MATH 110 Review Problem Set 1.1 Solutions

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1. We have

$$\begin{aligned}
f(2) &= 3(2)^2 - 2 + 2 = 3 \cdot 4 = 12 \\
f(-2) &= 3(-2)^2 - (-2) + 2 = 3(4) + 2 + 2 = 12 + 4 = 16 \\
f(a) &= 3a^2 - a + 2 \\
f(-a) &= 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2 \\
f(a+1) &= 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a - 1 + 2 = 3a^2 + 6a + 3 - a + 1 = 3a^2 + 5a + 4 \\
2f(a) &= 2(3a^2 - a + 2) = 6a^2 - 2a + 4 \\
f(2a) &= 3(2a)^2 - (2a) + 2 = 3(4a^2) - 2a + 2 = 12a^2 - 2a + 2 \\
f(a^2) &= 3(a^2)^2 - (a^2) + 2 = 3a^4 - a^2 + 2 \\
(f(a))^2 &= (3a^2 - a + 2)^2 = 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4 = 9a^4 - 6a^3 + 12a^2 - 4a + 4 \\
f(a+h) &= 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 + 6ah + 3h^2 - a - h + 2
\end{aligned}$$

   For this question, I will accept correct answers that do not involve simplification, but you will need to be able to do those simplifications when we calculate derivatives in chapters 2 and 3.

2. We can build up the calculation in steps. We have $f(x) = 1/x$, $f(a) = 1/a$, so the numerator is

$$f(x) - f(a) = \frac{1}{x} - \frac{1}{a} = \frac{a}{ax} - \frac{x}{ax} = \frac{a-x}{ax}$$

   where I subtracted fractions by putting them over a common denominator. It follows that the difference quotient is

$$\frac{f(x) - f(a)}{x - a} = (f(x) - f(a)) \cdot \frac{1}{x-a} = \frac{a-x}{ax} \cdot \frac{1}{x-a} = \frac{-1}{ax}$$

3. We first graph the line $y = x + 2$ on the domain $x \le -1$ by finding two points on the line. We select $x = -1$ which gives $y = x + 2 = -1 + 2 = 1$, so we plot the point $(x, y) = (-1, 1)$. Next we choose another x value ≤ -1, say $x = -3$ which gives $y = x + 2 = -3 + 2 = -1$, so we plot the point $(x, y) = (-3, -1)$. We draw a line segment connecting those two points, which graphs the function for $x \le -1$.

   Next, we graph the parabola $y = x^2$ on the domain $x > -1$. That is a standard parabola curve, and you should know that it opens upwards (or you can see that because the coefficient of x^2 is $+1$, a positive number); that it has vertex $(0, 0)$; and that it passes through a collection of points $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(2, 4)$, and so on. Note that $(-1, 1)$ isn't on the parabola, which has domain $x > -1$, so technically we should draw an open circle at $(-1, 1)$ on the parabola; however, the point $(-1, 1)$ is on the linear part of the curve. Because the curves join together, we don't have to draw either the open dot that goes with the parabola nor the closed dot that goes with the linear curve.

   See Figure 1 for the final result.

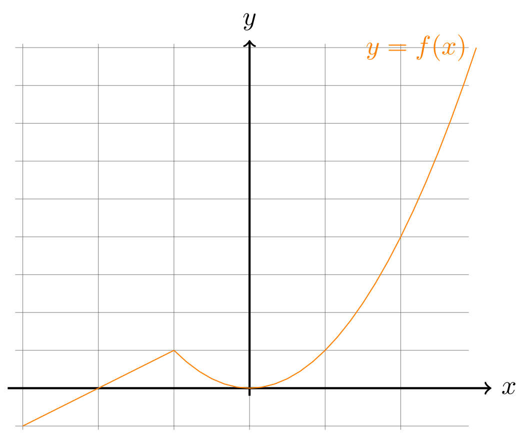

Figure 1: Graph of $y = f(x)$ from question 3