 \end{aligned}$$

- the filtering distribution at time $t$ as $p(x_t|y_{1:t}) \propto p(x_t|y_{1:t-1})p(y_t|x_t)$

$$\begin{aligned} x_t|y_{1:t} &\sim N(m_t, C_t) \\ m_t &= \big(1 - \frac{R_t}{R_t+\sigma^2}\big)a_t + \frac{R_t}{R_t+\sigma^2}y_t \\ C_t &= \frac{R_t}{R_t+\sigma^2}\sigma^2 \end{aligned}$$

```r
DLM<-function(data,sig2,tau2,m0,C0){
  n  = length(data)
  m  = rep(0,n)
  C  = rep(0,n)
  for (t in 1:n){
    if (t==1){
      a = m0
      R = C0 + tau2
    }else{
      a = m[t-1]
      R = C[t-1] + tau2
    }
    A = R/(R+sig2)
```

```
    m[t] = (1-A)*a + A*y[t]
    C[t] = A*sig2
  }
  return(list(m=m,C=C))
}
```

**Sequential Importance Sampling**

Let $\{(x_0, w_0)^{(i)}\}_{i=1}^N$ summarizes $p(x_0|y_0)$ such that, for example, $E(g(x_0)|y_0) \approx \frac{1}{N}\sum_{i=1}^N w_0^{(i)} g(x_0^{(i)})$. For $t = 1, ..., n$ where $n$ is the length of the sample, at any iteration

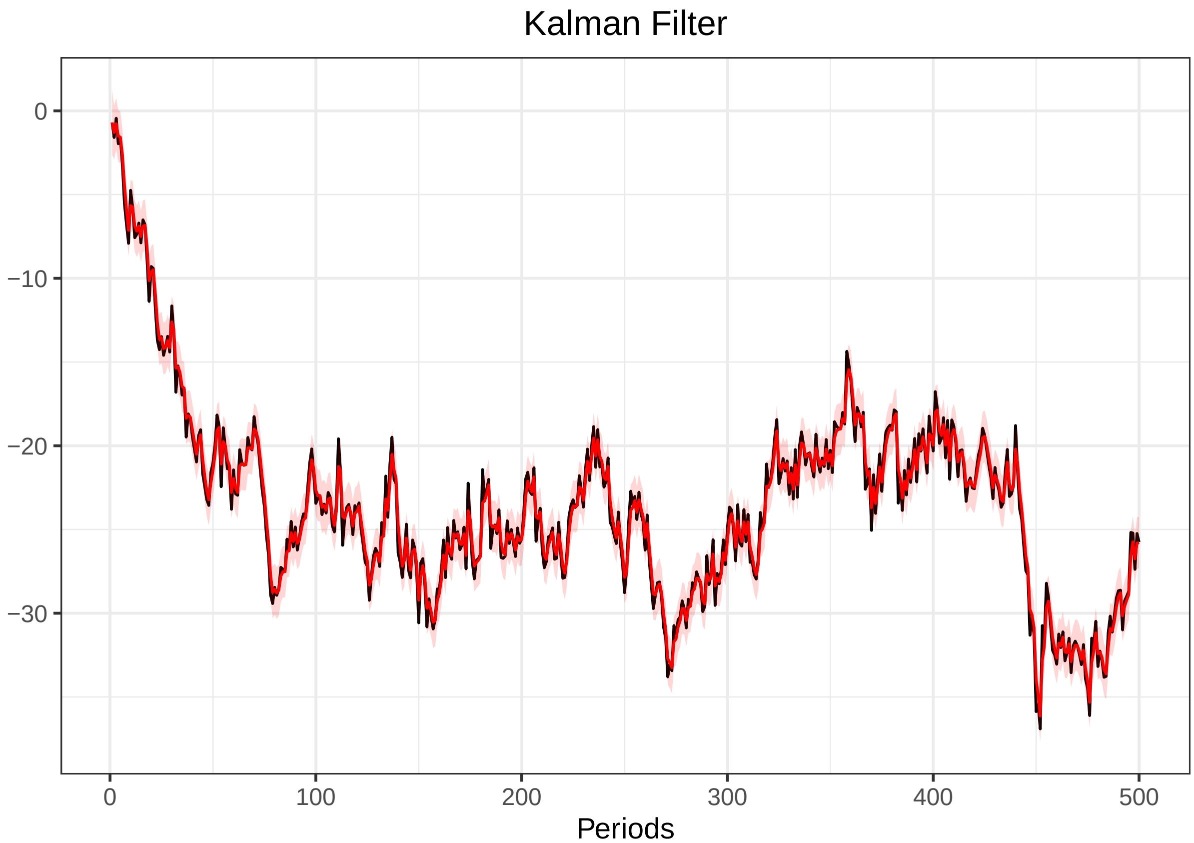
- Draw $x_t^{(i)} \sim N(x_{t-1}^{(i)})$ $\ i = 1, ..., N$ such that $\{(x_t, w_{t-1})^{(i)}\}_{i=1}^N$ summarizes $p(x_t|y_{t-1})$
- Set $w_t^{(i)} = w_{t-1}^{(i)} f_N(y_t; x_t^{(i)}, \sigma^2)$ $\ i = 1, ..., N$ such that $\{(x_t, w_t)^{(i)}\}_{i=1}^N$ summarizes $p(x_t|y_t)$

```
#Sequential Importance Sampling
#------------------------------
SISfun<-function(data,N,m0,C0,tau,sigma){
  xs<-NULL
  ws<-NULL
  ess<-NULL
  x  = rnorm(N,m0,sqrt(C0))
  w  = rep(1/N,N)
  for(t in 1:length(data)){
    x    = rnorm(N,x,tau)                   #sample from N(x_{0},tau)
    w    = w*dnorm(data[t],x,sigma)         #update weight
    xs = rbind(xs,x)
    ws = rbind(ws,w)

    wnorm= w/sum(w)                         #normalized weight
    ESS  = 1/sum(wnorm^2)                   #effective sample size

    ess =rbind(ess,ESS)
  }

  return(list(xs=xs,ws=ws,ess=ess))
}
```


Kalman Filter
0
−10
−20
−30
0
100
200
300
400
500
Periods