Transverse field Ising Model

Abstract

Study on the spectrum of the quantum transverse Ising Model.

1 Theory

The aim of this assignment was to study the N bodies transverse-field Ising Model. With hamiltonian:

$$\hat{H} = \lambda \sum_{i=1}^{N} \sigma_z^i + \sum_{i=1}^{N-1} \sigma_x^{i+1} \sigma_x^i \tag{1}$$

Where λ is a parameter linked to the intensity of an external field, and $\sigma_{x/z}^i$ are the operators obtained by

$$\sigma_{x/z}^i = \mathbb{1}_{2^{i-1}} \otimes \sigma_{x/z} \otimes \mathbb{1}_{2^{N-i}} \tag{2}$$

with $\sigma_{x/z}$ the Pauli matrices on the x or z axis.

2 Code Development

To obtain the $\hat{H}$ operator, it is necessary to compute the Kronecker product, as stated in Eq.[2], for all the cases prescribed by the i index. To perform this operation efficiently a function that exploits the properties of the identity matrices in the such operation has been developed. As input it needs the total dimension of the operator, the index of the Pauli matrix in the product, and the 2×2 Pauli matrix in form a 2D complex*16 array.

```
function getbigmat(paulimat, index, N) result(bigmat)
        integer :: index, N, ii,jj, kk1, kk2
        integer :: helpidx(2), blocksize
        double complex , dimension(2,2) :: paulimat
        double complex, dimension(2**N,2**N) :: bigmat
        blocksize = (2**(N-index))
        bigmat = 0.0
        do ii = 1, 2
```

```
            do jj = 1, 2
                do kk1 = 0,2**(index-1)-1
                    helpidx(1)= (ii)+2*kk1
                    helpidx(2)= (jj)+2*kk1
                    do kk2 = 1, blocksize
                        bigmat((helpidx(1)-1)*blocksize+kk2
                            ,(helpidx(2)-1)*blocksize+kk2) &
                            = paulimat(ii,jj)
                    end do
                end do
            end do
        end do
    end function
```

After computing the components of the Hamiltonian operator, I proceeded computing its eigenvalues and eigenvectors. This has been done using a function that interfaces more easily with the *zheev* subroutine of the **lapack** package.

3 Results

The lowest five eigenvalues have been evaluated for different number of particles in the system. The results of this test can be seen in Fig.[1]. Here for $\lambda = 0$ we can appreciate the degeneracy of the two ground states, and of the first excited level. As λ increases the degeneracy is broken, since the energy is now dependent also on the field alignment. Then the lowest eigenvalue for different number of particles, normalized with two different prescriptions, have been compared with the mean field solution of this Hamiltonian (Fig.[2]). The two kinds of normalization highlight different aspects of the energies of the system. The N-normalization is obtained by dividing the Hamiltonian eigenvalues by the number of particles, and represent the per-body energy (e). While the second one, $(N-1)$-normalization, is obtained by dividing the eigenvalues by the number of particles (N) minus one. The first normalization is found to be in better agreement with the mean field approximation for bigger values of λ, while the second one for lower values. This fact is well explained since in the mean field approach every particle is supposed to interact with the others in the same way. While for the Hamiltonian in Eq.[1] this is not true because of the missing couplings of the first and last particle. To overcome this issue and be able to investigate, at least visually, the potentiality of the mean field approach, I decided to study also the spectrum of the following Hamiltonian H'.

$$\hat{H}' = \hat{H} + \sigma_x^N \sigma_x^1 = \lambda \sum_{i=1}^{N} \sigma_z^i + \sum_{i=1}^{N-1} \sigma_x^i \sigma_x^{i+1} + \sigma_x^N \sigma_x^1 \tag{3}$$

With this the symmetry for the interaction term is introduced, and the comparison of the results with the mean field solution are now independent by the normalization prescription since both interaction and field term scales with the number of particles.

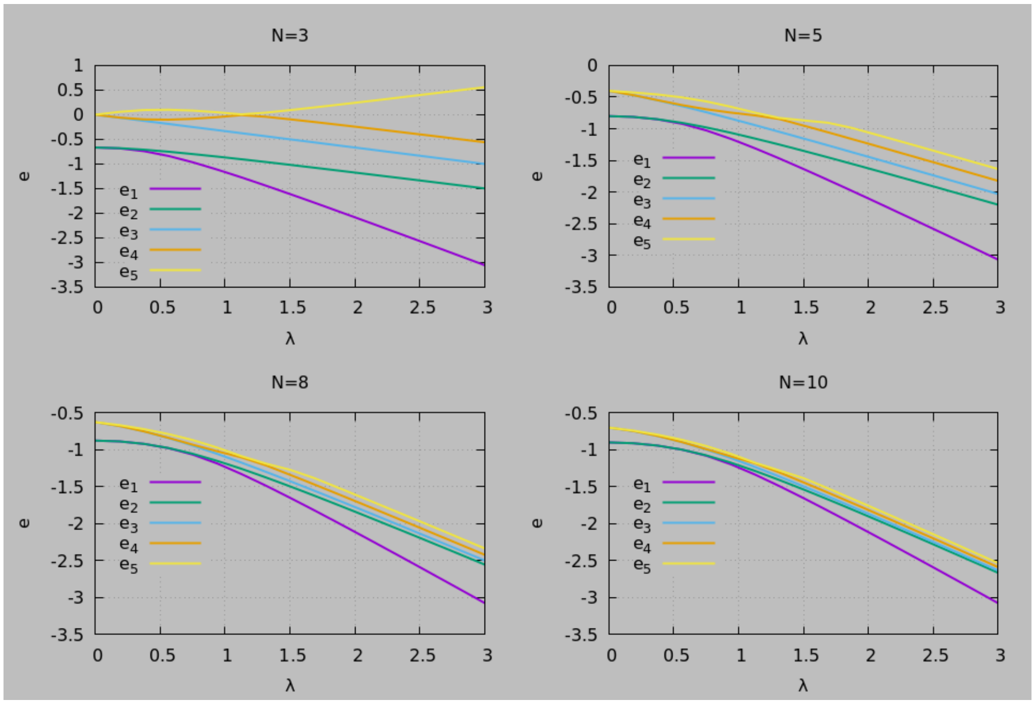

Figure 1: Plots showing the eigenvalues of $\hat{H}$ divided by the number of bodies(N) as function of the λ parameter. The results are presented for different number of particles.

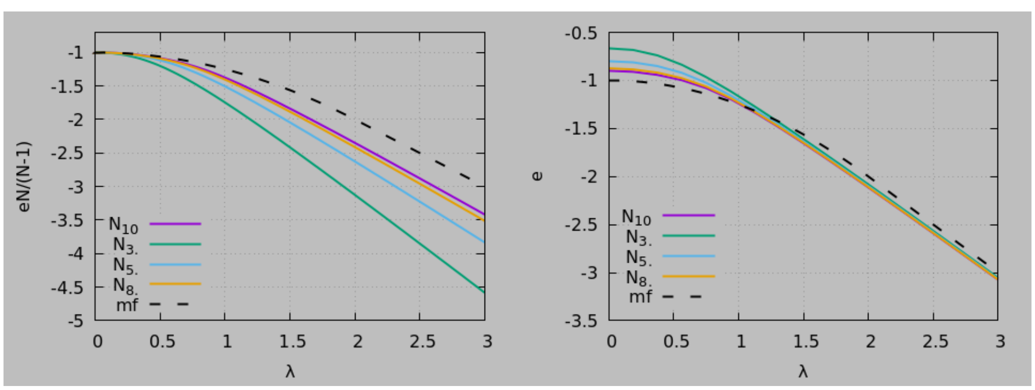

Figure 2: Plots showing the lowest energy per body for different number of particles in the system, with two different normalizations.

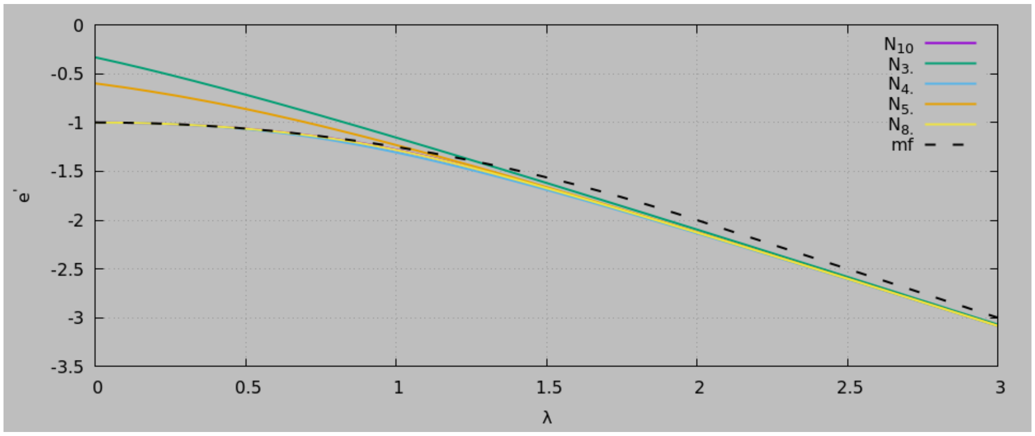

Figure 3: Plots showing the lowest energy per body for different number of particles in the system, with two different normalizations for H' Hamiltonian.

$H^{'}$ however suffer another issue: for low values of λ and using an odd number of particles, a perfect series of alternating spins is forbidden. And such states have a greater energies with respect to the ones with an even number of particles. The results of the study of $H^{'}$ are showed in Fig.[3]. Here, it is possible to directly see the difference of the ground states of even and odd numbers of particle. Moreover for even sets the overall correspondence with the mean field solution is improved in all the simulated domain of $lambda$. Indeed the mean field approach give an acceptable estimate of the solution.

4 Self Development

I found this project quite interesting, in fact it helped me to deepen by knowledge about the transverse field Ising Model and its mean field approximated solution. Further studies about this argument could focus on understanding the condition for the mean field solution to be a better picture of the exact eigenvalues of the Hamiltonian.