MATH 110 Problem Set 4.4 Solutions

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1. (a) Note that
$$v(v^2+2)^2 = v(v^4+4v^2+4) = v^5+4v^3+4v$$
so
$$\int v(v^2+2)^2\,dv = \int (v^5+4v^3+4v)\,dv = \frac{1}{6}v^6+v^4+2v^2+C$$
(b) We have
$$\int \sec t(\sec t+\tan t)\,dt = \int (\sec^2 t+\sec t\tan t)\,dt = \tan t+\sec t+C$$
(c) Recall the double angle trig identity
$$\sin 2x = 2\sin x\cos x$$
so
$$\int \frac{\sin 2x}{\sin x}\,dx = \int \frac{2\sin x\cos x}{\sin x}\,dx = 2\int \cos x\,dx = 2\sin x+C$$

2. (a) Let's work on the integrand a bit before we try to evaluate the integral. We have
$$\frac{\sin\theta+\sin\theta\tan^2\theta}{\sec^2\theta} = \frac{\sin\theta+\sin\theta\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = \frac{\sin\theta\cos^2\theta+\sin\theta\sin^2\theta}{1}$$
where we have multiplied through by $\cos^2\theta$. Factoring and using the Pythagorean identity,
$$\sin\theta\cos^2\theta+\sin\theta\sin^2\theta = \sin\theta(\cos^2\theta+\sin^2\theta) = \sin\theta$$
So the integral is
$$\int_0^{\pi/3} \frac{\sin\theta+\sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta = \int_0^{\pi/3}\sin\theta\,d\theta = -\cos\theta\Big|_0^{\pi/3} = -\cos(\pi/3)+\cos(0) = \frac{1}{2}$$
(b) We expand the integrand using the binomial theorem:
$$(1+x^2)^3 = 1^3+3\cdot 1^2\cdot x^2+3\cdot 1\cdot (x^2)^2+(x^2)^3 = 1+3x^2+3x^4+x^6$$
Then the integral is
$$\int_0^1 (1+x^2)^3\,dx = \int (1+3x^2+3x^4+x^6)\,dx\Big|_0^1 = x+x^3+\frac{3}{5}x^5+\frac{1}{7}x^7\Big|_0^1 = 1+1+\frac{3}{5}+\frac{1}{7} = \frac{96}{35}$$
(c) Again, we simplify the integrand:
$$\frac{x-1}{\sqrt[3]{x^2}} = \frac{x-1}{x^{2/3}} = \frac{x}{x^{2/3}} - \frac{1}{x^{2/3}} = x^{1/3}-x^{-2/3}$$
Therefore
$$\int_1^8 \frac{x-1}{\sqrt[3]{x^2}}\,dx = \int x^{1/3}-x^{-2/3}\,dx\Big|_1^8 = \frac{3}{4}x^{4/3}-3x^{1/3}\Big|_1^8 = \frac{3}{4}\cdot 8^{4/3}-3\cdot 8^{1/3}-\frac{3}{4}\cdot 1^{4/3}+3\cdot 1^{1/3} = 12-6-\frac{3}{4}+3 = \frac{33}{4}$$

3. (a) The displacement is

$$\int_1^6 v(t)\,dt = \int_1^6 (t^2-2t-8)\,dt = \frac{1}{3}t^3 - t^2 - 8t\Big|_1^6 = 72-36-48-\frac{1}{3}+1+8 = -\frac{10}{3}$$

meters. The distance traveled is

$$\int_1^6 |v(t)|\,dt$$

In order to evaluate that integral we have to break it into pieces depending on whether $v(t) \geq 0$ or $v(t) < 0$. In order to do that, we need to find the times when $v(t) = 0$:

$$v(t) = 0 \implies t^2 - 2t - 8 = 0 \implies (t+2)(t-4) = 0$$

So $v(t)$ is positive up to $t = -2$, then switches to negative between $t = -2$ and $t = 4$, then becomes positive again after $t = 4$. Our integral becomes

$$\int_1^6 |v(t)|\,dt = \int_1^4 -v(t)\,dt + \int_4^6 v(t)\,dt$$

Evaluating the first integral on the right above,

$$\int_1^4 (-t^2+2t+8)\,dt = -\frac{1}{3}t^3 + t^2 + 8t\Big|_1^4 = -\frac{64}{3}+16+32+\frac{1}{3}-1-8 = 18$$

The distance traveled in any interval should be a non-negative number. If we had gotten -18, we would know that we had made a mistake.

Evaluating the second integral on the right above,

$$\int_4^6 (t^2-2t-8)\,dt = \frac{1}{3}t^3 - t^2 - 8t\Big|_4^6 = 72-36-48-\frac{64}{3}+16+32 = \frac{44}{3}$$

which is again a non-negative number. Putting the results together,

$$\int_1^6 |v(t)|\,dt = \int_1^4 -v(t)\,dt + \int_4^6 v(t)\,dt = 18 + \frac{44}{3} = \frac{98}{3}$$

which is the distance traveled in meters.

(b) Let's work through this situation carefully. Let $s(t)$ be the position of the particle at time t. Then $s'(t) = v(t)$ is the velocity at time t and $v'(t) = a(t)$ is the acceleration at time t. We are going to solve this problem in two steps: we are first going to find $v(t)$ integrating $a(t)$. Then we are going to find the distance traveled by integrating $|v(t)|$.

To find $v(t)$ we use the Net Change Theorem:

$$\int_0^t v'(u)\,du = v(t) - v(0)$$

where we have used the dummy variable u in the integral because t is busy. We are given $v'(u) = a(u) = 2u+3$ and $v(0) = -4$:

$$\int_0^t (2u+3)\,dt = v(t) - (-4) \implies u^2 + 3u\Big|_0^t = v(t) + 4 \implies t^2 + 3t - 4 = v(t)$$

which solves the first half of our problem.

For the second half of the problem we find the distance traveled between $t = 0$ and $t = 3$ by evaluating the integral

$$\int_0^3 |v(t)|\, dt$$

Note that $v(t) = (t+4)(t-1)$ so $v(t) < 0$ on the interval $[0,1]$ and $v(t) > 0$ on the interval $[1,3]$. Therefore

$$\int_0^3 |v(t)|\, dt = \int_0^1 -v(t)\, dt + \int_1^3 v(t)\, dt$$

For the first integral,

$$\int_0^1 (-t^2 - 3t + 4)\, dt = -\frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t \Big|_0^1 = -\frac{1}{3} - \frac{3}{2} + 4 = \frac{13}{6}$$

a positive number. The second integral is

$$\int_1^3 (t^2 + 3t - 4)\, dt = \frac{1}{3}t^3 + \frac{3}{2}t^2 - 4t \Big|_1^3 = 9 + \frac{27}{2} - 12 - \frac{1}{3} - \frac{3}{2} + 4 = \frac{38}{3}$$

Altogether, the distance traveled is

$$\int_0^3 |v(t)|\, dt = \int_0^1 -v(t)\, dt + \int_1^3 v(t)\, dt = \frac{13}{6} + \frac{38}{3} = \frac{89}{3}$$

meters.

4. Let $V(t)$ be the volume of water that has flowed out of the tank at time t. We have $V'(t) = r(t) = 200 - 4t$, so by the Net Change Theorem

$$\int_0^{10} V'(t)\, dt = V(10) - V(0)$$

Implicit in the problem is that no water has flowed out of the tank at time $t = 0$, so $V(0) = 0$. Then

$$V(10) = \int_0^{10} (200 - 4t)\, dt = 200t - 2t^2 \Big|_0^{10} = 2000 - 200 = 1800$$

So 1800 liters flow out of the tank in the first 10 minutes.