MATH 110 Review Problem Set 0.A Solutions

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1. (a) $|8| - |-11| = 8 - 11 = -3$

   (b) $|\pi - 4| = -(\pi - 4)$ because $\pi < 4$ so $\pi - 4$ is negative. You can simplify the answer: $-(\pi - 4) = 4 - \pi$.

   (c) $\big| |-3| - |-5| \big| = |3 - 5| = |-2| = 2$.

   (d) $x \geq 2 \implies x - 2 \geq 0 \implies 2 - x \leq 0 \implies |2 - x| = -(2 - x)$. You can simplify the answer: $-(2 - x) = x - 2$.

2. (a) $4x - 10 > 5 \implies 4x > 15 \implies x > \frac{15}{4}$. See Figure 1.

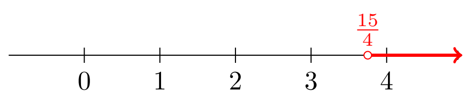

Figure 1: Graph of the solution set of $4x - 10 > 5$

   (b) $6 - 5x \leq 8 \implies -5x \leq 2 \implies x \geq -\frac{2}{5}$. (Note how the direction of the inequality sign changed!) See Figure 2.

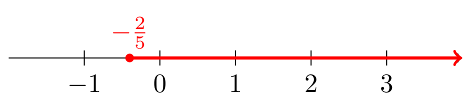

Figure 2: Graph of the solution set of $6 - 5x \leq 8$

   (c) $3x - 1 < 2 - x \implies 4x < 3 \implies x < \frac{3}{4}$. See Figure 3. (Alternatively you could have move the x's to the RHS: $3x - 1 < 2 - x \implies -3 < -4x$, and at this point you must remember to switch the direction of the inequality sign when you divide through by -4. It is best to avoid altogether the possibility of making an error by initially moving the x's to whichever side makes the coefficient positive, as in the first method.)

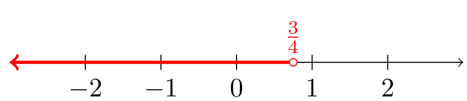

Figure 3: Graph of the solution set of $3x - 1 < 2 - x$

   (d) This is really two inequalities. In general, we solve two inequalities separately, but in this case we can solve them simultaneously. Adding -3 everywhere we have

$$-5 \leq -2x \leq 3$$

Dividing through by -2 and *reversing the sign of the inequalities* we have

$$\frac{5}{2} \geq x \geq -\frac{3}{2}$$

which I prefer to write using the $\leq$ symbol:

$$-\frac{3}{2} \leq x \leq \frac{5}{2}$$

See Figure 4.

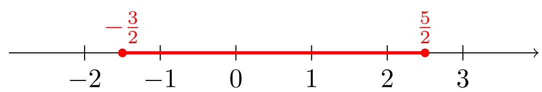

Figure 4: Graph of the solution set of $-2 \leq 3 - 2x \leq 6$

3. (a) $|4x| = 1$ means "$4x = -1$ or $4x = 1$". In the first case, we have $x = -\frac{1}{4}$. In the second case, we have $x = \frac{1}{4}$. Altogether, the solution set is $\{-\frac{1}{4}, \frac{1}{4}\}$. You should check that each number in the set really is a solution to the given equation.

   (b) $|2x - 1| = 4$ means $2x - 1 = -4$ or $2x - 1 = 4$. In the first case, we have $2x = -3$, $x = -\frac{3}{2}$. In the second case, we have $2x = 5$, $x = \frac{5}{2}$. Altogether the solution set is $\{-\frac{3}{2}, \frac{5}{2}\}$. You should check that each number in the solution set really is a solution to the given equation.

4. (a) $|x| \geq 5$ means $x \leq -5$ or $5 \leq x$. See Figure 5.

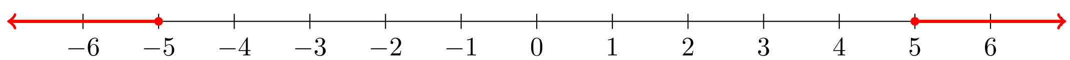

Figure 5: Graph of the solution set of $|x| \geq 5$

   (b) $|x - 2| \leq 0.5$ means

$$-0.5 \leq x - 2 \leq 0.5$$

   Solving the two inequalities simultaneously by adding 2 everywhere,

$$1.5 \leq x \leq 2.5$$

   See Figure 6.

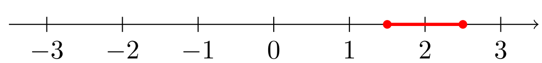

Figure 6: Graph of the solution set of $|x - 2| \leq 0.5$

   (c) $|x + 1| \geq 2$ means $x + 1 \leq -2$ or $2 \leq x + 1$. Solving the two inequalities simultaneously by subtracting 1 everywhere, $x \leq -3$ or $1 \leq x$. See Figure 7.

   (d) $|3x - 1| < 4$ means $-4 < 3x - 1 < 4$. Solving both inequalities simultaneously,

$$-4 < 3x - 1 < 4 \implies -3 < 3x < 5 \implies -1 < x < \frac{5}{3}$$

   See Figure 8.

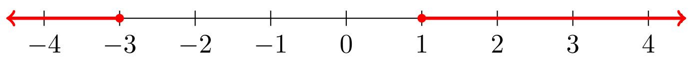

Figure 7: Graph of the solution set of $|x - 2| \leq 0.5$

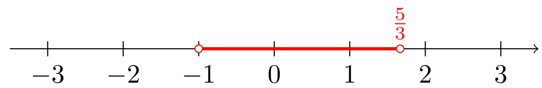

Figure 8: Graph of the solution set of $|3x - 1| < 4$

5. We could rewrite the absolute value expressions using the cases construction discussed in the lectures but there is a tidier way:

   (a) $|1 - 2x| = \sqrt{(1 - 2x)^2}$

   (b) $|x^2 - 1| = \sqrt{(x^2 - 1)^2}$

6. (a) Since there are x's in multiple locations in the given inequality $4x - 5 < 3x - 2 < 5x + 2$, rather than just one x, we can't simultaneously solve both inequalities, so we must work on them one at a time. Solving the first inequality,

$$4x - 5 < 3x - 2 \implies x < 3$$

Solving the second inequality,

$$3x - 2 < 5x + 2 \implies -4 < 2x \implies -2 < x$$

(note how I avoided having to divide through by a negative number by carefully chosing which side of the inequality to put my x's). Recombining the results, we have the solution $-2 < x < 3$. See Figure 9.

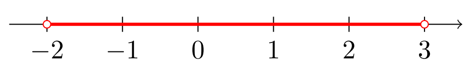

Figure 9: Graph of the solution set of $4x - 5 < 3x - 2 < 5x + 2$

   (b) In this case we factor the quadratic to obtain

$$(x + 1)(3x - 4) \geq 0$$

Note that I have ordered the factors by increasing root: the root of $x + 1$ is $x = -1$, and the root of $3x - 4$ is $x = \frac{4}{3}$. We now develop a table as discussed in the lectures; see Table 1. From the last column of the table it follows that the cases in which $(x + 1)(3x - 4) \geq 0$ are $x < -1$, $x = -1$, $x = \frac{3}{4}$, or $\frac{3}{4} < x$. We can combine those possibilities into a two inequalities describing the solution set: "$x \leq -1$ or $\frac{3}{4} \leq x$". See Figure 10.

   (c) First, we move everything to the LHS leaving just 0 in the RHS:

$$x^3 - 5x^2 + 6x < 0$$

Next, we attempt to factor the cubic. There is a common factor of x in every term so we take that out:

$$x(x^2 - 5x + 6) < 0$$

I	$(x+1)$	$(3x-4)$	$(x+1)(3x-4)$
$x < -1$	$-$	$-$	$+$
$x = -1$	0	$-$	0
$-1 < x < 3/4$	$+$	$-$	$-$
$x = 3/4$	$+$	0	0
$3/4 < x$	$+$	$+$	$+$

Table 1: Determining the sign of $(x+1)(3x-4)$

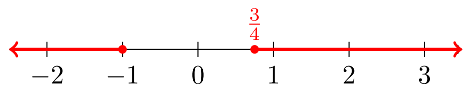

Figure 10: Graph of the solution set of $3x^2 - x - 4 \geq 0$

Now we factor the quadratic:

$$x(x-2)(x-3) < 0$$

Note that the factors are written in order of increasing root. In order to determine the sign of the LHS we build Table 2. From the last column of the table, we see that $x(x-2)(x-3) < 0$ when $x < 0$ or when $2 < x < 3$, which is the solution set for our inequality. See Figure 11.

(d) This problem is rather tricky. The temptation is to multiply by x, but remember that if x is negative, we must reverse the direction of the inequality signs. The problem is that we don't know anything about x; it could be positive *or* negative.

One way out of this is to make a series of assumptions that cover all the cases. The assumptions provide a context within which we can make progress. We are going to look at the cases $x < 0$, $x = 0$, and $x > 0$ which cover all the possibilities for x. We can quickly dispense with the case $x = 0$ because $1/x$ is not defined for $x = 0$.

Next let us assume that $x < 0$. Then $1/x \leq 3$ is automatically satisfied because $1/x < 0 < 3$. To solve $-1 < 1/x$ we multiply both sides by the negative number x *and reverse the sign of the inequality* to obtain $-x > 1$. We multiply both sides by -1 and reverse the inequality sign a second time to obtain $x < -1$.

Conversely, if $x < -1$ then $x < 0$ for sure, so $1/x < 0 < 3$, so $1/x \leq 3$ is satisfied; and we obtain $-1 < 1/x$ by dividing $x < -1$ through by $-x$ (which is a positive number).

Finally let us assume that $0 < x$. Then $-1 < 1/x$ is automatically satisfied because $-1 < 0 < 1/x$, and we just have to ensure $1/x \leq 3$. Because $x > 0$ we can apply the reciprocal rule, and so we have $\frac{1}{3} \leq x$.

Conversely, if $\frac{1}{3} \leq x$, then $0 < x$ so we can apply the reciprocal rule to obtain $1/x \leq 3$ and we also have $-1 < 1/x$ because $-1 < 0 < 1/x$, so the inequality $-1 < 1/x \leq 3$ is satisfied.

I	x	$(x-2)$	$(x-3)$	$x(x-2)(x-3)$
$x < 0$	$-$	$-$	$-$	$-$
$x = 0$	0	$-$	$-$	0
$0 < x < 2$	$+$	$-$	$-$	$+$
$x = 2$	$+$	0	$-$	0
$2 < x < 3$	$+$	$+$	$-$	$-$
$x = 3$	$+$	$+$	0	0
$3 < x$	$+$	$+$	$+$	$+$

Table 2: Determining the sign of $x(x-2)(x-3)$

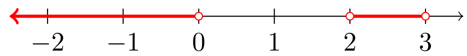

Figure 11: Graph of the solution set of $x^3 + 6x < 5x^2$

In summary, the inequality $-1 < 1/x \leq 3$ is satisfied when $x < -1$ or when $\frac{1}{3} \leq x$. See Figure 12. We will learn another way to solve this type of problem later in the course.

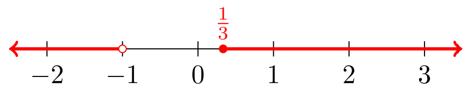

Figure 12: Graph of the solution set of $-1 < 1/x \leq 3$

7. (a) The temperature T in degrees Celsius at height h in meters is given by

$$T = 20 - \frac{1}{100}h,\ 0 \leq h \leq 12000$$

That says we start with a temperature of 20, then *decrease* ($-$) at a *rate of 1 degree per 100* ($\frac{1}{100}$) meters (h). We also note that the formula is only applicable above the ground ($h \geq 0$) and below a height of about 12000 meters ($h \leq 12000$).

(b) We know that $0 \leq h \leq 11000$ in this part of the problem, and we want to find an inequality for T. We modify the inequality for h until the expression in the middle looks more like T. First we multiply throughout by $-\frac{1}{100}$:

$$0 \leq h \leq 11000 \implies 0 \geq -\frac{1}{100}h \geq -110$$

Note that we have reversed the sign of the inequalities because $-\frac{1}{100}$ is negative. Now we add 20 throughout:

$$-110 \leq -\frac{1}{100}h \leq 0 \implies -90 \leq 20 - \frac{1}{100}h \leq 20$$

So the range of temperatures we can expect on such a flight is

$$-90 \leq T \leq 20$$

8. (a) There are two sets of absolute value symbols in the problem. We can reduce the number of absolute value symbols as follows. We have

$$\frac{|2x+1|}{|x-1|} = 3 \implies \left|\frac{2x+1}{x-1}\right| = 3 \implies \frac{2x+1}{x-1} = -3 \text{ or } \frac{2x+1}{x-1} = 3$$

In the first case we have

$$\frac{2x+1}{x-1} = -3 \implies 2x+1 = -3(x-1) \implies 2x+1 = -3x+3 \implies 5x = 2 \implies x = \frac{2}{5}$$

In the second case we have

$$\frac{2x+1}{x-1} = 3 \implies 2x+1 = 3(x-1) \implies 2x+1 = 3x-3 \implies 4 = x$$

So our solution set is $\{\frac{2}{5}, 4\}$. You should check that both of those numbers satisfy the original problem.

(b) Solving this problem benefits from some creative thinking. One way to do it might be to break it up into four cases, e.g., $x+4<0$ and $3x-2<0$ would be one case. Another way would be to break the line up into five pieces ($x<-4$, $x=-4$, $-4<x<\frac{2}{3}$, $x=\frac{2}{3}$, $\frac{2}{3}<x$) like we did when solving inequalities with quadratics. However, in this case I recommend dividing both sides of the equation by $|x+4|$. In order to do so, we must first check that $x=-4$ is not a solution to the equation (it is not) so we know that $|x+4|\neq 0$ and we can divide through by $|x+4|$:

$$1=\frac{|3x-2|}{|x+4|} \implies 1=\left|\frac{3x-2}{x+4}\right|$$

where we have used the absolute value rule $|a|/|b|=|a/b|$ as in the previous problem. Now there is only one absolute value sign in the expression and we can use the usual method for solving absolute value equalities. We have

$$\left|\frac{3x-2}{x+4}\right|=1 \iff \frac{3x-2}{x+4}=-1 \text{ or } \frac{3x-2}{x+4}=1$$

In the first case we have

$$\frac{3x-2}{x+4}=-1 \implies 3x-2=-(x+4) \implies 3x-2=-x-4 \implies 4x=-2 \implies x=-\frac{1}{2}$$

In the second case we have

$$\frac{3x-2}{x+4}=1 \implies 3x-2=x+4 \implies 2x=6 \implies x=3$$

So the solution set is $\{-\frac{1}{2},3\}$. You should check that both of those numbers actually are solutions to the equation.

9. (a) The most straightforward way to see this is to consider the cases $2x<0$, $2x=0$, and $2x>0$ separately. The case $2x=0$ is obviously eliminated because it implies $x=0$ which clearly does not satisfy the inequality.

In the first case, $2x<0$ implies that $|2x|=-2x$ by the definition of absolute value so the inequality becomes

$$3\leq -2x\leq 5 \implies -\frac{3}{2}\geq x\geq -\frac{5}{2} \implies -\frac{5}{2}\leq x\leq -\frac{3}{2}$$

where I have just re-written the last inequality using my preference of $\leq$ over $\geq$. You should check that conversely, $-\frac{5}{2}\leq x\leq -\frac{3}{2}$ implies that $3\leq |2x|\leq 5$.

In the remaining case, $2x>0$ implies that $|2x|=2x$ so the inequality becomes

$$3\leq 2x\leq 5 \implies \frac{3}{2}\leq x\leq \frac{5}{2}$$

You should check that conversely, $\frac{3}{2}\leq x\leq \frac{5}{2}$ implies that $3\leq |2x|\leq 5$.

In summary, the solution set is x either in the interval $[-\frac{5}{2},-\frac{3}{2}]$ or in the interval $[\frac{3}{2},\frac{5}{2}]$. In other words, the solution set is the union $[-\frac{5}{2},-\frac{3}{2}]\cup[\frac{3}{2},\frac{5}{2}]$. See Figure 13.

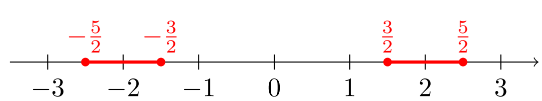

Figure 13: Graph of the solution set of $3\leq |2x|\leq 5$

(b) This problem is similar to the previous, but the outcome is slightly different. We again consider two separate cases, $x-4 \geq 0$ and $x-4<0$. In the first case we have

$$0<|x-4|<1 \implies 0<x-4<1 \implies 4<x<5$$

In the second case we have $|x-4|=-(x-4)$ and so

$$0<|x-4|<1 \implies 0<-(x-4)<1 \implies 0>x-4>-1 \implies 4>x>3 \implies 3<x<4$$

Altogether we have x must satisfy $3<x<4$ or $4<x<5$. Note that x cannot equal 4 in either case. See Figure 14. The solution set is the interval $(3,5)$ with the point 4 removed from it; such

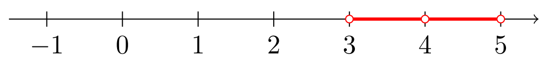

Figure 14: Graph of the solution set of $0 \leq |x-4| \leq 1$

a solution set is known as a *punctured interval*.

10. (a) It doesn't matter whether b is positive or negative, we can subtract b from both sides of the inequality in either case without changing the direction of the inequality, so we have

$$ax+b<c \implies ax<c-b$$

Now, because we know that a is negative, we can divide both sides of the inequality by a provided we switch the direction of the inequality sign:

$$ax<c-b \implies x>\frac{c-b}{a}$$

which is the solution to the original inequality.

(b) We solve this problem in a similar manner to the previous:

$$c(ax+b) \leq b \implies ax+b \leq \frac{b}{c} \implies ax \leq \frac{b}{c}-b \implies x \geq \frac{1}{a}\left(\frac{b}{c}-b\right) \implies x \geq \frac{b}{ac}-\frac{b}{a}$$

where the last simplification was optional. You could also add fractions if you wanted.

11. (a) Just try some numbers for x with the assistance of your calculator. If you're lucky or pick your numbers carefully, you don't even need a calculator. Let's try $x=4$. Then the LHS is

$$\sqrt{x^2+9}=\sqrt{4^2+9}=\sqrt{16+9}=\sqrt{25}=5$$

while the RHS is

$$|4|+3=4+3=7$$

The LHS and the RHS disagree in this case, so the equation is certainly not true for all values of x. (By the way, this once again shows that $\sqrt{x^2+9} \neq \sqrt{x^2}+\sqrt{9}$.)

(b) If $a=1$, the equation certainly is true for all values of x, because in that case the LHS and the RHS are exactly the same. If a is any other number, i.e., $a \neq 1$ and hence $a-1 \neq 0$, then we have

$$\frac{a}{a+x}=\frac{1}{1+x} \implies a(1+x)=1(a+x) \implies a+ax=a+x \implies (a-1)x=0 \implies x=0$$

where we are allowed to divide by $a-1$ because we're assuming $a \neq 1$. So if $a \neq 1$, the original equation is true only if $x=0$, which means it's not true for all values of x.