MATH 110 Review Problem Set 0.C Solutions

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1. (a) According to the formula from the lectures, the equation is

$$(x-4)^2 + (y-3)^2 = 5^2$$

(b) This question is nearly identical to the previous, except that we have a negative coordinate in the center. The equation is

$$(x+2)^2 + (y-7)^2 = 3^2$$

(c) Since the cirlce has center at the origin, its equation must be of the form

$$x^2 + y^2 = r^2$$

Since $(-5, 12)$ is a point on the circle it must satisfy the equation of the circle so we must have

$$(-5)^2 + (12)^2 = r^2 \implies 25 + 144 = r^2 \implies 169 = r^2 \implies r = 13$$

In summary, the equation of the circle is

$$x^2 + y^2 = 13^2$$

Alternatively, you could note that the radius of the circle is the distance from the center to any point on the circle. In this case the distance from the center to the point $(-5, 12)$ on the circle is

$$\sqrt{(0-(-5))^2 + (0-12)^2} = \sqrt{25+144} = \sqrt{169} = 13$$

So the radius $r = 13$.

(d) This is similar to the previous, except now the form of the equation is

$$(x-3)^2 + (y+8)^2 = r^2$$

Since the origin $(0, 0)$ is a point on the circle we must have

$$(0-3)^2 + (0+8)^2 = r^2 \implies 9 + 64 = r^2 \implies 73 = r^2$$

So we can write the equation of the circle as

$$(x-3)^2 + (y+8)^2 = 73$$

2. (a) We complete the square. We have

$$x^2 + 2x + y^2 = 0 \implies x^2 + 2x + 1 + y^2 = 1 \implies (x+1)^2 + (y-0)^2 = 1^2$$

so the equation is of the form of a circle with center $(-1, 0)$ and radius 1.

(b) Completing the square,

$$x^2 + 10x + y^2 - 6y + 8 = 0 \implies x^2 + 10x + 25 + y^2 - 6y + 9 + 8 = 25 + 9$$

Simplifying,

$$(x+5)^2 + (y-3)^2 = 25 + 9 - 8 = 26$$

so the equation is of the form of a circle with center $(-5, 3)$ and radius $r = \sqrt{26}$.

(c) Completing the square,

$$x^2 + y^2 - y + \left(-\frac{1}{2}\right)^2 = 6 + \left(-\frac{1}{2}\right)^2 \implies (x-0)^2 + \left(y - \frac{1}{2}\right)^2 = 6 + \frac{1}{4} = \frac{25}{4}$$

so the equation is of the form of a circle with center $(0, 1/2)$ and radius $5/2$.

(d) It's probably best to divide through by 3 to get the problem into a more familiar form. We have

$$x^2 + \frac{1}{3}x + y^2 - \frac{2}{3}y = \frac{5}{3} \implies x^2 + \frac{1}{3}x + \left(\frac{1}{6}\right)^2 + y^2 - \frac{2}{3}y + \left(-\frac{1}{3}\right)^2 = \frac{5}{3} + \frac{1}{36} + \frac{1}{9}$$

Simplifying,

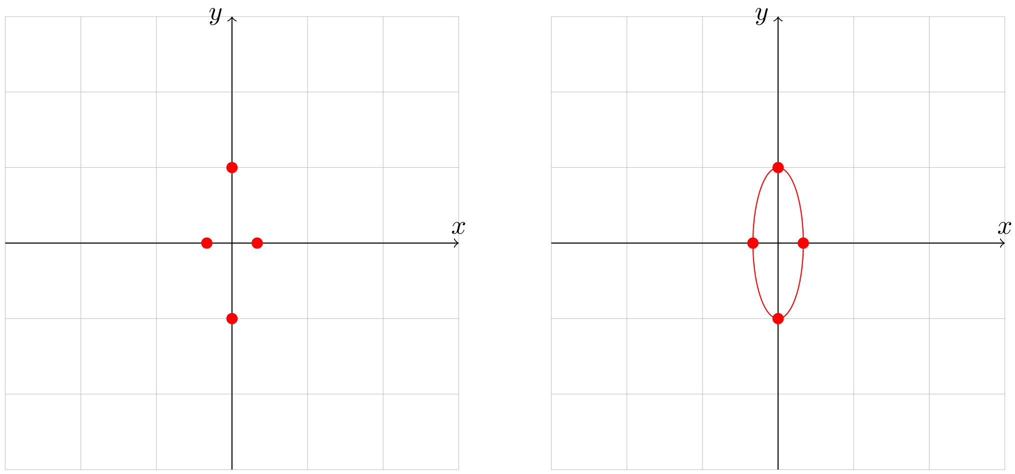
$$\left(x + \frac{1}{6}\right)^2 + \left(y - \frac{1}{3}\right)^2 = \frac{65}{36}$$

which is the equation of a circle with center $(-1/6, 1/3)$ and radius $\sqrt{65}/6$.

3. (a) In standard form, the equation is

$$\frac{x^2}{(1/3)^2} + \frac{y^2}{1^2} = 1$$

which is an ellipse in standard position with x-intercepts $(-1/3, 0)$ and $(1/3, 0)$, and y-intercepts $(0, -1)$ and $(0, 1)$. See Figure 1



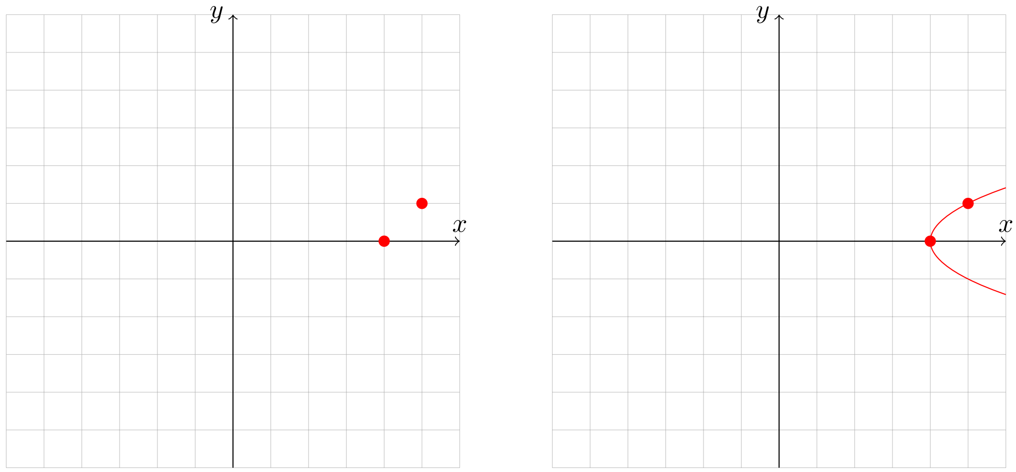
(a) intercepts (b) completed

Figure 1: Graphing $9x^2 + y^2 = 1$

(b) Rearranging,

$$x = y^2 + 4 \implies x - 4 = y^2$$

which is the equation of a parabola with vertex $(4, 0)$ opening to the right. Picking $y = 1$ gives $(5, 1)$ as another point on the parabola. See Figure 2.



(a) vertex, point (b) completed

Figure 2: Graphing $x = y^2 + 4$

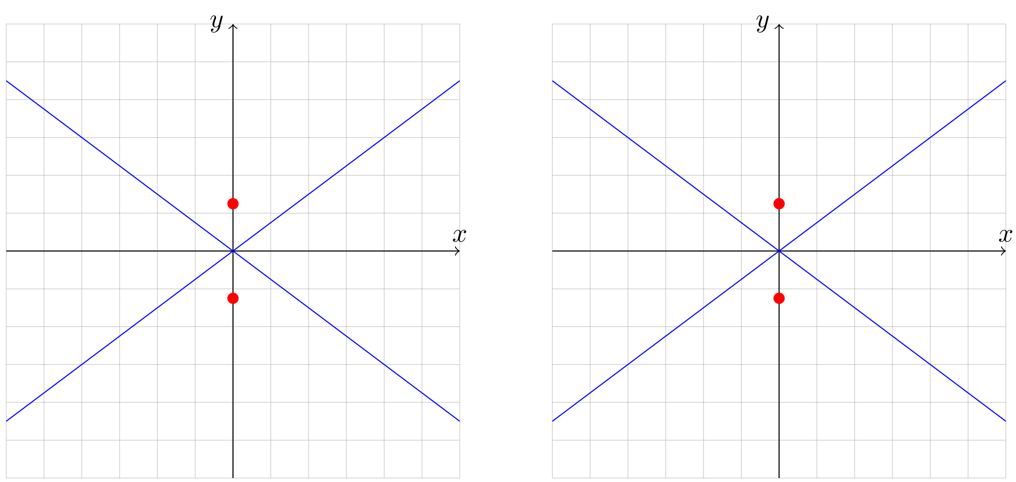
(c) Dividing through by 25,

$$-\frac{9x^2}{25} + \frac{16y^2}{25} = 1 \implies -\frac{x^2}{(5/3)^2} + \frac{y^2}{(5/4)^2} = 1$$

This is the equation of a hyperbola opening upward and downward, with y-intercepts $(-5/4, 0)$ and $(5/4, 0)$ and asymptotes

$$\frac{x}{5/3} + \frac{y}{5/4} = 0, \qquad -\frac{x}{5/3} + \frac{y}{5/4} = 0$$

See Figure 3.



(a) vertices, asymptotes (b) completed

Figure 3: Graphing $16y^2 - 9x^2 = 25$

(d) Completing the square,

$$-x^2 + y^2 + 4y + 4 = 14 \implies -x^2 + (y+2)^2 = 14 \implies -\frac{x^2}{(\sqrt{14})^2} + \frac{(y+2)^2}{(\sqrt{14})^2} = 1$$

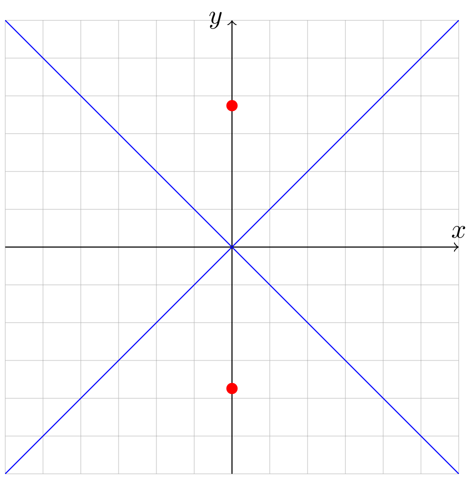
We begin by graphing the hyperbola

$$-\frac{x^2}{(\sqrt{14})^2} + \frac{y^2}{(\sqrt{14})^2} = 1$$

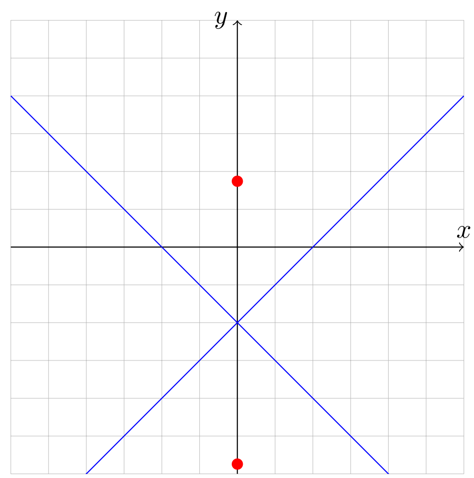
which opens upward and downward, has y-intercepts $(0, -\sqrt{14})$ and $(0, \sqrt{14})$, and has asymptotes

$$\frac{x}{\sqrt{14}} + \frac{y}{\sqrt{14}} = 0, \qquad -\frac{x}{\sqrt{14}} + \frac{y}{\sqrt{14}} = 0$$

in other words $x + y = 0$ and $-x + y = 0$. See Figure 4(a). Finally, we shift the diagram so that the center moves from $(0,0)$ to $(0,-2)$ to get the answer to the question. See Figure 4(b).



(a) Graph of $-x^2 + y^2 = 14$



(b) completed

Figure 4: Graphing $y^2 - x^2 + 4y = 10$

4. (a) A quick sketch (with a large-enough scale on the y-axis) shows that there seem to be two intersection points. See Figure 5(a). To find the coordinates of the intersection points, solve

$$-x^2 = y = 5x \implies -x^2 = 5x \implies x^2 + 5x = 0 \implies (x+5)x = 0$$

with roots $x = -5$, $x = 0$ and corresponding y-values $y = 5(-5) = -25$ and $y = 5(0) = 0$. So the intersection points are $(-5, -25)$ and $(0, 0)$. See Figure 5(b).

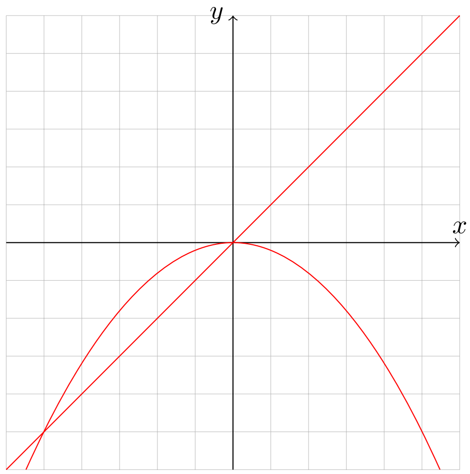
(b) A quick sketch of the curves again indicates that there should be two intersection points. See Figure 6(a). To find the intersection points, solve

$$x^2 - 1 = y = 2x + 2 \implies x^2 - 1 = 2x + 2 \implies x^2 - 2x - 3 = 0 \implies (x+1)(x-3) = 0$$

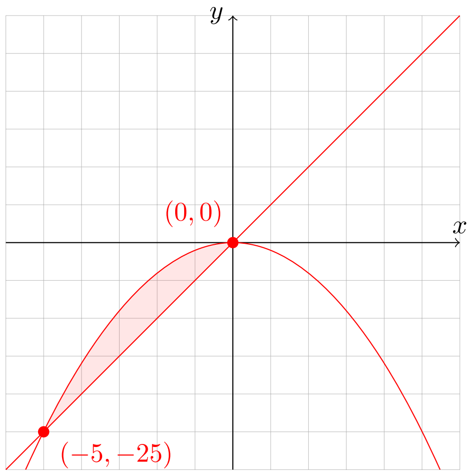
so the intersection points are $(-1, 0)$ and $(3, 8)$. See Figure 6(b).

5. (a) Completing the square,

$$x^2 + 4(y^2 - 8y + 16 - 16) = 16 \implies x^2 + 4(y-4)^2 = 80$$

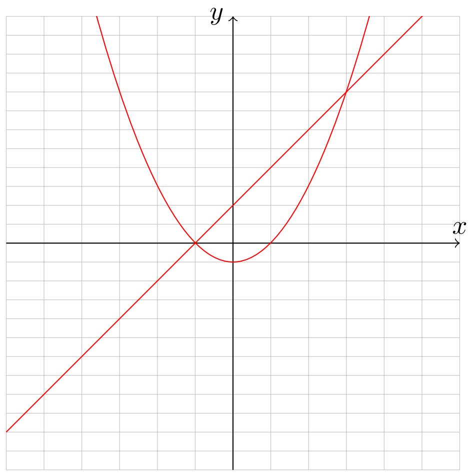


(a) Sketch of curves

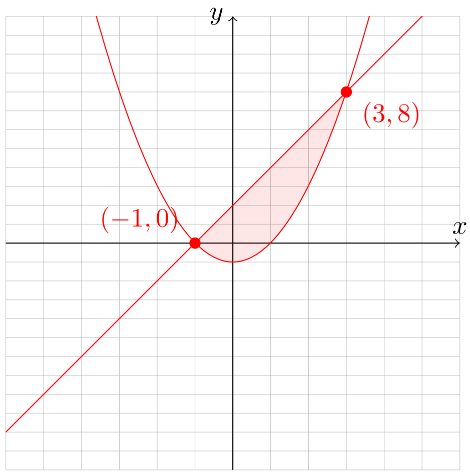


(b) Intersection points and region

Figure 5: Graphing region between $y = -x^2$ and $y = 5x$



(a) Sketch of curves



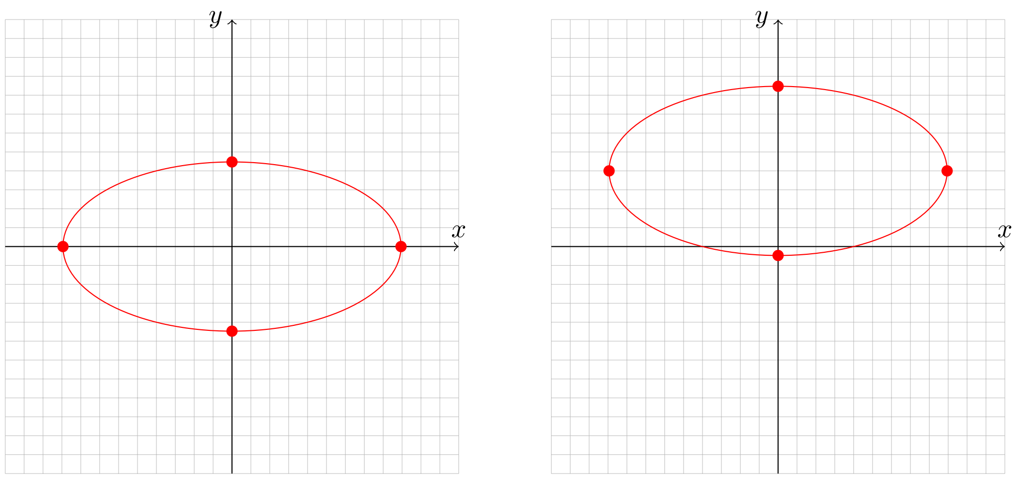
(b) Intersection points and region

Figure 6: Graphing the region between $y = x^2 - 1$ and $y = 2x + 2$

Putting the equation in standard form,

$$\frac{x^2}{(\sqrt{80})^2} + \frac{(y-4)^2}{(\sqrt{20})} = 1$$

The figure is an ellipse. We first graph $x^2/(\sqrt{80})^2 + y^2/(\sqrt{20})^2 = 1$ by locating the vertices, namely $(-\sqrt{80}, 0)$, $(\sqrt{80}, 0)$, $(0, -\sqrt{20})$, $(0, \sqrt{20})$ and joining them by appropriate arcs. See Figure 7(a). Then we shift the resulting figure so the center moves from $(0, 0)$ to $(0, 4)$. See Figure 7(b).



(a) Graph of $x^2/80 + y^2/20 = 1$ (b) Center shifted to $(0, 4)$

Figure 7: Graphing $x^2 + 4y^2 - 32y = 16$

(b) Completing the square,

$$9\left(x^2 - \frac{2}{3}x + \frac{1}{9} - \frac{1}{9}\right) - 4y^2 = -5 \implies 9\left(x - \frac{1}{3}\right) - 4y^2 = -4$$

Putting the equation into standard form,

$$-\frac{(x-1/3)^2}{(2/3)^2} + \frac{y^2}{1^2} = 1$$

We first graph $-x^2/(2/3)^2 + y^2/1^2 = 1$. It is a hyperbola opening upwards and downwards, with y-intercepts $(0, -1)$ and $(0, 1)$ and asymptotes

$$\frac{x}{2/3} + \frac{y}{1} = 0, \qquad -\frac{x}{2/3} + \frac{y}{1} = 0$$

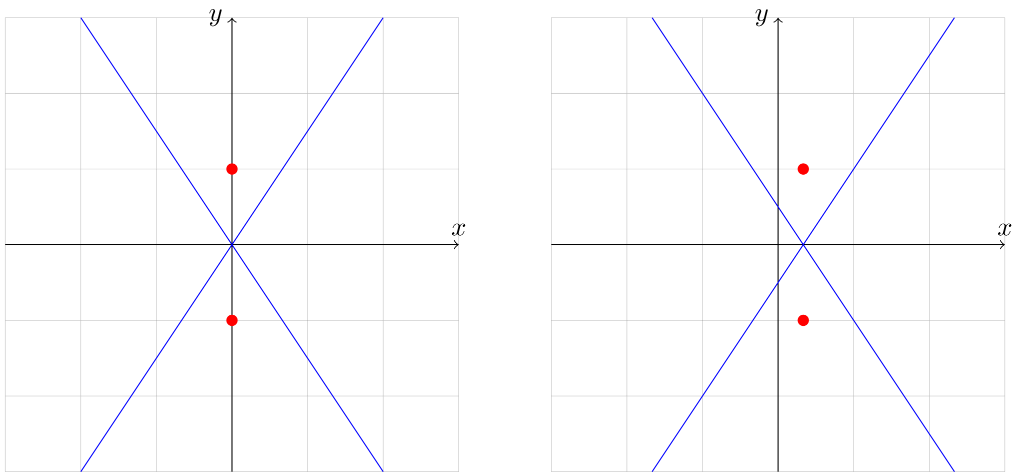
See Figure 8(a). Then we shift the graph so that the center moves from $(0, 0)$ to $(1/3, 0)$. See Figure 8(b).

6. A parabola in standard position which opens downward has equation

$$y = -ax^2$$

Shifting the vertex from $(0, 0)$ to $(1, 5)$, the equation becomes

$$y - 5 = -a(x-1)^2$$



(a) Graph of $-x^2/(2/3)^2 + y^2/1^2 = 1$

(b) Center shifted to $(1/3, 0)$

Figure 8: Graphing $9x^2 - 4y^2 - 6x + 5 = 0$

Since the parabola passes through $(-1, -4)$, the values $(x, y) = (-1, -4)$ must satisfy the equation so we have

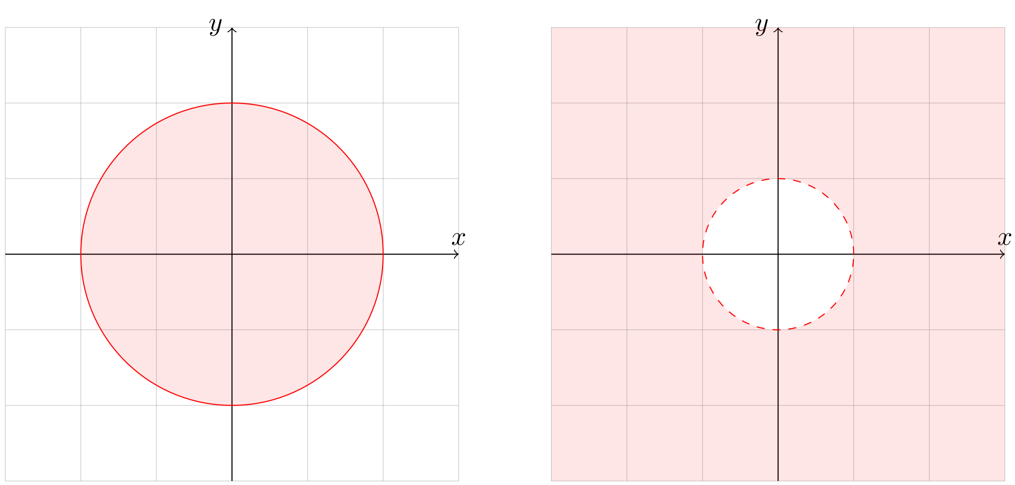
$$-4 - 5 = -a(-1-1)^2 \implies -9 = -a(2)^2 \implies a = \frac{9}{4}$$

So the parabola has equation

$$y - 5 = -\frac{9}{4}(x-1)^2$$

You might want to check the correctness of that equation by graphing it.

7. (a) The solution set is the set of all points the distance from the origin of which is less than $\sqrt{4}$, in other words the inside of a circle of center $(0, 0)$ and radius 2. See Figure 9(a).



(a) Graph of $x^2 + y^2 \leq 4$

(b) Graph of $x^2 + y^2 > 1$

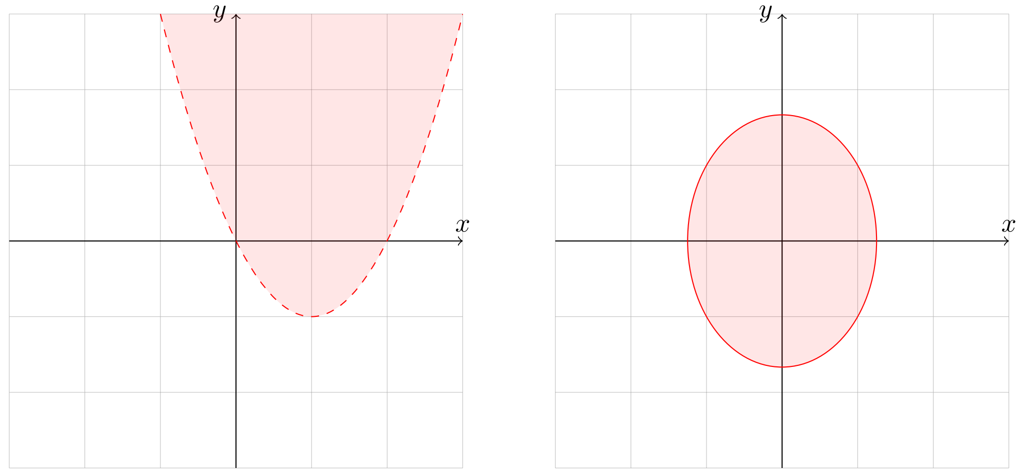
Figure 9: Graphing some inequalities

(b) This question is similar to the previous, however the set of points is the set that is not inside or on the unit circle. See Figure 9(b).

(c) Completing the square we have

$$y + 1 > (x - 1)^2$$

The region is the set of all points above a parabola with vertex $(1, -1)$ opening upwards.



(a) Graph of $y > x^2 - 2x$ (b) Graph of $16x^2 + 9y^2 \leq 25$

Figure 10: Graphing some other inequalities

(d) In standard form, the equation is

$$\frac{x^2}{(5/4)^2} + \frac{y^2}{(5/3)^2} \leq 1$$

so the boundary of the region is an ellipse in standard position with vertices $(-5/4, 0)$, $(5/4, 0)$, $(0, -5/3)$, $(0, 5/3)$. Since the ellipse is a flattened circle, and the ≤ 1 condition means everything inside and on the circle in the circle case, we graph all points inside and on the ellipse.

8. Completing the square,

$$x^2 + ax + \frac{a^2}{4} - \frac{a^2}{4} + 4\left(y^2 + \frac{b}{4}y + \frac{b^2}{64} - \frac{b^2}{64}\right) = -c$$

Simplifying,

$$\left(x + \frac{a}{2}\right)^2 + 4\left(y + \frac{b}{8}\right)^2 = \frac{a^2}{4} - \frac{b^2}{16} - c$$

Since the LHS is a sum of squares, it must always be ≥ 0. So for the equation to represent an ellipse, we must have the RHS ≥ 0 (what happens if the RHS $= 0$?). So the condition for the equation to represent an ellipse is

$$\frac{a^2}{4} - \frac{b^2}{16} - c > 0$$

When that condition is satisfied, the center of the ellipse is

$$\left(-\frac{a}{2}, -\frac{b}{8}\right)$$