MATH 110 Problem Set 1.5b Solutions

Edward Doolittle

Thursday, January 15, 2026

1. The first limit statement means that $f(x)$ can be made arbitrarily close to 3 by taking x close to (but less than) 2. The second limit statement means that $f(x)$ can be made arbitrarily close to 5 by taking x close to (but greater than) 2. It is not possible for $\lim_{x \to 2} f(x)$ to exist, because the two one-sided limits at 2 are not equal.

2. The limit statement means that $f(x)$ can be made arbitrarily large negative by taking x close to (but not equal to) -1.

3. See Figure 1 for the graph. We see from the graph that almost everywhere the left hand limit is equal to the right hand limit, except at $x = -2$. So the only value of a for which $\lim_{x \to a} f(x)$ does not exist is $a = -2$.

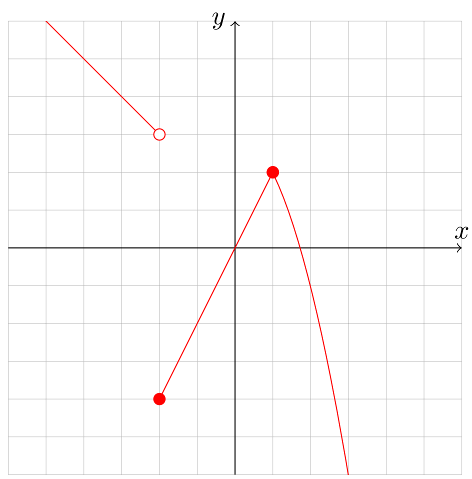
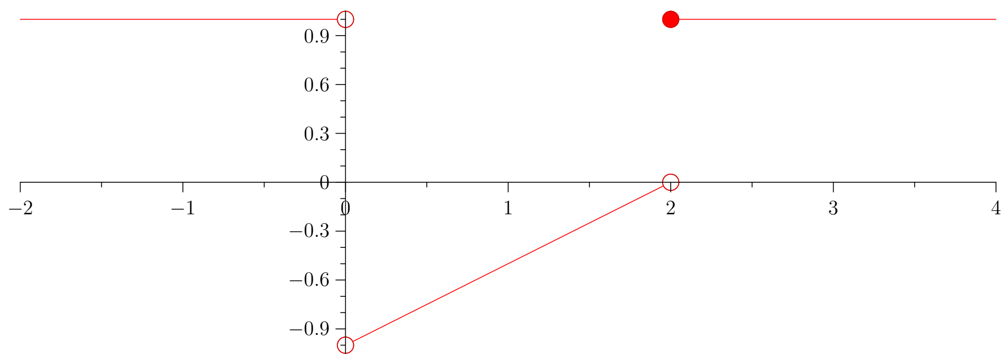
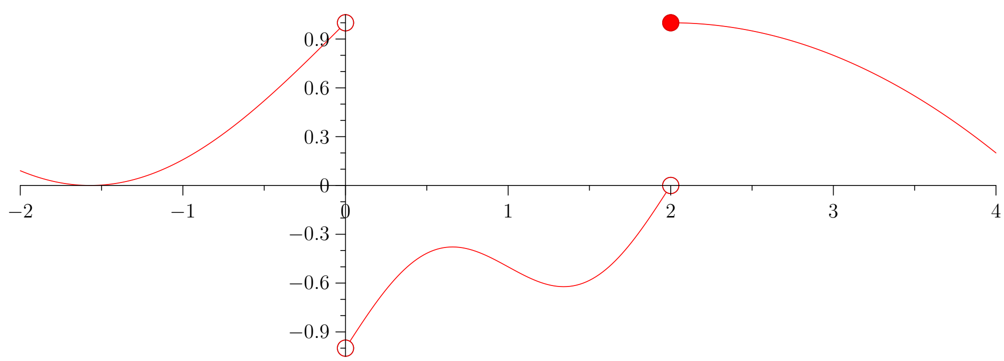


Figure 1: Graph of $y = f(x)$ for question 3

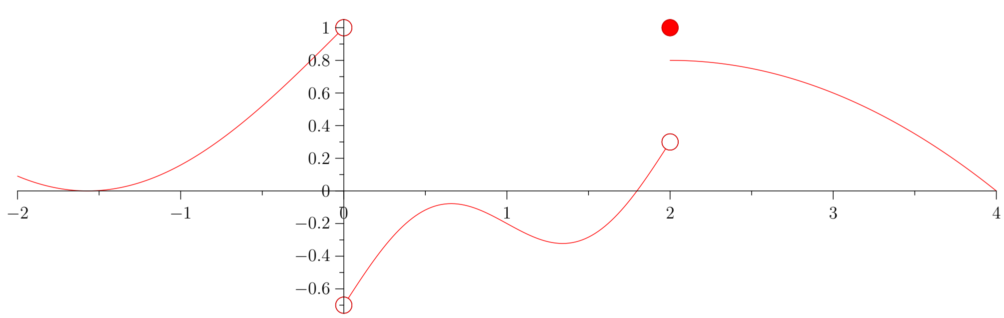
4. The following graph is perhaps the simplest answer to the question. Many others are possible.



The important features of the graph are the four points drawn with open or solid dots at $(0, 1)$, $(0, -1)$, $(2, 0)$, and $(2, 1)$, and the portions of the curves coming out of those points. The exact shapes of the curves are not important, so the following graph is also a correct answer to the question:



However the following graph is not a correct answer to the question because most of the curves don't approach the correct points so the limits don't satisfy the conditions of the problem:



5.

6.

7. (a) The answer is $\displaystyle\lim_{x\to -3^-} \frac{x+2}{x+3} = +\infty$. To obtain the answer, you can draw the graph in Figure 2, or you can argue as follows. Since $x \to -3^-$ we can assume that x is close to -3, so the numerator $x + 2$ is close to -1, a negative number. Furthermore, we can assume that x is slightly less than -3, so $x + 3$ is a small negative number. Dividing -1 by a small negative number gives a large positive number, so $\displaystyle\lim_{x\to -3^-} \frac{x+2}{x+3} = +\infty$.

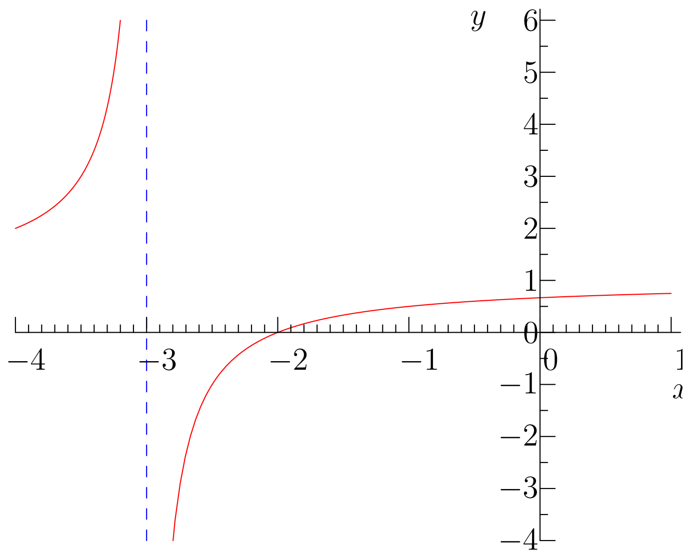


Figure 2: Graph of $y = (x+2)/(x+3)$

(b) You can obtain the answer $\lim_{x\to 0} \frac{x-1}{x^2(x+2)} = -\infty$ from the graph in Figure 3, or you can argue as follows. As $x \to 0$, we can assume x is close to 0, so we have $x-1$ close to -1, $x+2$ close to 2, so $\frac{x-1}{x+2}$ is close to $-1/2$, a negative number. Now let's look at the one-sided limits separately. As $x \to 0^-$ we have x^2 a very small positive number, so $\lim_{x\to 0^-} \frac{x-1}{x^2(x+2)} = -\infty$. Similarly, as $x \to 0^+$ we have x^2 again a very small positive number, so $\lim_{x\to 0^+} \frac{x-1}{x^2(x+2)} = -\infty$. Since both one-sided limits agree, the two sided limit exists and we can say $\lim_{x\to 0} \frac{x-1}{x^2(x+2)} = -\infty$.

The x^2 factor in the denominator ensures that the two-sided limit exists. What if x^2 were replaced by x, x^3, or x^4?

(c) Recall that $\cot x = \frac{\cos x}{\sin x}$. Since $x \to \pi^-$ we can assume that x is close to π so $\cos x$ is close to -1 and $\sin x$ is close to 0. Furthermore, we can assume that x is slightly less than π, so $\sin x$ is small and positive. A negative number divided by a small positive number gives a large negative number, so we conclude that $\lim_{x\to\pi^-} \cot x = -\infty$.

(d) It is generally a good idea in these sorts of problems to factor the numerator and denominator if possible. In this case we have $\frac{x^2-2x}{x^2-4x+4} = \frac{x(x-2)}{(x-2)^2}$. Since we are taking the limit $x \to 2^-$ we can assume that $x \neq 2$ so we can cancel a factor of $x-2$ from the numerator and denominator, giving $\lim_{x\to 2^-} \frac{x^2-2x}{x^2-4x+4} = \lim_{x\to 2^-} \frac{x}{x-2}$.

Now we are in a situation very similar to that of part (a). An analysis similar that of part (a) tells us that the numerator x is close to 2, a positive number, the denominator $x-2$ is close to 0 and negative, so the quotient is a large negative number, i.e. $\lim_{x\to 2^-} \frac{x}{x-2} = -\infty$.

8. The function tends to a finite limit everywhere except possibly where the denominator is 0. So our candidates for asymptotes are where $3x - 2x^2 = 0$, i.e., $x = 0$ and $x = 3/2$.

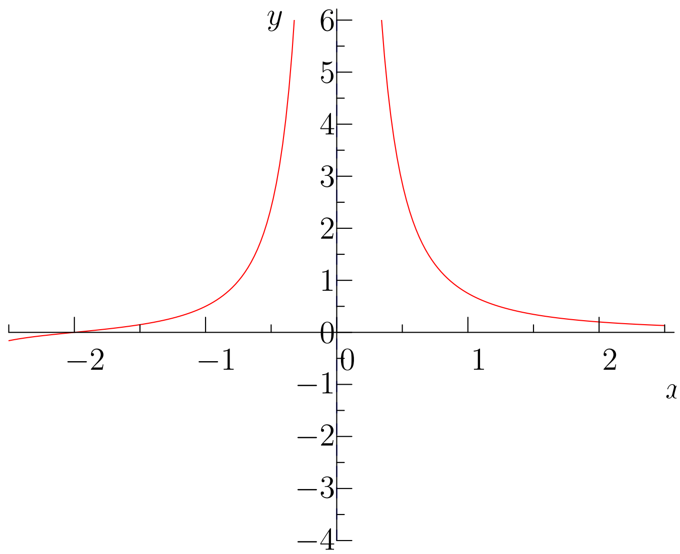


Figure 3: Graph of $y = (x-1)/(x^2(x+2))$

Just to the left of $x = 0$ the numerator is close to 1, a positive number, and the denominator $x(3-2x)$ is close to $x(3)$, a small negative number, so we have $\lim_{x \to 0^-} \frac{x^2+1}{3x-2x^2} = -\infty$. Just to the right of $x = 0$ the numerator is still close to 1 and the denominator $x(3-2x)$ is close to $x(3)$, a small positive number, so $\lim_{x \to 0^+} \frac{x^2+1}{3x-2x^2} = +\infty$.

Just to the left of $x = 3/2$, the numerator is positive and the denominator $x(3-2x)$ is small and positive, so $\lim_{x \to \frac{3}{2}^-} \frac{x^2+1}{3x-2x^2} = +\infty$, and just to the right of $x = 3/2$, the numerator is still positive and the denominator $x(3-2x)$ is small and negative, so $\lim_{x \to \frac{3}{2}^+} \frac{x^2+1}{3x-2x^2} = -\infty$.

In summary, the function $\frac{x^2+1}{3x-2x^2}$ has asymptotes at $x = 0$ and $x = 3/2$.

9.