

MATH 110 Problem Set 2.7

Edward Doolittle

Tuesday, February 24, 2026

The following problems based on Section 2.7 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

1. (Based on 2.7.8–9) If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is $s = 80t - 16t^2$.
 - (a) What is the maximum height reached by the ball?
 - (b) What is the velocity of the ball when it is 96 ft above the ground on the way up? On the way down?
2. (Based on 2.7.17) The mass of the part of a metal rod that lies between its left end and a point x meters to the right is $3x^2$ kg. Find the linear density when x is (a) 1 m, (b) 2 m, and (c) 3 m.
3. (Based on 2.7.29–30) The cost function for production of a commodity is $C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3$.
 - (a) Find and interpret $C'(100)$
 - (b) Compare $C'(100)$ with the exact marginal cost at production level 100, i.e., the cost $C(101) - C(100)$ of producing the 101st item after 100 items have been produced.
4. (Based on 2.7.24) Consider the chemical reaction $A + B \rightarrow C$ in which one molecule of reactant A combines with one molecule of reactant B to form one molecule of product C. If the initial concentrations of A and B are the same, say a moles/L, it turns out that the concentration of C as a function of time is given explicitly by the formula $[C] = a^2kt/(akt + 1)$ where k is a constant depending on the type of reaction.
 - (a) Find the rate of reaction at time t .
 - (b) Show that if $x(t) = [C]$ then $dx/dt = k(a - x)^2$.
5. (Based on 2.7.36) In a fish farm, a population of fish is introduced into a pond and harvested regularly. A model for the rate of change of the fish population is given by the equation

$$\frac{dP}{dt} = r_0 \left(1 - \frac{P(t)}{P_c}\right) P(t) - \beta P(t)$$

where r_0 is the birth rate of the fish, P_c is the maximum population that the pond can sustain (called the *carrying capacity*), and β is the percentage of the population that is harvested.

- (a) What value of dP/dt corresponds to a stable population?
- (b) If the pond can sustain 10,000 fish, the birth rate is 5%, and the harvesting rate is 4%, find the stable population level.
- (c) What happens if the harvesting rate β is raised to 5%?

You may find the following additional exercises from Section 2.7 helpful.

- 2.7 C-level: 1–4, 6–10, 17–18, 22, 27–30;
B-level: 11–16, 19–21, 23–24, 25–26;
A-level: 31–36