

MATH 110 Problem Set 3.2

Edward Doolittle

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The following problems based on Section 3.2 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

1. (Based on 3.2.5–8) Verify that the following functions satisfy the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

(a) $f(x) = x^3 - x^2 - 6x + 2$, $[0, 3]$

(b) $f(x) = \cos 2x$, $[\pi/8, 7\pi/8]$

2. (Based on 3.2.11–14) Verify that the following functions satisfy the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c which satisfy the conclusion of the Mean Value Theorem.

(a) $f(x) = x^3 + x - 1$, $[0, 2]$

(b) $f(x) = \frac{x}{x+2}$, $[1, 4]$

3. (Based on 3.2.9) Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no number c in $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?
4. (Based on 3.2.17–18) Let $f(x) = (x - 3)^{-2}$. Show that there is no number c in $(1, 4)$ such that $f(4) - f(1) = f'(c)(4 - 1)$. Why does this not contradict the Mean Value Theorem?
5. (Based on 3.2.19–20) Show that the equation $2x - 1 - \sin x = 0$ has exactly one real root.
6. (Based on 3.2.31) Use the Mean Value Theorem to prove the inequality $|\sin a - \sin b| \leq |a - b|$ for all a and b .

You may find the following additional exercises from Section 3.2 helpful.

3.2 C-level: 1–20, 34;

B-level: 25–27, 29, 33, 35;

A-level: 21–22, 23–24, 28, 30–32, 36