

# MATH 110 Problem Set 3.4

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The following problems based on Section 3.4 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

1. (Based on 3.4.57–60) Sketch the graph of a function  $f$  that satisfies all of the given conditions.

$$\lim_{x \rightarrow -\infty} f(x) = 1$$

$$\lim_{x \rightarrow \infty} f(x) = -3$$

$$\lim_{x \rightarrow 4^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 4^+} f(x) = 2$$

2. (Based on 3.4.7–8) Evaluate the following limit, justifying each step by indicating the appropriate properties of limits.

$$\lim_{x \rightarrow \infty} \frac{3x^2 - 8x + 2}{x^2 + 6x - 5}$$

3. (Based on 3.4.9–32) Find the following limits.

$$(a) \lim_{t \rightarrow -\infty} \frac{t^2 + 2}{t^3 + t^2 - 1}$$

$$(c) \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$$

$$(b) \lim_{x \rightarrow \infty} \frac{x + 2}{\sqrt{9x^2 + 1}}$$

$$(d) \lim_{x \rightarrow -\infty} \left( x + \sqrt{x^2 + 2x} \right)$$

4. (Based on 3.4.35–40) Find the horizontal and vertical asymptotes of each of the following curves. Check your work by graphing the curve on a graphing calculator or computer and estimating the asymptotes.

$$(a) y = \frac{3x - 1}{x + 2}$$

$$(b) y = \frac{x^2 + x - 6}{x^2 - 3x + 2}$$

$$(c) y = \frac{1 + 2x^3}{x^3 - x}$$

5. (Based on 3.4.6) Use a graph of the function

$$f(x) = \left( 1 - \frac{1}{x} \right)^x$$

to estimate the value of  $\lim_{x \rightarrow \infty} f(x)$  to two decimal places. Support your estimate by using a table of values.

6. (Based on 3.4.42) Calculate the limits

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{2x - 3} \quad \text{and} \quad \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 1}}{2x - 3}$$

Check your result by graphing the function. Can you explain any discrepancy you might notice?

7. (Based on 3.4.45–46) Find a formula for a function with vertical asymptotes  $x = -2$  and  $x = 3$  and horizontal asymptote  $y = 5/2$ .
8. (Based on 3.4.52–56) Find the limits as  $x \rightarrow \infty$  and  $x \rightarrow -\infty$ . Use that information, together with intercepts, to give a rough sketch of the graph of the function as was done in the textbook.

$$y = f(x) = x^2(x+2)^3(1-x)$$

You may find the following additional exercises from Section 3.4 helpful.

- 3.4 C-level: 1–40, 44–46, 48–51, 57–60;  
B-level: 41–43, 47, 52–56, 61–64;  
A-level: 65–74