MATH 110 Review Problem Set 0.1 Solutions

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Tuesday, January 6, 2026

1. (a) We have

$$7(t^2-1)-2(t+3)-5t(t-4)=7t^2-7-2t-6-5t^2+20t=2t^2+18t-13$$

(b) By the binomial theorem and the distributive law,

$$x(x-2)^2=x(x^2-4x+4)=x^3-4x^2+4x$$

2. (a) We put the fractions over a common denominator and add:

$$\frac{1}{x+2}+\frac{1}{x-1}=\frac{x-1}{(x+2)(x-1)}+\frac{(x+2)}{(x+2)(x-1)}=\frac{2x+1}{(x+2)(x-1)}$$

You could expand the denominator, but experience tells me that it's best to leave it in factored form.

(b) Invert and multiply:

$$\frac{x/a}{y/b}=\frac{x}{a}\div\frac{y}{b}=\frac{x}{a}\times\frac{b}{y}=\frac{bx}{ay}$$

3. (a) Each term has a factor of 5, and a factor of a, and a factor of x, so we take out a common factor of $5ax$ to obtain

$$5ax-35ax^2=5ax(1-7x)$$

(b) Let's first try guessing a root. A good place to start would be the factors of the constant term (-3), which has factors 1, -1, 3, and -3. (This approach isn't guaranteed, but when it does, it saves us some work.) Substituting in $x=1$ we get $2(1)^2-5(1)-3=-6\neq 0$, so $x=1$ is not a root. Substituting in $x=-1$, we get $2(-1)^2-5(-1)-3=2+5-3=4\neq 0$, so $x=-1$ is not a root. Substituting $x=3$ we get $2(3)^2-5(3)-3=18-15-3=0$, so $x=3$ is a root!

Now we use the factor theorem to write

$$2x^2-5x-3=(x-3)(ax+b)$$

We don't know what a and b are, but we can find out by polynomial long division or short division. I'll use short division. We expand the RHS to obtain

$$2x^2-5x-3=ax^2+bx-3ax-3b=ax^2+(b-3a)x-3b$$

comparing the coefficients of x^2, we see $2=a$. Comparing the constant terms, we see $-3=-3b$ which implies $b=1$. Altogether we should have

$$2x^2-5x-3=(x-3)(2x+1)$$

You should check that result by expanding the RHS.

4. (a) We factor the numerator and denominator, in search of common factors that can be canceled:

$$\frac{x^2+2x-3}{x^2-9} = \frac{(x+3)(x-1)}{(x+3)(x-3)} = \frac{x-1}{x-3}$$

(b) Again, we factor the numerator using the technique of question 3b, which won't be reproduced here. The denominator is harder to factor, but note that for this problem to go anywhere, there has to be a factor of $x+2$ or a factor of $5x-1$ in the denominator. (If there isn't, then we can't simplify any further and we stop.) After some experimentation we have

$$\frac{5x^2+9x-2}{10x^2-17x+3} = \frac{(x+2)(5x-1)}{(5x-1)(2x-3)} = \frac{x+2}{2x-3}$$

5. (a) We divide the coefficient of the x term by 2:

$$x^2+6x-10 = (x+6/2)^2 + \text{junk} = x^2+6x+9+\text{junk}$$

After cancellation we see that "junk" has to be -19. In summary,

$$x^2+6x-10 = (x+3)^2 - 19$$

You should check by expanding and gathering the RHS.

(b) Dealing with "leading coefficients" like the 3 in this case is a pain in the neck. See the textbook supplement for one approach. However, in this case I would use a slightly more creative approach. Write

$$3x^2-18x+7 = 3x^2-18x+6+1 = 3(x^2-6x+2)+1$$

Now we just have to complete the square inside the bracket. Similar to the previous problem we have

$$x^2-6x+2 = (x-3)^2+\text{junk} = x^2-6x+9+\text{junk}$$

Cancelling and solving we have junk $= -7$. Substituting that result into the previous, we have

$$3x^2-18x+7 = 3\left((x-3)^2-7\right)+1$$

That's pretty good, but we can gather the constants all together:

$$3x^2-18x+7 = 3(x-3)^2-21+1 = 3(x-3)^2-20$$

You should check by expanding the RHS. The approach in the textbook supplement is more straightforward but not as much fun.

6. We try to solve the equations by factoring.

(a) Factoring, we have

$$x^2+x-6=0 \implies (x+3)(x-2)=0$$

In order for the product $(x+3)(x-2)$ to be zero, one of the factors must be zero, so we have $x+3=0$ which implies $x=-3$, or $x-2=0$ which implies $x=2$. You should check that $x=-3$ and $x=2$ are both solutions to the original equation.

(b) As before, we try to solve by factoring. We guess 1, -1, 3 and -3 as roots, but none of those work. You could spend more time playing around with various attempts at factoring, but at some point you'll have to give up and try the quadratic formula instead. In the equation $3x^2-7x+3=0$ we have $a=3$, $b=-7$, and $c=3$, so plugging in to the quadratic formula we get

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{7\pm\sqrt{49-4(3)(3)}}{2(3)} = \frac{7+\sqrt{13}}{6} \text{ and } \frac{7-\sqrt{13}}{6}$$

It's a lot of work to check those answers by substituting into the original equation, but it would be a great exercise if you have time.

7. We calculate the discriminant $\Delta = b^2 - 4ac$:

   (a)
   $$5x^2 - 8x + 4 \implies a = 5,\ b = -8,\ c = 4 \implies \Delta = 64 - 4(5)(4) = -16$$
   Since the discriminant is negative, the quadratic has no roots so it cannot be factored, i.e., it is irreducible.

   (b)
   $$7x^2 + 3x - 3 \implies a = 7,\ b = 3,\ c = -3 \implies \Delta = 9 - 4(7)(-3) = 93$$
   Since the discriminant is positive, the quadratic has two (distinct) roots so it can be factored, i.e., it is reducible, not irreducible.

8. (a) Using the rule $\sqrt{a}\sqrt{b} = \sqrt{ab}$ we have
   $$\sqrt{45}\sqrt{20} = \sqrt{45 \times 20} = \sqrt{3^2 \times 5 \times 5 \times 2^2} = \sqrt{3^2 \times 5^2 \times 2^2} = 3 \times 5 \times 2 = 30$$

   (b) Using the rule $\sqrt{a}/\sqrt{b} = \sqrt{a/b}$ we have
   $$\frac{\sqrt{32x^4}}{\sqrt{2}} = \sqrt{\frac{32x^4}{2}} = \sqrt{16x^4} = 4x^2$$

9. (a) Using the laws $a^m a^n = a^{m+n}$ and $a^m/a^n = a^{m-n}$ we have
   $$\frac{a^2 \times a^{2n-2}}{a^{n+1} \times a^{n-1}} = \frac{a^{2+2n-2}}{a^{n+1+n-1}} = \frac{a^{2n}}{a^{2n}} = a^{2n-2n} = a^0 = 1$$

   (b) It may help if we re-write $x^{-1} = 1/x$ and $y^{-1} = 1/y$ and then work on the numerator and denominator separately, putting them over a common denominator:
   $$\frac{x^{-1} + y}{x + y^{-1}} = \frac{\frac{1}{x} + y}{x + \frac{1}{y}} = \frac{\frac{1}{x} + \frac{xy}{x}}{\frac{xy}{y} + \frac{1}{y}} = \frac{\frac{1+xy}{x}}{\frac{xy+1}{y}}$$
   Now we invert and multiply:
   $$\frac{\frac{1+xy}{x}}{\frac{xy+1}{y}} = \frac{1+xy}{x} \times \frac{y}{xy+1} = \frac{(1+xy)y}{x(xy+1)} = \frac{y}{x}$$
   where we have cancelled the common factor $1 + xy$. In summary,
   $$\frac{x^{-1} + y}{x + y^{-1}} = \frac{y}{x}$$
   One way to partially check that result might be evaluating the left and right hand sides for various numerical choices of x and y.

10. (a) We multiply and divide by the "conjugate radical" $2 + \sqrt{5}$:
   $$\frac{1}{2 - \sqrt{5}} = \frac{1}{2 - \sqrt{5}} \times \frac{2 + \sqrt{5}}{2 + \sqrt{5}} = \frac{2 + \sqrt{5}}{(2 - \sqrt{5})(2 + \sqrt{5})}$$
   The denominator is the product of a sum and a difference, so is a difference of squares like $(a - b)(a + b) = a^2 - b^2$, and we have
   $$\frac{1}{2 - \sqrt{5}} = \frac{2 + \sqrt{5}}{4 - 5} = \frac{2 + \sqrt{5}}{-1} = -2 - \sqrt{5}$$

(b) As in the previous question, we multiply and divide by the conjugate radical $\sqrt{2+h}+\sqrt{2-h}$:

$$\frac{4}{\sqrt{2+h}-\sqrt{2-h}} = \frac{4}{\sqrt{2+h}-\sqrt{2-h}} \times \frac{\sqrt{2+h}+\sqrt{2-h}}{\sqrt{2+h}+\sqrt{2-h}} = \frac{4(\sqrt{2+h}+\sqrt{2-h})}{(2+h)-(2-h)}$$

Simplifying the denominator and cancelling common factors,

$$\frac{4}{\sqrt{2+h}-\sqrt{2-h}} = \frac{4(\sqrt{2+h}+\sqrt{2-h})}{2h} = \frac{2(\sqrt{2+h}+\sqrt{2-h})}{h}$$

11. (a) By the binomial theorem and the distributive law,

$$(x-2)^2+2x(x+2)(x-4) = (x^2-4x+4)+2x(x^2-4x+2x-8) = x^2-4x+4+2x(x^2-2x-8)$$

Applying the distributive law again and gathering,

$$(x-2)^2+2x(x+2)(x-4) = x^2-4x+4+2x^3-4x^2-16x = 2x^3-3x^2-20x+4$$

(b) We could apply the binomial theorem twice in a clever way, but it is more straightforward just to multiply everything out in one shot:

$$(1-2x+x^2)^2 = (1-2x+x^2)(1-2x+x^2) = 1-2x+x^2-2x+4x^2-2x^3+x^2-2x^3+x^4$$

Gathering,

$$(1-2x+x^2)^2 = 1-4x+6x^2-4x^3+x^4$$

Another, trickier way to solve this problem is to note that $1-2x+x^2=(1-x)^2$ so

$$(1-2x+x^2)^2 = ((1-x)^2)^2 = (1-x)^4 = 1-4x+6x^2-4x^3+x^4$$

by the binomial theorem.

12. (a) We could answer this question like we did with question 9b, but let's try a different method this time known as *clearing fractions*. We multiply the numerator and denominator by whatever is required to eliminate the fraction in the numerator, i.e.,

$$\frac{1+\frac{1}{x-1}}{1+\frac{1}{x+1}} = \frac{1+\frac{1}{x-1}}{1+\frac{1}{x+1}} \times \frac{x-1}{x-1} = \frac{x-1+1}{(x-1)+\frac{x-1}{x+1}} = \frac{x}{(x-1)+\frac{x-1}{x+1}}$$

Similarly, we can clear the fraction in the denominator:

$$\frac{x}{(x-1)+\frac{x-1}{x+1}} = \frac{x}{(x-1)+\frac{x-1}{x+1}} \times \frac{x+1}{x+1} = \frac{x(x+1)}{(x-1)(x+1)+(x-1)} = \frac{x^2+x}{x^2+x-2}$$

(b) Structures like this are known as "continued fractions". We can simplify by starting at the bottom and working our way back up:

$$1-\frac{1}{1-\frac{1}{1-\frac{1}{1-x}}} = 1-\frac{1}{1-\frac{1}{\frac{1-x}{1-x}-\frac{1}{1-x}}} = 1-\frac{1}{1-\frac{1}{\frac{-x}{1-x}}} = 1-\frac{1}{1+\frac{1-x}{x}}$$

Again, we add by rewriting 1 as a fraction with a common denominator:

$$1-\frac{1}{1+\frac{1-x}{x}} = 1-\frac{1}{\frac{x}{x}+\frac{1-x}{x}} = 1-\frac{1}{\frac{1}{x}} = 1-x$$

13. (a) Note that $8 = 2^3$ is a perfect cube, so we can use the difference of cubes pattern $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ to obtain

$$x^3 - 8 = (x-2)(x^2+2x+4)$$

Check by multiplying the RHS. Now we should continue by attempting to factor the quadratic x^2+2x+4. However, note that the discriminant is $\Delta = (2)^2 - 4(1)(4) = 4 - 16 = -12$ so the quadratic is irreducible and we can go no further.

(b) It is generally very difficult to factor cubics unless there is something special about the cubic. In the previous case, the cubic was a difference of cubes. In this case, we use the method we used in question 3b. We test the integer factors of the constant term to see whether any of them is a root of the cubic. The factors of 30 are ±1, ±2, ±3, ±5, ±6, ±10, ±15, and ±30. We quickly find that -2 is a root of the cubic, so we use polynomial division to extract the factor $x+2$:

$$x^3+4x^2-11x-30 = (x+2)(x^2+bx+c) = x^3+bx^2+cx+2x^2+2bx+2c = x^3+(b+2)x^2+(c+2b)x+2c$$

Comparing coefficients, we see that $b = 2$ and $c = -15$, so we have

$$x^3+4x^2-11x-30 = (x+2)(x^2+2x-15)$$

Now we try to factor the remaining quadratic. It is easy to guess factors in this case, numbers that add to 2 and multiply to -15:

$$x^3+4x^2-11x-30 = (x+2)(x+5)(x-3)$$

It doesn't matter how you order the factors, but I prefer to order the factors in order of increasing root:

$$x^3+4x^2-11x-30 = (x+5)(x+2)(x-3)$$

Check by expanding the RHS!

14. (a) We must factor the numerator and the denominator. A factor of x comes immediately out of the numerator to give

$$x^3+x^2-6x = x(x^2+x-6) = (x+3)x(x-2)$$

(remember, I like to order factors by increasing order of the corresponding root). Now to factor the denominator, we should start by trying the roots of the numerator. (If the numerator and denominator don't have a root in common, they don't have a factor in common, and we can't simplify so we could just stop.) Immediately we find that $x = 2$ is a root of the denominator, and extracting that root by polynomial division gives

$$3x^2-8x+4 = (x-2)(ax+b) = ax^2+(b-2a)x-2b$$

which tells us that $a = 3$ and $b = -2$. You should check that

$$3x^2-8x+4 = (3x-2)(x-2)$$

In summary,

$$\frac{x^3+x^2-6x}{3x^2-8x+4} = \frac{(x+3)x(x-2)}{(3x-2)(x-2)} = \frac{(x+3)x}{3x-2}$$

You could expand the numerator, but there's no particular reason to do so, so I would just leave it alone.

(b) We often find that we can make simplifications if we can factor the polynomials involved in expressions, particularly polynomials in the denominator. Note that $x^2 - 6x + 5 = (x-1)(x-5)$ so we have

$$\frac{x}{x-1} + \frac{1}{x^2-6x+5} = \frac{x}{x-1} + \frac{1}{(x-1)(x-5)} = \frac{x(x-5)}{(x-1)(x-5)} + \frac{1}{(x-1)(x-5)} = \frac{x^2-5x+1}{(x-1)(x-5)}$$

Neither $x = 1$ nor $x = 5$ is a root of the numerator, so we cannot simplify any further. You could try factoring the numerator, but in my opinion that's unnecessarily messy.

15. (a) As usual with cubics, we should try the factors of the constant term as roots. The factors of 2 are ± 1, ± 2, and we find that $x = 1$ is a root, so we get

$$x^3 - 3x + 2 = (x-1)(x^2+bx+c) = x^3 + bx^2 + cx - x^2 - bx - c = x^3 + (b-1)x^2 + (c-b)x - c$$

Comparing coefficients gives $b = 1$ and $c = -2$; factoring the resulting quadratic gives

$$x^3 - 3x + 2 = (x-1)(x^2+x-2) = (x-1)(x+2)(x-1) = (x+2)(x-1)^2$$

(Check by expanding the RHS.) So the equation can be written

$$x^3 - 3x + 2 = 0 \implies (x+2)(x-1)^2 = 0$$

from which we see that the solutions to the equation are $x = -2$ and $x = 1$ (which is a "double root").

(b) Again, we try the factors of the constant terms as roots of the cubic. The factors are ± 1, ± 3, and we soon see that $x = 1$ works, so we have

$$x^3 - 6x^2 + 8x - 3 = (x-1)(x^2+bx+c) = x^3 + bx^2 + cx - x^2 - bx - c = x^3 + (b-1)x^2 + (c-b)x - c$$

Comparing coefficients gives $b = -5$, $c = 3$ so

$$x^3 - 6x^2 + 8x - 3 = 0 \implies (x-1)(x^2 - 5x + 3) = 0$$

Unlike the previous problem, in this case we can't factor the remaining quadratic, so we need to use the quadratic formula to get an expression for the roots. In summary, the solutions to $x^3 - 6x^2 + 8x - 3$ are

$$x = 1, \ \frac{5-\sqrt{13}}{2}, \ \frac{5+\sqrt{13}}{2}$$

16. (a) Plugging $n = 5$ into the binomial theorem we get

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

(b) By the binomial theorem we have

$$(x^2-1)^3 = (x^2)^3 + 3(x^2)^2(-1) + 3(x^2)(-1)^2 + (-1)^3$$

By the laws of exponents we have

$$(x^2-1)^3 = x^6 - 3x^4 + 3x^2 - 1$$

17. (a) Multiplying and dividing by the conjugate radical gives

$$\sqrt{x^2+2} - x = (\sqrt{x^2+2} - x) \times \frac{\sqrt{x^2+2}+x}{\sqrt{x^2+2}+x} = \frac{x^2+2-x^2}{\sqrt{x^2+2}+x} = \frac{2}{\sqrt{x^2+2}+x}$$

(b) Again, multiplying and dividing by the conjugate radical gives

$$\sqrt{2x^2+3x}-\sqrt{2x^2-3x}=(\sqrt{2x^2+3x}-\sqrt{2x^2-3x})\times\frac{\sqrt{2x^2+3x}+\sqrt{2x^2-3x}}{\sqrt{2x^2+3x}+\sqrt{2x^2-3x}}$$

Multiplying the numerators,

$$\sqrt{2x^2+3x}-\sqrt{2x^2-3x}=\frac{(2x^2+3x)-(2x^2-3x)}{\sqrt{2x^2+3x}+\sqrt{2x^2-3x}}=\frac{6x}{\sqrt{2x^2+3x}+\sqrt{2x^2-3x}}$$