TMA373 Assignment 3

Edoardo Mangia
edoardom@student.chalmers.se

Bibiana Farinha
bibiana@student.chalmers.se

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Contents

1 Task 1

a) Malthusian growth model

We consider the Malthusian growth model defined by the differential equation:

$$\frac{dp}{dt} = k\,p(t), \quad p(0) = p_0, \tag{1}$$

where the initial number of parasites is $p_0 = 5$ and the growth parameter is $k = 0.1 + \texttt{epsSTUD}$ (with `epsSTUD` here being $47 * 10^{-}3)$. The exact solution is given by:

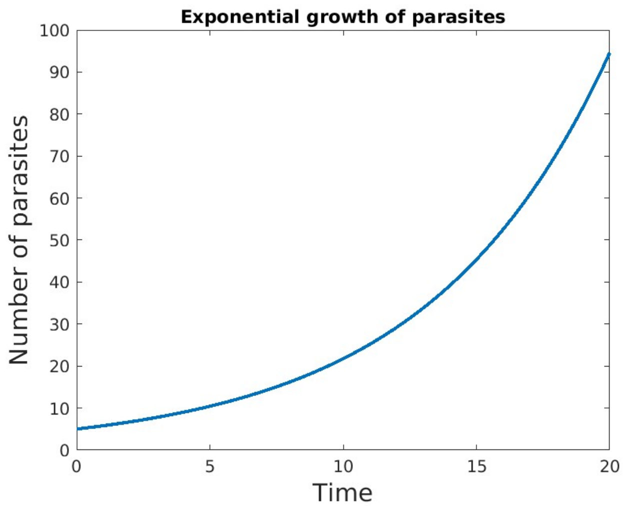
$$p(t) = p_0\ \exp(k\,t). \tag{2}$$

This solution is plotted over the time interval $[0, 20]$.

Implementation

```
epsSTUD = 47/10^3

p0 = 5;                     % initial number of parasites
k = 0.1 + epsSTUD;          % population growth rate
tExact = [0:0.05:20];       % time interval
pExact = p0*exp(k*tExact);

figure;
plot(tExact, pExact, 'LineWidth', 2);

...
```

Visualization



b) Euler method

Now, for the same ODE, we implemented the explicit Euler method and compared it in a plot against the exact solution. The latter method is given by:

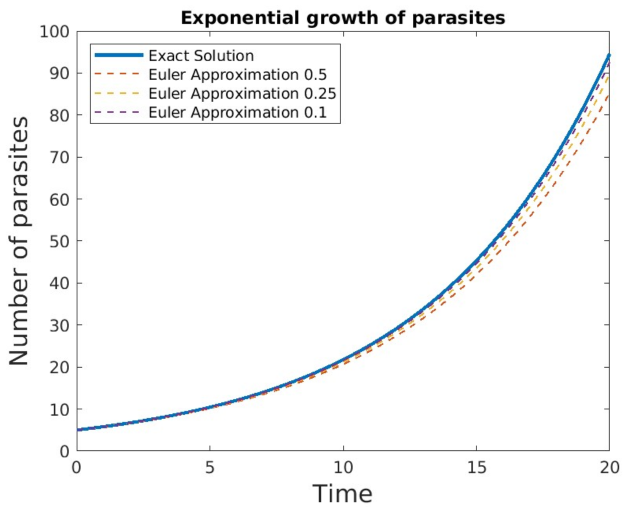
$$p_{n+1} = p_n + h\,k\,p_n, \tag{3}$$

in this case, we used step sizes $h = 0.5$, 0.25, and 0.1.

Implementation

```
t0 = 0;
y0 = p0;
pEnd = []; % auxiliar for task c)

for h = [0.5, 0.25, 0.1]
    N = 20/h;
    tEuler = zeros(1, N+1);
    pEuler = zeros(1, N+1);
    tEuler(1) = t0;
    pEuler(1) = y0;
    for n =1:N
        tEuler(n+1) =  tEuler(n) + h;
        pEuler(n+1) = pEuler(n) + h * k * pEuler(n);
    end
    pEnd = [pEnd, pEuler(end)]; % auxiliar for task c)
    hold on
    plot(tEuler , pEuler, '--', 'LineWidth', 1)
end

...
```

Visualization



As the plot shows, the approximations approach the exact solution as the step size decreases.

c) Convergence property

The explicit Euler method is a numerical integrator of order 1, meaning that the error at time T satisfies:

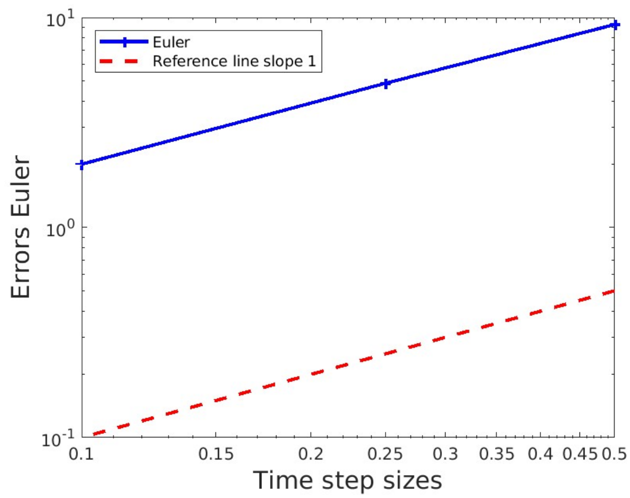
$$|p_N - p(T)| \leq C\,h, \tag{4}$$

where $h = T/N$.

Implementation

```
hplot = [0.5 0.25 0.1];
pEuler = pEnd;
pExact = pExact(end);
pExact = p0*exp(k*20);
Errplot = abs(pEuler - pExact);

...
```

Visualization



A log-log plot transforms a power relationship such as Error $\sim h^1$ in a straight line with slope p.

The Euler method is first order, with $p = 1$, therefore we can see the error plotted as a straight line with slope 1.

2 Task 2

a) Proof

In this task, we want to prove that total population, we will denote it by $P(t)$, is always constant in time.

The total population is given by the sum of humans, zombies and removed:

$$P(t) = H(t) + Z(t) + R(t).$$

Differentiating both sides, we get:

$$P'(t) = H'(t) + Z'(t) + R'(t).$$

From the given system of equations, can expand the right side of the equality above:

$$P'(t) = -\beta H(t)Z(t) + \beta H(t)Z(t) + \zeta R(t) - \alpha H(t)Z(t) + \alpha H(t)Z(t) - \zeta R(t).$$

By doing basic arithmetic operations, we can simplify the above:

$$P'(t) = 0.$$

The derivative of $P(t)$ indicates that the function remains constant, since the derivative of a constant is always zero.

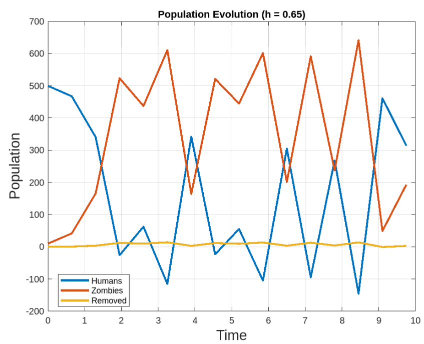
b) Euler Method

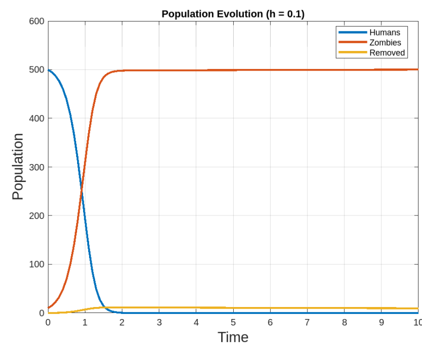
Below is the implementation of the Euler method, with step sizes $h = 0.65$ and $h = 0.1$ to approximate the solution over the time interval $[0, 10]$.

Implementation

```
alpha = 0.005 * epsSTUD;
beta = 0.01;
zeta = 0.02;

H0 = 500;
Z0 = 10;
R0 = 0;
T_end = 10;

h_values = [0.65, 0.1];
colors = {'b', 'r'};

for j = 1:length(h_values)
    h = h_values(j);
    N = round(T_end/h);
    t = 0:h:T_end;
    H = zeros(1, length(t));
    Z = zeros(1, length(t));
    R = zeros(1, length(t));

    H(1) = H0;
    Z(1) = Z0;
    R(1) = R0;

    for n = 1:N
        H(n+1) = H(n) + h * (-beta * H(n) * Z(n));
        Z(n+1) = Z(n) + h * (beta * H(n) * Z(n) + zeta * R(n) -
            alpha * H(n) * Z(n));
        R(n+1) = R(n) + h * (alpha * H(n) * Z(n) - zeta * R(n));
    end
```

Visualization





Comparing the plots for $h = 0.65$ and $h = 0.1$, it can be seen how the larger step size induce more instability in the results, while with a smaller step size, a more plausible trend is plotted. Instantly, the total population count remain 510, even with the fluctuations for $h = 0.65$.