MATH 110 Problem Set 2.7 Solutions

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1. (a) This question can be answered with or without calculus. Without calculus, we can complete the square to obtain

$$s = -16(t^2 - 5t) = -16\left(t^2 - 2\frac{5}{2}t + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right) = -16\left(t - \frac{5}{2}\right)^2 + 16\left(\frac{5}{2}\right)^2 = -16\left(t - \frac{5}{2}\right)^2 + 100$$

which has a maximum value of 100, so the ball reaches a maximum height of 100 ft.

With calculus, note that the ball reaches a maximum height when it is no longer moving upwards, i.e., when $s' = 0$, which occurs when $80 - 32t = 0$, i.e., $t = 5/2$ seconds. At that time, the ball is at a height of $s(5/2) = 80(5/2) - 16(5/2)^2 = 200 - 100 = 100$ ft.

   (b) When the ball is 96 ft above the ground we have $s(t) = 80t - 16t^2 = 96$ which gives $16t^2 - 80t + 96 = 0$, $t^2 - 5t + 6 = 0$ $(t-2)(t-3) = 0$, so the ball is 96 ft above the ground when $t = 2$ seconds and when $t = 3$ seconds. Note that the velocity of the ball is the derivative of s so $s'(t) = 80 - 32t$; at $t = 2$ we have $s'(2) = 80 - 32(2) = 80 - 64 = 16$, so the ball is moving upwards (positive is up in this problem) at 16 ft/s. When $t = 3$ we have $s'(3) = 80 - 32(3) = 80 - 96 = -16$, so the ball is moving downwards at 16 ft/s.

2. Based on the formula for linear density in the textbook, the linear density is $m'(x) = 6x$ kg/m. (a) When $x = 1$ the linear density is $m'(1) = 6$ kg/m; (b) 12 kg/m; (c) 18 kg/m.

3. (a) We have $C'(x) = 25 - 0.18x + 0.0012x^2$ so $C'(100) = 25 - 18 + 12 = 19$. We interpret this as an approximation to the marginal cost, i.e., the cost of producing one more unit of the commodity after we have produced 100 is approximately 19 monetary units. (The problem doesn't state what those units are; they could be dollars, thousands of dollars, millions of dollars, etc.)

   (b) The exact cost of producing one more item after 100 have been produced is

$$\begin{aligned} M &= C(101) - C(100) \\ &= 339 + 25(101) - 0.09(101)^2 + 0.0004(101)^3 - (339 + 25(100) - 0.09(100)^2 + 0.0004(100)^3) \\ &= 19.0304 \end{aligned}$$

We can see that the marginal cost is very close to the approximation in the previous part; the relative error is $0.0304/19 = 0.0016$, or 0.16%.

4. (a) The rate of the reaction is

$$d[C]/dt = \frac{d}{dt}\frac{a^2kt}{akt+1} = \frac{(akt+1)(a^2k) - a^2kt(ak)}{(akt+1)^2} = \frac{a^3k^2t + a^2k - a^3k^2t}{(akt+1)^2} = \frac{a^2k}{(akt+1)^2}$$

   (b) We have worked out the left side of the purported identity above. The right side is

$$k(a-x)^2 = k\left(a - \frac{a^2kt}{akt+1}\right)^2 = k\left(\frac{a^2kt + a - a^2kt}{akt+1}\right)^2 = k\frac{a^2}{(akt+1)^2}$$

which is the same as the left side and we are done.

5. (a) By a stable population, we mean a population which doesn't change in number (of course, the particular individuals will change from year to year, but the number of them won't). That means P is constant throughout time, so $dP/dt = 0$.

   (b) We are given $P_c = 10,000$, $r_0 = 0.05$, and $\beta = 0.04$; furthermore, if the population is stable, we have $dP/dt = 0$. Substituting those values into the equation, we have

$$0 = 0.05\left(1 - \frac{P(t)}{10,000}\right)P(t) - 0.04P(t)$$

   Clearing fractions by multiplying through by $10,000$ and factoring,

$$0 = P(t)(0.05\,(10,000 - P(t)) - 400) = P(t)(100 - 0.05P(t))$$

   with solutions $P(t) = 0$ and $P(t) = 2000$. The stable population levels are $P(t) = 0$ and $P(t) = 2000$; if we have no fish, we will continue to have no fish; on the other hand, if we have any other number of fish, the population will in the long run tend to $2,000$ fish, meaning we'll only be able to harvest about 80 fish per year in the long run.

   (c) As with the previous, we have $P_c = 10,000$, $r_0 = 0.05$, and $dP/dt = 0$, but now we have $\beta = 0.05$ instead of 0.04, which could happen, say, if we tried to harvest 100 fish per year at the $2,000$ population level. In that case our equation becomes

$$0 = 0.05\left(1 - \frac{P(t)}{10,000}\right)P(t) - 0.05P(t)$$

   Clearing fractions by multiplying through by $10,000$ and factoring,

$$0 = P(t)(0.05\,(10,000 - P(t)) - 500) = P(t)(-0.05P(t))$$

   with only the solution $P(t) = 0$. That means that no matter how many fish we start with, in the long run we'll have no fish under those conditions.