

MATH 110 Problem Set 4.5

Edward Doolittle

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The following problems based on Section 4.5 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

1. (Based on 4.5.7–14) Evaluate the following indefinite integrals using an appropriate substitution.

(a) $\int x^2(x^3 + 5)^9 dx$ (b) $\int (3t + 2)^{2.4} dt$ (c) $\int \frac{x}{(x^2 + 1)^2} dx$ (d) $\int \frac{1}{(5t + 4)^{2.7}} dt$

2. (Based on 4.5.15–22) Evaluate the following indefinite integrals using an appropriate substitution.

(a) $\int \sec 2\theta \tan 2\theta d\theta$ (b) $\int \sqrt{x} \sin(1 + x^{3/2}) dx$ (c) $\int \frac{\cos(\pi/x)}{x^2} dx$ (d) $\int (1 + \tan \theta)^5 \sec^2 \theta d\theta$

3. (Based on 4.5.23–30) Evaluate the following indefinite integrals using an appropriate substitution.

(a) $\int \frac{dt}{\cos^2 t \sqrt{1 + \tan t}}$ (b) $\int \sin t \sec^2(\cos t) dt$ (c) $\int \frac{x^2}{\sqrt{1-x}} dx$ (d) $\int x^3 \sqrt{x^2 + 1} dx$

4. (Based on 4.5.35–51) Evaluate the following definite integrals using an appropriate substitution.

(a) $\int_0^{\pi/2} \cos x \sin(\sin x) dx$ (b) $\int_0^a x \sqrt{a^2 - x^2} dx$ (c) $\int_0^4 \frac{x}{\sqrt{1+2x}} dx$ (d) $\int_0^{T/2} \sin(2\pi t/T - \alpha) dt$

5. (Based on 4.5.55–56)

- (a) Evaluate $\int_{-2}^2 (x + 3) \sqrt{4 - x^2} dx$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.
- (b) Evaluate $\int_0^1 x \sqrt{1 - x^4} dx$ by making a substitution and interpreting the resulting integral in terms of an area.

6. Evaluate the following integrals.

(a) $\int \cos^3 x \sin^6 x dx$ (b) $\int_{-1}^1 \frac{x^2 \tan x}{x^4 + 1} dx$

7. (Based on 4.5.60) If f is continuous and $\int_0^9 f(x) dx = 4$, find $\int_0^3 xf(x^2) dx$.

8. (Based on 4.5.64) If f is continuous on $[0, \pi]$, use the substitution $u = \pi - x$ to show that

$$\int_0^\pi xf(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$$

You may find the following additional exercises from Section 4.5 helpful.

4.5 C-level: 1–51;
B-level: 52–60;
A-level: 61–66