MATH 110 Review 0.C

Review of Graphs of Second Degree Equations

Edward Doolittle

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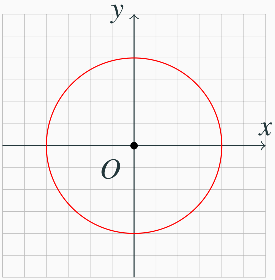
Department of Indigenous Knowledge and Science
First Nations University of Canada

Contents

Graphs of Second-Degree Equations

Circles centered at the Origin

- A *circle centered at the origin of radius* r is the set of all points at distance r from the origin.



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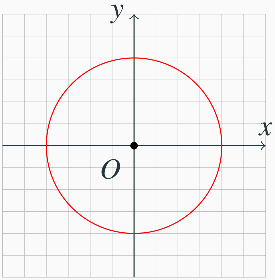

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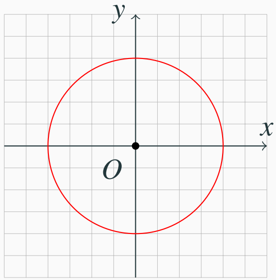
$$\sqrt{(x-0)^2+(y-0)^2}=r$$



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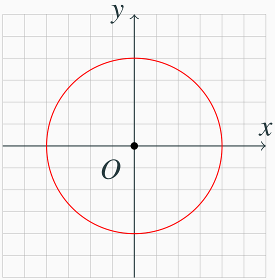
- Usually we simplify and square both sides.

$$x^2+y^2=r^2$$



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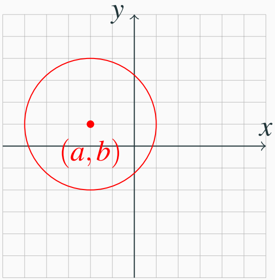

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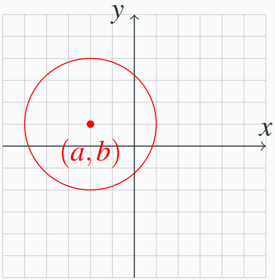

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- Simplifying,

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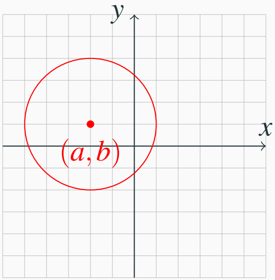
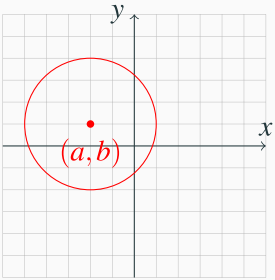


Expanding an Equation of a Circle

- Consider a circle centered at $(-2, 1)$ of radius 3.

$$(x+2)^2 + (y-1)^2 = 3^2$$



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- We can expand to obtain

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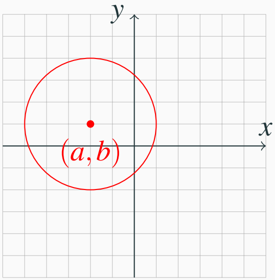

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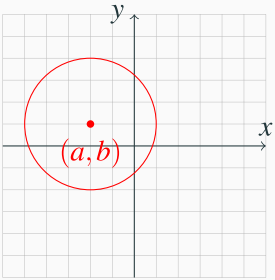
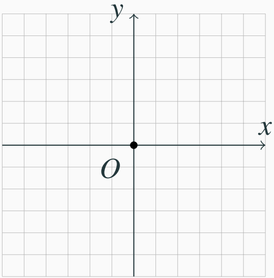
- Moving constants, another way to write the equation of that circle is

$$x^2 + y^2 + 4x - 2y = 4$$



Completing the Square

- On the other hand, sometimes we are given a circle in expanded form and must determine its center and radius.



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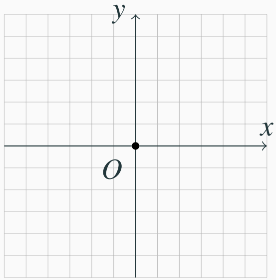

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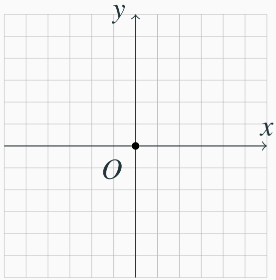

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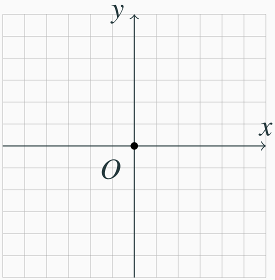
$$x^2 - 4x + 4 + y^2 + 2y + 1 = 11 + 4 + 1$$
$$(x-2)^2 + (y+1)^2 = 16 = 4^2$$



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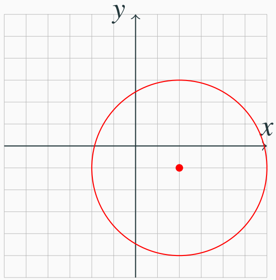
- The circle has center $(2, -1)$ and radius 4



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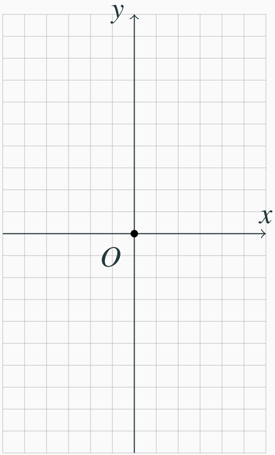

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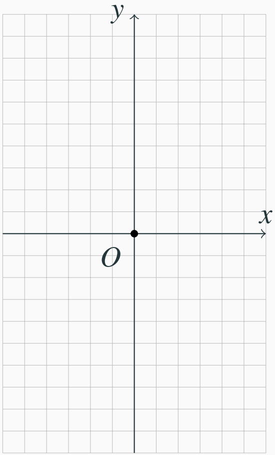

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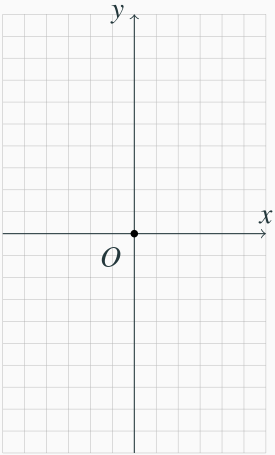

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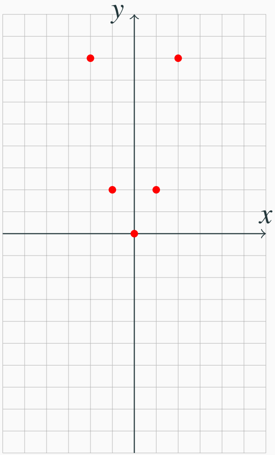

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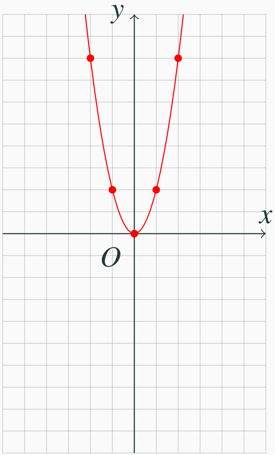
- So we can pick values for x and get a corresponding value for y.
- When $x = 0$, $y = 0$; when $x = 1$, $y = 2$; etc.
- Connect the dots smoothly to get the curve.



Opening Upward/Downward

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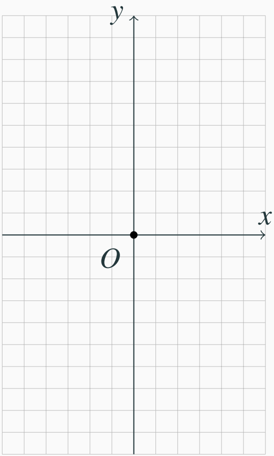

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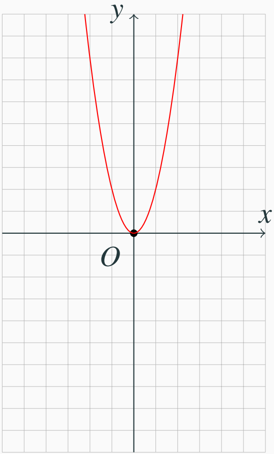

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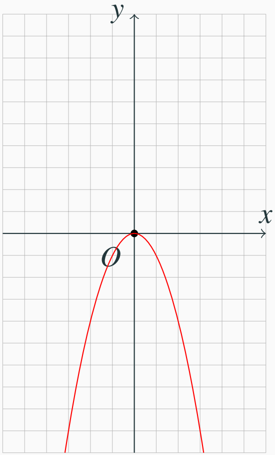

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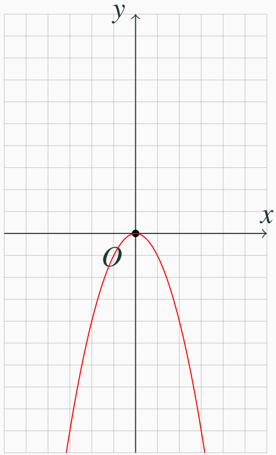

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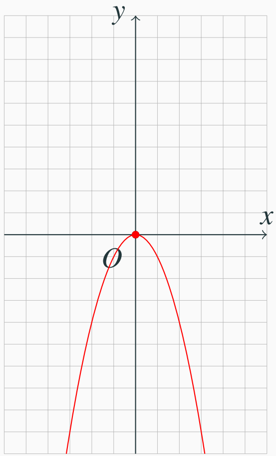

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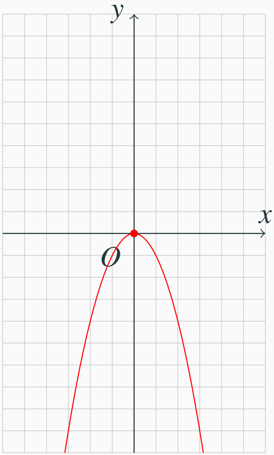

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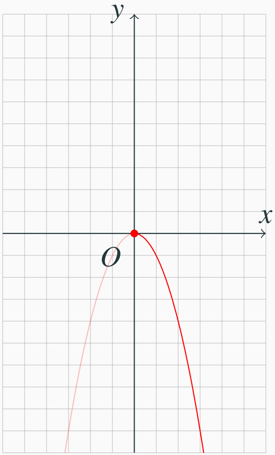

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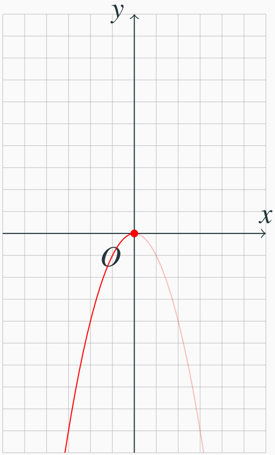

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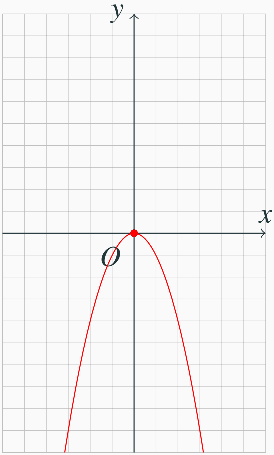

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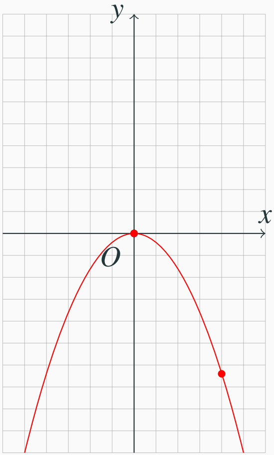

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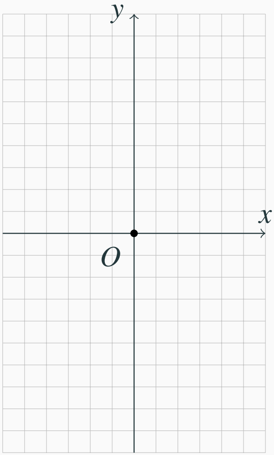

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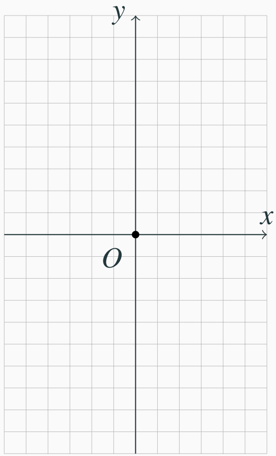

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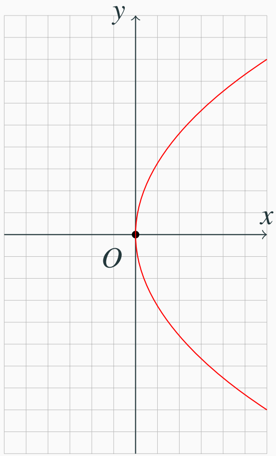

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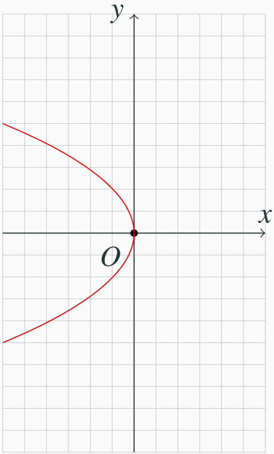

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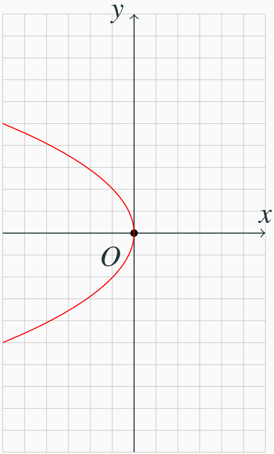

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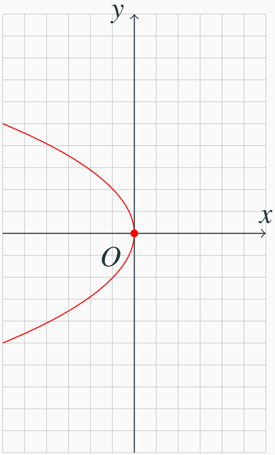

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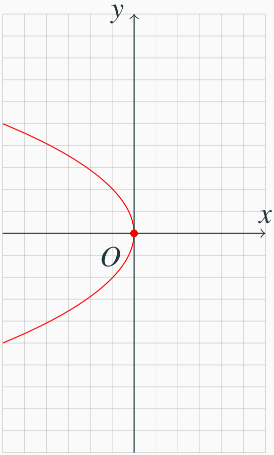

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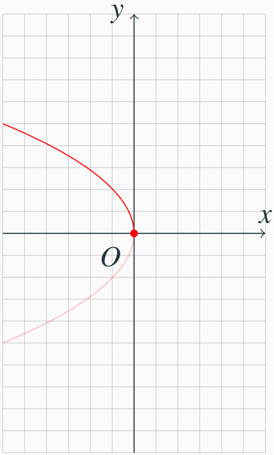

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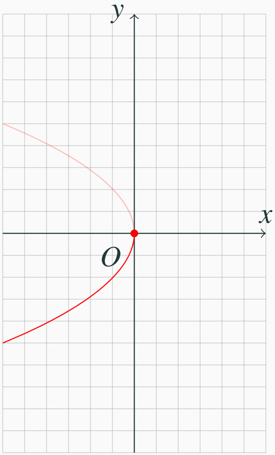

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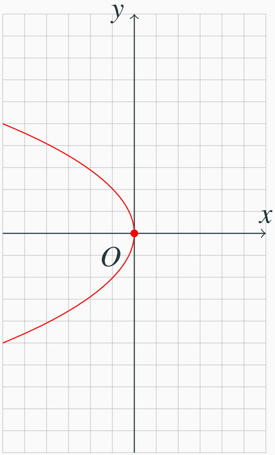

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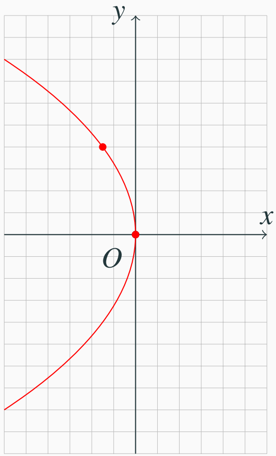

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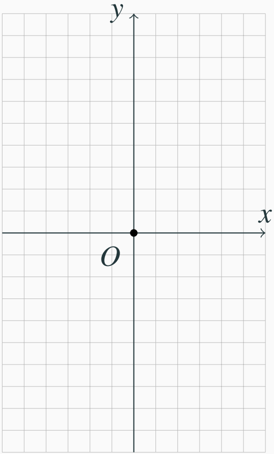

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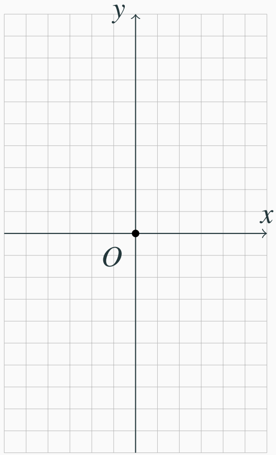

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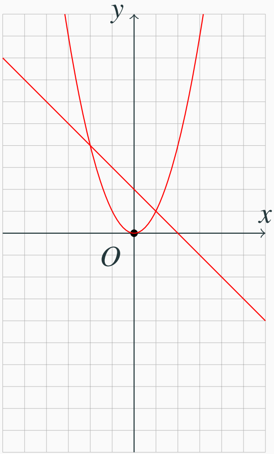

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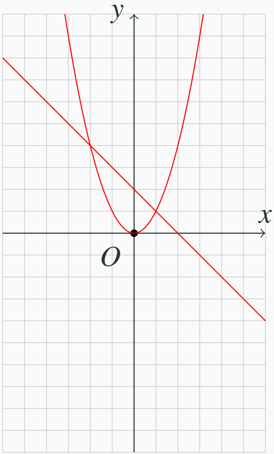

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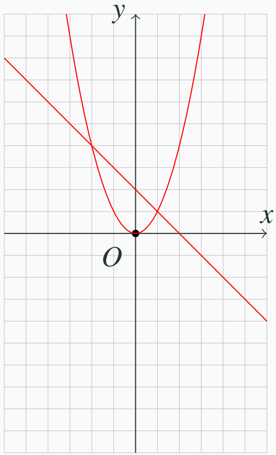

Intersection between Line and Parabola

- We will need to graph lines and parabolas on the same graph.
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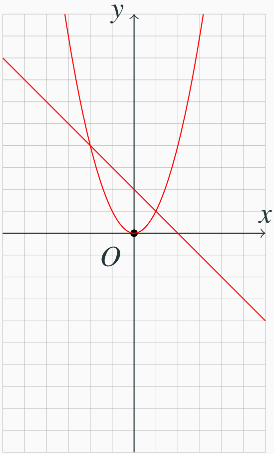
- Use $y = 2 - x$ to figure out the corresponding y-values: $y = 4$ or $y = 1$



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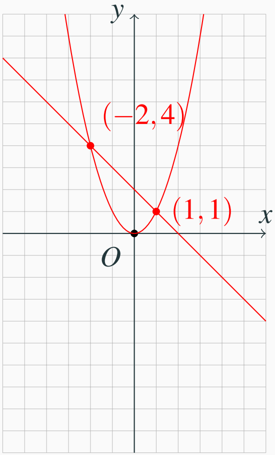
- Use $y = 2 - x$ to figure out the corresponding y-values: $y = 4$ or $y = 1$
- The intersections are $(-2, 4)$ and $(1, 1)$



Ellipses Centered at $(0, 0)$

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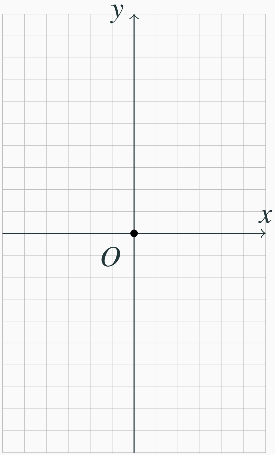

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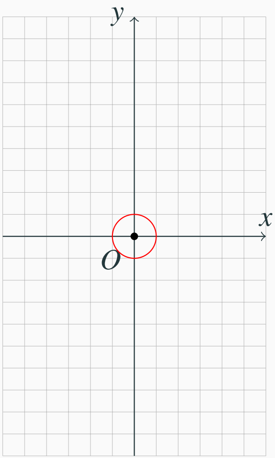

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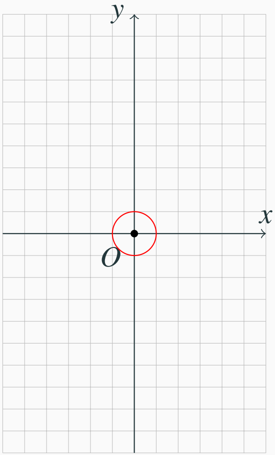

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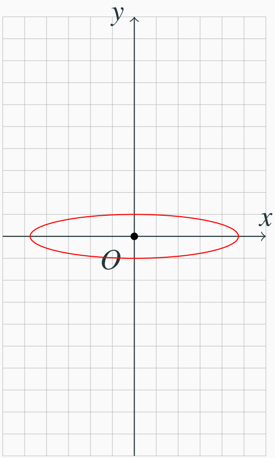

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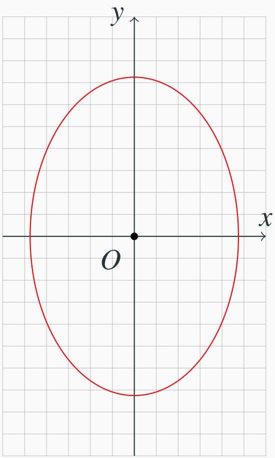

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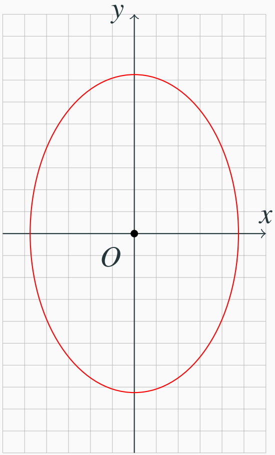
- The equation is usually written

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Intercepts of an Ellipse

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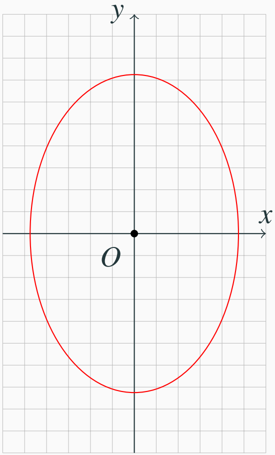

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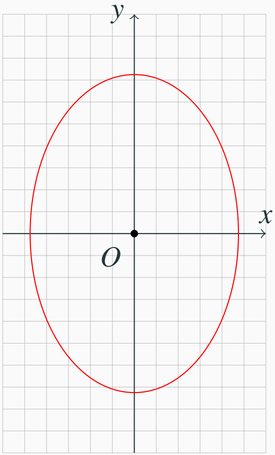

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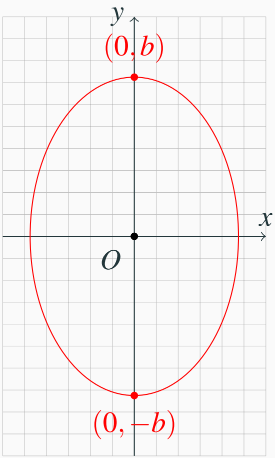

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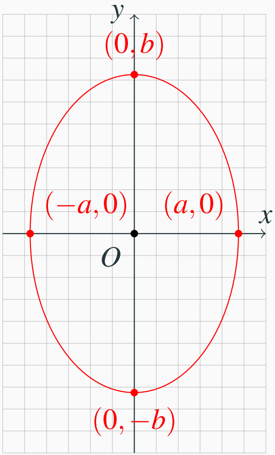
- Similarly the x-intercepts are $(-a, 0)$ and $(a, 0)$.



Example: $9x^2 + 16y^2 = 144$

- Consider the ellipse given by

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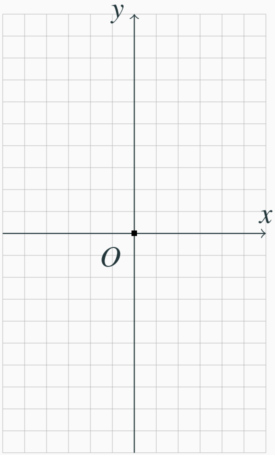

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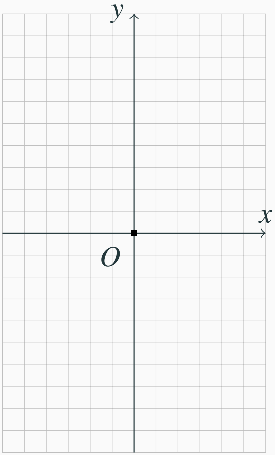

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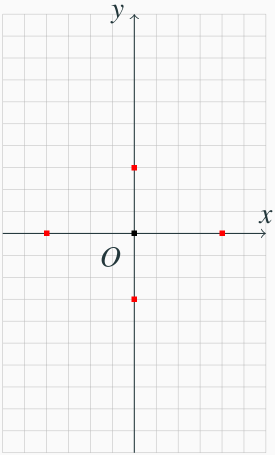

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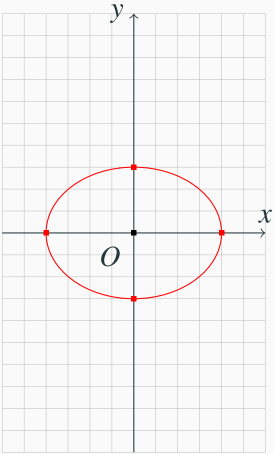
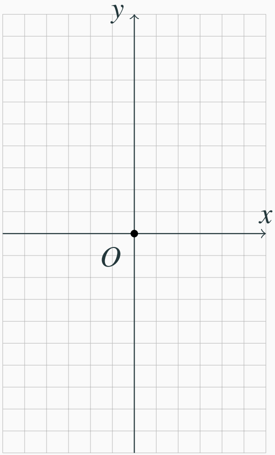
- By the method previous slide, the intercepts are $(0, -3)$, $(0, 3)$, $(-4, 0)$, $(4, 0)$
- We can now draw a rough graph of the ellipse.



Hyperbola in Standard Position

- The equation of a hyperbola is obtained from an ellipse by changing $+$ to $-$:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



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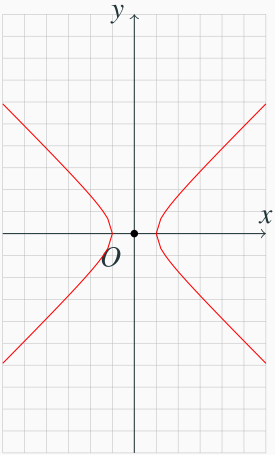

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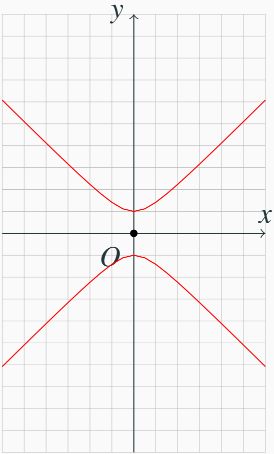
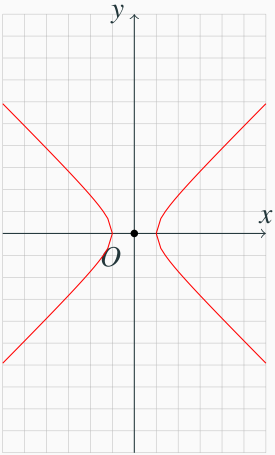
- We get a different hyperbola if we put the $-$ in front of x and $+$ in front of y:

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Intercepts of a Hyperbola

- Unlike an ellipse in standard position, a hyperbola in standard position only has two intercepts.



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- Unlike an ellipse in standard position, a hyperbola in standard position only has two intercepts.
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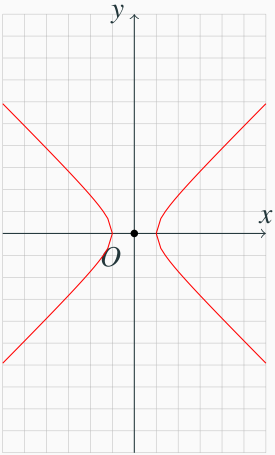

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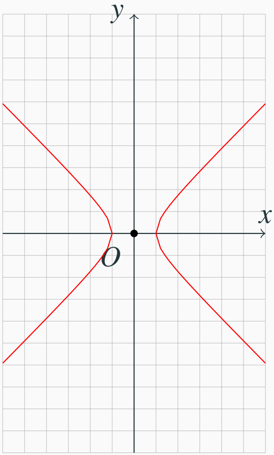

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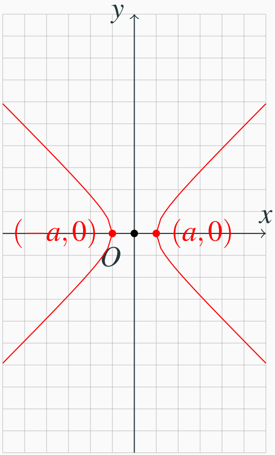
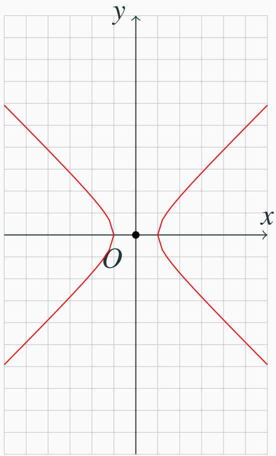
- The hyperbola in standard position has two intercepts, $(-a, 0)$ and $(a, 0)$.



Asymptotes of a Hyperbola

- Unlike an ellipse or a circle or a parabola, a hyperbola has two branches.
- To graph a hyperbola, it helps to first draw its asymptotes.



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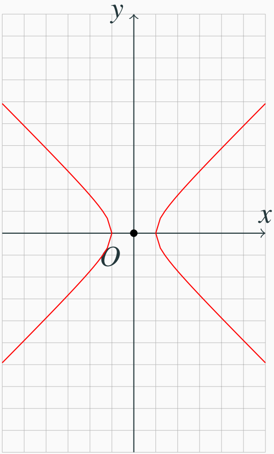
- To find the asymptotes, we change the 1 on the RHS of the equation to a 0 and factor:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \implies \left(\frac{x}{a} + \frac{y}{b}\right)\left(\frac{x}{a} - \frac{y}{b}\right) = 0$$



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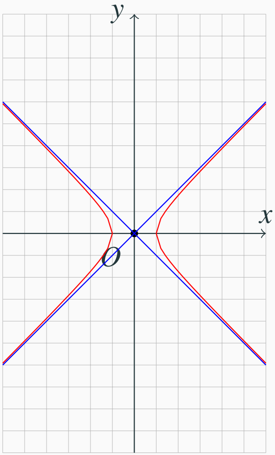
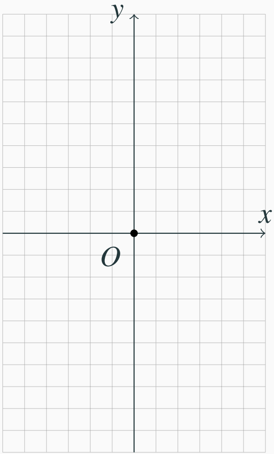
- That gives us two lines, $x/a + y/b = 0$ and $x/a - y/b = 0$.



Example: $4x^2 - 9y^2 = 36$

- For example, graph $4x^2 - 9y^2 = 36$



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- Write in standard form by dividing through by 36:

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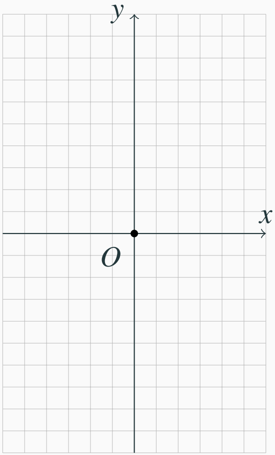

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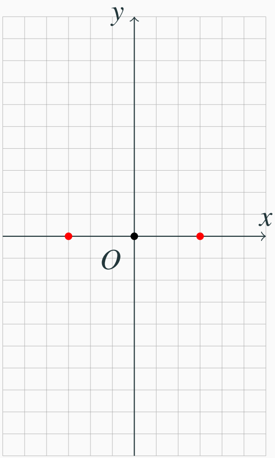

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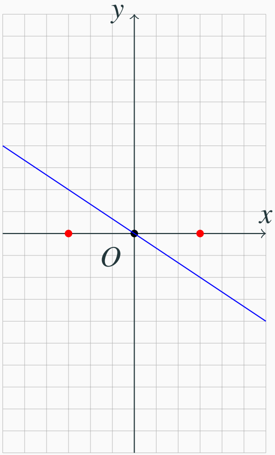

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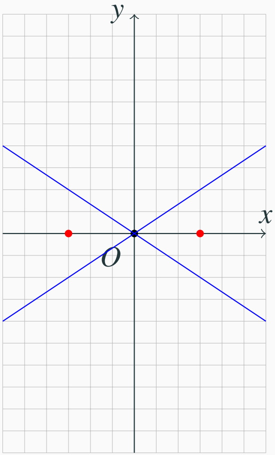

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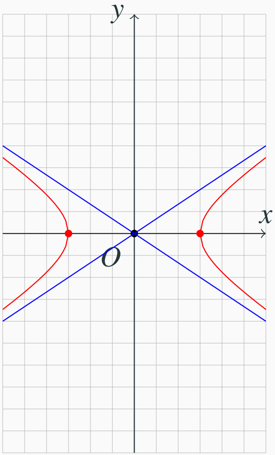
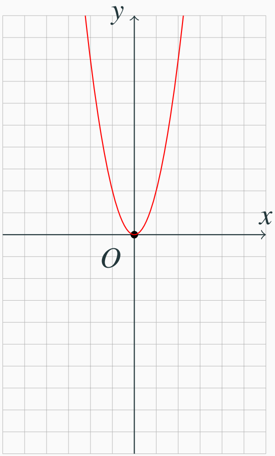
- Now graph it in the "frame" provided by the asymptotes.



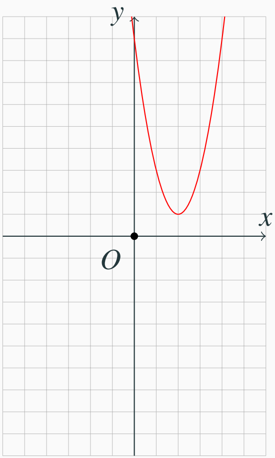
Shifing Parabolas

- We start with a parabola in standard position: $y = 2x^2$



Shifing Parabolas

- We start with a parabola in standard position: $y = 2x^2$
- To shift the vertex two units to the right and one unit up, we *subtract* 2 from x and 1 from y in the equation: $(y-1) = 2(x-2)^2$



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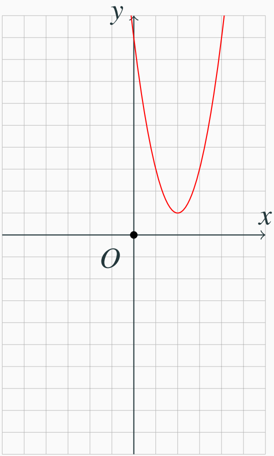
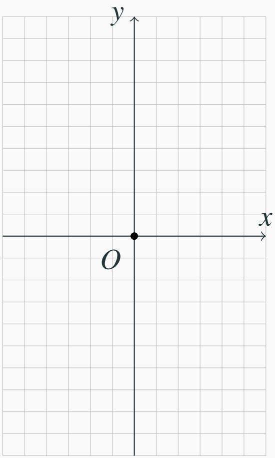
- We can simplify the previous equation: $y - 1 = 2(x^2 - 4x + 4) \implies y = 2x^2 - 8x + 9$



Completing the Square

- Conversely, given an equation like $y = 2x^2 - 8x + 9$ we can reverse those steps to put it in standard form.



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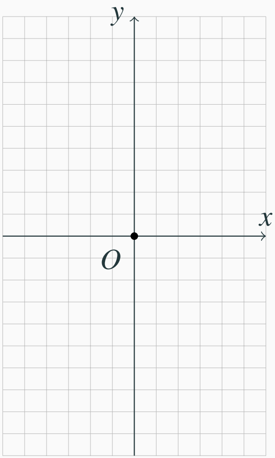
- Completing the square: $y = 2(x^2 - 4x) + 9 \implies y = 2(x^2 - 4x + 4 - 4) + 9 \implies y = 2(x^2 - 4x + 4) - 8 + 9$



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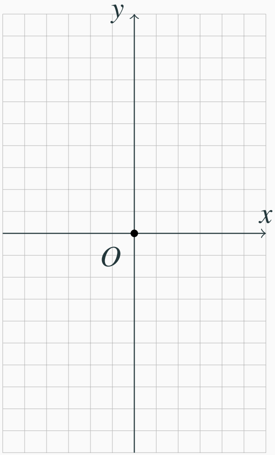
- Finally, we have

$$y - 1 = 2(x - 2)^2$$



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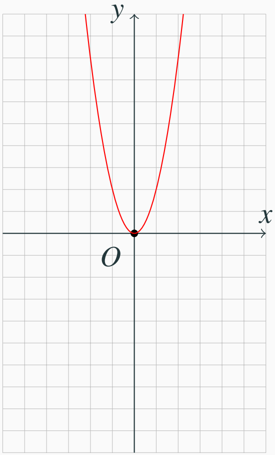
- To graph it, we start with $y = 2x^2$



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- Completing the square: $y = 2(x^2 - 4x) + 9 \implies y = 2(x^2 - 4x + 4 - 4) + 9 \implies y = 2(x^2 - 4x + 4) - 8 + 9$
- Finally, we have

$$y - 1 = 2(x - 2)^2$$

- To graph it, we start with $y = 2x^2$
- Then shift it 1 unit up and 2 units to the right.