

# MATH 110 Problem Set 4.5

Edward Doolittle

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The following problems based on Section 4.5 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

1. (Based on 4.5.7–14) Evaluate the following indefinite integrals using an appropriate substitution.

$$(a) \int x^2(x^3 + 5)^9 dx \quad (b) \int (3t + 2)^{2.4} dt \quad (c) \int \frac{x}{(x^2 + 1)^2} dx \quad (d) \int \frac{1}{(5t + 4)^{2.7}} dt$$

2. (Based on 4.5.15–22) Evaluate the following indefinite integrals using an appropriate substitution.

$$(a) \int \sec 2\theta \tan 2\theta d\theta \quad (b) \int \sqrt{x} \sin(1+x^{3/2}) dx \quad (c) \int \frac{\cos(\pi/x)}{x^2} dx \quad (d) \int (1+\tan \theta)^5 \sec^2 \theta d\theta$$

3. (Based on 4.5.23–30) Evaluate the following indefinite integrals using an appropriate substitution.

$$(a) \int \frac{dt}{\cos^2 t \sqrt{1+\tan t}} \quad (b) \int \sin t \sec^2(\cos t) dt \quad (c) \int \frac{x^2}{\sqrt{1-x}} dx \quad (d) \int x^3 \sqrt{x^2+1} dx$$

4. (Based on 4.5.35–51) Evaluate the following definite integrals using an appropriate substitution.

$$(a) \int_0^{\pi/2} \cos x \sin(\sin x) dx \quad (b) \int_0^a x \sqrt{a^2 - x^2} dx \quad (c) \int_0^4 \frac{x}{\sqrt{1+2x}} dx \quad (d) \int_0^{T/2} \sin(2\pi t/T - \alpha) dt$$

5. (Based on 4.5.55–56)

(a) Evaluate  $\int_{-2}^2 (x+3)\sqrt{4-x^2} dx$  by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

(b) Evaluate  $\int_0^1 x\sqrt{1-x^4} dx$  by making a substitution and interpreting the resulting integral in terms of an area.

6. Evaluate the following integrals.

$$(a) \int \cos^3 x \sin^6 x dx \quad (b) \int_{-1}^1 \frac{x^2 \tan x}{x^4 + 1} dx$$

7. (Based on 4.5.60) If  $f$  is continuous and  $\int_0^9 f(x) dx = 4$ , find  $\int_0^3 xf(x^2) dx$ .

8. (Based on 4.5.64) If  $f$  is continuous on  $[0, \pi]$ , use the substitution  $u = \pi - x$  to show that

$$\int_0^\pi xf(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$$

You may find the following additional exercises from Section 4.5 helpful.

- 4.5 C-level: 1–51;
- B-level: 52–60;
- A-level: 61–66