MATH 110 Quiz 1 Section 003 Solutions

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1. (a) The table is as follows. The $x - 16$ column is of course trivial, as is the $f(x)$ column once the other two are done (just shifting decimal points).

x	$4 - \sqrt{x}$	$x - 16$	$f(x)$
17.00	-0.1231	1.0000	-0.1231
16.10	-0.01248	0.1000	-0.1248
16.01	-0.001250	0.0100	-0.1250
15.99	0.001250	-0.0100	-0.1250
15.90	0.01252	-0.1000	-0.1252
15.00	0.1270	-1.0000	-0.1270

   (b) Based on the above table, a reasonable guess for the limit would be -0.1250 to four decimal places. (Using limit theorems, we can now show that that is the exact answer.)

2. Candidates for vertical asymptotes of rational functions are vertical lines over x values at which the denominator goes to 0. Factoring the denominator, we have $y = \dfrac{x+1}{(x+3)(x-2)}$ so our candidates for asymptotes are the lines $x = -3$ and $x = 2$. The rational function may have a removable discontinuity at those x values rather than an infinite discontinuity, however, so we must check (one-sided) limits as x approaches those candidate values.

   For $\lim_{x \to -3^-}$ we have $x + 1$ and $x - 2$ both negative and non-zero, and $x + 3$ negative and close to zero. Three negatives make a negative, and a number in the denominator close to zero gives a large overall result, so $\lim_{x \to -3^-} \dfrac{x+1}{(x+3)(x-2)} = -\infty$.

   For $\lim_{x \to -3^+}$, $x + 1$ and $x - 2$ are again both negative and non-zero, but $x + 3$ is positive and close to zero, giving the limit $\lim_{x \to -3^+} \dfrac{x+1}{(x+3)(x-2)} = +\infty$.

   For $\lim_{x \to 2^-}$, $x + 1$ and $x + 3$ are both positive, and $x - 2$ is negative and close to zero, so overall we have a limit of $-\infty$. For $\lim_{x \to 2^+}$ we have a limit of $+\infty$.

   In summary, there are vertical asymptotes at $x = -3$ and $x = 2$.