MATH 110-004 200730 Quiz 6 Solutions

Edward Doolittle

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1. We follow the standard procedure for optimizing a continuous function on a closed interval. (Why is f continuous?)

   (a) The critical numbers are where $f'(x) = 4x^3 - 4x$ is zero or doesn't exist. Since f is a polynomial the derivative always exists, so the critical numbers are exactly where $f'(c) = 4c^3 - 4c = 4c(c-1)(c+1) = 0$, namely $c = 0$, $c = 1$, and $c = -1$. Since all of those critical numbers are inside the interval $[-2,3]$ we keep them all.

   (b) Evaluating f at the critical numbers we have $f(0) = 3$, $f(-1) = 2$, $f(1) = 2$.

   (c) Evaluating f at the endpoints, $f(-2) = 16 - 8 + 3 = 11$ and $f(3) = 81 - 18 + 3 = 66$.

   (d) The smallest of the above function values is 2, so f takes on a minimum value of 2. The largest of the above function values is 66, so f takes on a maximum value of 66 in $[-2,3]$.

2. First, we guess an interval on which $f(x) = 5x - 3\sin x + 1$ changes sign. One guess might be $[-1,1]$. Note that $f(-1) = -5 - 3\sin(-1) + 1 < -4 + 3 < 0$ and $f(1) = 5 - 3\sin(1) + 1 > 6 - 3 > 0$, so f changes sign on $[-1,1]$. (It is acceptable to evaluate $f(-1)$ and $f(1)$ numerically instead of using inequalities.) Since f is continuous, the IVT says that it must have a root between -1 and 1, so f has at least one root. (Intervals other than $[-1,1]$ could work, too.)

   Now assume that f has at least two roots, e.g., a and b. Since f is continuous and differentiable and $f(a) = f(b)$, Rolle's theorem tells us there is a c between a and b such that $f'(c) = 0$. However, $f'(c) = 5 - 3\sin c$, and

   $$\sin c \leq 1 \implies -3 \leq -3\sin c \implies 5 - 3 \leq 5 - 3\sin c \implies 0 < 2 \leq 5 - 3\sin c = f'(c)$$

   contradicting $f'(c) = 0$. Therefore our assumption that f has two or more roots is wrong, and we conclude that f has exactly one root.