MATH 110 Problem Set 2.2

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The following problems based on Section 2.2 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

1. (Based on 2.2.19–29) Find the derivatives of the following functions from first principles. State the domain of the function and the domain of the derivative.

   (a) $f(x) = \frac{3}{2}x^2 - x + \frac{37}{10}$ (b) $f(x) = x + \sqrt{x}$ (c) $f(x) = \frac{3+x}{1-3x}$ (d) $g(t) = \frac{1}{\sqrt{t}}$

2. (Based on 2.2.31–32) If $f(x) = x^3 - 2x^2$, find $f'(x)$. Check to see that your answer is reasonable by graphing f and f'.

3. (Based on 2.2.51–53) Given $f(x) = x^2 - 6x + 5$, use the definition of derivative to find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$. Graph f, f', f'', and f''' on a common screen. Are the graphs consistent with the geometric interpretations of these derivatives?

4. (Based on 2.2.16) Make a careful sketch of the graph of $f(x) = \cos x$ and below it sketch the graph of f'. Can you guess a formula for $f'(x)$ from its graph?

5. (Based on 2.2.43–44) Graph the function $f(x) = x^{2/3}$. Zoom in repeatedly, first toward the point $(8,4)$ and then toward the origin. What is different about the behavior of f in the vicinity of those two points? What do you conclude about the differentiability of f?

6. (Based on 2.2.57–60) Show that the function $f(x) = |x| - x$ is not differentiable at $x = 0$. Find a formula for $f'(x)$ and sketch its graph.

7. (Based on 2.2.55–56) Let $f(x) = \sqrt[3]{x}$.

   (a) If $a \neq 0$, find $f'(a)$ using the power rule for derivatives.
   (b) Show that $f'(0)$ does not exist.
   (c) Show that $f(x)$ has a vertical tangent at $(0,0)$.

8. (Based on 2.2.65) Let l be the tangent line to the curve $y = x^2 - 3x + 3$ at the point $(1,1)$ on the curve. The *angle of inclination* of l is the angle ϕ that l makes with the positive direction of the x-axis. Calculate ϕ in radians correct to two decimal points.

You may find the following additional exercises from Section 2.2 helpful.

2.2 C-level: 1–2, 3, 4–11, 12–15, 16, 19–29, 31–32, 33–36, 37–38, 39–42, 47–50, 51–53;
    B-level: 17–18, 30, 43–44, 45–46, 54, 55-56, 57–60, 63–64;
    A-level: 61, 62, 65;