# MATH 110 Problem Set 2.5 Solutions

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1. (a) We can write $g(x) = u^7$ where $u = 5x + x^3$. Differentiating, we have

$$\frac{dg}{du} = 7u^6$$
$$\frac{du}{dx} = 5 + 3x^2$$

so by the chain rule

$$\frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} = 7u^6(5 + 3x^2) = 7(5x + x^3)^6(5 + 3x^2)$$

(b) By the product rule,

$$H'(t) = \left(\frac{d}{dt}t^2\right)\cos kt + t^2\left(\frac{d}{dt}\cos kt\right) = 2t\cos kt + t^2\frac{d}{dt}\cos kt$$

Now we apply the chain rule to $\cos kt$. Let $u = kt$; then

$$\frac{d}{dt}\cos kt = \frac{d}{du}\cos u \cdot \frac{d}{dt}(kt) = -\sin u \cdot k = -k\sin kt$$

Putting it all together,

$$H'(t) = 2t\cos kt - kt^2\sin kt$$

(c) By the product rule,

$$\frac{d}{dz}(1+2z)^{10}(3-2z)^8 = \left(\frac{d}{dz}(1+2z)^{10}\right)(3-2z)^8 + (1+2z)^{10}\left(\frac{d}{dz}(3-2z)^8\right)$$

By the chain rule applied twice,

$$\begin{aligned}\frac{d}{dz}(1+2z)^{10}(3-2z)^8 &= 10(1+2z)^9\left(\frac{d}{dz}(1+2z)\right)(3-2z)^8 + (1+2z)^{10}\cdot 8(3-2z)^7\left(\frac{d}{dz}(3-2z)\right) \\ &= 10(1+2z)^9(2)(3-2z)^8 + (1+2z)^{10}(8)(3-2z)^7(-2)\end{aligned}$$

Further simplification is possible but is not necessary at this time.

(d) First we write $W(t)$ in terms of powers:

$$W(t) = \left(\frac{t}{t^2+4}\right)^{1/2}$$

By the chain rule,

$$W'(t) = \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-1/2} \cdot \frac{d}{dt}\frac{t}{t^2+4}$$

By the quotient rule,

$$W'(t) = \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-1/2} \cdot \frac{\left(\frac{d}{dt}t\right)(t^2+4) - t\left(\frac{d}{dt}(t^2+4)\right)}{(t^2+4)^2} = \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-1/2} \cdot \frac{(t^2+4) - t(2t)}{(t^2+4)^2}$$

2. By the product rule

$$y'(x) = \left(\frac{d}{dx}x\right)\sin(3x-\pi) + x\left(\frac{d}{dx}\sin(3x-\pi)\right) = \sin(3x-\pi) + x\left(\frac{d}{dx}\sin(3x-\pi)\right)$$

By the chain rule

$$\frac{d}{dx}\sin(3x-\pi) = \sin'(3x-\pi)\cdot\frac{d}{dx}(3x-\pi) = \cos(3x-\pi)\cdot(3) = 3\cos(3x-\pi)$$

Putting it all together,

$$y'(x) = \sin(3x-\pi) + 3x\cos(3x-\pi)$$

Evaluating at $x = \pi/2$,

$$y'(\pi/2) = \sin\left(\frac{3\pi}{2} - \pi\right) + \frac{3\pi}{2}\cos\left(\frac{3\pi}{2} - \pi\right) = \sin(\pi/2) + \frac{3\pi}{2}\cos(\pi/2) = 1 + \frac{3\pi}{2}(0) = 1$$

which is the slope of the tangent line. The slope of the normal line is the negative reciprocal $-1/1 = -1$. The equation of the tangent line is

$$y - \frac{\pi}{2} = 1\left(x - \frac{\pi}{2}\right) \implies y = x$$

and the equation of the normal line is

$$y - \frac{\pi}{2} = -1\left(x - \frac{\pi}{2}\right) \implies y = -x + \pi$$

3. The first derivative is found by applying the product rule then the chain rule:

$$y' = \left(\frac{d}{dx}x^2\right)\cos 3x + x^2\left(\frac{d}{dx}\cos 3x\right) = 2x\cos 3x + x^2(-\sin 3x)\frac{d}{dx}(3x) = 2x\cos 3x - 3x^2\sin 3x$$

The second derivative is found by taking the derivative of each term of the first derivative:

$$y'' = 2\cos 3x + 2x\cdot(-3\sin 3x) - 6x\sin 3x - 3x^2\cdot 3\cos 3x = (2-9x^2)\cos 3x - 12x\sin 3x$$

4. The derivative is

$$y' = 2\cos x + 2\sin x\cos x$$

To find horizontal tangents we must solve $y' = 0$, which we do by factoring:

$$\begin{aligned} 2\cos x + 2\sin x\cos x &= 0 \\ 2\cos x(1+\sin x) &= 0 \end{aligned}$$

The equation is true if and only if $\cos x = 0$ or $1 + \sin x = 0$, i.e., if $x = \pi/2 + k\pi$, $k = 0, \pm 1, \pm 2, \ldots$ or $x = 3\pi/2 + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$.

5. By the chain rule, we have

$$F'(x) = f'(g(x))\cdot g'(x)$$

Evaluating the derivative at $x = 2$ and substituting the values we know,

$$F'(2) = f'(g(2))\cdot g'(2) = f'(-1)\cdot 5 = 4\cdot 5 = 20$$

6. By the product rule we have

$$\frac{d}{dx}f(x) = \frac{d}{dx}(xg(x^2)) = \left(\frac{d}{dx}x\right)g(x^2) + x\left(\frac{d}{dx}g(x^2)\right) = g(x^2) + x\left(\frac{d}{dx}g(x^2)\right)$$

By the chain rule we have

$$\frac{d}{dx}f(x) = g(x^2) + xg'(x^2)\cdot 2x = g(x^2) + 2x^2g'(x^2)$$

Taking the second derivative,

$$\begin{aligned}\frac{d^2}{dx^2}f(x) &= \frac{d}{dx}\left(g(x^2) + 2x^2g'(x^2)\right)\\ &= \frac{d}{dx}g(x^2) + \left(\frac{d}{dx}2x^2\right)g'(x^2) + 2x^2\left(\frac{d}{dx}g'(x^2)\right)\\ &= 2xg'(x^2) + 4xg'(x^2) + 2x^2g''(x^2)(2x) = 6xg'(x^2) + 4x^3g''(x^2)\end{aligned}$$

7. Differentiating by the product and chain rules,

$$y' = 2A\cos 2t - 2B\sin 2t$$

Differentiating again,

$$y'' = -4A\cos 2t - 4B\sin 2t$$

So we have

$$\begin{aligned}y'' + 4y &= -4A\cos 2t - 4B\sin 2t + 4(A\sin 2t + B\cos 2t)\\ &= -4A\cos 2t - 4B\sin 2t + 4A\sin 2t + 4B\cos 2t\\ &= 0\end{aligned}$$

8. We take a few derivatives and look for a pattern. By the chain rule we have

$$\begin{aligned}y &= 2\cos 2x\\ y' &= -2\sin 2x\\ y'' &= -2^2\cos 2x\\ y^{(3)} &= 2^3\sin 2x\\ y^{(4)} &= 2^4\cos 2x\end{aligned}$$

Note that the coefficient in $y^{(n)}$ is $2^n$. Also, note that the functions are cos, $-$sin, $-$cos, sin, repeating in a cycle of 4. So the 23rd derivative should have the same function as the 19th, 15th, 11th, 7th, and 3rd derivatives. Therefore we should have

$$y^{(23)} = 2^{23}\sin 2x$$

9. We have

$$L'(t) = 4.25\cos\left[\frac{2\pi}{365}(t-80)\right]\cdot\frac{d}{dt}\frac{2\pi}{365}(t-80) = \frac{8.5\pi}{365}\cos\left[\frac{2\pi}{365}(t-80)\right]$$

Now we need to figure out the $t$ values for the various dates given. On April 1, the number of days already past (in a non-leap year) is $31+28+31=90$, so April 1 is $t=91$. On June 21, we have

$t = 31 + 28 + 31 + 30 + 31 + 21 = 172$. On December 1, we have $t = 365 - 31 + 1 = 335$. So on April 1 we have

$$L'(91) = \frac{8.5\pi}{365} \cos\left[\frac{2\pi}{365}(91 - 80)\right] = 0.072$$

i.e., the number of hours of daylight is increasing by 0.072 hours per day; on June 21 we have

$$L'(172) = \frac{8.5\pi}{365} \cos\left[\frac{2\pi}{365}(172 - 80)\right] = -0.001$$

i.e., the length of the day is changing hardly at all, and on December 1 we have

$$L'(335) = \frac{8.5\pi}{365} \cos\left[\frac{2\pi}{365}(335 - 80)\right] = -0.023$$

i.e., the amount of daylight is decreasing by about 0.023 hours per day. When do you think the amount of daylight is changing the fastest? When is it hardly changing at all?

10. (a) We have a semicircle is $\pi$ radians or 180 degrees, so if an angle measures $\theta$ radians it must measure

$$t = \frac{180}{\pi}\theta$$

degrees. (Try a few values for $\theta$ to convince yourself that that is correct.)

(b) It follows from the above that

$$\cos^\circ t = \cos\theta = \cos\frac{\pi}{180}t$$

(c) Taking a derivative of the above by the chain rule we have

$$\frac{d}{dt}\cos^\circ t = \frac{d}{dt}\cos\frac{\pi}{180}t = -\frac{\pi}{180}\sin\frac{\pi}{180}t = -\frac{\pi}{180}\sin^\circ t$$

Note the extra factor of $\pi/180$ in comparison with the derivative of the trig functions in radians. Higher derivatives would include higher powers of that factor, which would be somewhat inconvenient.