

Anomaly detection interpretation based on Mahalanobis distance decomposition

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Outline:

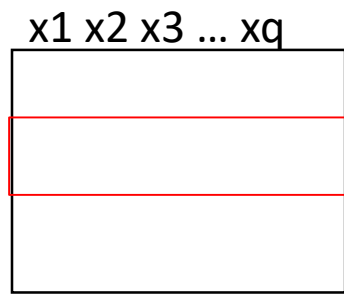
- 0) Introduction
- 1) Training
- 2) Gaussian mixture Fitting
- 3) Mahalanobis distance computation
- 4) Minimum Mahalanobis distance computation
- 5) Check if the distance is under the threshold
- 6) Minimum Mahalanobis distance decomposition
- 7) Define the starting variable more responsible for the anomaly

0) Introduction

- **PCA** can be seen as a simpler autoencoder
- Going in another space(components), coming back(starting variables) and computing the ***error in reconstruction*** can detect anomalies (when there is a big error we raise an alert)
- Fitting the nominal data with a ***set of Gaussian*** allows to create a more reliable model
- Decomposing the ***mahalanobis distance*** it possible to retrieve the initial variables

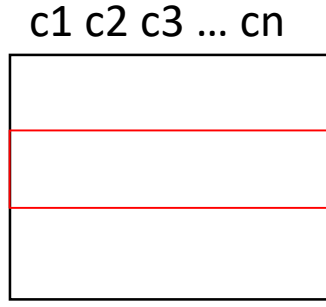
1) Training

Dataset Training X_train



q variables

Apply PCA



n components

$$C_1 = \sum_{i=1}^q K_{1i} * x_i$$

⋮

$$C_n = \sum_{i=1}^q K_{ni} * x_i$$

PCA loggings


| | | |
|-----|-----|-----|
| K11 | ... | K1q |
| ⋮ | ... | ⋮ |
| Kn1 | ... | Knq |

OUTPUT 1

2) Gaussian mixture fitting

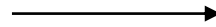
Dataset Training Transformed C

c1 c2 c3 ... cn



| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Fit mixture with m Gaussian



$i = 1 \dots m$

$$\mu_i = [\mu_{1_i}, \dots, \mu_{c_i}]$$

$$\Sigma_i = \text{diag}(\sigma_{1_i}, \dots, \sigma_{c_i})$$

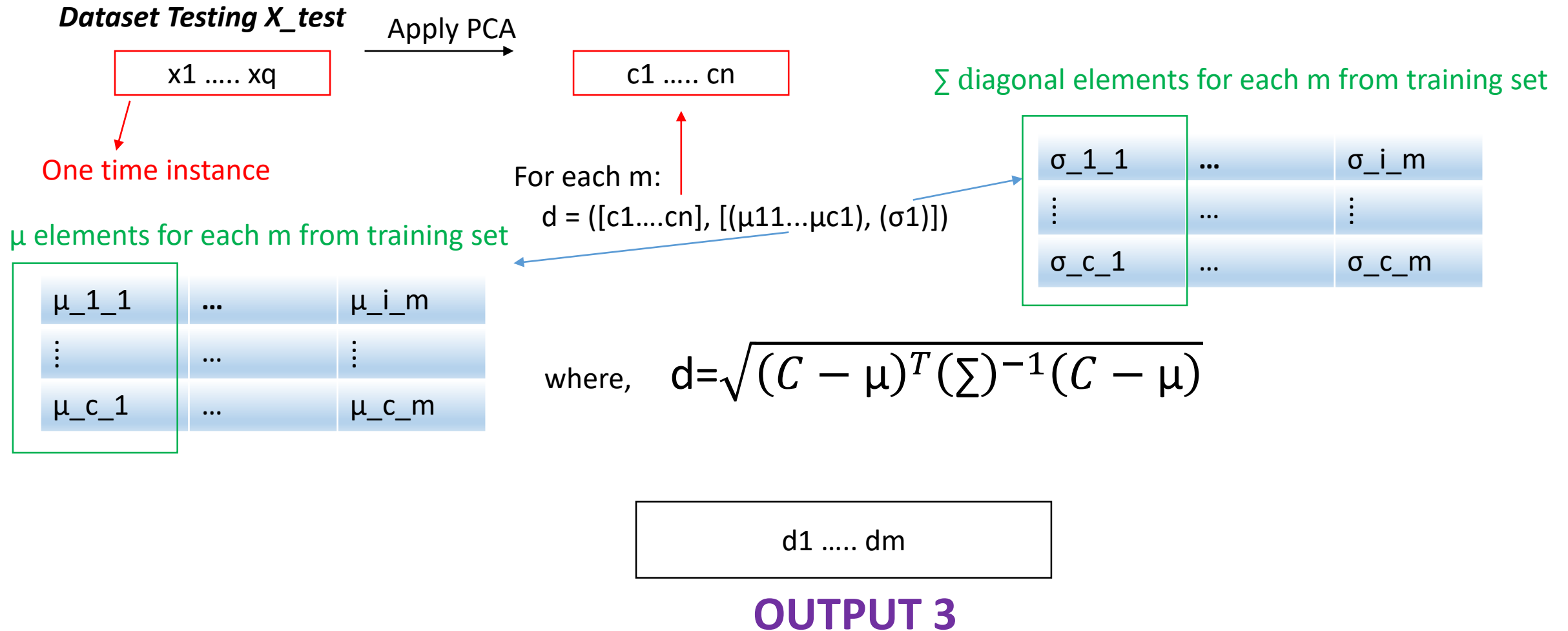
One time instance

| | | |
|-------------|-----|-------------|
| μ_{1_1} | ... | μ_{i_m} |
| \vdots | ... | \vdots |
| μ_{c_1} | ... | μ_{c_m} |

| | | |
|----------------|-----|----------------|
| σ_{1_1} | ... | σ_{1_m} |
| \vdots | ... | \vdots |
| σ_{c_1} | ... | σ_{c_m} |

OUTPUT 2

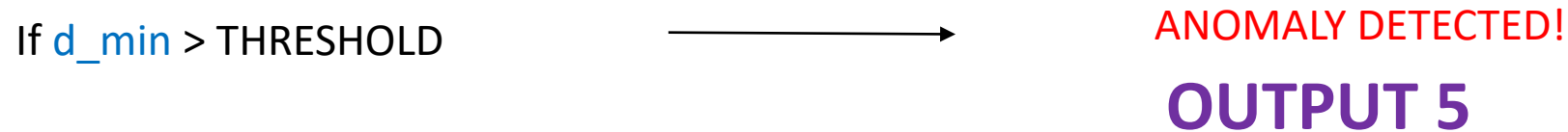
3) Mahalanobis distance computation



4) Minimum Mahalanobis distance computation



5) Check if the distance is under the threshold



How did we decompose the distance?

Test set

Training set

$$C_1 = \sum_{i=1}^q K_{1i} * x_i$$
$$d_{\min} = \sqrt{(C - \mu)^T (\Sigma)^{-1} (C - \mu)}$$

Relative to the nearest gaussian

$$d_{\min}^2 = \frac{(c_1 - \mu_1)^2}{\sigma_1} + \dots + \frac{(c_n - \mu_n)^2}{\sigma_n}$$
$$d_{\min}^2 = f(x) + f(\mu)$$

6) Minimum Mahalanobis distance decomposition

$$d_min_finale = d_min^2 - \sum_{i=1}^q \mu_i^2$$

$A_{x_i^2}$

For each $i=1, \dots, q$

OUTPUT 6

A_{x_i}

For each $i=1, \dots, q$

$A_{x_i x_j}$

For each $i=1, \dots, q$ with $i \neq j$

7) Define the starting variable more responsible for the anomaly

Search the bigger $|A_*|$ and define the related x_i

OUTPUT 7

How do we decide if the $|A_*|$ is enough big?

For each $|A_*|$ is used a threshold based on ***mean*** and ***standard deviation*** computed on nominal $|A_*|$.

Example

q_variables = 4, n_components = 3, mixture=5 ANOMALY → q=4

K components

```
[[-0.46182718 -0.5591836  0.6551106 -0.21180051]
 [-0.33681882 -0.41828449 -0.34570756  0.76946569]
 [ 0.82035696 -0.49709971  0.21379657  0.18492528]]
```

| | VEL LIN | LS | LC | LD |
|--------------|-------------|------------|--------|-------------|
| DATASET TEST | [0.26173008 | 6.54661097 | 0.6605 | 5.02211103] |

DATASET TEST AFTER PCA [-4.412617 0.809501 -1.969678]

RESULT

$x_1^2 - x_2^2 - x_3^2 - x_4^2$

[0.008695936764374215, 48.24616801989206, 0.5818851660326526, 124.30340628217398]

$x_1 - x_2 - x_3 - x_4$

[-0.6186189056274868, 79.90366203485536, -0.6952100181715678, -108.64032194935119]

$x_1 x_2 - x_1 x_3 - x_1 x_4 - x_2 x_3 - x_2 x_4 - x_3 x_4$

[-0.20991945107618623, 0.03354080910481509, -0.14702507822337377, -2.264383404426474,
-81.03373527567052, -9.631450904620603]

Example

| | | | | |
|-------------|----------------------|----------------------|---------------------|-----------------------|
| X1^2 | 0.005376593658132822 | 0.01406528277371126 | 564.3575118429884 | 0.002196104282187567 |
| X2^2 | 0.040022072775998334 | 320.10897652642507 | 0.7930072409584044 | 0.36852173286333867 |
| X3^2 | 193.25887720279718 | 0.3754555508489652 | 0.5067394335545415 | 0.22720295203060326 |
| X4^2 | 0.45034457752432877 | 1.5386103458720952 | 0.2849199979105846 | 340.83840098506346 |
| X1 | 0.054911344699514836 | 0.0888142607471055 | -33.38083180975043 | 0.0658485898436691 |
| X2 | -0.08925692813758927 | -7.98253945611004 | 5.232223899516044 | 3.566804419599815 |
| X3 | -54.17268150656344 | 2.3877555988058226 | -0.6171273657751212 | -0.4132277125747382 |
| X4 | -2.3068927546455718 | 4.26402026740884 | -3.1143331960345266 | -107.7154173109028 |
| X1X2 | 0.008552019287260062 | 1.2370533667432237 | [2.684127741816416 | -0.003609486991530586 |
| X1X3 | -0.7532365886426685 | 0.05369842807563012 | -0.5898713514139415 | 0.0007791503364421645 |
| X1X4 | -0.00803356603166589 | 0.024017115638546523 | -2.6145577138339178 | 0.17838582884391493 |
| X2X3 | -1.2928885479482584 | 5.096473437770478 | 0.5090851031898308 | 0.2323795995184279 |
| X2X4 | 0.17111227973470042 | -28.286041254294155 | -0.4534104177139992 | -10.690481434188866 |
| X3X4 | 11.209345586500177 | 0.9132331182282317 | 0.4657955320045161 | 10.787548116360082 |

Example (summing up the terms)

1
2
3
4

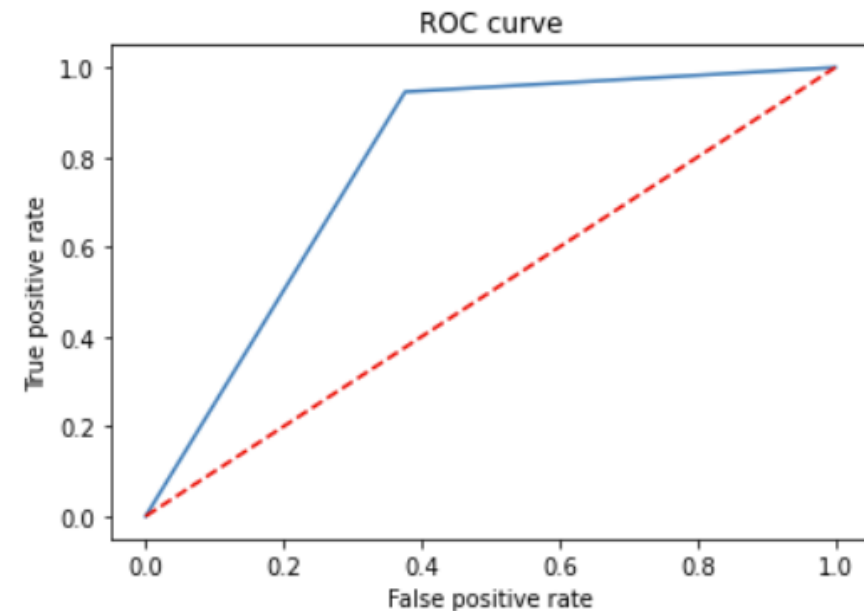
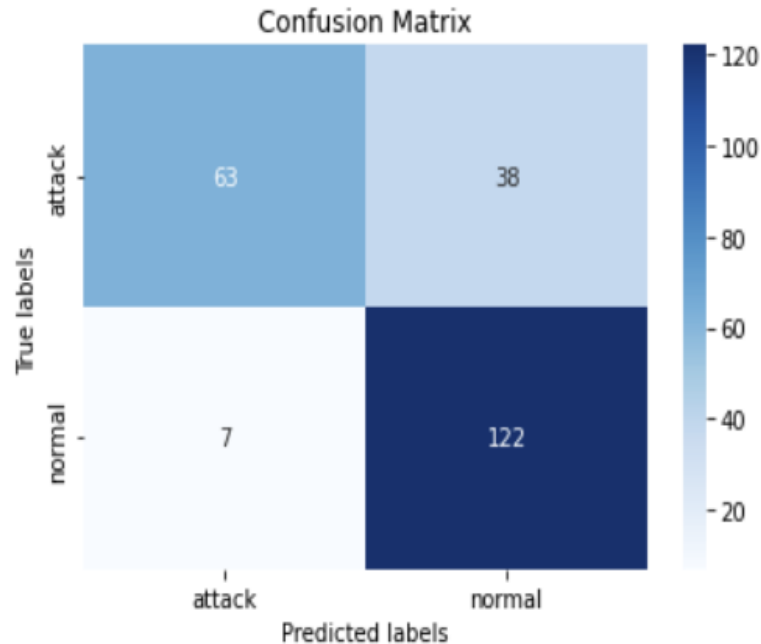
0.06028794
-0.04923486
139.0861957
- 1.85654818

0.10287954
312.12643707
2.76321115
5.80263061

530.97668003
6.02523114
-0.11038793
-2.8294132

0.06804469
3.93532615
-0.18602476
233.12298367

Overall result



Results and next challenges

- The accuracy is around 80%
- In most cases the system is able to give the right interpretation to the anomaly
- Find a smarter criteria for the THRESHOLD
- Investigate how the number of components are related to the interpretation, is there a limit?
- Define the nominal model with another kind of distribution

Thank you for your attention

Question??