

# MATH 110 Problem Set 2.7

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The following problems based on Section 2.7 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

- (Based on 2.7.8–9) If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after  $t$  seconds is  $s = 80t - 16t^2$ .
  - What is the maximum height reached by the ball?
  - What is the velocity of the ball when it is 96 ft above the ground on the way up? On the way down?
- (Based on 2.7.17) The mass of the part of a metal rod that lies between its left end and a point  $x$  meters to the right is  $3x^2$  kg. Find the linear density when  $x$  is (a) 1 m, (b) 2 m, and (c) 3 m.
- (Based on 2.7.29–30) The cost function for production of a commodity is  $C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3$ .
  - Find and interpret  $C'(100)$
  - Compare  $C'(100)$  with the exact marginal cost at production level 100, i.e., the cost  $C(101) - C(100)$  of producing the 101st item after 100 items have been produced.
- (Based on 2.7.24) Consider the chemical reaction  $A + B \rightarrow C$  in which one molecule of reactant A combines with one molecule of reactant B to form one molecule of product C. If the initial concentrations of A and B are the same, say  $a$  moles/L, it turns out that the concentration of C as a function of time is given explicitly by the formula  $[C] = a^2kt/(akt + 1)$  where  $k$  is a constant depending on the type of reaction.
  - Find the rate of reaction at time  $t$ .
  - Show that if  $x(t) = [C]$  then  $dx/dt = k(a - x)^2$ .
- (Based on 2.7.36) In a fish farm, a population of fish is introduced into a pond and harvested regularly. A model for the rate of change of the fish population is given by the equation

$$\frac{dP}{dt} = r_0 \left( 1 - \frac{P(t)}{P_c} \right) P(t) - \beta P(t)$$

where  $r_0$  is the birth rate of the fish,  $P_c$  is the maximum population that the pond can sustain (called the *carrying capacity*), and  $\beta$  is the percentage of the population that is harvested.

- What value of  $dP/dt$  corresponds to a stable population?
- If the pond can sustain 10,000 fish, the birth rate is 5%, and the harvesting rate is 4%, find the stable population level.
- What happens if the harvesting rate  $\beta$  is raised to 5%?

You may find the following additional exercises from Section 2.7 helpful.

- 2.7 C-level: 1–4, 6–10, 17–18, 22, 27–30;  
B-level: 11–16, 19–21, 23–24, 25–26;  
A-level: 31–36