TITLE OF THE PAPER

EDUARDO SILVA
ESILVA@DIM.UCHILE.CL

Abstract. abstract

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Introduction

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We show:

Theorem ??. *Every countable group G has a non-empty, strongly aperiodic subshift on the alphabet $\{0, 1\}$.*

Theorem ??. *Every finitely generated group G has a non-empty G-effectively closed strongly aperiodic subshift.*

1. Preliminaries

Throughout this article the groups G considered will always be countable; we denote their identity element 1_G. When G is finitely generated we associate a finite set $S \subset G$ of generators and the undirected right Cayley graph $\Gamma(G, S) = (G, \{\{g, gs\} \mid g \in G, s \in S\})$ so that (G, d) is a metric space where d is the distance induced on G by $\Gamma(G, S)$. If two words w_1, w_2 in S^* represent the same element in G, we write $w_1 =_G w_2$. We shall denote by $B(g, n) = \{h \in G \mid d(g, h) \leq n\}$ the ball of size n centered in $g \in G$. In general we denote $B_\Gamma(v, n)$ the ball of size n centered in v of an arbitrary graph Γ. For $g \in G$, we denote by $|g|$ the length of a shortest path from 1_G to g in $\Gamma(G, S)$, that is to say $|g| = d(1_G, g)$. We also denote by $\mathtt{WP}(G) := \{w \in (S \cup S^{-1})^* \mid w =_G 1_G\}$ the set of words which can be written using elements from S and their inverses which are equal to 1_G in the group G. If $\mathtt{WP}(G)$ is a decidable language we say that G has decidable word problem. For more references see [17].

We now give some basic definitions of symbolic dynamics. For a more complete introduction the reader may refer to [15, 8]. Let $\mathcal{A}$ be a finite alphabet and G a countable group. The set $\mathcal{A}^G = \{x : G \to \mathcal{A}\}$ equipped with the left group action $\sigma : G \times \mathcal{A}^G \to \mathcal{A}^G$ defined by $(\sigma_g(x))_h = x_{g^{-1}h}$ is the *G-fullshift*. The elements $a \in \mathcal{A}$ and $x \in \mathcal{A}^G$ are called *symbols* and *configurations* respectively. By taking the discrete topology on $\mathcal{A}$ we obtain that the set of configurations $\mathcal{A}^G$ is compact and metrizable. In the case of a countable group, given an enumeration $1_G = g_0, g_1, \dots$ of G, the topology is generated by the metric $d(x, y) = 2^{-\inf(\{n \in \mathbb{N} \mid x_{g_n} \neq y_{g_n}\})}$. If E is a subset of $\mathcal{A}^G$, we denote by $\overline{E}$ its topological closure. In the case of a finitely generated group another possibility which is more practical is $d(x, y) = 2^{-\inf\{|g| \mid g \in G: x_g \neq y_g\}}$. This topology is generated by a clopen basis given by the *cylinders* $[a]_g = \{x \in \mathcal{A}^G | x_g = a \in \mathcal{A}\}$. A *support* is a finite subset $F \subset G$. Given a support F, a *pattern with support* F is an element p of $\mathcal{A}^F$, i.e. a finite configuration and we write $supp(p) = F$. We also denote the cylinder generated by p centered in g as $[p]_g = \bigcap_{h\in F}[p_h]_{gh}$ One says that a pattern $p \in \mathcal{A}^F$ *appears* in a configuration $x \in \mathcal{A}^G$ if there exists $g \in G$ such that for any $h \in F$, $x_{gh} = p_h$, said otherwise, if there exists g such that $x \in [p]_g$. In this case we write $p \sqsubset x$. We denote the set of finite patterns over G as $\mathcal{A}^*_G := \bigcup_{F\subset G, |F|<\infty} \mathcal{A}^F$.

Definition 1.1. *A subset X of $\mathcal{A}^G$ is a* G-subshift *if it is σ-invariant – $\sigma(X) \subset X$ – and closed for the cylinder topology. Equivalently, X is a G-subshift if and only if there exists a set of forbidden patterns $\mathcal{F} \subset \mathcal{A}^*_G$ that defines it.*

$$X = X_{\mathcal{F}} := \left\{ x \in \mathcal{A}^G \mid \forall p \in \mathcal{F}, p \not\sqsubset x \right\} = \bigcap_{p\in\mathcal{F}, g\in G} \mathcal{A}^G \setminus [p]_g.$$

That is, a G-subshift is a shift-invariant subset of $\mathcal{A}^G$ which can be written as the complement of a union of cylinders. If it is clear from the context, we will drop the G and simply refer to a subshift. A subshift $X \subseteq \mathcal{A}^G$ is *of finite type* – G-SFT for short – if there exists a finite set of forbidden patterns $\mathcal{F}$ such that $X = X_{\mathcal{F}}$.

Consider a group which is generated by a finite set S. A *pattern coding* c is a finite set of tuples $c = (w_i, a_i)_{1\leq i\leq n}$ where $w_i \in (S \cup S^{-1})^*$ and $a_i \in \mathcal{A}$. We say that a pattern coding is *consistent* if for every pair of tuples such that $w_i =_G w_j$ (w_i and w_j represent the same element under G) then $a_i = a_j$. We say a consistent pattern coding c *codifies* a pattern P if every w_i represents an element of $supp(P)$ and for every $g \in supp(P)$ there exists a tuple $(w_i, a_i) \in c$ such that $g =_G w_i$ and $P_g = a_i$.

Definition 1.2. *For a finitely generated group G we say a subshift $X \subseteq \mathcal{A}^G$ is* G-effectively closed *if there exists a Turing machine with oracle* $\texttt{WP}(G)$ *which recognizes a set of pattern codings such that the consistent ones codify a set of patterns $\mathcal{F}$ such that $X = X_{\mathcal{F}}$. If the same property is valid without the oracle we say X is* effectively closed.

Being G-effectively closed is a generalization of the same concept for $\mathbb{Z}$-subshifts where the set of forbidden patterns is a recursively enumerable set of words.

Let $x \in \mathcal{A}^G$ be a configuration. The *orbit* of x is the set of configurations $orb_\sigma(x) = \{\sigma_g(x) \mid g \in G\}$, and the *stabilizer* of x is the set of group elements $stab_\sigma(x) = \{ g \in G \mid \sigma_g(x) = x\}$.

Definition 1.3. *A G-subshift $X \subseteq \mathcal{A}^G$ is* weakly aperiodic *if for every configuration $x \in X$, $|orb_\sigma(x)| = \infty$. A G-subshift $X \subseteq \mathcal{A}^G$ is* strongly aperiodic *if for every configuration $x \in X$, $stab_\sigma(x) = \{1_G\}$.*

For infinite groups the weak concept of aperiodicity is relevant and implied by strong aperiodicity.

2. Non-empty strongly aperiodic subshifts

2.1. A non-empty strongly aperiodic subshift over $\{0, 1\}$ in any countable group.

2.2. A graph-oriented construction and some computational properties.

we still don't know if this kind of construction is always possible. To our knowledge the following question remains open:

Question. *dssad?*

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