MATH 110 Quiz 1 Solutions

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1. (a) The table is as follows. The $9 - x$ column is trivial, but you might want to double check with a calculator just in case.

x	$9 - x$	$\sqrt{x} - 3$	$f(x)$
10.00	-1.0000	+0.1623	-6.1623
9.10	-0.1000	+0.01622	-6.0166
9.01	-0.01000	+0.001666	-6.0017
8.99	+0.01000	-0.001667	-5.9983
8.90	+0.1000	-0.01671	-5.9833
8.00	+1.0000	-0.1716	-5.8284

Note that to have four decimal points of accuracy in the final column, I had to keep more than four decimal points in the intermediate calculations; I had to keep four significant figures (five would have been better).

(b) Based on the above table, a reasonable guess for the limit would be a number about halfway between -6.0017 and -5.9983, i.e., about -6.0000 to four decimal places. (Using limit theorems, we can now show that that is the exact answer.)

2. Candidates for vertical asymptotes of rational functions are vertical lines over x values at which the denominator goes to 0. The denominator goes to 0 when $x - 3 = 0$, so our candidates for an asymptote is the line $x = 3$. The rational function may have a removable discontinuity at that x value rather than an infinite discontinuity, however, so we must check (one-sided) limits as x approaches that candidate value.

For $\lim\limits_{x\to 3^-}$ we have $x - 3$ slightly smaller than 0 and the denominator $(x+3)(x-2)$ close to 6×1 which is positive, so $\lim\limits_{x\to 3^-} \dfrac{(x+3)(x-2)}{x-3}$ is $-\infty$.

For $\lim\limits_{x\to 3^+}$, again the numerator is close to 6, a positive number, and the denominator tends to 0 from above, so is positive and small, so the limit $\lim\limits_{x\to 3^+} \dfrac{(x+3)(x-2)}{x-3}$ is $+\infty$.

In summary, there is one and only one vertical asymptote, at $x = 3$.