MATH 110 Lecture 5.1

Areas Between Curves

Edward Doolittle

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Department of Indigenous Knowledge and Science
First Nations University of Canada

Contents

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- Now we look at a more sophisticated version of the area problem: the area between two curves.
- Next section we will look at a non-geometric application of integration.

The Area Between Curves

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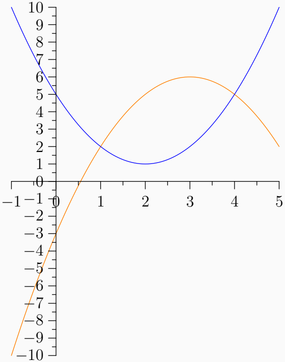

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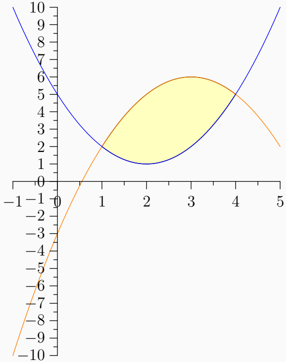

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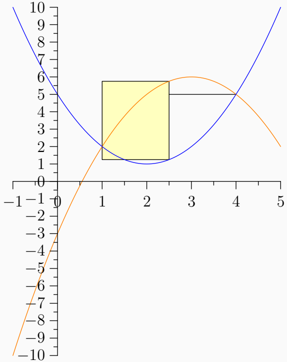

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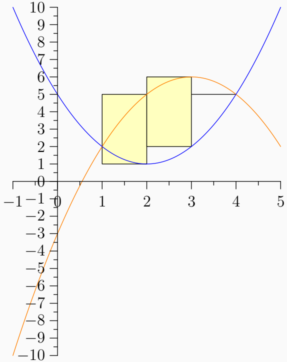

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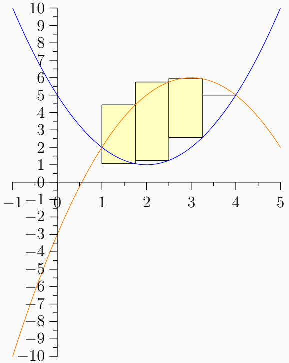

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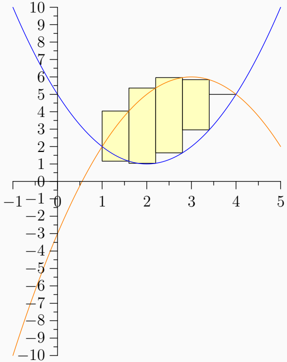

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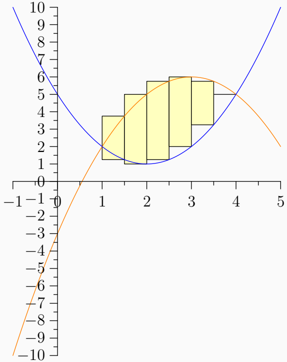

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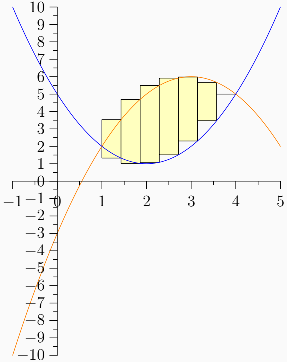

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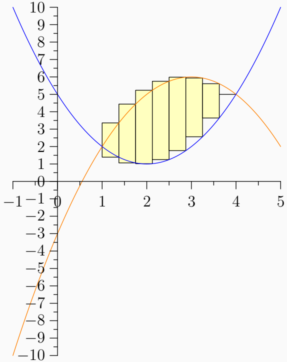

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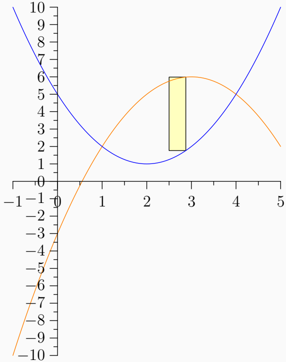
- Here the width of the rectangles are equal, and their heights are given by the height of the region at the right-hand side of the rectangles.
- We would like to be able to express the area of the region as an integral.

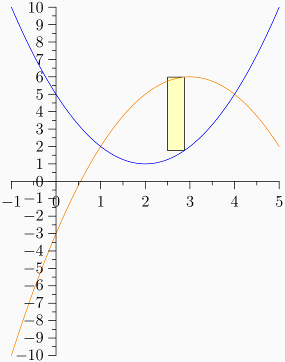
The Area Between Two Curves as an Integral

- In order to construct an integral for the area between two curves, we first find the area of one of the “area elements”.



The Area Between Two Curves as an Integral

- In order to construct an integral for the area between two curves, we first find the area of one of the "area elements".
- The width of such a rectangle is Δx which is $1/n$ times the width of the region.



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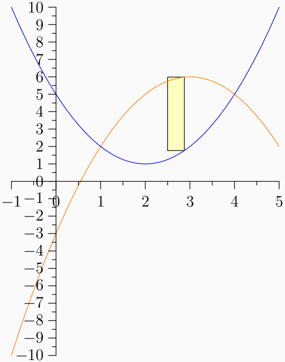

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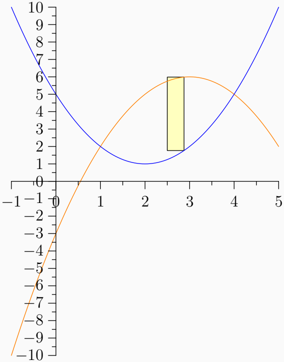
- The area of an area element is $(f(x_i) - g(x_i))\Delta x$.



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- The area of an area element is $(f(x_i) - g(x_i))\Delta x$.
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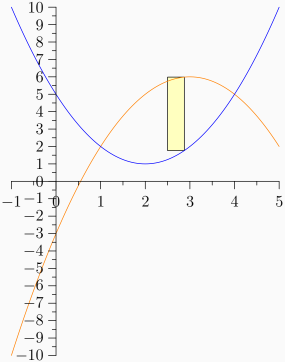

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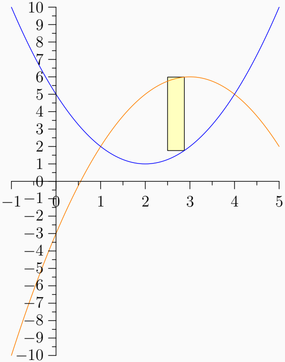
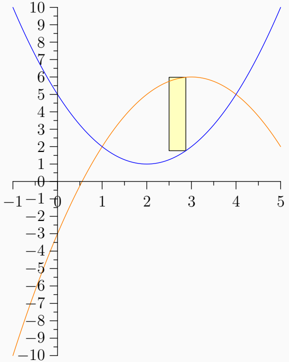
- So the area of the region should be



Evaluating The Area Between Two Curves as an Integral

- In the previous slide we concluded that the area of the region bounded by f and g is $\int_a^b (f(x) - g(x))\,dx$.



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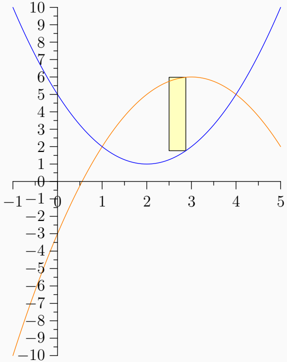

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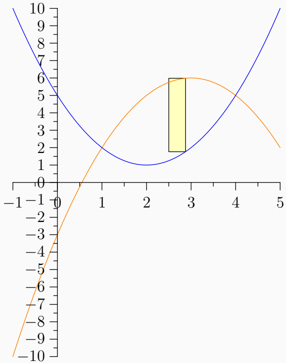

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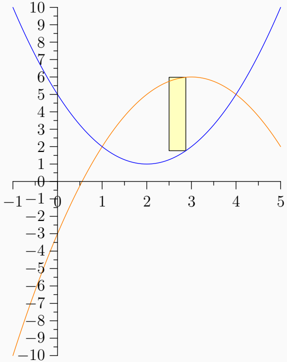
- We do this by solving the equation $f(x) = g(x)$.
- In this case, $f(x) = -x^2 + 6x - 3$ and $g(x) = x^2 - 4x + 5$, so $f(x) = g(x)$ becomes $-x^2 + 6x - 3 = x^2 - 4x + 5$ or $x^2 - 5x + 4 = 0$ with solutions $x = 1$ and $x = 4$.



Areas Between Curves with Several Intersections

EJD

Integrating Against y

EJD

Examples

1. Find the area of the region bounded by the given curves.
   1.1 $y = 20 - x^2$, $y = x^2 - 12$
   1.2 $x + y = 0$, $x = y^2 + 3y$
   1.3 $y = \sqrt{x}$, $y = x^2$, $x = 2$
   1.4 $y = \sin(\pi x/2)$, $y = x^2 - 2x$
2. Find the area of the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$.
3. Approximate the area of the region bounded by the curves $y = \tan(x^2)$, $x = 1$, and $y = 0$.

Exercises

Now you should work on Problem Set 5.1. After you have finished it, you should try the following additional exercises from Section 5.1:

5.1 C-level: 1–28, 29, 30–32;
B-level: 33–38, 46–50;
A-level: 53–60.