FIRST NATIONS UNIVERSITY OF CANADA
DEPARTMENT OF SCIENCE
MATH 110–S01–S02 200830

MATH 110–S01–S02 Final Examination 200830

Time: 3 hours
Instructor:
Dr. Edward Doolittle

Name: ____________________
Student #: ____________________
Section: ____________________

(marks)

You have 3 hours to do all of the following questions. The test is worth a total of 100 marks. Please justify your conclusions and show all your work. A non-programmable calculator of the type mentioned in the course outline is permitted; no other aids are permitted. Use the backs of the pages for rough work.

(10) 1. Find $\dfrac{dy}{dx}$ in each of the following cases. You do not have to simplify your answer.

(a) $y = x^5 - \dfrac{1}{x^{1/2}} + \tan(x^2)$

(b) $y = \dfrac{\cos(\sqrt{x})}{\sin x}$

(c) $x = \sin(xy) + y$

(d) $y = \displaystyle\int_2^x \frac{1}{t^2+1}\,dt$

(10) 2. Evaluate the following limits if they exist.

(a) $\lim_{x \to 0} \frac{x}{\sin \pi x}$

(b) $\lim_{x \to -\infty} \frac{4x^3 - 9x}{(3-x)(x^2+3)}$

(c) $\lim_{x \to 2^+} \frac{x^2 - 4}{x^2 - 4x + 4}$

(d) $\lim_{x \to \infty} x - \sqrt{x^2 + 2x}$

(10) 3. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is a maximum? (Hint: the area S of a square in terms of its perimeter p is $A = (p/4)^2 = p^2/16$; the area T of a circle in terms of its circumference q is $T = \pi(q/(2\pi))^2 = q^2/(4\pi)$.)

(10) 4. Two cars start moving from the same point. One travels south at 60 km/h and the other travels west at 25 km/h. At what rate is the distance between the cars increasing two hours later?

(12) 5. Consider the function f given below along with its first and second derivatives.

$$f(x) = -\frac{(5x+7)(x+11)}{(1-x)^2} \qquad f'(x) = -\frac{72(x+3)}{(1-x)^3} \qquad f''(x) = -\frac{144(x+5)}{(1-x)^4}$$

Fill in the blanks for parts (a) and (b). Use the space below to show your work. If more space is required use the back of the previous page and indicate that you have done so.

(a) Identify all (if any)
Intercepts ____________
Local extrema (max/min) ____________
Inflection points ____________
Asymptotes ____________

(b) Determine the intervals on which f is
Increasing ____________
Decreasing ____________
Concave up ____________
Concave down ____________

(c) Use the information in parts (a) and (b) on the previous page to sketch a graph of f.

(10) 6. Evaluate the following integrals.

(a) $\displaystyle\int \frac{(x+3)^2}{x^5}\,dx$

(b) $\displaystyle\int \frac{\sin x}{(4-\cos x)^2}\,dx$

(c) $\displaystyle\int_1^9 \sqrt{2x+7}\,dx$

(d) $\displaystyle\int_0^1 \frac{x}{\sqrt{x^2+3}}\,dx$

(10) 7. Sketch the region bounded by the curves $x+y=3$ and $y=x^2-2x-3$ and then find the area of the region.

(8) 8. Find the tangent line to the curve

$$x^3 + y^3 + (x - y)^3 = 20$$

at the point $(1, 3)$ on the curve.

(10) 9. (a) Calculate $\int_0^3 (x^2+x)\,dx$ from first principles, i.e., using the definition of a definite integral. (Note: you will receive no credit for using the Fundamental Theorem of Calculus in this problem.) You may find some of the following formulas useful:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

(b) Find a Riemann sum which estimates the integral in part (a). The sum should use $n=3$ equal intervals and should use the right-hand endpoints of the intervals for sample points.

(10) 10. (a) Show that the equation $x^5 + x + 1 = 0$ has **exactly** one root in the interval $[-2, 1]$.

(b) List the x values at which the function graphed below is not continuous.

(c) List the x values at which the function graphed below is continuous but not differentiable.