MATH 110 Review 0.1

Review of Algebra

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Contents

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- That is our first result in algebra.

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- In general, we write $(x\times y)\times z = x\times(y\times z)$, or in short form $(xy)z = x(yz)$.

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- Order of operations (BEDMAS) says that multiplication is done before addition so we can drop the brackets on the right hand side (RHS).
- The *distributive law* is usually written $x(y+z) = xy + xz$.

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- Apply it again: $(s+3)t + (s+3)2 = st + 3t + (s+3)2$.
- And again: $st + 3t + (s+3)2 = st + 3t + 2s + 6$.

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- Applying the commutative law of addition, we have $4 + (-3 \times x) + 6 = 4 + 6 + (-3 \times x)$.
- We can add the 4 and 6 to get 10: $4 + 6 + (-3 \times x) = 10 + (-3 \times x)$.

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- By Order of Operations we can drop the brackets: $10+(-3\times x)=10-3x$.
- In summary, $4-3x+6=10-3x$.

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- We just *multiply the numerators and multiply the denominators* to get the answer.

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- Sometimes we see the same procedure in slightly different notation:

$$\frac{a/b}{c/d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

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- So all three of the following are negatives of $2/3$:

$$-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$$

Equivalent Fractions

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$$1 = \frac{1}{1} = \frac{2}{2} = \frac{7}{7} = \frac{-3}{-3} = \cdots$$
- Multiplying any fraction by any of those representations of 1 gives an equivalent fraction:

$$\frac{2}{3} = \frac{4}{6} = \frac{14}{21} = \frac{-6}{-9} = \cdots$$

Adding Fractions I

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- For example, to add $2/3$ and $4/7$, we find fractions equivalent to $2/3$, and fractions equivalent to $4/7$, looking for a common denominator.

$$\frac{2}{3} = \frac{4}{6} = \cdots = \frac{14}{21} = \cdots$$

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- We found a common denominator, 21.

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- Note that we can always find a common denominator:

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$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

- That is a general formula for adding fractions.

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- We find a common denominator and then subtract only the numerators:
$$\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad - bc}{bd}$$
- It's better to remember the procedure for adding and subtracting fractions, rather than the formula.

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Expanding and Factoring

- The opposite of *expanding* is *factoring*.
- If I start with an expression like $2x(x+3)$ I can expand it to get $2x^2+6x$
- On the other hand, if I start with $2x^2+6x$ I can factor it to get $2x(x+3)$.
- Sometimes expanding is appropriate, and sometimes factoring is appropriate.
- It's hard to give a rule to tell you which is better in a given situation. You need to use your judgment.

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- For the example x^2+5x+6, you would eventually guess 2 and 3: their sum is 5 and their product is 6.
- So we guess $x^2+5x+6=(x+2)(x+3)$.
- Check by expanding the RHS.

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- We can check by expanding the RHS:

$$(x-a)(x^2+ax+a^2) = x^3 + ax^2 + a^2x - ax^2 - a^2x - a^3 = x^3 - a^3$$

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$$(x-a)(x^2+ax+a^2) = x^3 + ax^2 + a^2x - ax^2 - a^2x - a^3 = x^3 - a^3$$

- For example, $x^3 - 8 = (x-2)(x^2+2x+4)$. (Check!)

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- Putting back $-b$ for a we have $x^3 + b^3 = (x+b)(x^2 - bx + b^2)$. Check!

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- That means that $x - 2$ should be a factor of $x^2 - 5x + 6$.
- Indeed it is: check $x^2 - 5x + 6 = (x - 2)(x - 3)$

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- Check that $x^2 - 5x + 6 = (x-2)(x-3)$

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- The first term is correct. For the second term to be correct we need $2a = 6$, $a = 6/2 = 3$.

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- Now that the quadratic is in that form, we can do other useful things with it, e.g., factoring it as a difference of squares.
- See the supplement for the procedure to follow when the coefficient of x^2 is not 1, e.g., for $2x^2 + 8x + 6$.

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- That is the quadratic formula, and it can save you a lot of work if you memorize it.

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- If that expression is zero, the quadratic equation has two identical roots.
- If that expression is positive, the quadratic equation has to different roots.
- Because the expression $b^2 - 4ac$ helps us discriminate between those three cases, it is called the discriminant of the quadratic equation.

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- $x/x = 1$ so x^0 is 1.
- Similarly, x^{-1} is $1/x$, x^{-2} is $(1/x)/x = 1/x^2$, and so on.

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- There is a pattern to these results called the *binomial theorem*. See the textbook supplement for details.

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- In general, $(xy)^m = x^m y^m$.
- If you think about it, you'll see that those results hold for negative values of m and/or n as well.

- We know what x^n means for $n = 0, 1, 2, \ldots$ and $n = -1, -2, -3, \ldots$.

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- In other words, $x^{1/2}$ should be $\sqrt{x}$.
- It turns out that everything works out fine if we say $x^{1/2} = \sqrt{x}$, $x^{1/3} = \sqrt[3]{x}$, and so on.

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- (You could handle $\sqrt{x+y}$ using an extended version of the binomial theorem, but that is beyond our powers at the moment.)

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$$\frac{\sqrt{x}+\sqrt{y}}{4} = \frac{\sqrt{x}+\sqrt{y}}{4} \times \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{x-y}{4(\sqrt{x}-\sqrt{y})}$$