

# **MATH 110 Lecture 3.5**

## Summary of Curve Sketching

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Department of Indigenous Knowledge and Science  
First Nations University of Canada

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- A Domain
- B Intercepts

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- C Symmetry

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- B Intercepts
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- E Intervals of Increase/Decrease

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- F Local Max/Min

# Guidelines for Sketching a Curve

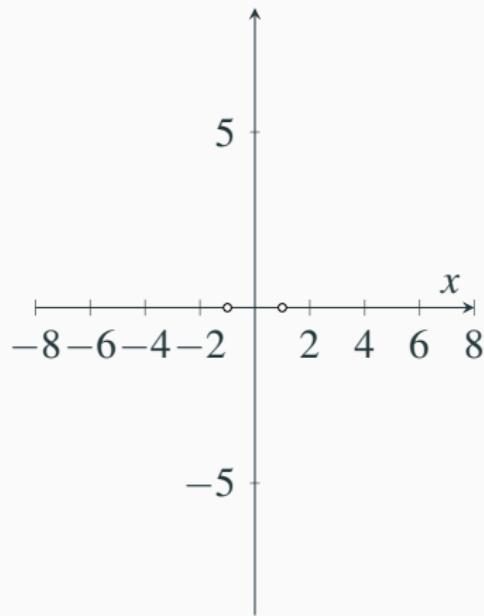
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- G Concavity and Inflection Points

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- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of Increase/Decrease
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- H Sketch the Curve

**Sketch**  $y = \frac{2x^2}{x^2 - 1}$ ,  $y = \frac{2}{1 - 1/x^2}$ ,  $y' = \frac{-4x}{(x^2 - 1)^2}$ ,  $y'' = \frac{12x^2 + 4}{(x^2 - 1)^3}$

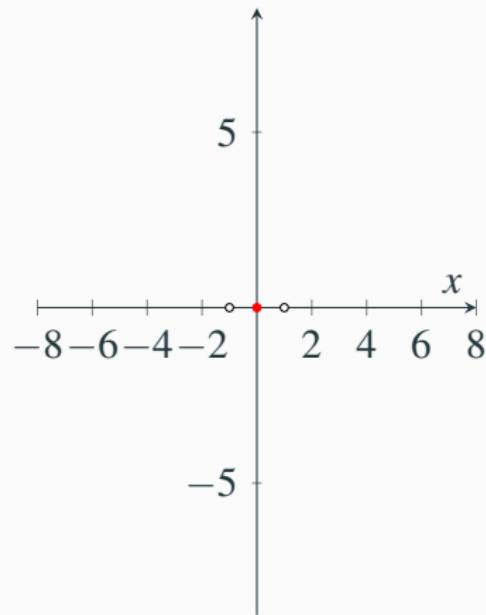
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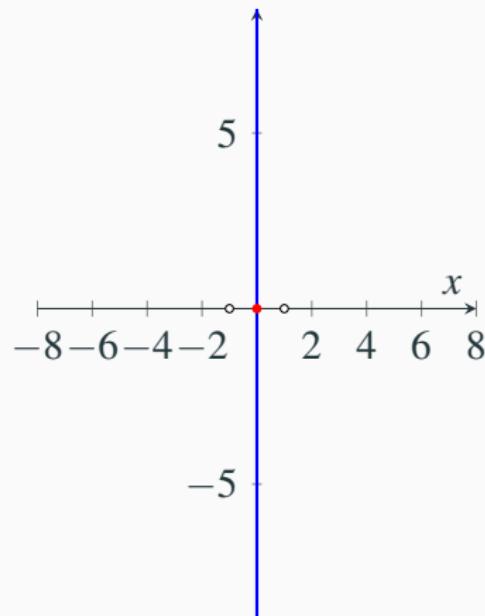
A Domain

B Intercepts



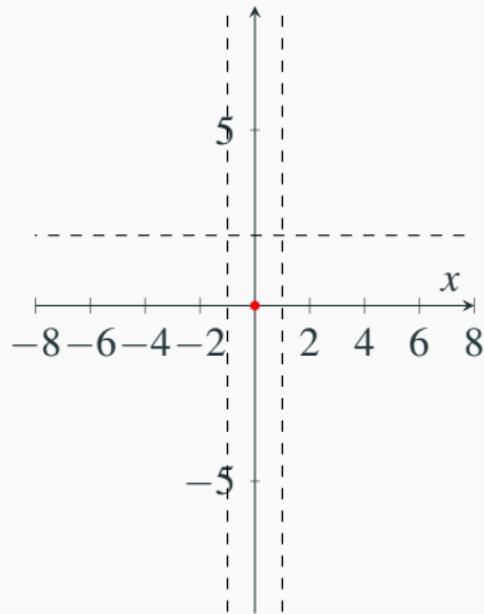
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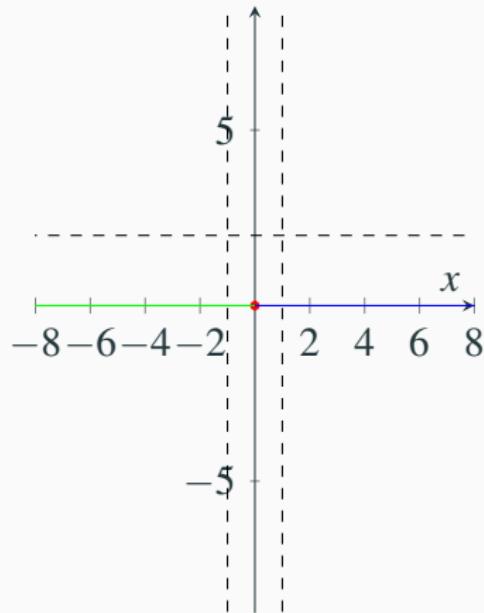
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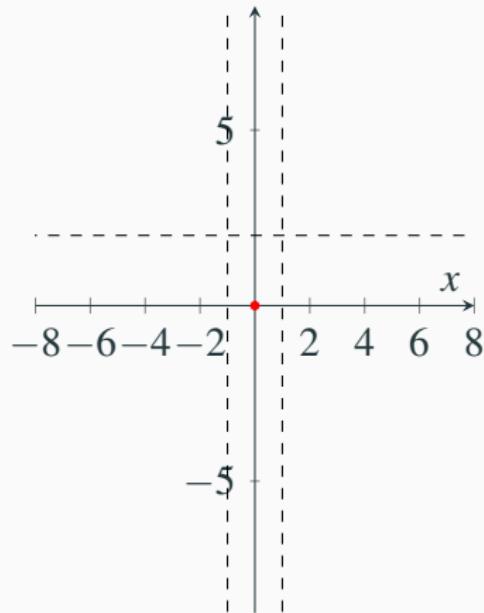
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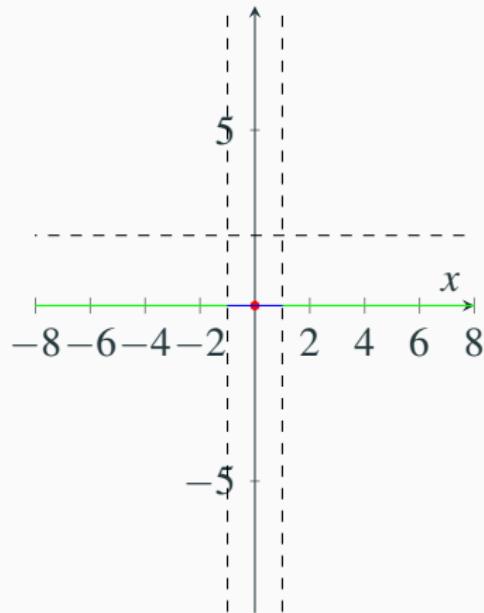
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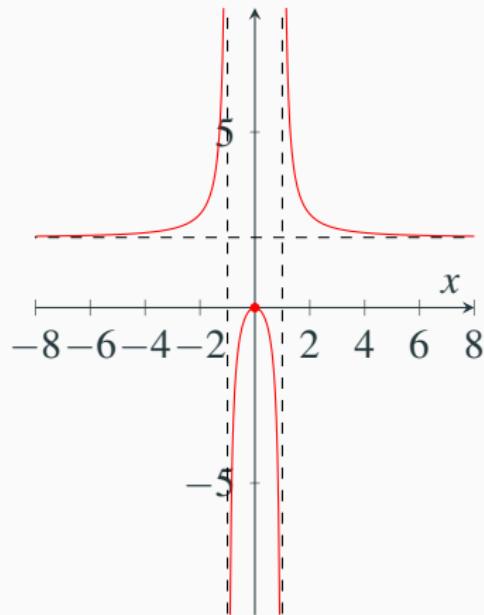
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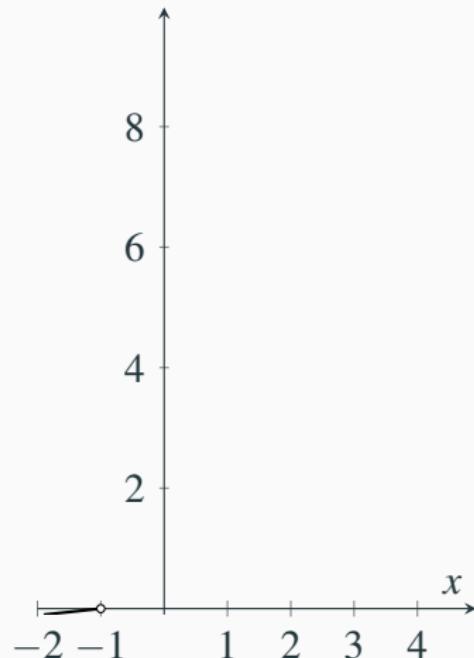
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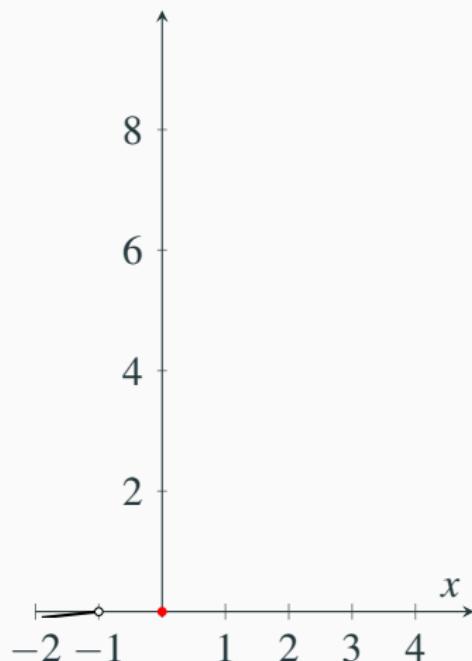
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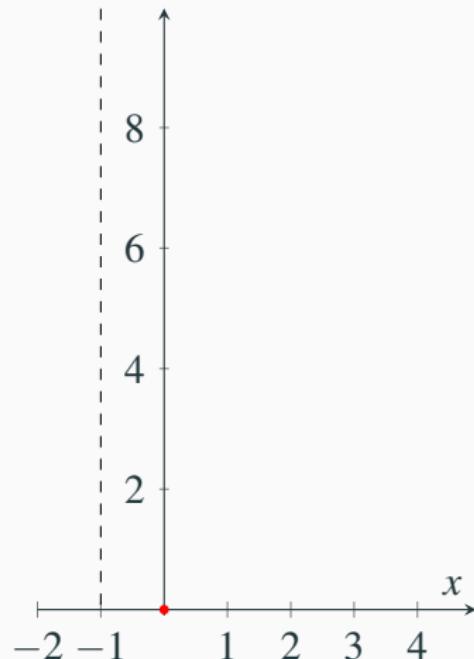
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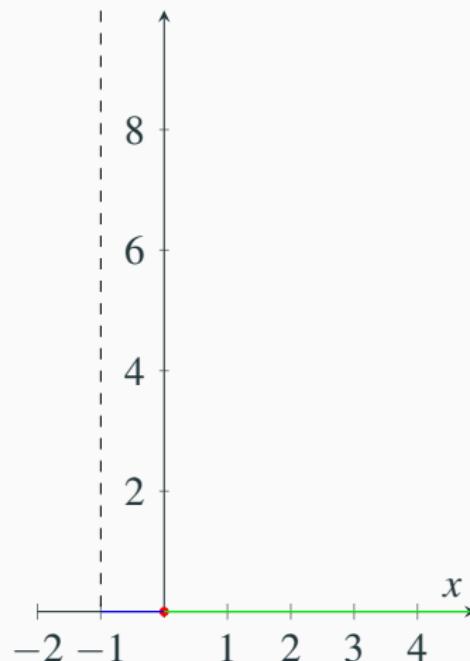
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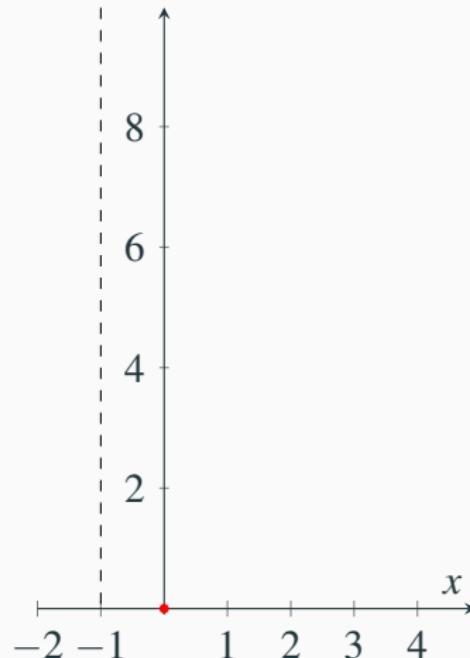
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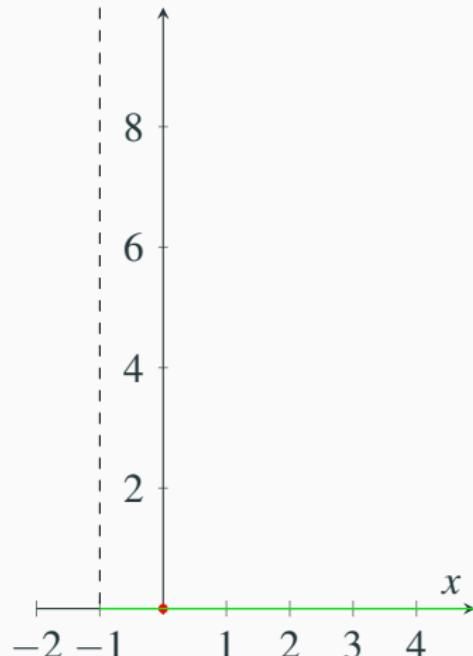
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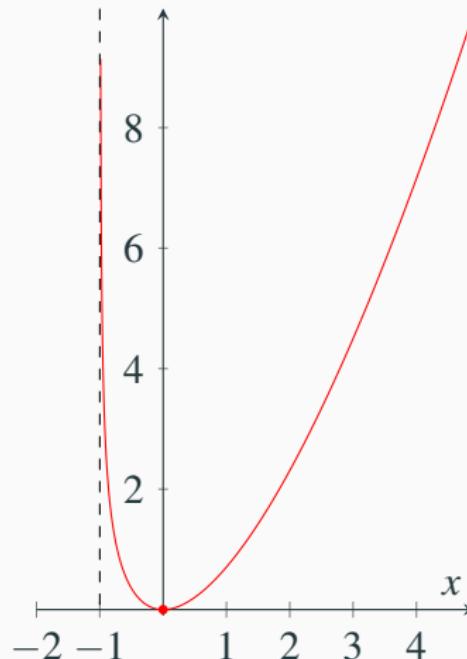
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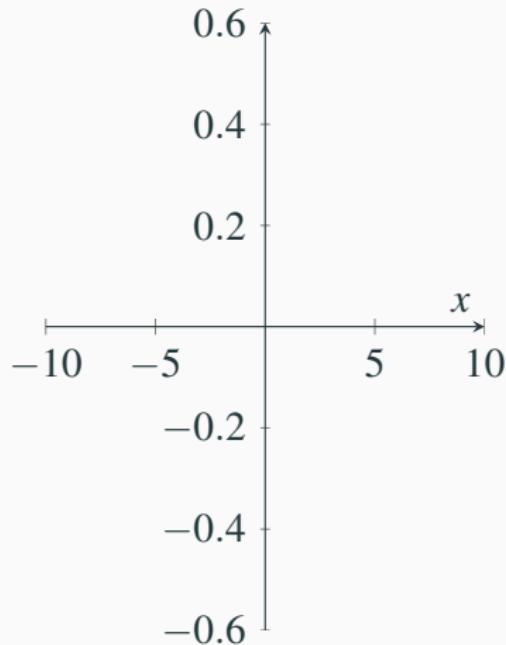
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**Sketch**  $y = \frac{\cos x}{2+\sin x}$ ,  $y' = -\frac{2\sin x + 1}{(2+\sin x)^2}$ ,  $y'' = \frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

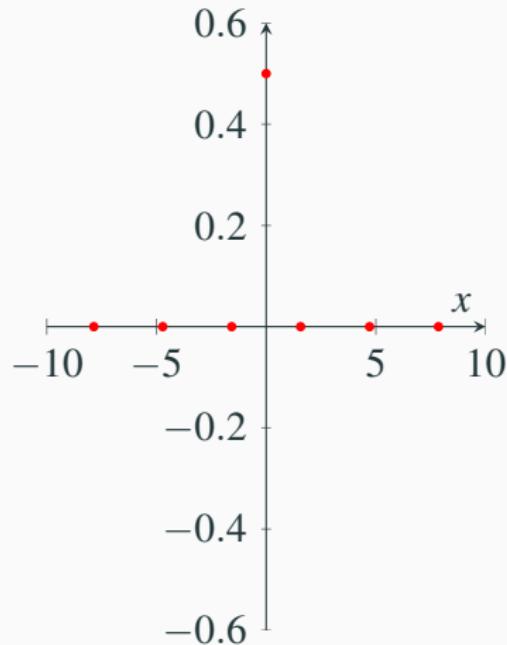
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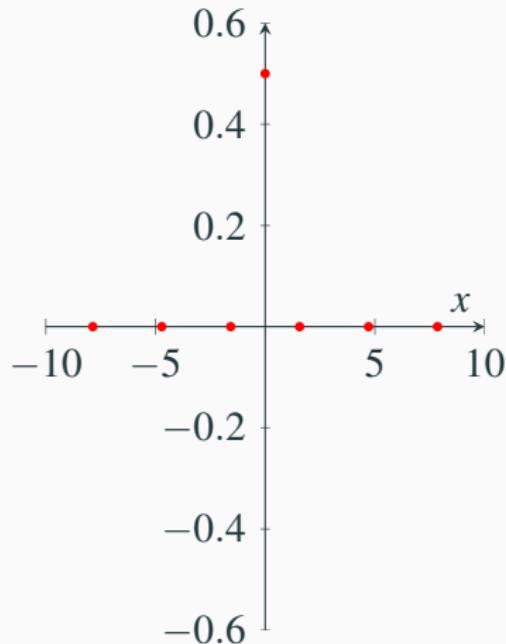
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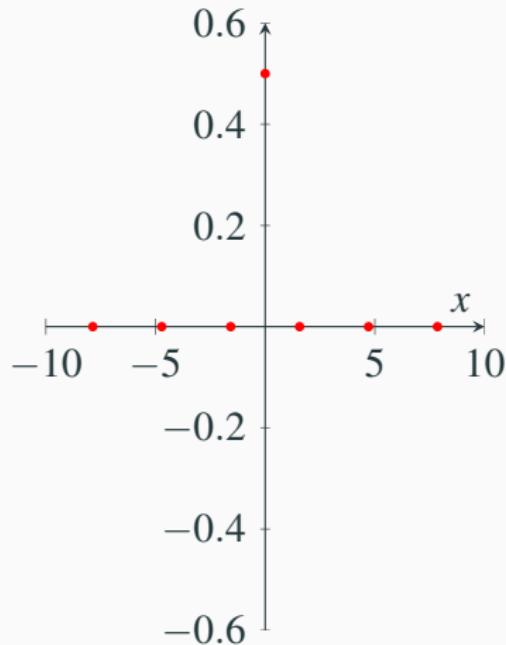
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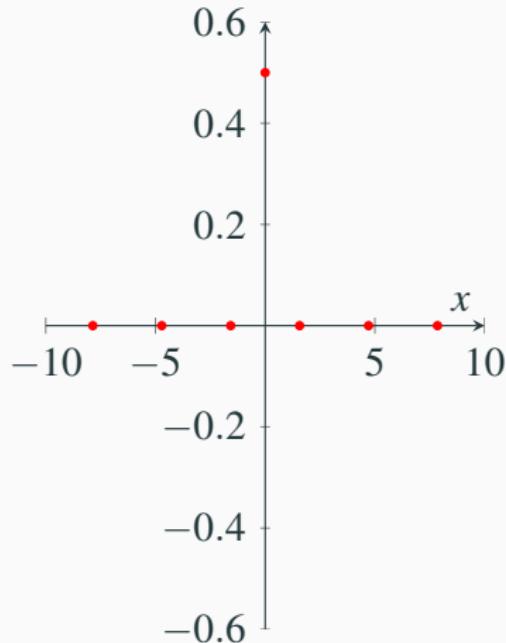
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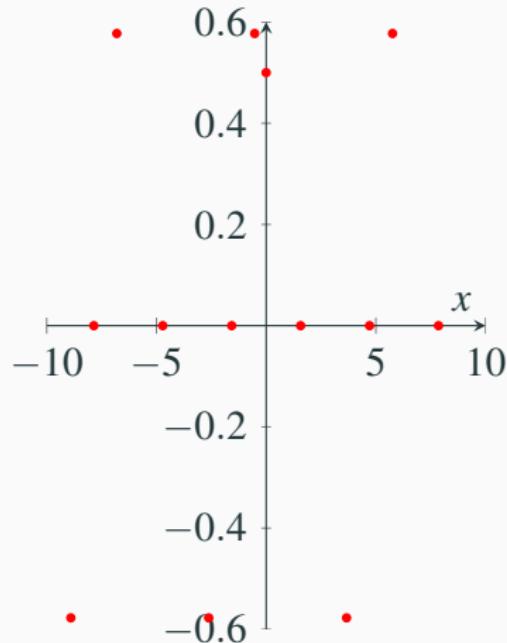
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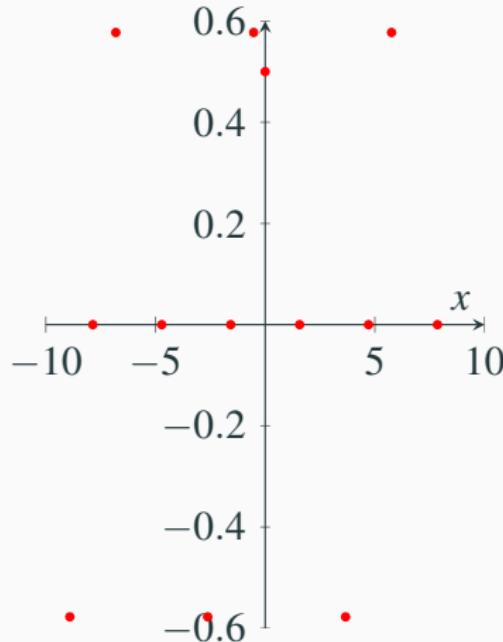
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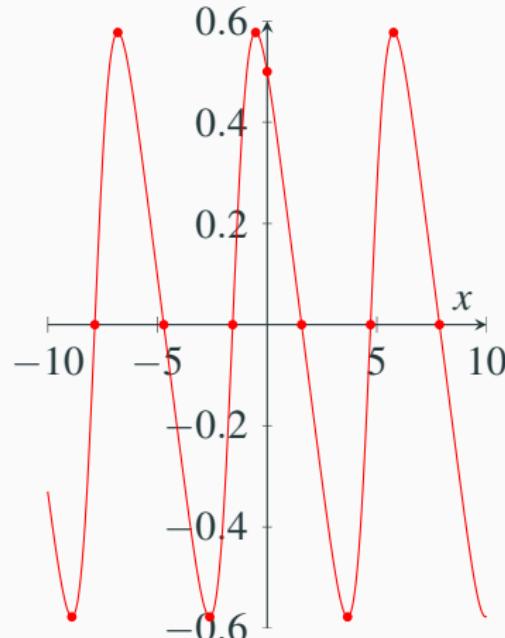
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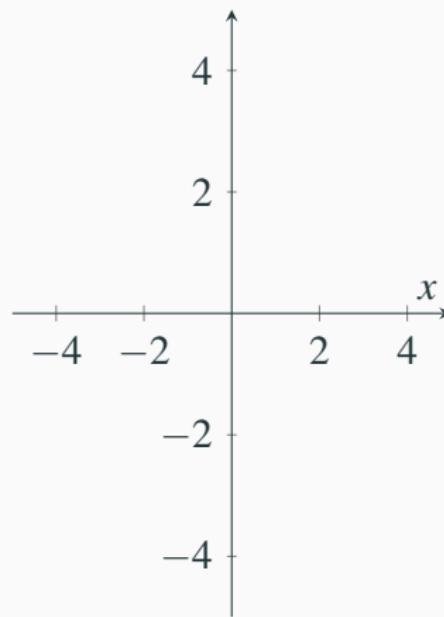
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# Slant Asymptotes

**Sketch**  $y = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}$ ,  $y' = \frac{x^2(x^2+3)}{(x^2+1)^2}$   $y'' = \frac{2x(3-x^2)}{(x^2+1)^3}$

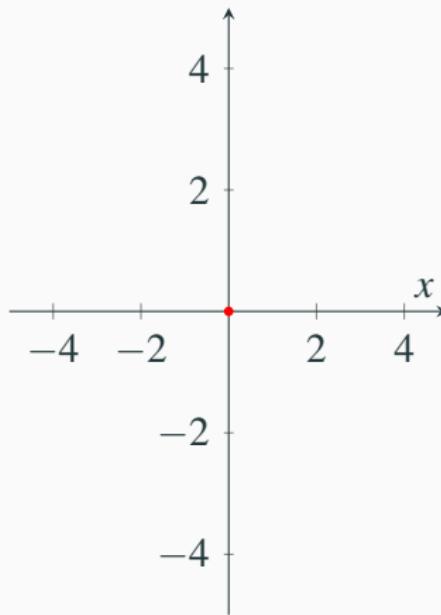
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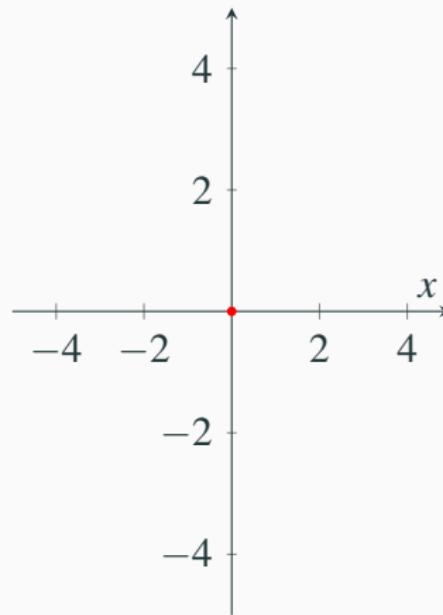
A Domain

B Intercepts



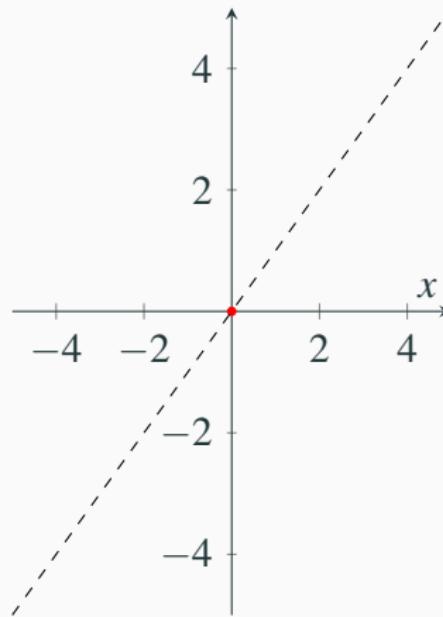
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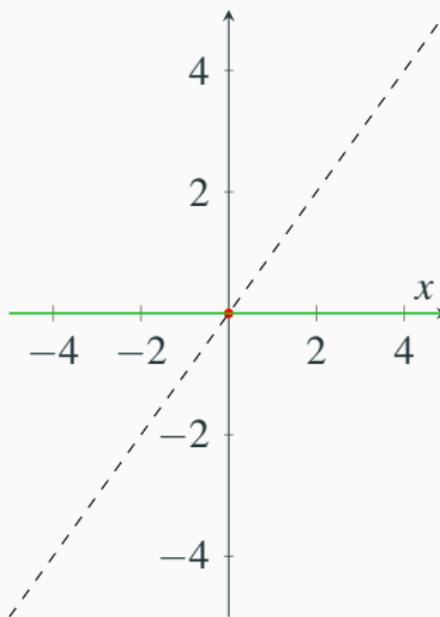
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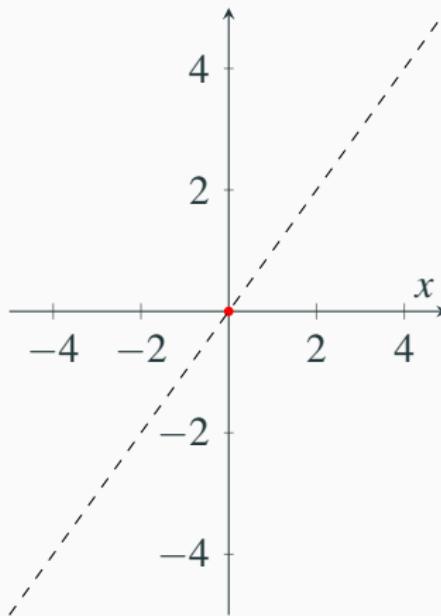
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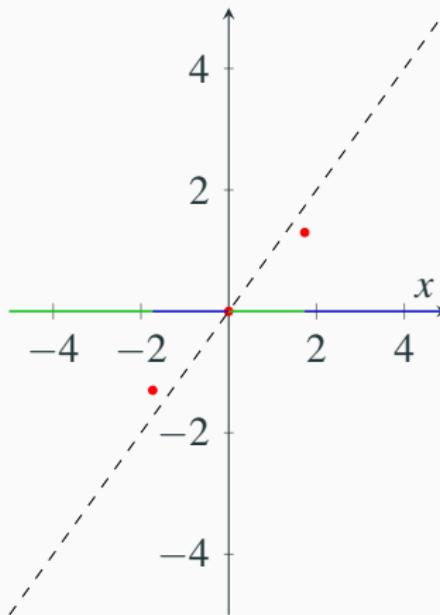
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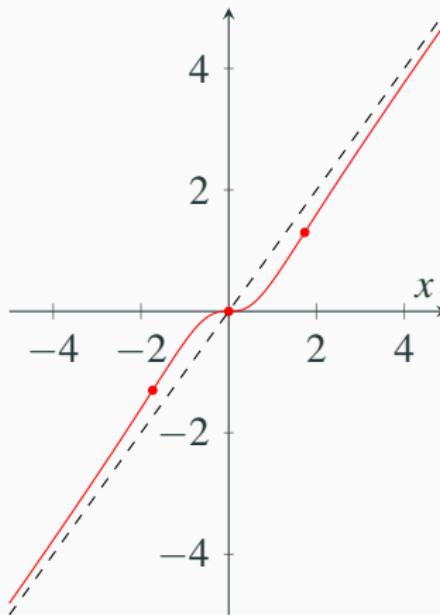
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## Examples and Exercises

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## Exercises

Now you should work on Problem Set 3.5. After you have finished it, you should try the following additional exercises from Section 3.5:

3.5 C-level: 1–20, 45–48, 49–54;

B-level: 21–40;

A-level: 41–44, 55–60