UNIVERSITY OF REGINA
Department of Mathematics and Statistics

Mathematics 110 (Calculus I)
Final Exam

17 December 2014

Time: 3 hours **Full Name:**__________________________________

Pages: 11 **Student Number:**____________________________

Instructor:

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INSTRUCTIONS

1. Full credit is awarded only for well-presented, correct solutions in which all of your work is shown.
2. Your solution and final answer should appear on the right side of the pages in this exam booklet; the left side pages are to be used as scrap paper and are provided for rough work only. If you require more space for your solution than the right side allows, then use the left side (facing) page and indicate on your solution that there is work to be found on the facing page.
3. The marks allocated for each question are found to the left of the question.
4. Graphing calculators, tablets, and phones are not permitted.

For instructor use only:

Page:	2	3	4	5	6	8	9	10	11	Total
Marks:	9	13	12	14	12	10	12	8	10	100
Score:										

Marks

(9) 1. Evaluate the given limit; if a limit does not exist, then state so.

(a) $\lim\limits_{x \to 2} \dfrac{8x - 2x^3}{x^2 + x - 6}$

(b) $\lim\limits_{x \to \infty} \dfrac{3x + 5x^2 + 1}{3 - 2x^2}$

(c) $\lim\limits_{x \to 0} \dfrac{3\sin(5x)}{2x}$

(6) 2. Consider the function $f(x) = \left\{ \begin{array}{ll} x^2 - a^2 & \text{if } x < 4 \\ ax + 20 & \text{if } x \geq 4 \end{array} \right\}.$

Find all values of the constant a such that $f(x)$ is continuous for all real numbers x.

(7) 3. (a) State the limit definition of the derivative function $f'(x)$.

(b) Use the limit definition to find $f'(x)$ if $f(x) = \dfrac{1}{x+1}$

(12) 4. Find the indicated derivative. You do not have to simplify your answer.

(a) $f(x) = \frac{1}{2}x^6 - 3x^4 + x$ $f''(2) = ?$

(b) $g(v) = \left(\dfrac{v}{v^3 + 1}\right)^6$ $g'(v) = ?$

(c) $y = x\sin(\frac{1}{x})$ $y'(x) = ?$

(d) $f(t) = (3t - 1)^4(2t + 1)^{-3}$ $f'(t) = ?$

(6) 5. Find the equation of the tangent line to the graph of $y = f(x)$ of the function $f(x) = 3x^{1/2} + 1$ at the point $(4, 7)$.

(8) 6. Two runners start moving from the same point. One runs north at 8 km/hr and the other runs east at 6 km/hr. At what rate is the distance between the runners increasing two hours later?

(12) 7. Consider the function $f(x) = \dfrac{x^2}{x^2+9}$. You may use the fact that the first and second derivatives of $f(x)$ are $f'(x) = \dfrac{18x}{(x^2+9)^2}$ and $f''(x) = \dfrac{54(3-x^2)}{(x^2+9)^3}$. Remember to show your work.

(a) Identify (if applicable):

Domain:

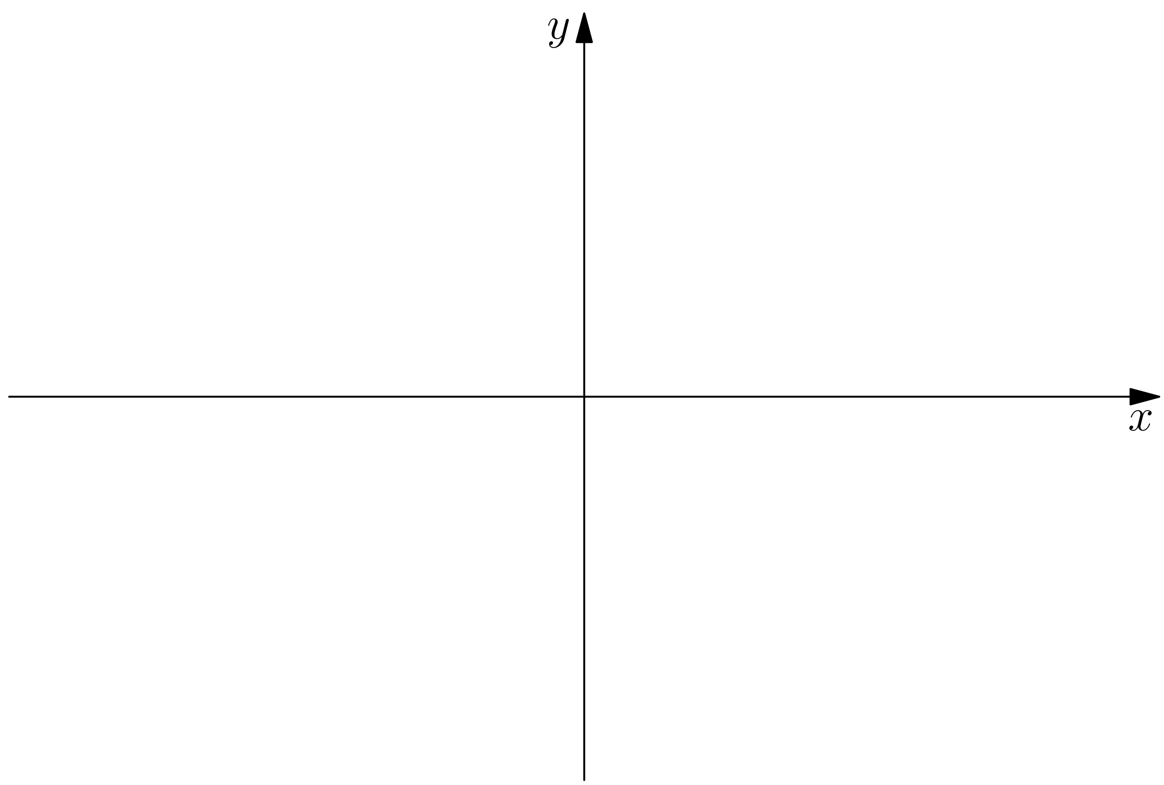
All x- and y-intercepts:

Horizontal and Vertical Asymptotes (show the relevant limits in your work):

Local (relative) Maxima and Minima (give x and y coordinates):

Inflection Points (give x and y coordinates):

(b) Use the information in part (a) to sketch the graph of $y = f(x)$.



(10) 8. A rectangular area of 3200 m^2 is to be fenced off. Two opposite sides will use fencing costing \$1 per metre and the remaining sides will use fencing costing \$2 per metre. Use optimization techniques to find the dimensions and the total cost of the cheapest rectangle.

(12) 9. Evaluate each of the following indefinite and definite integrals:

(a) $\displaystyle\int \left(\sqrt[4]{x} + \frac{2}{x^2} + 3 \right) dx$

(b) $\displaystyle\int_1^2 \frac{x+1}{x^3}\, dx$

(c) $\displaystyle\int \sin^3 x \cos x \, dx$

(d) $\displaystyle\int \frac{x}{(3x^2-1)^2}\, dx$

(4) 10. Differentiate implicitly and solve for $\dfrac{dy}{dx}$ if $\cos(xy) = 1 + \sin(y)$.

(4) 11. Evaluate the integral $\displaystyle\int_0^1 x^3\sqrt{1-x^2}\,dx$.

(10) 12. Find the area of the region between the curves $y = 1 - x^2$ and $y = \dfrac{x}{2} + \dfrac{1}{2}$. Include a sketch of the relevant region as part of your solution.