MATH 110 Problem Set 2.1

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The following problems based on Section 2.1 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

1. (Based on 2.1.1–4)
   (a) Find the slope of the tangent line to the parabola $y = x^2 - 2x - 3$ at the point $(0, -3)$ on the parabola.
   (b) Find an equation for the tangent line.
   (c) Graph the parabola and the tangent line on a common screen.

2. (Based on 2.1.5–8) Find an equation of the tangent line to the given curve at the given point.

   (a) $y = \dfrac{1}{x+1}$, $(0,1)$ (b) $y = \sqrt{2x-1}$, $(5,3)$ (c) $y = x^3 - 4x$, $(-1,3)$ (d) $y = \dfrac{x^2}{2x-1}$, $(0,0)$

3. (Based on 2.1.9–10)
   (a) Find the slope of the tangent line to the curve $y = 10 + 3x^2 - x^3$ at the point where $x = a$ on the curve.
   (b) Use the above result to find the equations of the tangent lines to the curve at the points $(1,12)$ and $(3,10)$ on the curve.
   (c) Graph the curve and both tangents on a common screen.

4. (Based on 2.1.15–16) The displacement in meters of a particle moving in a straight line is given by the equation of motion $s = 1/t$, where t is measured in seconds. Find the velocity and the speed of the particle at times $t = a$, $t = 1$, $t = 2$, and $t = 3$.

5. (Based on 2.1.20–21) Sketch the graph of a function f for which $f(0) = 1$, $f'(0) = -1$, $f'(1) = 0$, and $f'(2) = 3$.

6. (Based on 2.1.31–36) Find the derivatives $f'(a)$ of the following functions from first principles.

   (a) $f(x) = 4x^2 - x^3$ (b) $f(t) = t^4 - 5t$ (c) $f(x) = \dfrac{x^2+1}{x-2}$ (d) $f(x) = \sqrt{3x+1}$

7. (Based on 2.1.13–14) If a ball is thrown straight upward into the air with a velocity of 5 m/s, its height (in meters) after t seconds is given by $y = 5t - 4.9t^2$. Find the velocity when $t = 0.5$.

8. (Based on 2.1.37–42) Each of the following limits represents the derivative of some function f at some number a. State such an f and a in each case.

   (a) $\displaystyle\lim_{h\to 0} \frac{(2+h)^5 - 32}{h}$ (b) $\displaystyle\lim_{h\to 0} \frac{\sqrt{16+h} - 4}{h}$ (c) $\displaystyle\lim_{t\to \pi/2} \frac{\sin t - 1}{t - \pi/2}$

9. (Based on 2.1.53–56) The cost of producing x widgets at a widget factory is $C = f(x)$ dollars.

(a) What is the meaning of the derivative $f'(x)$? What are its units?

(b) What does the statement $f'(100) = 12$ mean?

(c) Do you think the values of $f'(x)$ will increase or decrease for small increasing values of x? For large increasing values of x?

10. (Based on 2.1.59–60) Calculate $f'(0)$ for the function

$$f(x) = \begin{cases} x^2 \cos \dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

You may find the following additional exercises from Section 2.1 helpful.

2.1 C-level: 1–4, 5–8, 9–10, 15–16, 17–18, 20–23, 27–30, 31–36, 37–42, 43–44, 47–50;
B-level: 11–12, 13–14, 19, 24–26, 45–46, 51–52, 53–56, 57–58, 61;
A-level: 59–60