Some Important Formulas for Midterm Test 1

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1.5

- **The definition of a limit.** We write
$$\lim_{x\to a} f(x) = L$$
and say
the limit as x approaches a of $f(x)$ is L
if we can make the values of $f(x)$ as close to L as we like by taking x to be sufficiently close to (but not equal to) a.
- **The definition of a one-sided limit.** We write
$$\lim_{x\to a^-} f(x) = L$$
and say
the left-hand limit as x approaches a of $f(x)$ is L
if we can make the values of $f(x)$ as close to L as we like by taking x to be sufficiently close to (but less than) a.
There is a similar definition for the right-hand limit $\lim_{x\to a^+} f(x) = L$. (What is the definition?)
- **The connection between one-sided and two-sided limits.** We have
$$\lim_{x\to a} f(x) = L$$
if and only if
$$\lim_{x\to a^-} f(x) = L \text{ and } \lim_{x\to a^+} f(x) = L.$$
- **The definition of an infinite limit.** Let f be a function defined on both sides of a, except possibly at a itself. Then we write
$$\lim_{x\to a} f(x) = \infty$$
and say
the limit as x approaches a of $f(x)$ is infinity
or
$f(x)$ increases without bound as x approaches a
if $f(x)$ can be made as large as we like by taking x sufficiently close to **(but not equal to)** a.
There are similar definitions for $\lim_{x\to a} f(x) = -\infty$ and the corresponding one-sided limits.

- **The definition of a vertical asymptote.** The line $x = a$ is called a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \to a} f(x) = \infty \quad \lim_{x \to a^-} f(x) = \infty \quad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \quad \lim_{x \to a^-} f(x) = -\infty \quad \lim_{x \to a^+} f(x) = -\infty$$

In other words, if any of the limits as x approaches a from the left or the right is $\pm\infty$, we say that $y = f(x)$ has a vertical asymptote.

1.6 – **The limit laws.** Suppose $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then

1. $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a}[c \cdot f(x)] = c \cdot \lim_{x \to a} f(x)$ where c is a constant
4. $\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
5. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ **under the condition that** $\lim_{x \to a} g(x) \neq 0$.

If the condition in the last rule above is violated, we can't conclude anything about the limit without further work.

- **Bottom-level limit laws.** We use the above limit laws in conjunction with these basic limit laws to build up rules for limits of complex functions.

1. $\lim_{x \to a} x = a$
2. $\lim_{x \to a} c = c$

- **Limits of root functions.**
  * $\lim_{x \to a}[f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer.
  * $\lim_{x \to a} x^n = a^n$.
  * $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ under the condition that $a > 0$
  * $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ under the condition that $\lim_{x \to a} f(x) > 0$

When n is odd we can remove the condition $a > 0$.

- Limits of common functions.

1. If f is a polynomial, then $\lim_{x \to a} f(x) = f(a)$
2. If $f = g/h$ is a rational function (i.e., g and h are polynomials), **and** $h(a) \neq 0$, then $\lim_{x \to a} f(x) = \frac{g(a)}{h(a)}$.

- **Single discrepancy theorem.** If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$, provided the limits exist.
- **One-sided limits.** $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$.
- **Limits preserve inequalities**: If $f(x) \leq g(x)$ for all $x \neq a$, $\lim_{x \to a} f(x) = L$, and $\lim_{x \to a} g(x) = M$, then $L \leq M$.
- **Squeeze theorem.** If $f(x) \leq g(x) \leq h(x)$ for all $x \neq a$, and $\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x)$, then $\lim_{x \to a} g(x) = L$.

1.8 – **Definition of continuity.** We say a function f is **continuous at** a if the following statement is true:

$$\lim_{x \to a} f(x) = f(a)$$

- **Meaning of continuity.** The definition really says three things:
  1. $\lim_{x\to a} f(x)$ exists.
  2. $f(a)$ exists.
  3. The above two values are equal.
- **Types of discontinuities.**
  * **Removable discontinuities:** the limit $\lim\limits_{x\to a}$ exists but is not equal to $f(a)$.
  * **Jump discontinuities**: the one-sided limits exist but are not equal.
  * **Infinite discontinuities:** either or both of the one-sided limits are infinite.
  * All other cases: where either of the one-sided limits doesn't exist (i.e., the function oscillates rapidly near a).
- **Definition of continuity from the right.** We say a function f is **continuous from the right at** a if the following statement is true:
$$\lim_{x\to a^+} f(x) = f(a)$$
- **Definition of continuity from the left.** We say a function f is **continuous from the left at** a if the following statement is true:
$$\lim_{x\to a^-} f(x) = f(a)$$
- **Definition of continuity on an interval.** A function is called **continuous on an interval** if it is continuous at each point a in the interval.
- **Continuity on intervals with endpoints.** If the interval has an endpoint or endpoints, we understand *continuous* at the endpoint to mean *continuous from the right* or *continuous from the left* as appropriate.
- **Algebraic combinations of continuous functions.** The limit laws for combining functions give rules for combining continuous functions. Suppose f and g are continuous at a.
  1. The sum $f+g$ is continuous at a.
  2. The difference $f-g$ is continuous at a.
  3. The constant multiple cf is continuous at a.
  4. The product fg is continuous at a.
  5. The quotient f/g is continuous at a **under the condition that** $g(a) \neq 0$**.**

  In summary, any algebraic combination of continuous functions is continuous, provided we don't divide by zero.
- **Continuity of common functions.** Polynomials, rational functions, root functions, and trig functions are continuous on their domains.
- **Composition of functions.** The function $f\circ g$ defined by $(f\circ g)(x) = f(g(x))$ is called the composition of f and g. Its domain is a subset of the domain of g.
- **Limits of compositions.** If f is continuous at b and $\lim\limits_{x\to a} g(x) = b$, then
$$\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$$
- **Continuity at a point** a **of compositions.** If g is continuous at a and if f is continuous at $g(a)$, then the function $(f\circ g)(x) = f(g(x))$ is continuous at a.
- **Continuity on a domain of compositions.** If g and f are continuous on their domains, then the composition $(f\circ g)(x) = f(g(x))$ is continuous on its domain.
- **The intermediate value theorem.** Suppose that f is continuous on the closed interval $[a,b]$ and let N be any number satisfying $f(a) \leq N \leq f(b)$. Then there exists a number c in the interval (a,b) such that $f(c) = N$.