LAB #4

Solving the 1D steady state heat diffusion problem by FD

- Solve linear systems with Matlab
  - Direct solution to linear systems "\" (mldivide):
    x = *A**b*, where *A* is a square matrix and *b* is a column vector, gives the solution to the linear set of equations $Ax = b$.
    MATLAB automatically chooses the most appropriate algorithm for the solution of the system, depending on the structure of *A* (*e.g.* Gauss LU is used for generic full or sparse matrices, Thomas is used for tridiagonal sparse matrices).
- Special instructions
  - "spy(*A*)" plots the sparsity pattern of the matrix *A*.
  - "sparse(*A*)" converts a sparse or full matrix to sparse form by squeezing out any zero elements
  - "full(*A*)" converts a sparse matrix to a full one by introducing zero elements
  - "spdiags(*B*, *b*, *m*, *n*)" creates an m-by-n sparse matrix by taking the columns of *B* and placing them along the diagonals specified by the vector *b*. <u>BE CAREFUL</u>: if $m >= n$ spdiags takes elements of super-diagonals from the lower part of the column of *B*, and elements of sub-diagonals from the upper part of the column of *B*; if $m < n$, then super-diagonals are from the upper part of the column of *B*, and sub-diagonals from the lower part

EXERCISES

1) In an infinite slab of thickness δ = 50 mm (see Fig. 1) and thermal conductivity k = 5 W/m/K, a volumetric heat generation q''' = 500 kW/m^3 takes place. Its surface A is adiabatic, while its surface B is kept at a constant temperature T_0 = 300 K. Assuming the problem is 1D along x, compute analytically and plot the temperature distribution across the slab. Then re-compute the temperature distribution using the finite differences (FD) method and compare the numerical solution with the analytical one, using different line styles, to be explained in a clear legend.

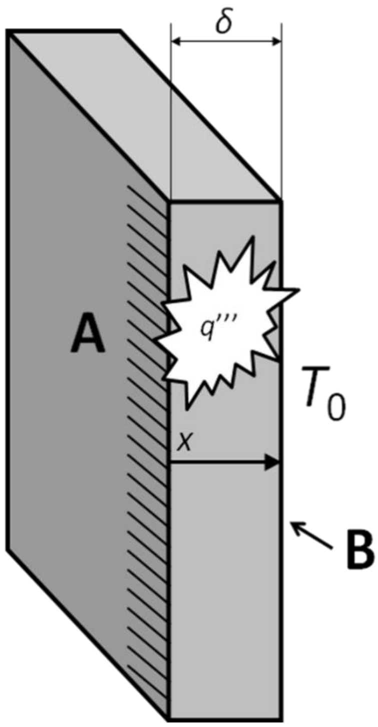

Fig. 1

2) In the combustion chamber of a large thermoelectric power plant there are several stainless steel (SS) pipes (thermal conductivity k = 15 W/m/K) having an internal diameter D_{int} = 24 mm and an external diameter D_{out} = 36 mm (Fig. 2). Inside the pipes, where the heat transfer coefficient is h_{in} = 1000 W/m^2/K, vaporizing water flows at the constant temperature T_w = 340 ℃. The external surface of the pipe, facing the flames, experiences a circumferentially uniform heat flux q'' = 5 kW/m^2. Compute and plot the steady state temperature distribution across the pipes (along the direction "r"). Save the script in a file to be named **EX2_*yoursurname.properextension*** and the plot in a file to be named **EX2_*yoursurname*_plot.*properextension***.

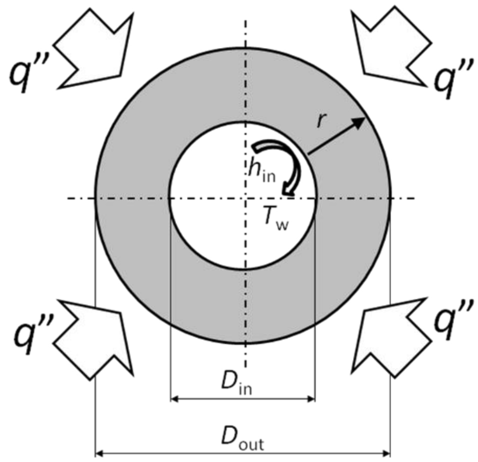

Fig. 2

3) A cylindrical copper rod (diameter D = 0.01 m, length L = 4 m) transports a current I = 1 kA. The rod (see Fig. 3) is immersed in (and cooled by) a liquid Nitrogen bath at T_b = 77 K (heat transfer coefficient h = 500 W/m^2/K). The two boundaries are kept at constant temperature T = T_b. Compute numerically the steady state temperature distribution along the rod in a script named **ES3_*yoursurname.properextension*** and plot the temperature along the rod in a figure, to be saved as **ES3_*yoursurname*_plot.*properextension***.
Cu electrical resistivity ρ_{el} = 1.75e-8 Ohm×m
Cu thermal conductivity k = 350 W/m/K

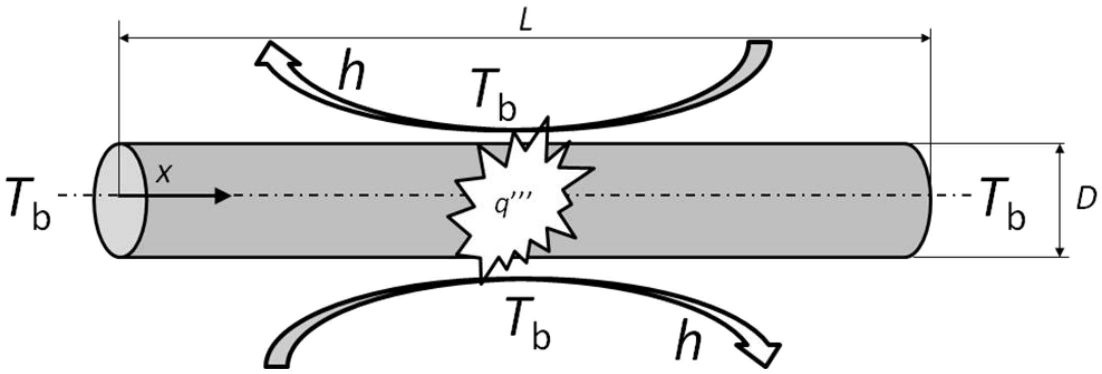

Fig. 3

4) In the same geometry of exercise 1, the insulation on the adiabatic wall is removed so that surface A experiences now heat transfer by convection to a fluid at T_f = 273 K (heat transfer coefficient h_f = 100 W/m2/K), see Fig. 4. Compute analytically and numerically (using FD) the temperature distribution in the slab, and compare the two solutions on the same plot.

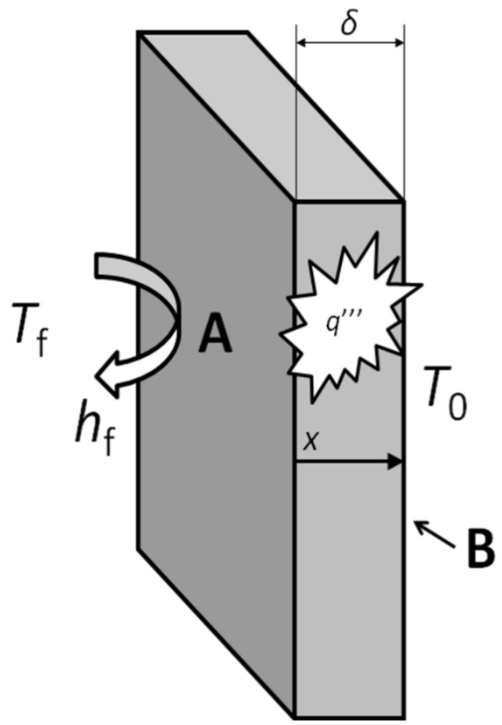

Fig. 4

5) A transport current of 50 A flows in the electrical conductor shown in Fig. 5 (laterally insulated, with length L = 1 m, resistivity ρ = 1.75e-8 Ωm, thermal conductivity k = 350 W/m/K), with diameter D = 4 mm. Compute the temperature distribution along the conductor, if the boundaries (x = 0, x = L) are kept at the given temperature T_0 = 300 K. Solve the problem numerically using FD.

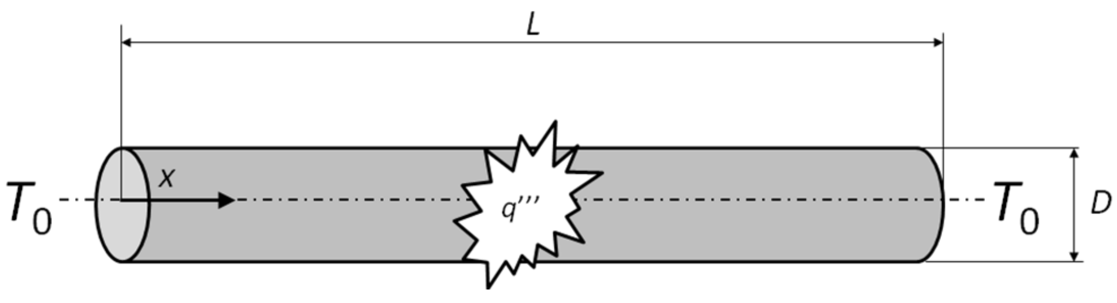

Fig. 5

6) Pressurized water at a constant temperature T_w = 400 K flows in a cylindrical pipe (inner diameter D_{in} = 20 mm, outer diameter D_{out} = 24 mm, conductivity k = 0.035 W/m/K, heat transfer coefficient h_{in} = 100 W/m^2/K). On the outer surface the pipe is cooled by air at T_a = 300 K (heat transfer coefficient h_{out} = 10 W/m^2/K), see Fig. 6. Compute analytically and numerically the radial temperature profile across the pipe wall cross section, using the FD method, and compare the two results in a plot with a proper legend.

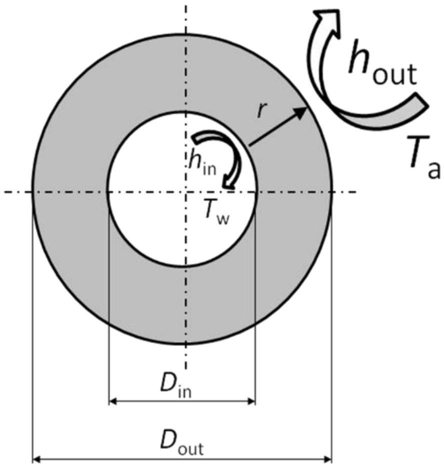

Fig. 6