An Agent based High-Order Naming Model

Andrea Caputo, Andrea Cota, Edoardo Grasso

Abstract

The Naming Game has been exploited throughout the years in order to produce models consisting of locally interacting individuals which spontaneously give rise to global coordination. Recent models based on the naming game have shown how minority groups can initiate social change in the emergence of new social conventions. In the present work, we study an agent based version of the network-based Naming Game presented in [1]. We will appreciate how our model produce comparable results with the cited one.

1 Introduction

In social phenomena the basic constituents are humans, i.e., complex individuals who interact with a limited number of peers, usually negligible compared to the total number of people in the system. Human behaviour is governed by many aspects, related to social context, culture, law and other factors. Opinions and believes are at the basis of behaviour, and can be seen as the internal state of individuals that drives a certain action. Opinion formation is a complex process depending on the information that we collect from peers or other external sources, among which mass media are certainly the most predominant. Traditionally studied by social science, opinion formation, as well as other social processes, have become increasingly appealing to scientists from other fields. A large amount of work is concentrated in building models of opinion dynamics, using tools borrowed from physics, mathematics and computer science. One of this models is the Naming model, a functional model in which the individuals perform pairwise interactions to negotiate the conventional forms to be associated with a set of meanings. The very first version of the model, described in [2], consisted simply of a set of individuals without any regard about the topology of the system. In this model, at each step two random individuals are selected among the set and an interaction occurs between them. The dynamic of the interaction is very simple: each individual has a vocabulary of words, in each interaction one individual is the speaker and the other one is the listener. The speaker picks a word from his vocabulary and if the listener has that word than the interaction succeed: both individuals delete all the words different from that one; else, if the listener does not have the selected word, the interaction fails: the listeners adds the spoken word to his vocabulary. Later has been proposed an high-order Naming Model [1] that introduced group interactions. In group interactions one speaker is selected and all the other components of the group are listeners, in this case an interaction succeeds only if every component of the group has the spoken word with the same results as described above[1]. The topology of this model is based on hypergraphs, at each step a random hyperlink of the network is selected, also a random node in this hyperlink is picked which will act as speaker, every other nodes in the selected hyperlink will be listeners. [1] also introduces the presence of a committed minority, that is a fraction of people that starts with a certain word and never changes it, nor acquires other words during interactions. The purpose of that article, actually, was to study the critical mass of committed minority that could be able to flip the whole system. Meaning that, in the beginning every agent in the model agrees on a certain word, called B, except for this fraction of committed minority that proposes the word A. The question is "how large should the committed minority be to make the system converge to a state where every agent agrees on the word A"? We did not address this specific question, but we rather focused on reproducing one particular scenario presented in that article with our agent based model, trying to obtain comparable results. We will start with a discussion of the minimal naming game which we also translated to a agent based model.

[1] there are many possible rules to choose whether an interaction succeeds or not, this one is also called the unanimity rule, that is the strictest rule possible

2 The Minimal Naming Game

The minimal Naming Game is played by a population of N agents that engage in pairwise interactions in order to negotiate conventions (i.e., associations between forms and meanings), and it is able to describe the emergence of a global consensus among them. Each agent disposes of an internal inventory, in which an a priori unlimited number of words can be stored. As initial conditions we require all inventories to be empty. At each time step (t = 1, 2, ..), a pair of neighboring agents is chosen randomly, one playing as "speaker", the other as "listener", and negotiate according to the following rules:

- the speaker randomly selects one of its words (or invents a new word if its inventory is empty) and conveys it to the listeners;
- if the hearer's inventory contains such a word, the two agents update their inventories so as to keep only the word involved in the interaction (success);
- otherwise, the listener adds the word to those already stored in its inventory (failure).

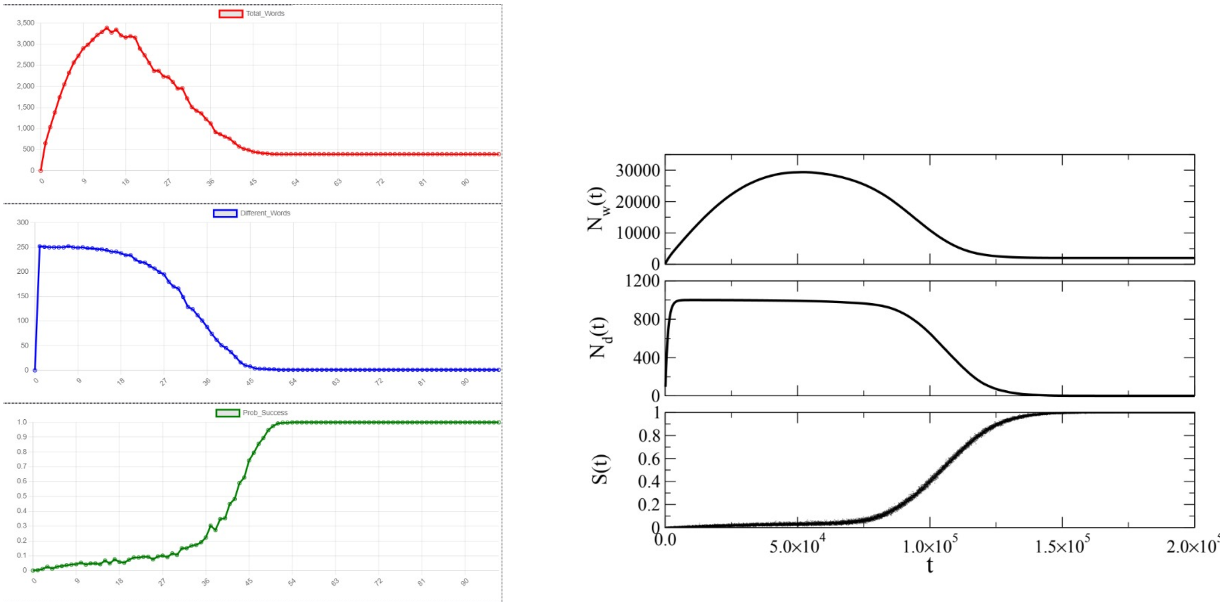
For the sake of simplicity the model does not take into account the possibility of homonymy, so that each newly invented word has never appeared before in the population. The main quantities that describe the system's evolution are:

- the total number $N_w(t)$ of words in the system at time t;
- the number of different words $N_d(t)$ in the system at time t;
- the average success rate $S(t)$, i.e., the probability that a certain agent gets involved in a successful interaction at a given time t.

For our model, we recreated at first the version with complete random interactions, and then extended it by adding a topology. So in this extended version agents are placed on a 30x30 grid and, at each step, every agent is asked to move to a neighboring cell with less than two occupants. Then, in each cell that contains two agents, the interaction occurs (the roles of speaker and listener are assigned randomly). The consensus state is obtained when $N_d(t) = 1$ and $N_w(t) = N$ (so that $S(t) = 1$). The temporal evolution of the three main quantities is presented in the figure.

If we take in consideration the version of the model with random interactions, our results agree with the paper as can be seen in figure 1: at first, every agent that is selected to act as the speaker creates a new word to fill its inventory, so they invent a large number of different words ($N/2$ on average).Then agents start spreading these words throughout the system thanks to failure events. The number of words decreases only by means of successful interactions, which are limited in the early stages. The S-shaped curve of the success rate summarizes the dynamics: initially, agents hardly understand each others ($S(t)$ is very low); then the inventories start to present significant overlaps, so that $S(t)$ rapidly increases until it reaches 1.

Finally, in figure 2 we can see an example of our result for the extended version of the model, with agents living on a two dimensional grid. We can see that it took much longer to reach consensus in this version of the model, that's presumably because we added a spatial constrain: the version of the model with random interactions represented the situation where a piece of information could travel any distance in a single time step without friction. We will not dig further on this basic model because it is not supposed to be the main focus of our work.



(a) Results from our model: single run with 400 agents

(b) Results from [2]: simulation with 2000 agents averaged over 300 runs

Figure 1: Comparison between our results and the original article's results

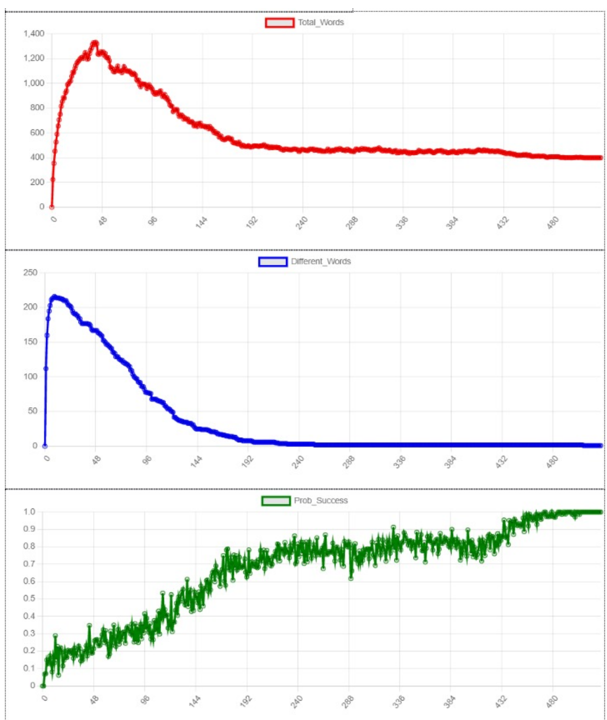


Figure 2: Single run simulation with agents living in a two dimensional grid

3 The Higher Order Naming Game

Once we found comparable results with the classic version of the base model, we moved on the advanced one, that includes group interactions and a committed minority. Group interactions are encoded through hyperlinks, that are links that can connect more than two nodes. A hyperlink of a hypergraph is basically a group of nodes that interact simultaneously.

The Network-Based Naming Game

The model presented in [1] works as follows:
At each time step a hyperlink is randomly chosen and a speaker agent is picked at random among the nodes in the selected hyperlink. All the other nodes present in the same hyperlink act as listeners. In the whole system there are two words (or opinions): a minority opinion A and a general opinion B, so each agent can have one between these two or both in its inventory. The speaker selects a random word from its inventory, and communicates it to the group. Then the result of the interaction is based on the unanimity rule, that is: only if every agent in the group has in its inventory the spoken word, the interaction can succeed, and if it does, and all the agents keep only this word in the inventory. Otherwise the interaction fails and all the agents that did not have the spoken word, add it to the inventory. Note that even though the unanimity rule is satisfied, the interaction could not succeed, that is because a new parameter $\beta \in [0, 1]$ is introduced into the model. Thus, if an agreement can be reached, two possibilities exist :

- with probability β all the nodes of the considered hyperlink agree on the chosen name A and erase all the other names from their vocabularies;
- with probability $1 - \beta$ there is no convergence but the nodes who did not have A add it to their vocabulary.

The parameter β modulates social influence, i.e. the propensity of individuals to change their behaviour to meet the demands of a social environment. The smaller the β the less the individuals participating are prone to change their views in spite of the social influence mechanism. Finally, is allowed the presence of a committed minority among the agents. These agents starts with only the minority opinion A and never modify their inventories, meaning that any interaction involving an agent that's part of the committed minority will always fail if the spoken word is B.

The Grid-Based Model

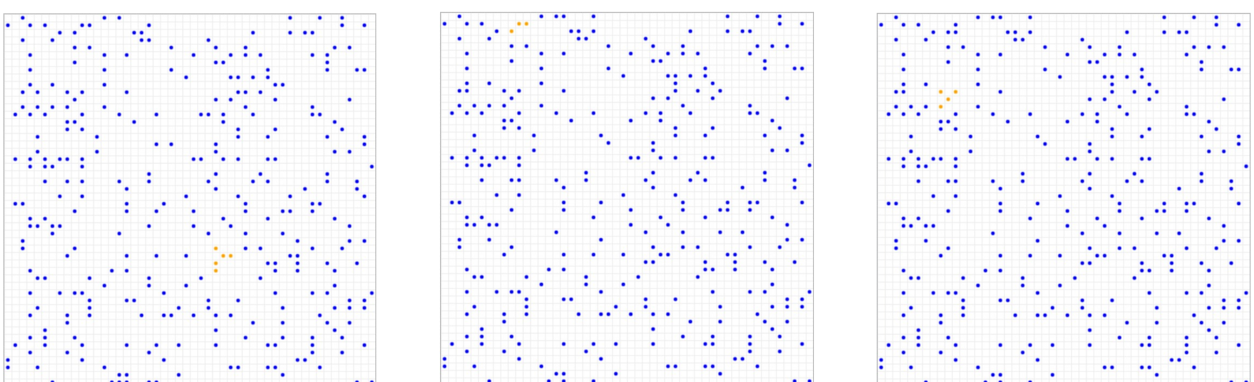


Figure 3: Example of activation of agents and group formation

Our model replaces the hypergraph structure with a 50x50 grid in which agents live. The dynamic is very similar to the original model: at each step one random agent is activated, so he moves towards one of its free neighboring cells, and if it moves near to other agents, a group of maximum 5 individuals is created and the interaction takes place (see figure 3). The result of the interaction depend on the unanimity rule and the presence of committed individuals. If the consensus is possible, a number in (0,1) is randomly extracted, if it's less than β, the interaction succeed, else the interaction fails, just like in the original model.

The Study Case

The objective of [1] was to study the mechanism for which the system filps, starting from a large number of individuals that thinks B and ending with the whole system thinking A. So the authors focused on the role of the committed minority and how the critical mass (that is the critical fraction of committed individuals that can end up flipping the system) depends on β and other parameters like the number of total individuals and the actual topology of the system. Our purpose instead was to simply reproduce with our agent-based model one of the particular cases studied in [1]. Therefore we fixed the parameters to adapt to such situation, in particular:

- $N = 327$, number of agents;
- $p_c = 0.003$, percentage of committed agents (results simply in 1 committed agent among the total 327);
- $\beta = 0.336$
- maximum group size = 5;

We set the size of the grid to 50x50, noticing that this way agents usually spread so as to create both high and low density areas, recalling the behaviour of a heterogeneous network.

Results

We say that a run has converged if all the agents agree about the opinion. Since there is one committed agent that thinks A (the minority opinion), it's not possible to converge with all agents thinking B (the general opinion). So we reach convergence only if every agent has just the opinion A in their inventory. When an agent has both the opinion A and B we say they have a mixed opinion.

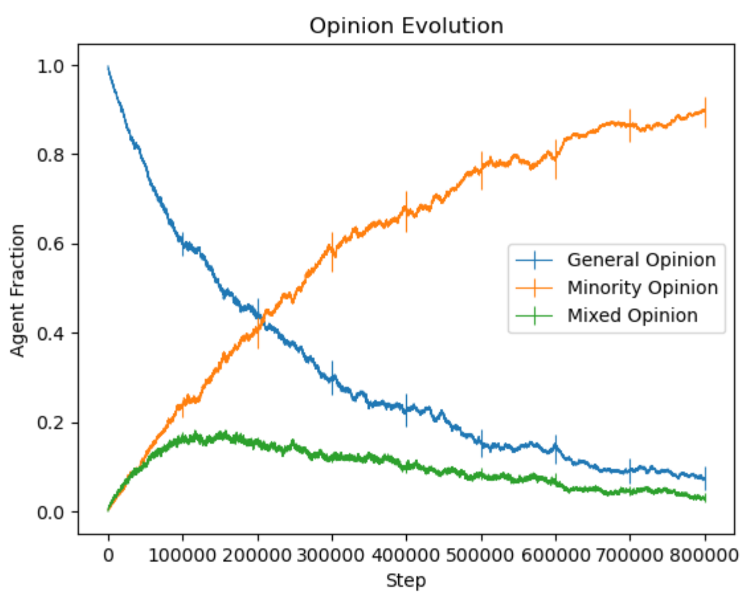


Figure 4: Batch-run results

We performed 50 different runs with 800000 steps each under the same initial conditions. This way we observed a stochastic nature for the convergence, sometimes it was reached very early, sometimes very late, and other never reached at all. In figure 4 we can see the evolution of the opinions over time. Each point shown is an average over the 50 runs we made, so the uncertainity of these points is estimated as the standard error of the mean defined as: $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N_r}}$ where in this case $N_r = 50$ and σ_x is the dispersion of the values in each step.

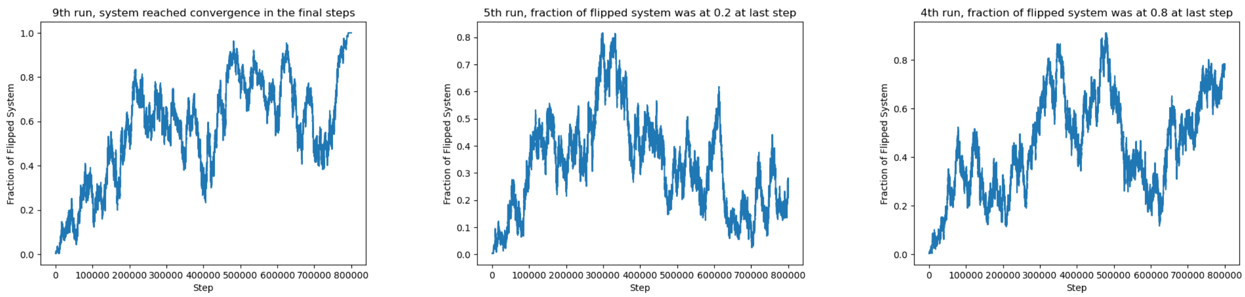


Figure 5: Examples of the behavior of the system throughout a single run

With further analysis we were able to tell that 39 out of 50 runs had reached convergence in 800000 steps. In those runs, the average number of steps needed to reach convergence is 447003. In non converged runs, the average percentage of flipped system at the last step is 0.53, conferming that some runs were on the verge of convergence while others were still very far as can be observed in figure 5. Figure 5 also reveals a very important feature of this system that is the oscillating behavior with the minority that enlarges and shrinks at alternating phases. With the provided set of parameters we know for sure that the minority is able to flip the system given enough time, the speed with which this happens depends on stochastic fluctuations.

4 Conclusions

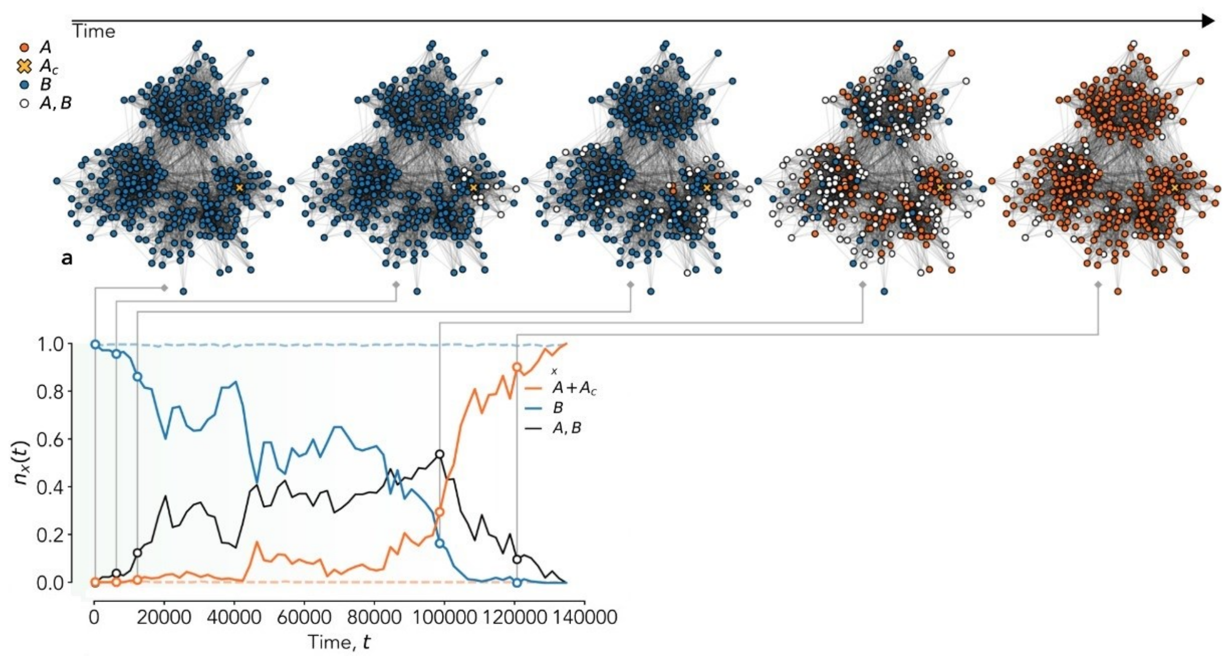


Figure 6: Original paper results with the same initial conditions we also used.

Standing to our results the naming model seems to be reproducible as an agent based model. By looking at the same study conducted in [1] we can see that this same system on their network-based model required at most 140000 steps to converge as it can be seen in figure 6, we believe that our much larger number of steps needed is due to the fact that we provided the agents the ability to move and thus of interacting groups to mutate, introducing a new (and very large) stochastic factor. Another difference is that their mixed opinion reach higher values than ours, this can be explained by the fact that in our model there's a high probability that the same group interacts twice without any changes in the composing individuals, while in their each node is connected to multiple hyperlinks (so it is part of multiple groups). When the same individuals interact twice the first time each agent will have both opinions and the second it is almost guarantee that they will reach an agreement[2]. This phenomenon can be attenuated by letting the agent move further than one cell at each step. Our attempts in changing the agent's behaviour this way resulted in a higher average mixed opinion but also in a drastic increase in stochastic fluctuations.

Further Works

With this project we just tried to replicate the results from [1], so it could be interesting to try to adapt this model to more realistic scenarios. Also, the study of the critical mass described in [1] could be replicated using our model as a further proof of the compatibility of the two.

References

[1] Iacopini, Iacopo, et al. "Group interactions modulate critical mass dynamics in social convention." Communications Physics 5.1 (2022): 64.

[2] Baronchelli, Andrea. "A gentle introduction to the minimal naming game." Belgian Journal of Linguistics 30.1 (2016): 171-192.

[2]The only case when this could not happen is when the committed agent is involved.