MATH 110 Problem Set 3.7

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The following problems based on Section 3.7 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

1. (Based on 3.7.12) A box with a square base and open top must have a volume of $32,000$ cm^3. Find the dimensions of the box that minimize the amount of material used. See Figure 1.

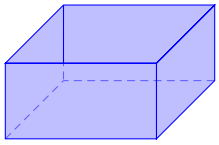

Figure 1: Box with Square Base and Open Top

2. (Based on 3.7.34) A Norman window has the shape of a rectangle surmounted by a semicircle. (See Figure 2.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.

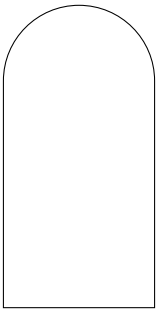

Figure 2: A Norman Window

3. (Based on 3.7.50) A woman at a point A on the shore of a circular lake with radius 2 mi wants to arrive at a point C diametrically opposite A on the other side of the lake in the shortest possible time. (See Figure 3.) She can walk at the rate of 4 mi/h and row a boat at 2 mi/h. How should she proceed?

4. (Based on 3.7.61–54) A manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager charge to maximize revenue?

5. (Based on 3.7.76) A rain gutter is to be constructed from a metal sheet of width 30 cm by bending up one-third of the sheet on each side through an angle of θ. (See Figure 4.) How should θ be chosen so that the gutter will carry the maximum amount of water?

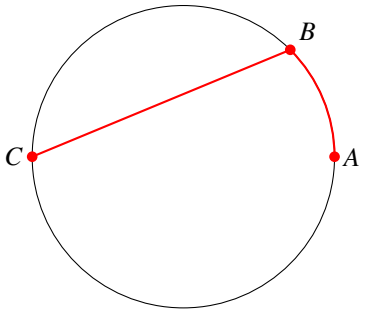

Figure 3: Path on a Circular Lake

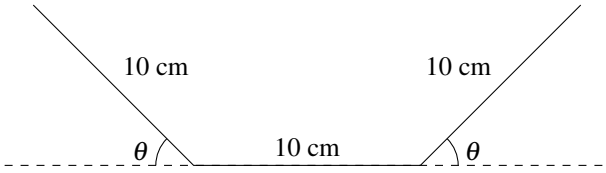

Figure 4: Cross Section of a Rain Gutter

6. (Based on 3.7.71) Let v_1 be the velocity of light in air and v_2 the velocity of light in water. A ray of light will travel from a point A in the air to a point B in the water by a path ACB that minimizes the time taken. Show that

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

where θ_1 (the angle of incidence) and θ_2 (the angle of refraction) are as shown in Figure 5. This equation is known as Snell's Law.

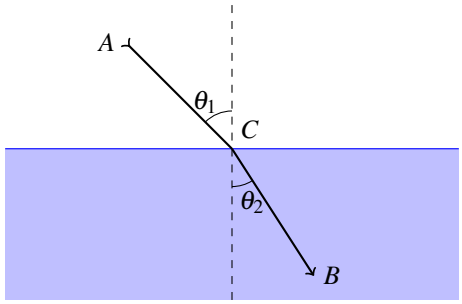

Figure 5: Ray of Light Passing from Air into Water

You may find the following additional exercises from Section 3.7 helpful.

3.7 C-level: 1–64;
B-level: 65–67, 69–70;
A-level: 71–80