Analysis of the vibrations of a beam

Laboratory experience

Course of Mechanical Vibrations

June 1, 2022

Important notes

This document contains the detailed assignments of the laboratory experience of the course of *Mechanical Vibrations*.

Important notes and FAQs:

- The report is individual, so every student has to write their own.
- You can write the report either in English or in Italian.
- The number of pages does not influence the final grade, what is important is that you answer to all the requests and give your personal interpretation of the results.
- The report has to be uploaded 3-4 days before the oral exam (an apposite form will be set up on the on-line community before every oral examination).
- You have to print the report and bring it with you at the oral exam.
- Most of the time, if your oral examination fails, you can keep the report without re-writing it from scratch.
- If you do not give the exam in this academic year, you can still write the report of the laboratory experience you attended this year (May 2022).
- Plots must include the units of measurement.
- Remember that angular frequency (rad/s) is different from frequency (Hz).
- Don't forget to check the hints in the last section of this document.
- Useful code for MATLAB is available at the end of the document. Anyway you are free to use the software you prefer.

For any question regarding the report, send an e-mail at edoardo.dallaricca@unitn.it.

Introduction

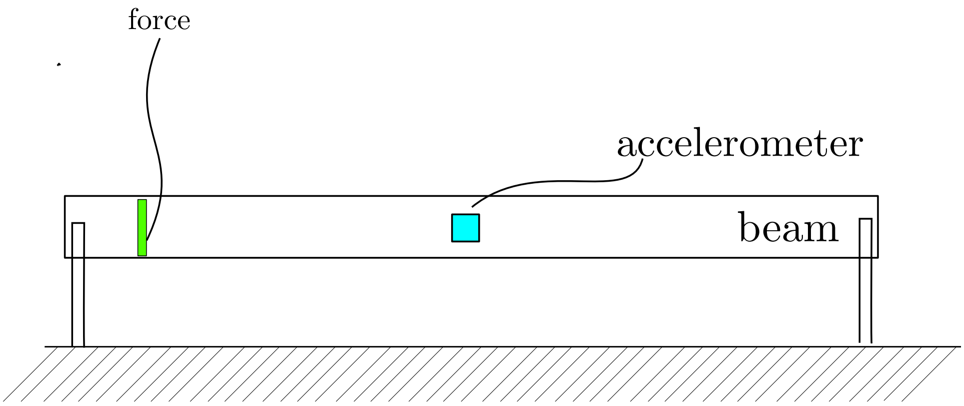

Figure 1: System sketch, with accelerometers and force sensor

The analysis will focus on the laboratory setup schematized in Figure 1. A beam (length L) is pinned to two rods, which are connected to the ground.
An hammer generates a force $f(t, x)$ on a specific point of the beam ($L/8$, green line in Figure 1). The force can be modeled as an impulse in the time domain, while in the space domain can be modeled as a concentrated force.
The force is measured thanks to a force sensor (which is included in the hammer), the accelerations of the beam and the cart are measured with two accelerometers positioned at $L/2$ (in Figure 1 you can see only one accelerometer, the other one is on the other side of the beam).
The collected data are stored in the file named *data* (the data were acquired with a sampling frequency of 51 200 Hz). The first column of the file is the acquisiton time, the second the force recorded by the hammer, the third and the fourth the accelerations recorded by the accelerometers.

Analytical model

The beam data are:

- Young modulus: E =206 GPa
- Mass density: ρ =7850 kg m^{-3}
- Beam cross-section area: $A = 111\,\mathrm{mm}^2$
- Beam cross-section moment of inertia: $J = 6370\,\mathrm{mm}^4$
- Beam length: L =0.7 m
- To build the analytical model and plot the frequency responses, use the following values of the damping for the modes: $\zeta_1 = 0.05$, $\zeta_2 = \zeta_3 = 0.01$

Given the configuration of the experiment, perform the following calculations using the beam data reported above:

- Referring to Figure 1, choose a reference frame and write the analytical model of the beam. Consider only the first three modes to descrie the system
- Consider the system as undamped. Calculate the first three natural frequencies and plot the mode shapes
- Consider the system as damped. Start from the generic differential equation of the forced damped modal coordinates:
    1. Calculate the modal projection of the impulse force $f(t, x)$
    2. Plot the analytical transfer functions between an impulse force applied at $L/8$ and the beam acceleration at $L/2$ up to 2000 Hz. How many peaks are present in the transfer function? Why?

Experimental methodology

Using the file given you:

- Load and plot the mean beam accelerations as a function of time. Pay attention to how you compute the mean signal. Not all the acquired samples are significant to our scope (for example, some are noise). So, extract only the meaningful time window
- Calculate the experimental transfer functions between the acquired impulse impulse force and mean acceleration. Compare it with the analytical transfer function and write some considerations.
- Using the half power points method, estimate the damping ratios of the natural frequencies

Useful hints

Experimental TFs

To compute the experimental TFs in MATLAB, you can use the command `tfestimate`. It is based on the Welch's method (`https://en.wikipedia.org/wiki/Welch%27s_method`).

```
% t   : time vector
% acc: acceleration vector
% fs  : sampling frequency
% Tf  : transfer function (complex)
% Fr  : vector of the frequencies
[Tf,Fr] = tfestimate(f,acc,[],[],[],fs);

% Tm : magnitude of the transfer function
Tm = abs(Tf); % real vector
```

Analytical TFs

To compute the analytical TFs in MATLAB

```


                    % L is the number of samples of the experimental
                        data

                    fr = samp_freq*(0:(L/2)-1)/L; %frequency scale

                    s = sqrt(-1)*fr; % definition of vector "s" as "
                        i" times "fr"
                    % "s" is a complex vector.


                    %% ================= Defining analytical TF
                        =====================

                    % Now we define an analytical TF. We don't want
                        not a constant
                    % vector, instead a function of some
                    % parameters ("a,b,c" in this case).

                    G_an = @(a,b,c) a.*s.^2 + b.*s + c./s.^3; % This
                        is not a
                    % vector, it is a
                    % vectorial function!

                    % NB: this is not the only way to define
                        functions in matlab
```

Half power point method

Let Tm be a discrete function, coming from experimental data (see 'Experimental TFs' for the function definition). Let g be a continuous constant function.
The first step in the half power point method is to find the intersection between the two functions, starting from a user-defined starting point.
See the MATLAB example below.

```
%% ================== Preliminary commands ========================

clc;          % clear command window (where you see text outputs)
close all;    % close all the opened figures
clear all;    % clear all the save variables




%% ========== Defining vectors of frequnecies ===================


fr = samp_freq*(0:(L/2)-1)/L; %frequency scale


sol = fsolve(@(x) interp1(fr,abs(Tm), x, 'spline') - g,
    starting_point);
```