Project in TMA947 / MMG621 – Nonlinear optimization Planning of electricity production and transmission

1 Introduction

In this project, you will derive a simplified model for planning production and transmission of electric power. Among other things, the model will only take into account power plants for which "production is plannable", i.e., production from power plants such as hydro, thermal, and nuclear, whereas power production from power plants such as solar and wind are not included in the planning.

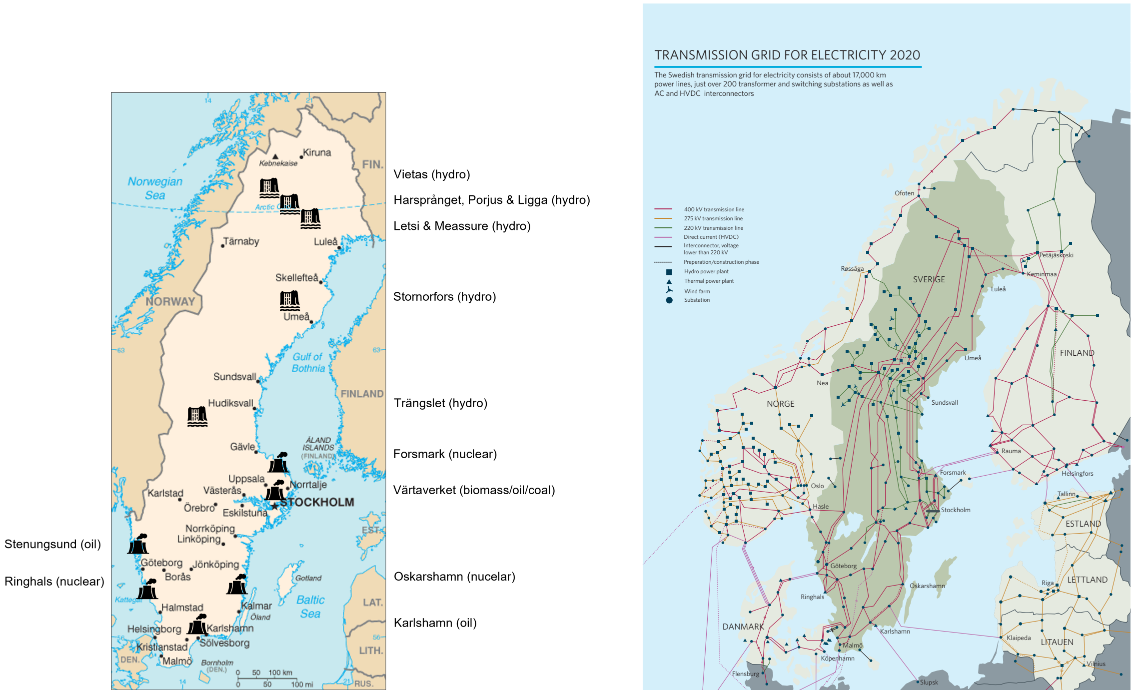
To put the problem in context, Table 1 contains information about the 14 largest power plants in Sweden for which "production is plannable". Moreover, in Figure 1a, the plants approximate geographic locations are shown, and Figure 1b shows the main transmission lines of the Swedish national electricity grid. As can be seen, this is a large-scale and complex system. In this project, we will, for simplicity, consider a simplified electric grid with fewer components. Moreover, the data in the project is also made-up. Nevertheless, the same type of modeling can be done for real electricity production and transmission.

Name	Capacity [MW]	Type of power plant
Forsmarks	3 271	nuclear
Ringhals	2 190	nuclear
Oskarshamn	1 450	nuclear
Harsprångets	977	hydro
Karlshamnsverket	662	oil
Stornorrfors	599	hydro
Värtaverket	579	biomass; oil; coal
Stenungsunds	520	oil
Letsi	486	hydro
Porjus	481	hydro
Messaure	446	hydro
Ligga	327	hydro
Vietas	306	hydro
Trängslet	300	hydro

Table 1: The 14 largest power plants in Sweden for which production is plannable, i.e., excluding wind and solar power. Data from `https://openinframap.org/stats/area/Sweden/plants`, with capacities for Ringhals and Oskarshamn updated from their corresponding Wikipedia page.

[1]Original map is in public domain, and taken from `https://commons.wikimedia.org/wiki/File:Sweden-CIA_WFB_Map.png`. Geographic location of the power plants from `https://openinframap.org/stats/area/Sweden/plants`.

[2]This is a cropped and resized version of the original image by Svenska Kraftnät from `https://www.svk.se/siteassets/english/national-grid/transmission-grid-for-electricity-map.pdf`.



(a) Map of power plants.

(b) Map of transmission grid.

Figure 1: (a) Map of Sweden with the approximate locations of the power plants given in Table 1 marked on the map. For convenience, some power plants that are geographically (relatively) close to each other have been marked together.[1](b) Map of the national grid for electricity transmission.[2]

2 A simplified model of a transmission grid

In an alternating current (AC) circuit, the direction of flow of the current is periodically reversed and the magnitude of the current changes continuously with time. The flow of the current is sinusoidal, i.e., the amplitude of the current in a fixed point in the grid follows a sine function in time. This is how electric energy is transported and consumed in most of the electric grids around the world. Also the Swedish electric grid uses alternating current, and the Swedish grid operates at 50Hz, meaning that the current changes direction 100 times per second.

In an electric grid that uses alternating current, there are several notions of power that are important. Instantaneous power at a point in the circuit is the rate of flow of energy past that point at a given time instance. However, not all instantaneous power results in net energy transfer when averaged over one cycle in the alternating current. This is due to that energy is stored in the transmission grid itself, in elements such as inductors and capacitors. The net energy transfer, average over the length of one cycle, is known as active power. When a generator transmits power in a transmission grid, the active power is the power that a receiving unit on the other side of the transmission grid can actually use. The remaining part of the energy in the system, which does not result in a net energy transfer, is known as reactive power.

When planning electric power production and transmission, one has to consider both the active and the reactive power. To this end, consider the network shown in Figure 2. To each node k in the network, there is an associated voltage amplitude v_k, and a voltage phase angle θ_k. The voltage amplitudes and the phase angles at two connected nodes determine how much power that flows in the transmission line (i.e., the edge) between the nodes. In the simplest model, for two neighboring nodes k and ℓ, the amount of active power that flows from node k to node ℓ (note the order of the indices!) is given by

$$p_{k\ell} = v_k^2 g_{k\ell} - v_k v_\ell g_{k\ell} \cos(\theta_k - \theta_\ell) - v_k v_\ell b_{k\ell} \sin(\theta_k - \theta_\ell),$$

and the amount of reactive power that flows from node k to node ℓ is given by

$$q_{k\ell} = -v_k^2 b_{k\ell} + v_k v_\ell b_{k\ell} \cos(\theta_k - \theta_\ell) - v_k v_\ell g_{k\ell} \sin(\theta_k - \theta_\ell).$$

Here, $b_{k\ell}$ and $g_{k\ell}$ are parameters describing the edges between the nodes.[3] The voltage angles are given radians, and must be kept between $-\pi$ and π. The voltage amplitudes are given in normalized voltage units (vu), and must be kept within 0.98 vu and 1.02 vu. The active and reactive powers are given in normalized power units (pu). The parameter values $b_{k\ell}$ and $g_{k\ell}$ are normalized accordingly, and the values of the parameters for the edges in Figure 2 are given in Table 2. Also note that for a link between node k and node ℓ, in general, one has that

$$p_{k\ell} \neq -p_{\ell k}.$$

This is due to energy losses in the network. The same is true for the reactive power, i.e., in general one has that $q_{k\ell} \neq -q_{\ell k}$.

Coeff. \ Edge (k,ℓ)	$(1,2)$	$(1,11)$	$(2,3)$	$(2,11)$	$(3,4)$	$(3,9)$	$(4,5)$	$(5,6)$	$(5,8)$	$(6,7)$	$(7,8)$	$(7,9)$	$(8,9)$	$(9,10)$	$(10,11)$
$b_{k\ell}$	-20.1	-22.3	-16.8	-17.2	-11.7	-19.4	-10.8	-12.3	-9.2	-13.9	-8.7	-11.3	-7.7	-13.5	-26.7
$g_{k\ell}$	4.12	5.67	2.41	2.78	1.98	3.23	1.59	1.71	1.26	1.11	1.32	2.01	4.41	2.14	5.06

Table 2: Values for the parameters describing the edges. Note that $b_{k\ell} = b_{\ell k}$ and $g_{k\ell} = g_{\ell k}$.

Each node in the network is nonempty, meaning that for each node there is either at least one generator, at least one consumer, or both generators and consumers. Each generator can generate a nonnegative amount of active power, up to some maximum capacity, and the price per generated unit power varies among the generators. Moreover, each consumer has a demand for a certain amount of active power, and if a generator and a consumer are located in the same node then the generator can provide active power to the consumer without having to transmit the power in the electric grid. We also assume that each generator can be used to either generate or absorb reactive power, and that the generators are the only objects in the network that can generate or absorb the reactive power.[4] The amount of reactive power a generator can generate is at most 3% of the maximum capacity of the generator, and the amount of reactive power a generator can absorb is at most -3% of the maximum capacity of the generator. There is no cost associated to generating or absorbing reactive power. In the network, there are nine generators, denoted $G_1, \dots, G_9$, and seven consumers, denoted $C_1, \dots, C_7$. Data for the generators is given in Table 3, and data for the consumers is given in Table 4.

[3] More specifically, they are the real and imaginary parts of the so-called shunt admittance, respectively. The admittance of a electronic component is a measure of how easily current can flow in the component. By definition, the admittance Y is $Y = 1/Z$ where Z is the impedance of the component, i.e., the combined effect of the resistance and reactance for a component that operates under alternating current. This model for active and reactive power flow is normally only used for shorter transmission lines.

[4] This assumption is based on the fact that reactive power cannot be used to extract energy. Therefore, consumers have no interest in reactive power, and therefore have no demand for it.

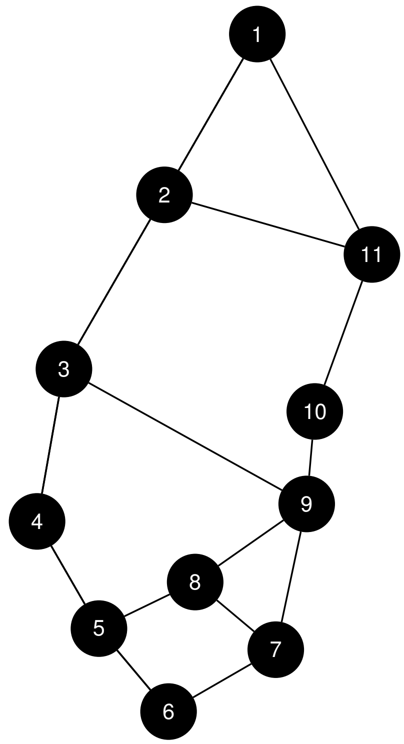


Figure 2: Illustration of the transmission grid as a network. Generators and consumers are located in the nodes of the network, and the transmission lines are the edges in the network.

3 Assignment

Your first task is to formulate an optimization problem for minimizing the cost of production of active power, while satisfying the consumers demand for active power. The nonlinear program should be based on the information and data given in the Section 2.

When you have formulated the model, your next task is to implement the model in a suitable programming language (see instructions below, in Appendix A), and to solve the minimization problem numerically.

Your final task is to take the results from the above two tasks and write a report. The report must be written in an appropriate text formatting tool (e.g., LaTeX or Word), and *must* look professional. The latter is very important, since one of the purposes of this assignment is that you should learn how to formulate a complex problem as an easily understandable nonlinear program, and how to present it to an audience of both experts and non-experts. Therefore, the report should look as if you were given the task to formulate and solve this model for a company, public institution, or NGO.

Generator	Location node	Maximum capacity [pu]	Energy production cost [SEK/pu]
G_1	2	0.02	175
G_2	2	0.15	100
G_3	2	0.08	150
G_4	3	0.07	150
G_5	4	0.04	300
G_6	5	0.17	350
G_7	7	0.17	400
G_8	9	0.26	300
G_9	9	0.05	200

Table 3: Parameters related to the 9 generators.

Generator	Location node	Demand active power [pu]
C_1	1	0.10
C_2	4	0.19
C_3	6	0.11
C_4	8	0.09
C_5	9	0.21
C_6	10	0.05
C_7	11	0.04

Table 4: Parameters related to the 7 consumers.

3.1 Further instructions for the report

In the industry, it is more likely that your model will be used if the managers understand it. To make the model easy to understand, even for people with just a basic knowledge of mathematics, the variables, the objective function, and the constraints of the model have to be clearly defined and explained. Moreover, the report should also contain answers to the following questions that are related to the numerical solution obtained from the implementation: What is the total cost of power production? How much active power does each generator produce? How much reactive power does each generator produce or absorb? What are the voltage amplitudes and the voltage phase angles in the nodes? How much active and reactive power is flowing along the edges of the network? You also have to be able to explain what you know about the obtained solution: Is the solution locally optimal? Is it globally optimal? Motivate this carefully.

We have the following requirements on your report:

- it shall include a figure illustrating the power flows in the problem;
- the variables must be clearly defined and connected to the figure;
- the objective function and the constraints should be clearly described;
- the report must contain answers to all questions above. However, the questions should be answered in a "natural way", and *not* be given as a "list of answers to the questions".
- the code you implement and use for completing the assignment should be added in the report as an appendix.

A suggested outline for the report is:

- title page;

- possibly a short abstract summarizing the content;
- mathematical formulation;
- results and analysis (where you interpret the numerical results);
- summary and conclusions;
- appendix with code.

The report should be written in groups of two or three students. Names and email addresses of all the group members should be clearly stated on the first page of the report. You may discuss the problem with other students. However, it is not allowed to use solutions made by others in any form. Moreover, each group must hand in their own solution. Finally, note that the report will be checked for plagiarism via `https://www.ouriginal.com/`.

Good luck!

A Instructions for implementation in julia

To implement the model you formulated, you should use the programming language `julia`, which is installed on the StuDat Linux computers. If you want to use your own laptop, follow the `julia` installation guide `https://julialang.org/downloads/`. In any case, on a machine where `julia` is installed, open the `julia` promp (also know as the REPL, often accessible by writing `julia` in a terminal) and run the following lines to install `JuMP` and the solver `Ipopt`:

```
import Pkg
Pkg.add("JuMP")
Pkg.add("Ipopt")
```

`JuMP` is a modeling language for mathematical optimization embedded in `julia`, and `Ipopt` is an optimization solver that works with `JuMP`. The latter is open source and available under the EPL license.[5] Other solvers can be found here `https://jump.dev/JuMP.jl/stable/installation/#Installation-Guide`.

A small example of how to use `julia` and `JuMP` is provided in the files `intro_example.jl` and `intro_data.jl`, which can be found on the course homepage. You can run the files in the terminal by moving to the folder in which they are located (use `cd` to change folder) and typing

```
julia intro_example.jl
```

Study these files carefully to get acquainted with the language and the package. Most of what you will need in order to implement your model is given in this example. Many more examples can also be found on the `JuMP` homepage `https://jump.dev/JuMP.jl/stable/`.

`julia` can conveniently be used together with the IDE Visual Studio Code. It should be installed on the StuDat Linux computers, but can also be found here `https://code.visualstudio.com/`. To use Visual Studio Code with `julia`, an extension to it must be installed via VS Code Marketplace (in Visual Studio Code, go to `View/Extensions` and search for julia; see instructions `https://code.visualstudio.com/docs/languages/julia`).

[5]The license text found here `https://github.com/coin-or/Ipopt/blob/stable/3.14/LICENSE`