Derive the expression for the hybrid matrix H.

$$\begin{bmatrix} f_m \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} H_{11} H_{12} \\ H_{21} H_{22} \end{bmatrix} \begin{bmatrix} \dot{x}_s \\ -f_s \end{bmatrix}$$

1.

$$f_m = H_{12}(-f_s) \qquad \dot{x}_s = 0$$

$$f_m(1+C_{mf}) - C_2 f_s = \dot{x}_m Z_{cm} \qquad where \qquad \begin{array}{l} Z_{cs} = Z_s + C_s, \qquad Z_{cm} = Z_m + C_m, \\ \dot{x}_m = \dfrac{\dot{x}_s Z_{cs} - f_m C_3}{C_1} \end{array}$$

$$f_m \frac{C_1 + C_1 C_{mf} + C_3 Z_{cm}}{C_1} = C_2 f_s$$

$$f_m = \frac{C_1 C_2 f_s}{C_1(1 + C_{mf}) + C_3 Z_{cm}}$$

$$H_{12} = \frac{-C_1 C_2}{C_1(1 + C_{mf}) + C_3 Z_{cm}}$$

2.

$$\dot{x}_m = H_{21}(\dot{x}_s) \qquad f_s = 0$$

$$C_4 \dot{x}_s = f_m(1 + C_{mf}) - \dot{x}_m Z_{cm} \qquad where f_m = \frac{\dot{x}_s Z_{cs} - C_1 \dot{x}_m}{C_3}$$

$$C_4 \dot{x}_s = \frac{\dot{x}_s Z_{cs} - C_1 \dot{x}_m}{C_3}(1 + C_{mf}) - \dot{x}_m Z_{cm}$$

$$C_3 C_4 \dot{x}_s = \dot{x}_s Z_{cs} - C_1 \dot{x}_m + C_{mf} \dot{x} s Z_{cs} - C_{mf} C_1 \dot{x}_m - C_3 \dot{x}_m Z_{cm}$$

$$\dot{x}_m(C_1 + C_1 C_{mf} + C_3 Z_{cm}) = \dot{x}_s(Z_{cs} + C_{mf} Z_{cs})$$

$$H_{21} = \frac{Z_{cs}(1 + C_{mf})}{C_1(1 + C_{mf}) + C_3 Z_{cm}}$$

3.

$$\dot{x}_m = H_{22}(-f_s) \qquad \dot{x}_s = 0$$

$$f_m(1+C_{mf}) - C_2 f_s = \dot{x}_m Z_{cm} \qquad where \qquad \begin{array}{l} Z_{cs} = Z_s + C_s, \qquad Z_{cm} = Z_m + C_m, \\ f_m = \dfrac{-\dot{x}_m C_1}{C_3} \end{array}$$

$$-\frac{\dot{x}_m C_1}{C_3} C_2 f_s = \dot{x}_m Z_{cm}$$

$$\dot{x}_m(C_3 Z_{cm} + C_1) = C_2 f_s C_3$$

$$H_{22} = \frac{-C_2 C_3}{C_3 Z_{cm} + C_1}$$

4.

$$f_m = H_{11}\dot{x}_s \qquad f_s = 0$$

$$f_m(1 + C_{mf}) - C_4\dot{x}_s - \dot{x}_m C_m = \dot{x}_m Z_m$$

$$C_4 x_s = f_m(1{+}C_{mf}){-}\dot{x}_m Z_{cm} \qquad \textit{where} \qquad \begin{array}{l} Z_{cs} = Z_s + C_s, \qquad Z_{cm} = Z_m + C_m, \\ f_m = \dfrac{\dot{x}_s Z_{cs} - f_m C_3}{C_1} \end{array}$$

$$C_4 C_1 \dot{x}_s = C_1 f_m(1 + C_{mf}) - \dot{x}_s Z_{cs} Z_{cm} - f_m C_3 Z_{cm}$$

$$(C_4 C_1 + Z_{cm} Z_{cs})\dot{x}_s = f_m(C_1 + C_1 C_{mf} - C_3 Z_{cm})$$

$$H_{11} = \frac{C_4 C_1 + Z_{cm} Z_{cs}}{C_1(1 + C_{mf}) - C_3 Z_{cm}}$$