Midterm Test 1

Time: 50 minutes   Name: _____

Instructors: Student #: _____

   Dr. Edward Doolittle   Section: _____

(marks) You have 60 minutes to do each of the following questions. The test is worth a total of 50 marks. Please justify your conclusions and show all your work. A calculator of approved type is permitted. No other aids are permitted. Use the backs of the pages for rough work.

1. Find the following limits.

(6)   (a) $\displaystyle\lim_{x\to-4}\frac{x^2+5x+4}{x^2+3x-4}$

(6)   (b) $\displaystyle\lim_{h\to9}\frac{\sqrt{h}-3}{h-9}$

2. Consider the functions

$$f(x) = x^2 + 1 \text{ and } g(x) = 3x - 1$$

(4)   (a) Show that $f \circ g \neq g \circ f$.

(4)   (b) Find a function h so that $g \circ h(x) = x$.

3. Find, with justification, the following infinite limits:

(4)   (a) $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x^2 - 6x + 9}$

(4)   (b) $\lim\limits_{x \to \pi^+} \cot x$

(6) 4. Find, with justification, the vertical asymptote(s) (if any) of the function

$$f(x) = \frac{x^2 + x}{x^2 - x - 2}$$

(10)   5. Consider the function

$$f(x) = \begin{cases} cx^3 + x & \text{if } -\infty < x < 2 \\ x^3 - cx & \text{if } 2 \le x < \infty \end{cases}$$

For what value(s) (if any) of the constant c is $f(x)$ continuous on the entire real line $(-\infty, \infty)$?

(6)   6. Is there a number a such that

$$\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

exists? If so, find the value of a and the value of the limit.