

# MATH 110-004 200730 Quiz 5 Solutions

Edward Doolittle

January 5, 2026

1. In Figure 1, ship *A* moves ‘west’ along the  $x$ -axis and ship *B* moves ‘north’ along the  $y$  axis. We have the Pythagorean relation among the variables  $x$ ,  $y$ , and  $z$ :

$$x^2 + y^2 = z^2$$

Differentiating with respect to time,

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

From the given data, at 4:00 pm we have  $dx/dt = -10$ ,  $x = -35 - 4(10) = -75$ ,  $dy/dt = 25$ ,  $y = 0 + 4(25) = 100$ . We can calculate  $z$  using the Pythagorean relation:  $z^2 = x^2 + y^2 = 75^2 + 100^2 = 125^2$ . Since the distance must be positive we have  $z = 125$ . Filling in that data we have

$$-75 \cdot -10 + 100 \cdot 25 = 125 \cdot \frac{dz}{dt} \implies \frac{dz}{dt} = 26$$

The ships are moving away from each other at a rate of 26 km/h at 4:00 pm.

2. **Solution 1:** The relations among the dimensions of the sphere are

$$C = 2\pi r \quad A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3$$

Differentiating,

$$dC = 2\pi dr \quad dA = 8\pi r dr \quad dV = 4\pi r^2 dr$$

We are given  $C = 84$  and  $dC = 0.5$ . Solving for  $r$  and  $dr$  using the first set of relations,  $r = 84/2\pi \approx 13.4$ ,  $dr = 0.5/2\pi \approx 0.080$ , so

$$dA = 8\pi r dr \approx 26.9 \quad dV = 4\pi r^2 dr \approx 180$$

So the maximum error in the calculated value of the surface area is about 26.9 cm<sup>2</sup> and the maximum error in the calculated value of the volume is about 180 cm<sup>3</sup>.

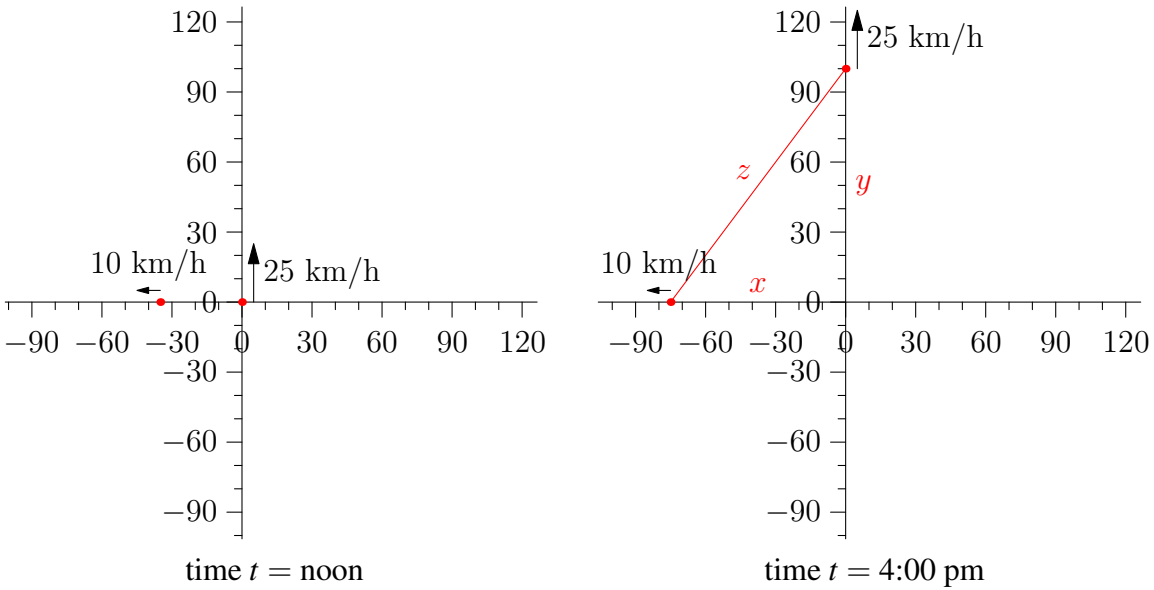


Figure 1: Position of ships at noon and 4:00 pm

Since  $A = 4\pi r^2 \approx 2260 \text{ cm}^2$ , the relative error in the value of  $A$  is about  $dA/A = 0.012$  or about 1.2%. Since  $V = (4/3)\pi r^3 \approx 10100 \text{ cm}^3$ , the relative error in  $V$  is about  $dV/V \approx 180/10100 \approx 0.018$  or about 1.8%.

**Solution 2:** We can find formulas for  $A$  and  $V$  in terms of  $C$  without  $r$ . We have  $r = C/2\pi$  so

$$A = 4\pi r^2 = 4\pi \left( \frac{C}{2\pi} \right)^2 = 4\pi \frac{C^2}{4\pi^2} = \frac{C^2}{\pi} \quad V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left( \frac{C}{2\pi} \right)^3 = \frac{4}{3}\pi \frac{C^3}{8\pi^3} = \frac{C^3}{6\pi^2}$$

Differentiating,

$$dA = \frac{2}{\pi}C dC \quad dV = \frac{1}{2\pi^2}C^2 dC$$

Substituting  $C = 84$  and  $dC = 0.5$  we have

$$dA = \frac{2}{\pi}84 \times 0.5 \approx 26.7 \quad dV = \frac{1}{2\pi^2}(84)^2 \times 0.5 \approx 179.$$

So the errors in the calculated values of the surface area and volume are about  $26.7 \text{ cm}^2$  and  $179 \text{ cm}^3$  respectively.

Since  $A = C^2/\pi \approx 2250 \text{ cm}^2$ , the relative error in  $A$  is approximately  $dA/A \approx 26.7/2250 \approx 0.012$  or 1.2%. Since  $V = C^3/6\pi^2 \approx 10000 \text{ cm}^3$ , the relative error in  $V$  is approximately  $dV/V \approx 0.018$  or about 1.8%.