MATH 110-003 200730 Quiz 7 Solutions

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1. **Method 1:** Finding antiderivatives, $f'(x) = C + 8x - 6x^2 + 15x^4$ and $f(x) = D + Cx + 4x^2 - 2x^3 + 3x^5$. (Check by differentiating!) Then we have $f'(0) = C$ on the one hand, and $f'(0) = 5$ by the given data, so $C = 5$. Similarly, we have $f(0) = D$ on the one hand, and $f(0) = -1$ by the given data, so $f(x) = -1 + 5x + 4x^2 - 2x^3 + 3x^5$.

   Method 2: In more complicated problems, it is probably best to try to figure out as much as possible about the constants after each antidifferentiation. In this case, it doesn't matter, and in some cases it doesn't help, but let's try it this way just for fun. We have $f'(x) = C + 8x - 6x^2 + 15x^4$ as above, but *now* we use the information $f'(0) = 5$ to obtain $f'(x) = 5 + 8x - 6x^2 + 15x^4$. Finding the antiderivative of that, $f(x) = D + 5x + 4x^2 - 2x^3 + 3x^5$. Now we use $f(0) = -1$ to obtain $D = -1$, so the final answer is $f(x) = -1 + 5x + 4x^2 - 2x^3 + 3x^5$.

2. (a) (8 marks) Let $f(x) = \dfrac{1}{x} - 9$. Then $f'(x) = -\dfrac{1}{x^2}$. The formula for Newton's method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\dfrac{1}{x_n} - 9}{-\dfrac{1}{x_n^2}} = x_n + \frac{\dfrac{x_n^2}{x_n} - 9x_n^2}{\dfrac{x_n^2}{x_n^2}}$$
$$= x_n + x_n - 9x_n^2 = 2x_n - 9x_n^2.$$

   (b) (2 marks) With $x_1 = 0.1$ we have

$$x_2 = 2(0.1) - 9(0.1)^2 = 0.2 - 0.09 = 0.11.$$

   (c) (1 mark) The number of digits of accuracy doubles, roughly, with each iteration of Newton's method, so a reasonable guess is that x_n is 2^{n-1} 1s after the decimal point. A quick check of the first few iterations should reinforce that guess. (It is possible to prove that that guess is correct using the expression

$$x_n = \frac{10^{2^{n-1}} - 1}{9 \cdot 10^{2^{n-1}}}$$

   and mathematical induction, but that is not necessary to get the bonus mark.)