MATH 110 Review 0.B

Review of Coordinate Geometry and Lines

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Tuesday, January 6, 2026

Department of Indigenous Knowledge and Science
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Contents

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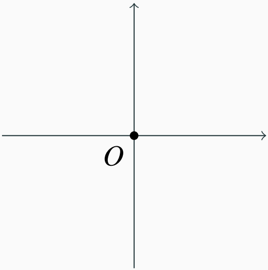

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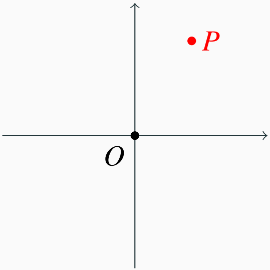

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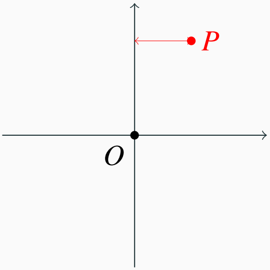

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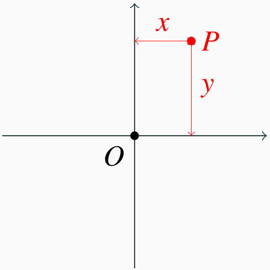

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- Given a point on the plane ...
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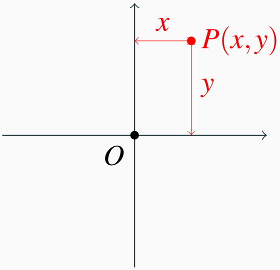
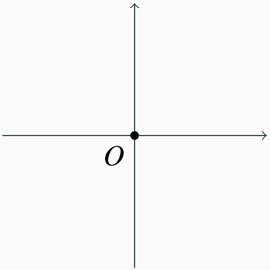
- and to the horizontal axis ...
- which become the (x, y) coordinates of the point.



Regions in the Cartesian Plane

- Each of the following four regions is called a *half plane*:



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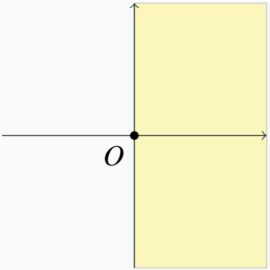

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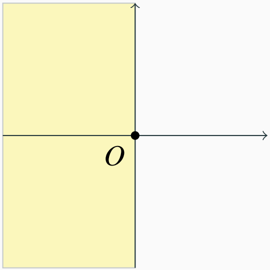

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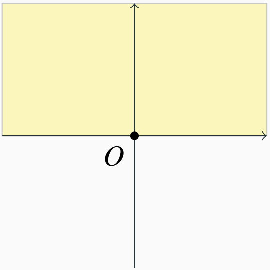

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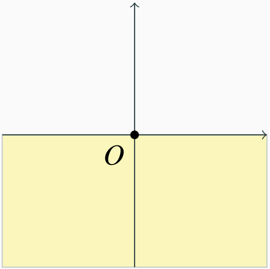

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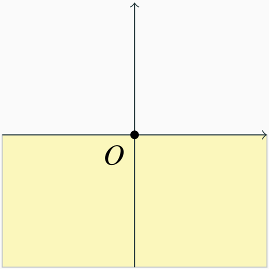

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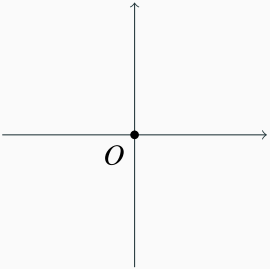

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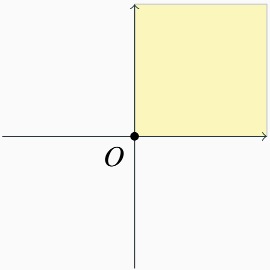

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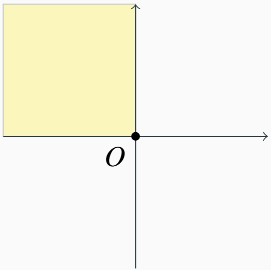

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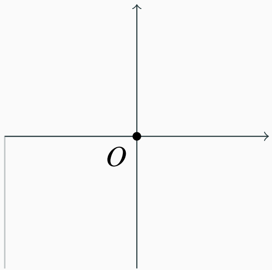

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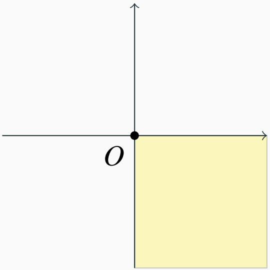

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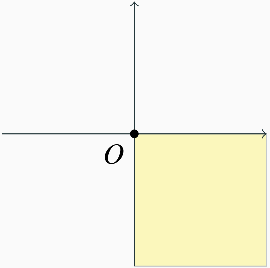

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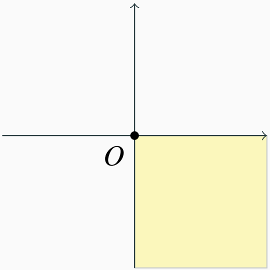
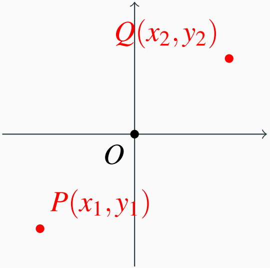
- Each of the following four "quarter planes" is called a *quadrant*: quadrant IV: $x > 0, y < 0$
- Note that each quadrant has a name (I, II, III, IV) and a description in terms of inequalities.
- Other regions in the plane can be described by more complicated inequalities.



Distance in the Cartesian Plane

- Consider two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the Cartesian plane.



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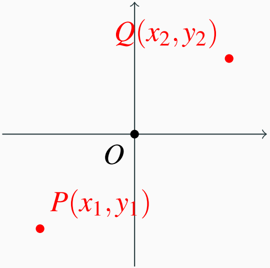

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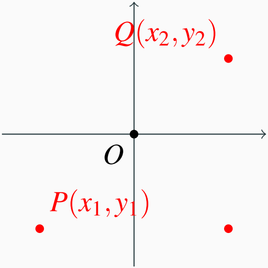

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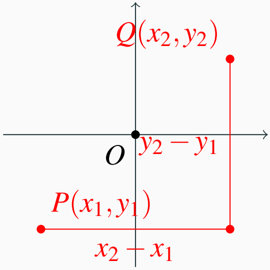

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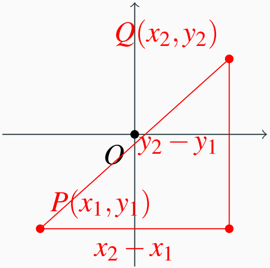
- We then use the Pythagorean Theorem to find the distance between P and Q.



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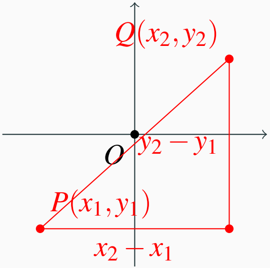
- The distance is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



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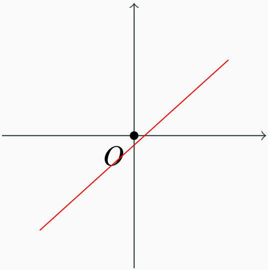

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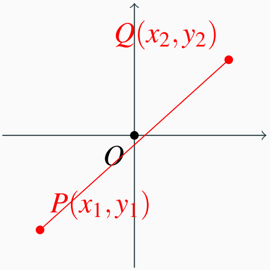

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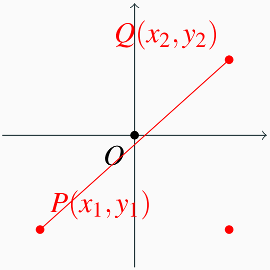

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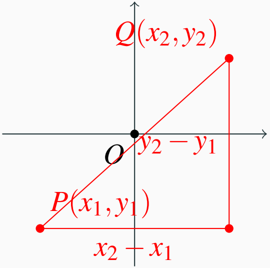

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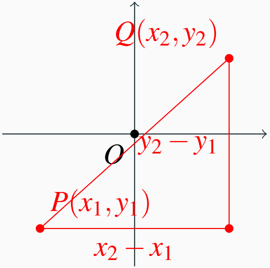
- This time, we calculate their ratio: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



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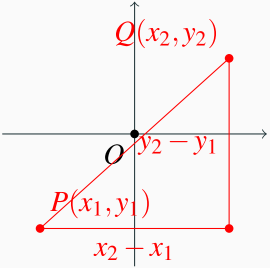

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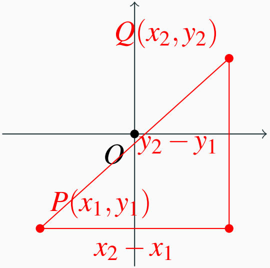
- We call m the *slope* of the line.



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- That is the *point-slope* form of the equation of a line.

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- We call that the *slope-intercept* form of the equation of a line.

Horizontal and Vertical Lines

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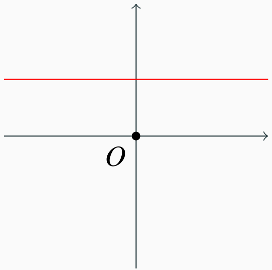

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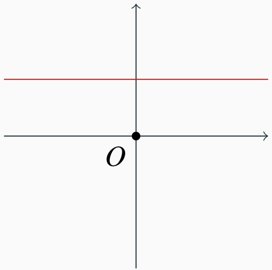

Horizontal and Vertical Lines

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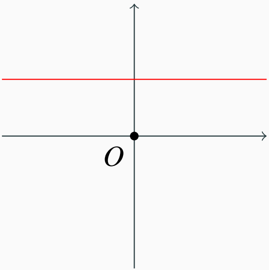

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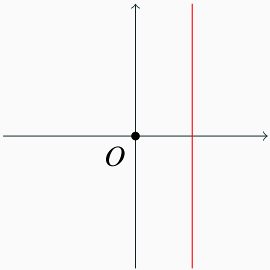

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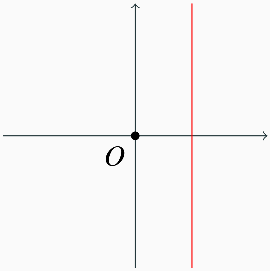
- So the equation of a horizontal line is $y - y_1 = 0$ (point-slope form) or $y = b$ (slope-intercept form).
- On the other hand, the slope of a vertical line is undefined (division by 0).
- However, by analogy with the previous case, we can write an equation for a vertical line: $x = a$



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- We must have $A \neq 0$ or $B \neq 0$. What happens if both $A = B = 0$?
- Given point-slope form we can solve for general form:

$$y - y_1 = m(x - x_1) \implies -mx + 1y + (mx_1 - y_1) = 0$$

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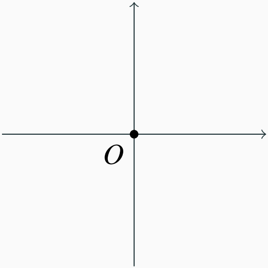
- Given general form, with $B = 0$, we can solve for vertical form:

$$Ax + 0y + C = 0 \implies x = -\frac{C}{A}$$

Graphing Lines

- To graph a line, 1) solve for y then 2) substitute two different numbers for x to get two points on the line.



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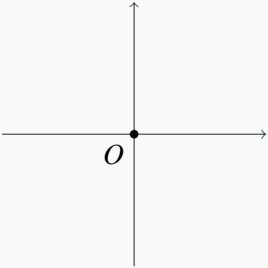

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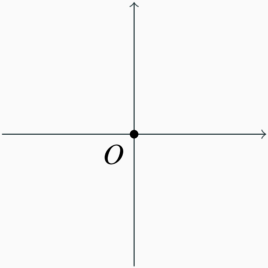

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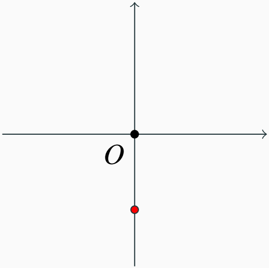

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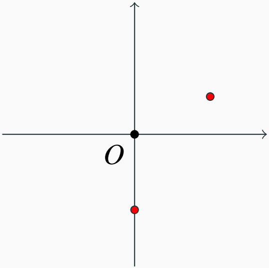

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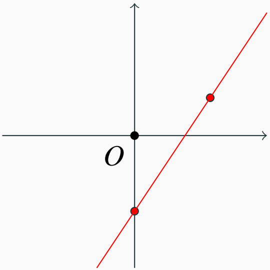
- Graph those points then draw a line through them.



Graphing Linear Inequalities

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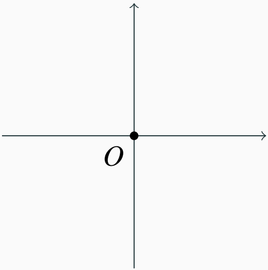

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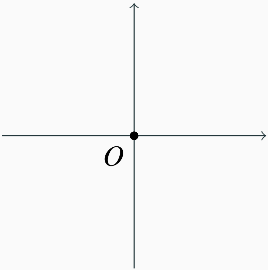

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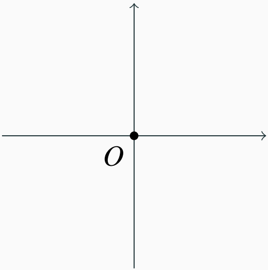

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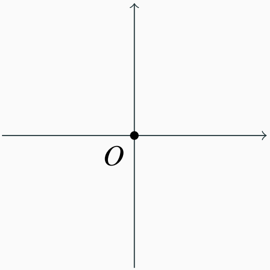

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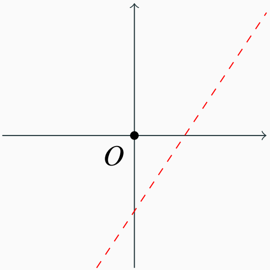

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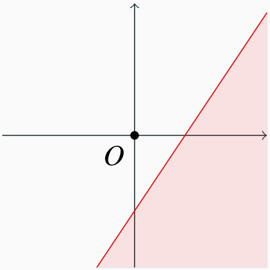

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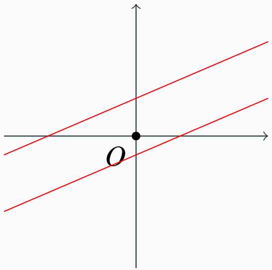

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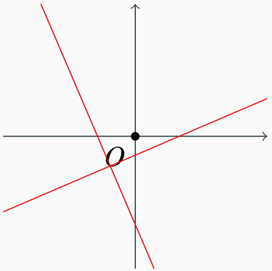

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