

MATH 110 Review Problem Set 0.D Solutions

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1. As with most unit conversions, the best way is to use

$$360^\circ = 2\pi \text{ radian} \implies 1 = \frac{2\pi \text{ radian}}{360^\circ}$$

- (a) $60^\circ = 60^\circ \times \frac{2\pi \text{ radian}}{360^\circ} = \frac{2\pi}{6} \text{ radian} = \frac{\pi}{3} \text{ radian}$
- (b) $330^\circ = 330^\circ \times \frac{2\pi \text{ radian}}{360^\circ} = \frac{11\pi}{6} \text{ radian}$
- (c) $-20^\circ = -20^\circ \times \frac{2\pi \text{ radian}}{360^\circ} = -\frac{\pi}{9} \text{ radian}$
- (d) $780^\circ = 780^\circ \times \frac{2\pi \text{ radian}}{360^\circ} = \frac{13\pi}{3} \text{ radian}$
2. (a) $3\pi \text{ radian} = 3\pi \text{ radian} \times \frac{360^\circ}{2\pi \text{ radian}} = 540^\circ$
- (b) $\frac{7\pi}{3} \text{ radian} = \frac{7\pi}{3} \text{ radian} \times \frac{360^\circ}{2\pi \text{ radian}} = 420^\circ$
- (c) $-\frac{5\pi}{6} \text{ radian} = -\frac{5\pi}{6} \text{ radian} \times \frac{360^\circ}{2\pi \text{ radian}} = -150^\circ$
- (d) $6 \text{ radian} = 6 \text{ radian} \times \frac{360^\circ}{2\pi \text{ radian}} = \frac{1020^\circ}{\pi} \approx 325^\circ$
3. (a) See Figure 1(a).
(b) See Figure 1(b).
(c) See Figure 2(a).
(d) See Figure 2(b).
4. (a) Draw the angle in standard position then draw a right triangle with the terminal arm of the angle as hypotenuse and base coincident with the x -axis. Because of the $\pi/3$ radian angle in the triangle it is a $1, 2, \sqrt{3}$ triangle, or to be more precise, a $-1, 2, \sqrt{3}$ triangle because it is in quadrant II. See Figure 3(a). From the diagram, it follows that the exact ratios are

$$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2} \quad \cos \frac{2\pi}{3} = \frac{-1}{2} = -\frac{1}{2} \quad \tan \frac{2\pi}{3} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

- (b) Draw the angle in standard position then draw a right triangle with the terminal arm of the angle as hypotenuse and base coincident with the x -axis. Because of the $\pi/4$ radian angle in the triangle it is a $1, 1, \sqrt{2}$ triangle. See Figure 3(b). It follows that the exact ratios are

$$\sin \frac{9\pi}{4} = \frac{1}{\sqrt{2}} \quad \cos \frac{9\pi}{4} = \frac{1}{\sqrt{2}} \quad \tan \frac{9\pi}{4} = \frac{1}{1} = 1$$

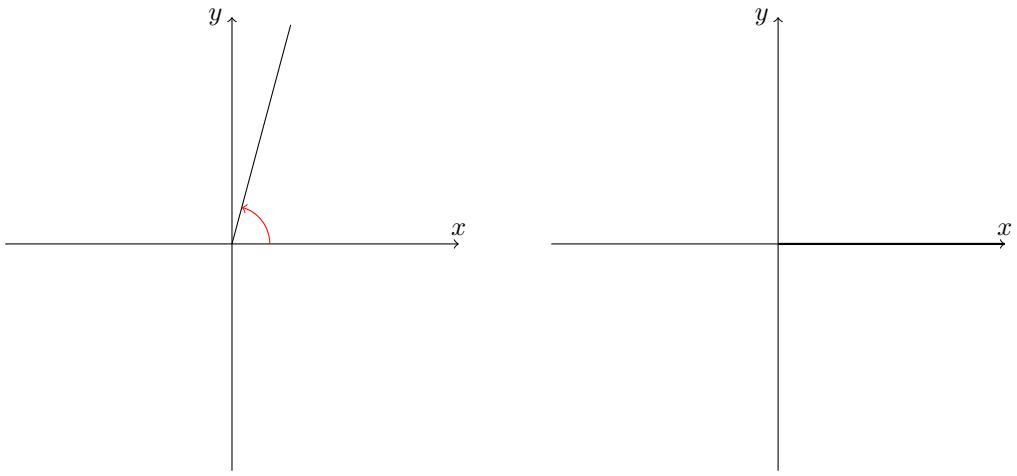


Figure 1: Angles in standard position for 3a and 3b

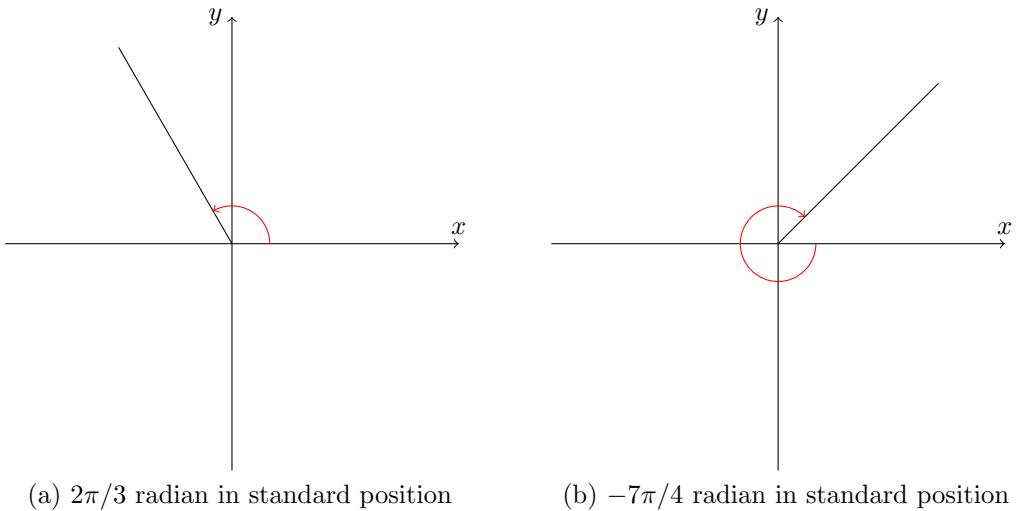
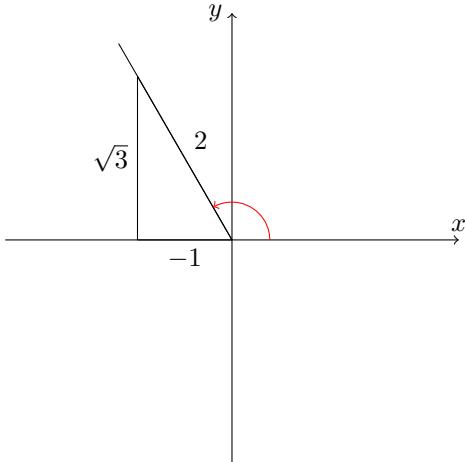
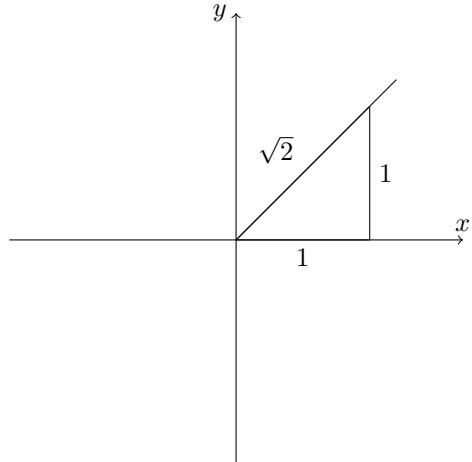


Figure 2: Angles in standard position for 3c and 3d



(a) $2\pi/3$ radian and triangle



(b) $9\pi/4$ radian and triangle

Figure 3: Angles in standard position for 4a and 4b

- (c) Draw the angle in standard position then draw a right triangle with the terminal arm of the angle as hypotenuse and base coincident with the x -axis. Because of the $\pi/6$ radian angle in the triangle it is a $-1, 2, -\sqrt{3}$ triangle. See Figure 4(a). It follows that the exact ratios are

$$\sin\left(-\frac{5\pi}{6}\right) = \frac{-1}{2} = -\frac{1}{2} \quad \cos\left(-\frac{5\pi}{6}\right) = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2} \quad \tan\left(-\frac{5\pi}{6}\right) = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

- (d) In this case the terminal arm of the angle is coincident with the positive y -axis so we can't really draw a right triangle as we have with the other problems. In this case we have a "degenerate" triangle with hypotenuse 1, adjacent leg (base) 0, and opposite leg (height) 1. The ratios are therefore

$$\sin\left(-\frac{11\pi}{2}\right) = \frac{1}{1} = 1 \quad \cos\left(-\frac{11\pi}{2}\right) = \frac{0}{1} = 0 \quad \tan\left(-\frac{11\pi}{2}\right) = \frac{1}{0} = \infty$$

I used the symbol ∞ for the tan ratio, but more properly the result should read "undefined".

5. We shall solve these problems by drawing an appropriate diagram reflecting the given data, and then finding the remaining trig ratios from the diagram.

Alternatively, you could solve the problem as follows: first use the Pythagorean identity to find the magnitude of another of the ratios. Then use the quadrant of the angle to determine the sign of that ratio. From there, use reciprocal and ratio identities to find the other ratios. However, I recommend drawing a diagram.

- (a) Because $0 < \theta < \pi/2$, draw an angle with terminal arm in quadrant I. Since $\sin \theta = 5/13 = O/H$, make the hypotenuse of the triangle 13 units and the opposite side 5 units. See Figure 5(a). Then the Pythagorean theorem says that the adjacent side is $\sqrt{13^2 - 5^2} = 12$ units. Now we know everything about the sides of the triangle so we can calculate the other trig ratios:

$$\cos \theta = \frac{12}{13} \quad \tan \theta = \frac{5}{12}$$

and the other ratios could be easily determined as reciprocals if desired (e.g., $\sec \theta = 1/\cos \theta = 13/12$).

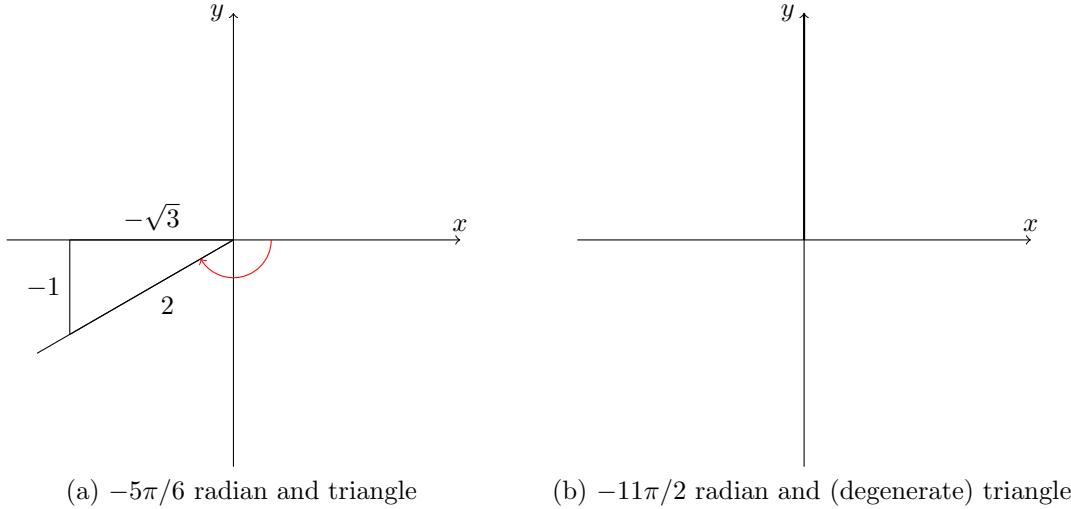


Figure 4: Angles in standard position for 4c and 4d

- (b) In this case, $\tan \theta$ is not given as a ratio, but we can easily make it into a ratio: $\tan \theta = 3/1 = O/A$. Again the terminal arm should be in the first quadrant. See Figure 5(b). Now the Pythagorean theorem says that the hypotenuse is $\sqrt{3^2 + 1^2} = \sqrt{10}$ units long so the remaining trig ratios are

$$\sin \theta = \frac{3}{\sqrt{10}} \quad \cos \theta = \frac{1}{\sqrt{10}}$$

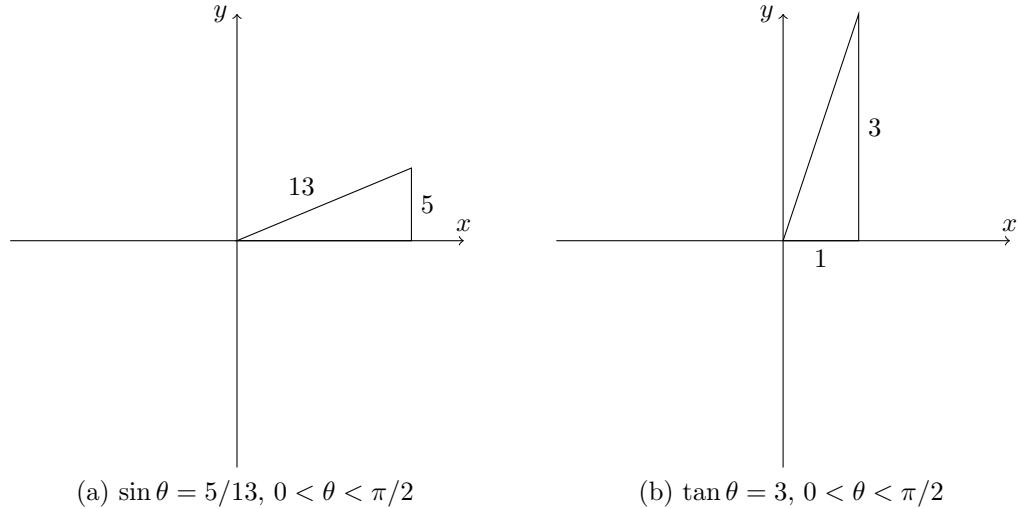


Figure 5: Triangles in standard position for 5a and 5b

- (c) In this case, the angle is in the second quadrant and we are given the ratio $\sec \theta = -2.25/1 = H/A$. The hypoteneuse can never be negative so we should rewrite our ratio as $\sec \theta = 2.25/-1 = H/A$. We draw a triangle in the second quadrant with hypotenuse 2.25 units and adjacent side -1 units long. See Figure 6(a). It follows by the Pythagorean theorem that the opposite side is $\sqrt{2.25^2 - (-1)^2} \approx 2.02$ units long. (We go to two decimal points of accuracy because that is the

level of accuracy with which the data -2.25 is given.) So the ratios are

$$\sin \theta \approx \frac{2.02}{2.25} \approx 0.90 \quad \cos \theta = \frac{-1}{2.25} \approx -0.44 \quad \tan \theta \approx \frac{2.02}{-1} = -2.02$$

- (d) The angle is in either quadrant III or quadrant IV. Writing $\cot \alpha$ as a ratio, we have $\cot \alpha = 3.5/1 = A/O$. There is no angle in quadrant III or quadrant IV with both A and O positive, so we have to rewrite the ratio as $\cot \alpha = -3.5/-1 = A/O$. See Figure 6(b). By the Pythagorean theorem the hypotenuse is then $\sqrt{(-3.5)^2 + (-1)^2} \approx 3.6$ and the other trig ratios are

$$\sin \alpha \approx \frac{-1}{3.6} \approx -0.27 \quad \cos \alpha \approx \frac{-3.5}{3.6} \approx -0.96 \quad \tan \alpha = \frac{-1}{-3.5} \approx 0.29$$

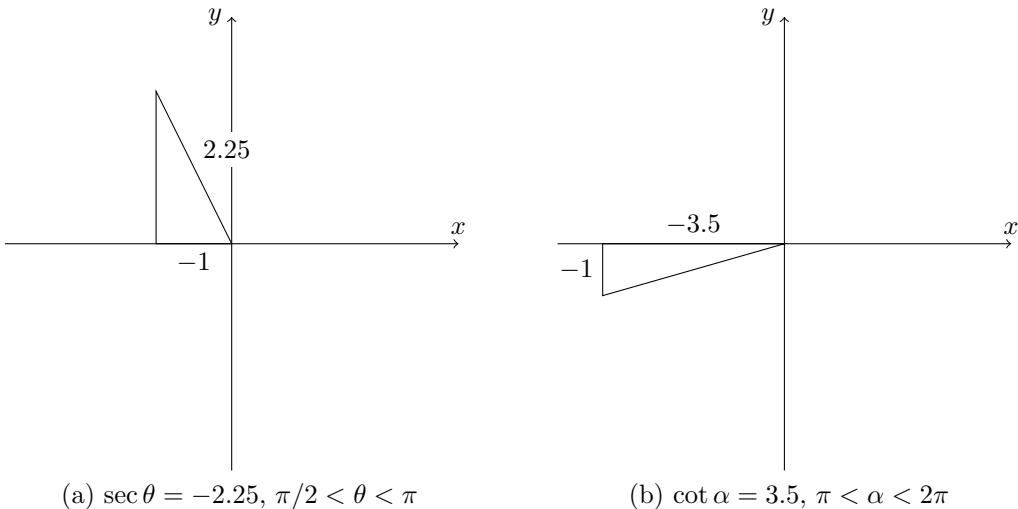


Figure 6: Triangles in standard position for 5c and 5d

6. You should stick to using the three most common trig ratios (\sin , \cos , \tan) to answer this question because working out the others (\sec , \csc , \cot) with your calculator introduces and additional unnecessary complication.

- (a) In this case we have expressions for the sides opposite (6 cm) and adjacent to (x) the angle, so the natural ratio to consider is \tan . We have

$$\tan \frac{2\pi}{5} = \frac{6}{x} \implies x = \frac{6}{\tan(2\pi/5)} \approx 1.95 \text{ cm}$$

- (b) In this case we have expressions for the opposite and hypotenuse so the best ratio is \sin :

$$\sin \frac{\pi}{3} = \frac{1}{x} \implies \frac{\sqrt{3}}{2} = \frac{1}{x} \implies x = \frac{2}{\sqrt{3}} \text{ in}$$

- (c) In this case we have expressions for the adjacent and hypotenuse so the best ratio is \cos :

$$\cos 40^\circ = \frac{x}{35} \implies x = 35 \cos 40^\circ \approx 26.8 \text{ meters}$$

- (d) In this case we have the opposite and adjacent so we use tan:

$$\tan 30^\circ = \frac{x}{2} \implies x = 2 \tan 30^\circ = \frac{2}{\sqrt{3}} \text{ km}$$

7. (a) Solving for $\sin x$,

$$2 \sin x - 1 = 0 \implies 2 \sin x = 1 \implies \sin x = \frac{1}{2}$$

Draw a triangle with hypotenuse 2 and opposite side 1. Note that that is possible in either quadrant I or II. In quadrant I we have a $1, 2, \sqrt{3}$ triangle, so the angle is $x = \pi/6$. In quadrant II, the angle is $x = 5\pi/6$. (Draw your own diagrams!)

- (b) The given equation implies $\tan x = \pm 1/\sqrt{3}$, so there are four possibilities, one in each quadrant (we basically don't care about the signs, anything goes). We again use $1, 2, \sqrt{3}$ triangles, one in each quadrant, and determine that $x = \pi/6, 5\pi/6, 7\pi/6$, or $11\pi/6$. (Draw your own diagrams!)
- (c) Here we have $\cos x = \pm\sqrt{3}/2$, so again we don't care about the quadrant or the signs, and again we use a $1, 2, \sqrt{3}$ triangle to obtain $x = \pi/3, 2\pi/3, 4\pi/3$, and $5\pi/3$. (Draw your own diagrams!)
- (d) The best way to proceed is using trig identities to somehow simplify the expression. I recommend using the double angle identity for $\cos 2x$ in terms of \cos so we don't end up mixing \cos and \sin in the same expression:

$$\cos 2x + \cos x = 0 \implies 2\cos^2 x - 1 + \cos x = 0 \implies 2\cos^2 x + \cos x - 1 = 0$$

Letting $u = \cos x$ we have a quadratic equation in u :

$$2u^2 + u - 1 = 0 \implies (2u - 1)(u + 1) = 0 \implies u = 1/2 \text{ or } u = -1$$

In the first case, $\cos x = 1/2 = A/H$. Since H is always positive, that means A must be positive as well, which means we are in quadrant I or IV. Drawing the appropriate $1, 2, \sqrt{3}$ triangles, we determine that $x = \pi/3$ or $5\pi/3$. In the second case, $\cos x = -1 = -1/1 = A/H$. The only way to have adjacent side equal to the hypotenuse is if the opposite side (height) is 0 leading to a degenerate triangle with $x = \pi$. (Draw your own diagrams!)

Alternatively, you may find the graph of cosine helpful in solving this problem.

8. We can use angles in standard position inside a unit circle to help answer these questions. Consider a point on the unit circle. We have $\cos \theta = A/H = A/1 = A$ and $\sin \theta = O/H = O/1 = O$, so we can read the basic trigonometric ratios from the diagram in Figure 7, and in fact the coordinates of the point are $(x, y) = (\cos \theta, \sin \theta)$. It follows that conditions on $\cos \theta$ become conditions on x in the graph, and conditions on $\sin \theta$ become conditions on y in the graph. (We will see about conditions on $\tan \theta$ below.)

- (a) The condition $\cos \theta \leq 1/2$ becomes $x \leq 1/2$ in our framework. First, graph the inequality $x \leq 1/2$ on the plane containing the unit circle we have been using. satisfy the condition; see Figure 8(a). Then find all corresponding points on the circle, and the corresponding angles; see Figure 8(b). We see from the latter diagram and a couple of $1, 2, \sqrt{3}$ triangles (not pictured) that the angles θ satisfying the inequality are given by $\pi/3 \leq \theta \leq 5\pi/3$.

- (b) Solve for $\sin \theta$:

$$-2 \sin \theta + 1 \geq 0 \implies -2 \sin \theta \geq -1 \implies \sin \theta \leq \frac{1}{2}$$

Note the reversal of the inequality!

Now, the condition $\sin \theta \leq 1/2$ translates to $x \leq 1/2$; see Figure 9(a). Then find all corresponding points on the circle, and the corresponding angles; see Figure 9(b). We see from the latter diagram and a couple of $1, 2, \sqrt{3}$ triangles (not pictured) that the angles θ satisfying the inequality are given by $0 \leq \theta \leq \pi/6$ or $5\pi/6 \leq \theta \leq 2\pi$.

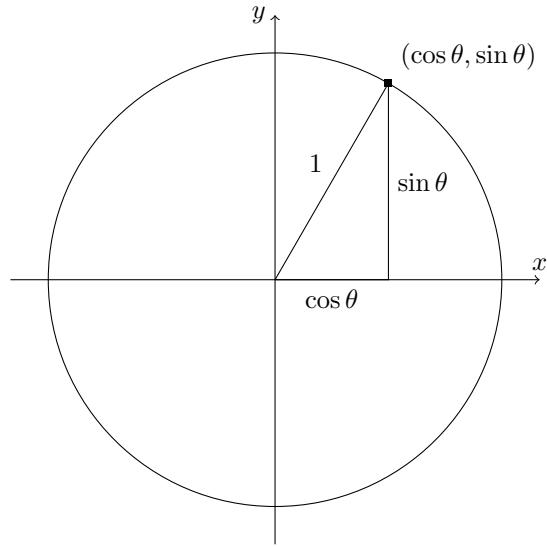


Figure 7: Trigonometric ratios in a unit circle

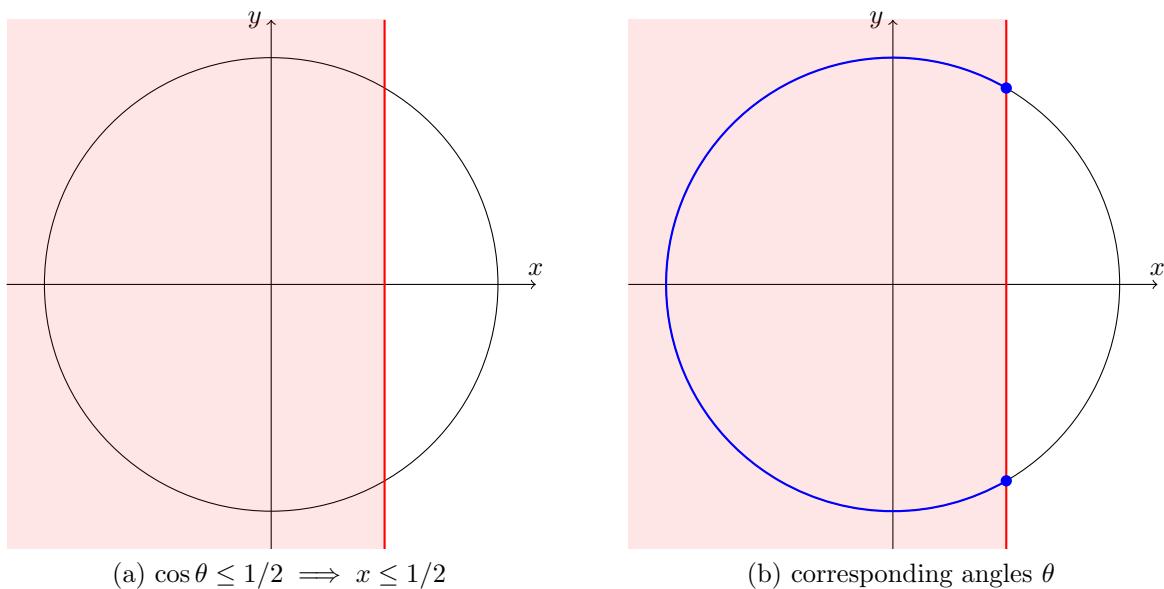


Figure 8: Solving the inequality $\cos \theta \leq 1/2$ for θ

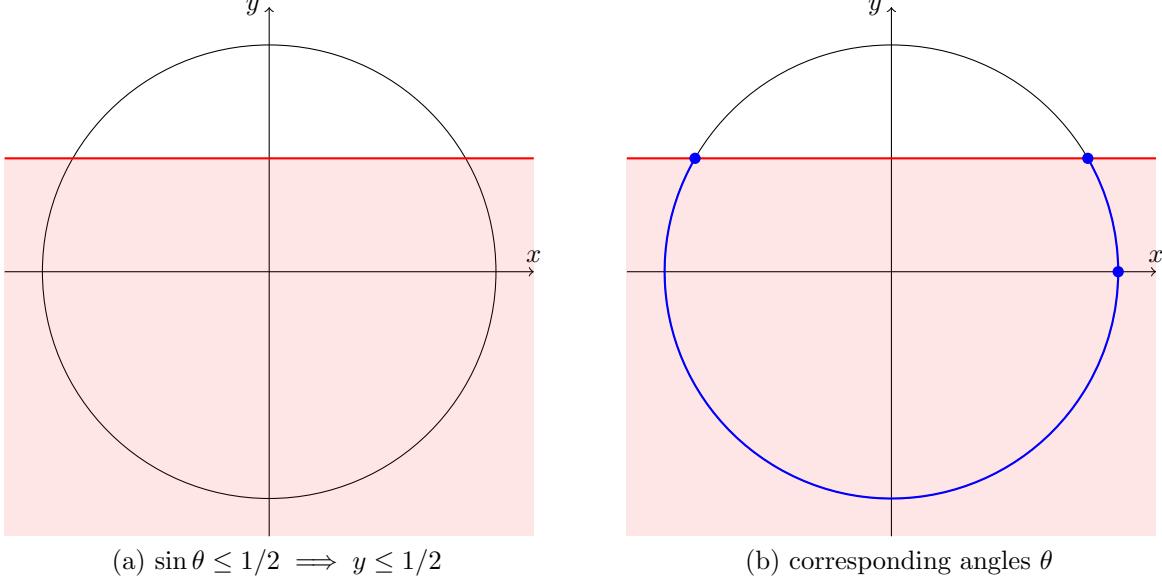


Figure 9: Solving the inequality $-2 \sin \theta + 1 \geq 0$ for θ

(c) In our framework, the inequality becomes

$$-\frac{1}{\sqrt{3}} < \frac{y}{x} < \sqrt{3}$$

which can be interpreted as a condition on the slope of a line through the origin. See Figure 10(a). Then find all corresponding points on the circle, and the corresponding angles; see Figure 10(b). We see from the latter diagram and four $1, 2, \sqrt{3}$ triangles (not pictured) that the angles θ satisfying the inequality are given by $0 \leq \theta < \pi/3, 5\pi/6 < \theta < 4\pi/3$, or $11\pi/6 < \theta \leq 2\pi$.

- (d) The given condition becomes simply $y < x$, which is easy to graph; see Figure 11(a). Then find all corresponding points on the circle, and the corresponding angles; see Figure 11(b). We see from the latter diagram and a couple of $1, 1, \sqrt{2}$ triangles (not pictured) that the angles θ satisfying the inequality are given by $0 \leq \theta < \pi/4$ or $5\pi/4 < \theta < 2\pi$.
- 9. We just use the formula for arc length = $r\theta$ where θ is the angle of the arc in radians. We convert 60° to radians: $60^\circ = 60^\circ \times 2\pi/360^\circ = \pi/3$, so the arc length is $r\theta = 18\pi/3 = 6\pi$ meters.
- 10. We have $10 = 30\theta$ which implies $\theta = 10/30 = 1/3$ radians or $1/3 \times 360/(2\pi) = 60/\pi \approx 19.1^\circ$.

11. (a) By the angle subtraction formula, the LHS is

$$\sin(2\pi - x) = \sin 2\pi \cos x - \cos 2\pi \sin x = 0 \cdot \cos x - 1 \sin x = -\sin x$$

which is identical to the RHS.

- (b) Squaring the RHS,

$$\sin^2 x - 2 \sin x \cos x + \cos^2 x = \sin^2 x + \cos^2 x - 2 \sin x \cos x = 1 - \sin 2x$$

by the Pythagorean identity and the double angle formula for sin.

- (c) Expanding the LHS we have

$$(\sin x + \sin y)(\sin x - \sin y) = \sin^2 x - \sin^2 y$$

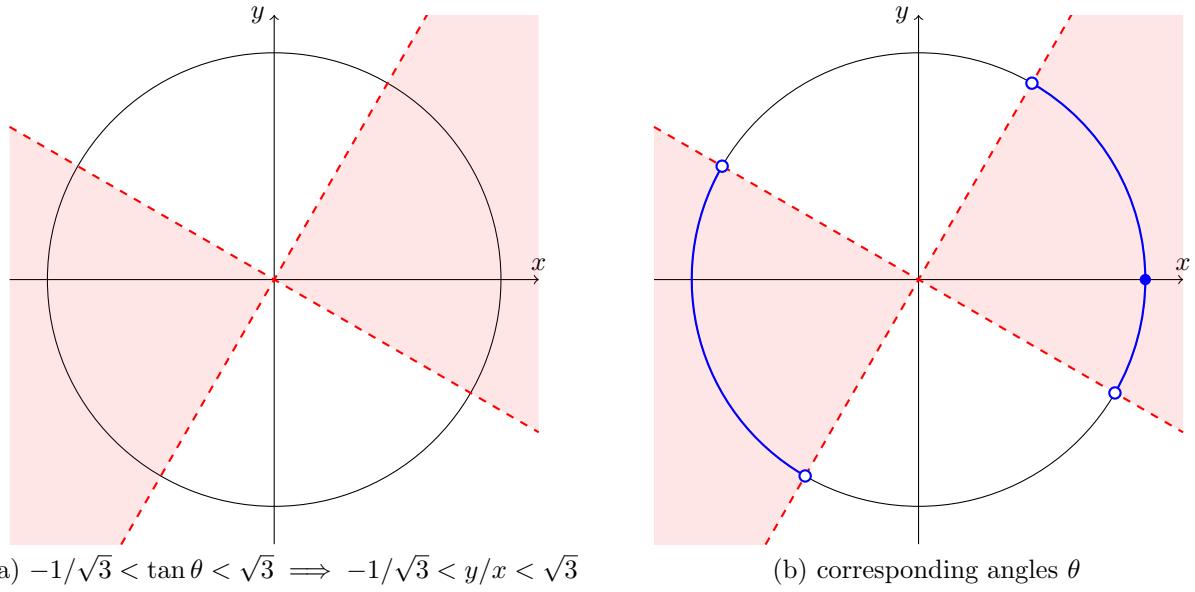


Figure 10: Solving the inequality $1/\sqrt{3} < \tan \theta < \sqrt{3}$ for θ

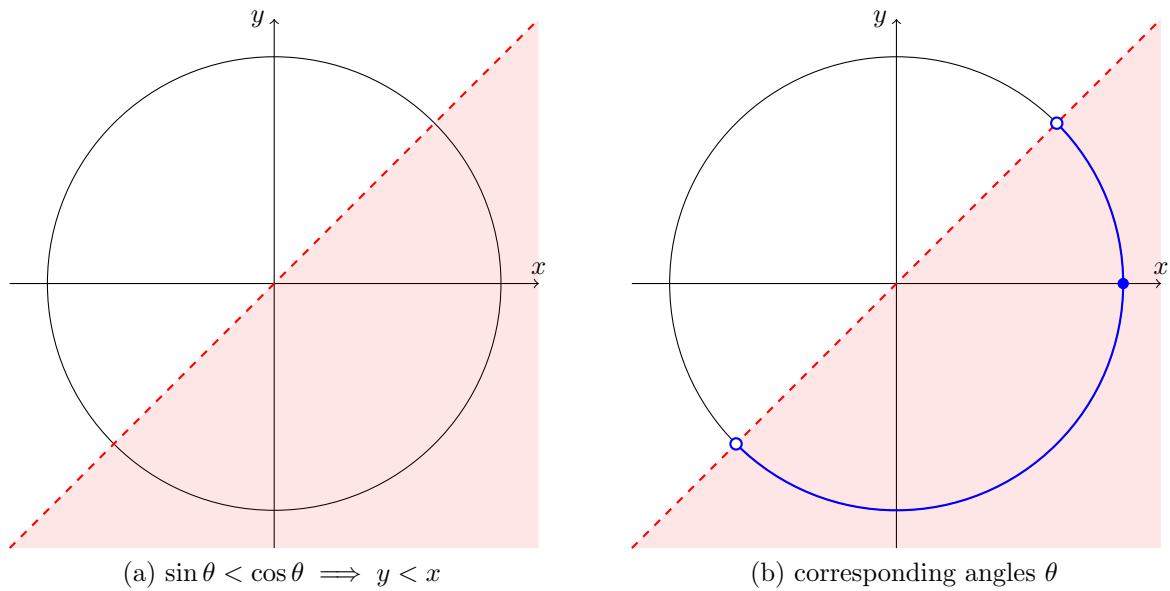


Figure 11: Solving the inequality $\sin \theta < \cos \theta$ for θ

Applying the product formula to the RHS,

$$\sin(x+y)\sin(x-y) = \frac{1}{2}(\cos((x+y)-(x-y)) - \cos((x+y)+(x-y)))$$

Simplifying, the RHS is

$$\frac{1}{2}(\cos 2y - \cos 2x) = \frac{1}{2}(1 - 2\sin^2 y - 1 + 2\sin^2 x) = \sin^2 x - \sin^2 y$$

which is identical with the LHS.

- (d) By the angle addition formula, the LHS is

$$\sin 3\theta = \sin(\theta + 2\theta) = \sin \theta \cos 2\theta + \cos \theta \sin 2\theta$$

By the double angle formulas, the LHS is

$$\sin \theta \cos 2\theta + \cos \theta \sin 2\theta = \sin \theta(1 - 2\sin^2 \theta) + \cos \theta \cdot 2\sin \theta \cos \theta$$

Simplifying and using $\cos^2 \theta = 1 - \sin^2 \theta$, the LHS is

$$\sin \theta - 2\sin^3 \theta + 2\cos^2 \theta \sin \theta = \sin \theta - 2\sin^3 \theta + 2(1 - \sin^2 \theta) \sin \theta = 3\sin \theta - 4\sin^3 \theta$$

which is identical to the RHS.

12. Since x and y are in the first quadrant, we can use the reasoning of problem 5 to determine

$$\begin{aligned}\sin x &= \frac{15}{17} & \tan x &= \frac{15}{8} \\ \sin y &= \frac{1}{2} & \cos y &= \frac{\sqrt{3}}{2} & \tan y &= \frac{1}{\sqrt{3}}\end{aligned}$$

- (a) By the angle addition formula for sin,

$$\sin(x+y) = \sin x \cos y + \cos x \sin y = \frac{15}{17} \cdot \frac{\sqrt{3}}{2} + \frac{8}{17} \cdot \frac{1}{2} = \frac{15\sqrt{3} + 8}{34}$$

- (b) By the angle subtraction formula for cos,

$$\cos(x-y) = \cos x \cos y + \sin x \sin y = \frac{8}{17} \frac{\sqrt{3}}{2} + \frac{15}{17} \frac{1}{2} = \frac{8\sqrt{3} + 15}{34}$$

- (c) By the double angle formula for cos,

$$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{8}{17}\right)^2 - 1 = -\frac{161}{289}$$

- (d) By the angle addition formula for tan,

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{15/8 + 1/\sqrt{3}}{1 - (15/8)(1/\sqrt{3})} = \frac{15\sqrt{3} + 8}{8\sqrt{3} - 15}$$

13. (a) This is a consequence of the angle addition formula

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

Letting $y = x$ we have

$$\sin(x+x) = \sin x \cos x + \cos x \sin x \implies \sin 2x = \sin x \cos x + \sin x \cos x = 2 \sin x \cos x$$

as required.

(b) By the double angle formula for cos, the RHS of the purported identity is

$$\frac{1 + \cos 2x}{2} = \frac{1 + (2 \cos^2 x - 1)}{2} = \frac{2 \cos^2 x}{2} = \cos^2 x$$

so is identical to the LHS.

14. We just replace every y in the subtraction formula with $-y$:

$$\sin(x - y) = \sin x \cos y - \cos x \sin y \implies \sin(x - (-y)) = \sin x \cos(-y) - \cos x \sin(-y)$$

Now applying rules for even and odd functions,

$$\sin(x + y) = \sin x \cos y - \cos x (-\sin y) = \sin x \cos y + \cos x \sin y$$

as required.