MATH 110 Problem Set 4.1 Solutions

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1. (a) By the definition of area, the area under the curve $y = x^3$ from $x = 0$ to $x = 1$ is

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} x_i^3 \, \Delta x$$

where $\Delta x = (b - a)/n = (1 - 0)/n = 1/n$ and $x_i = a + i\Delta x = 0 + i \cdot 1/n = i/n$:

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

(b) Evaluating the above limit, we have

$$\begin{aligned}
A &= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^3} \frac{1}{n} \\
&= \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} i^3 \\
&= \lim_{n\to\infty} \frac{1}{n^4} \frac{n^4 + 2n^3 + n^2}{4} \\
&= \lim_{n\to\infty} \frac{n^4 + 2n^3 + n^2}{4n^4} \\
&= \lim_{n\to\infty} \frac{(n^4 + 2n^3 + n^2)\frac{1}{n^4}}{4n^4 \frac{1}{n^4}} \\
&= \lim_{n\to\infty} \frac{1 + 2/n + 1/n^2}{4} \\
&= \frac{1 + 0 + 0}{4} = \frac{1}{4}
\end{aligned}$$

2. We need to make an educated guess about which part of the summand is Δx and which part is $f(x_i)$. Let's guess $\Delta x = 2/n$ and $f(x_i) = (5+2i/n)^{10}$. That would mean that $f(x) = x^{10}$ and $x_i = 5+2i/n = 5+i\Delta x$. Since $x_i = a+i\Delta x$ that means $a = 5$. Since $\Delta x = (b - a)/n = 2/n$ that means $b - a = 2$ which implies $b = a + 2 = 5 + 2 = 7$. Checking, the area is

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\, \Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} (a + i\Delta x)^{10}\, \Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(5 + i\frac{2}{n}\right)^{10} \frac{2}{n}$$

so our guesses were correct. Then the region which answers the question is the region bounded by the curve $y = x^{10}$ and the lines $x = 5$, $x = 7$, and $y = 0$.