

# MATH 110 Quiz 1 Section 004 Solutions

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1. (a) The table is as follows. The  $x - 16$  column is of course trivial, as is the  $f(x)$  column once the other two are done (just shifting decimal points).

$x$	$4 - \sqrt{x}$	$x - 16$	$f(x)$
17.00	-0.1231	1.0000	-0.1231
16.10	-0.01248	0.1000	-0.1248
16.01	-0.001250	0.0100	-0.1250
15.99	0.001250	-0.0100	-0.1250
15.90	0.01252	-0.1000	-0.1252
15.00	0.1270	-1.0000	-0.1270

- (b) Based on the above table, a reasonable guess for the limit would be  $-0.1250$  to four decimal places. (Using limit theorems, we can now show that that is the exact answer.)
2. Candidates for vertical asymptotes of rational functions are vertical lines over  $x$  values at which the denominator goes to 0. Factoring the denominator, we have  $y = \frac{x+1}{(x+3)(x-2)}$  so our candidates for asymptotes are the lines  $x = -3$  and  $x = 2$ . The rational function may have a removable discontinuity at those  $x$  values rather than an infinite discontinuity, however, so we must check (one-sided) limits as  $x$  approaches those candidate values.

For  $\lim_{x \rightarrow -3^-}$  we have  $x + 1$  and  $x - 2$  both negative and non-zero, and  $x + 3$  negative and close to zero. Three negatives make a negative, and a number in the denominator close to zero gives a large overall result, so  $\lim_{x \rightarrow -3^-} \frac{x+1}{(x+3)(x-2)} = -\infty$ .

For  $\lim_{x \rightarrow -3^+}$ ,  $x + 1$  and  $x - 2$  are again both negative and non-zero, but  $x + 3$  is positive and close to zero, giving the limit  $\lim_{x \rightarrow -3^+} \frac{x+1}{(x+3)(x-2)} = +\infty$ .

For  $\lim_{x \rightarrow 2^-}$ ,  $x + 1$  and  $x + 3$  are both positive, and  $x - 2$  is negative and close to zero, so overall we have a limit of  $-\infty$ . For  $\lim_{x \rightarrow 2^+}$  we have a limit of  $+\infty$ .

In summary, there are vertical asymptotes at  $x = -3$  and  $x = 2$ .