H = host reveals a goat

W_x = prize is behind X

Bayes: $P(W|H) = P(W) \cdot \frac{P(H|W)}{P(H)}$

$P(W)$: prior

$P(H|W)$: likelihood

(1) Prize behind A

$P(H|W_A) = 1$

(2) $P(H|W_B) = 1/2$

(3) $P(H|W_C) = 1/2$

Law of total prob. to compute $P(H)$:

$$P(H) = \sum_{i=A,B,C} P(W_i) \cdot P(H|W_i)$$

$$= \frac{1}{3} \cdot 1 + 2 \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{2}{3}$$

(1) $P(W|H) = \frac{1}{3} \cdot \frac{1}{2/3} = \frac{1}{2}$ 50 %.

(2) $P(W|H) = \frac{1}{3} \cdot \frac{1/2}{2/3} = \frac{1}{4}$ 25 %.

(3) $P(W|H) = \frac{1}{3} \cdot \frac{1/2}{2/3} = \frac{1}{4}$ 25 %.