

MATH 110-004 200730 Quiz 8 Solutions

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1. The translation is straightforward: in the limit Δx becomes dx ; the rest of the summand, $2 - 5(x_i^*)^2 + \cos(x_i^*)$, becomes the integrand $2 - 5x^2 + \cos x$; the summation symbol becomes the integration sign; and the bounds of integration are supplied by the interval over which you integrate. So an answer is

$$\int_0^\pi (2 - 5x^2 + \cos x) dx$$

You can check the answer by forming a Riemann sum for the above definite integral and comparing with the question.

2. The definition of a definite integral says

$$\int_0^1 (x^2 + 3) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n ((x_i^*)^2 + 3) \Delta x$$

If we divide the interval $[0, 1]$ into n equal subdivisions we have $\Delta x = (1 - 0)/n = 1/n$ and endpoints of the intervals given by $x_i = 0 + (i/n)(1 - 0) = i/n$. If we choose as sample points the right-hand endpoints of the intervals we have $x_i^* = x_i$. (Other choices are possible, but unless another choice is specifically requested, the right-hand endpoints are easiest to work with.) Putting all of that information in to the above formula, we have

$$\int_0^1 (x^2 + 3) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\left(\frac{i}{n} \right)^2 + 3 \right) \frac{1}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i^2}{n^3} + \frac{3}{n} \right)$$

Using rules for evaluating summations, and the supplied formulas, we have

$$\int_0^1 (x^2 + 3) dx = \lim_{n \rightarrow \infty} \left(\frac{1}{n^3} \sum_{i=1}^n i^2 + \frac{1}{n} \sum_{i=1}^n 3 \right) = \lim_{n \rightarrow \infty} \left(\frac{1}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{1}{n} 3n \right) = \frac{2}{6} + 3 = \frac{10}{3}$$