MATH 110 Review Problem Set 0.B Solutions

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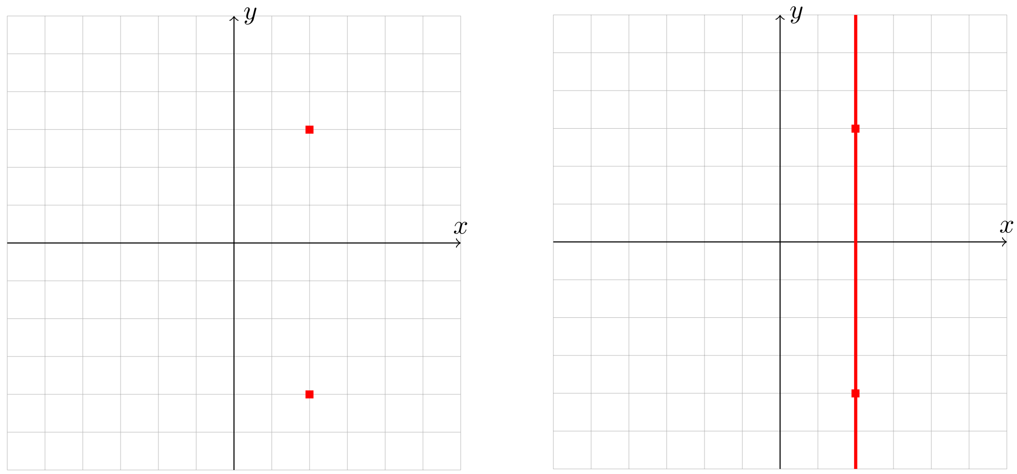
1. (a) We have $(x_1, y_1) = (1, 2)$, $(x_2, y_2) = (4, 6)$, so $x_2 - x_1 = 3$, $y_2 - y_1 = 4$, and the distance between the points is
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$
   (b) We have to be careful with the negative numbers. $x_2 - x_1 = -3 - 2 = -5$, $y_2 - y_1 = 7 - (-5) = 12$, and the distance is
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-5)^2 + (12)^2} = 13$$
   (c) The distance is
$$\sqrt{(-2 - (-1))^2 + (6 - (-3))^2} = \sqrt{(-1)^2 + (9)^2} = \sqrt{82}$$
   (d) The distance is $\sqrt{(a - b)^2 + (b - a)^2}$. You will usually see expressions like that simplified as follows: note that $(b - a) = -(a - b)$ so $(b - a)^2 = (-(a - b))^2 = (a - b)^2$ and we have
$$\sqrt{(a - b)^2 + (b - a)^2} = \sqrt{2(a - b)^2} = \sqrt{2}\,\sqrt{(a - b)^2} = \sqrt{2}\,|a - b|$$
   Note the appearance of the absolute value symbol.

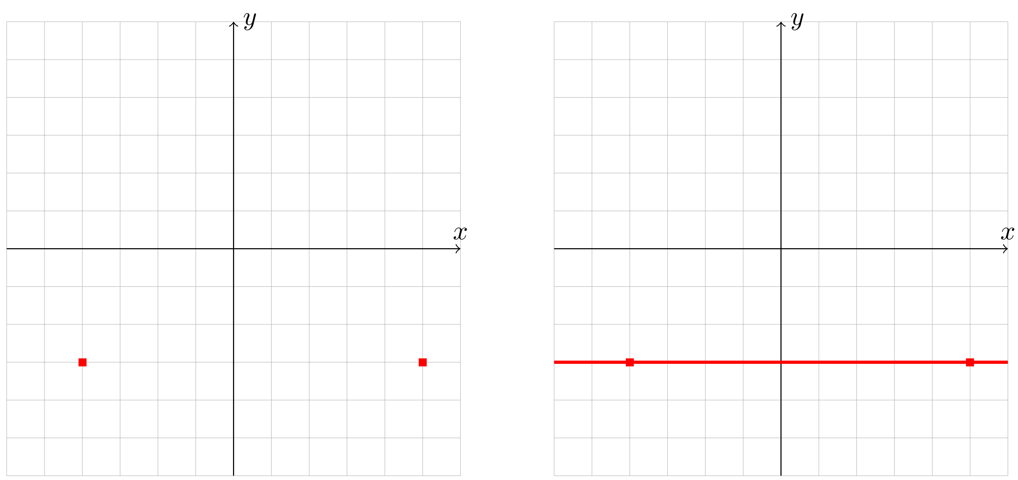
2. (a) As before we have $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (5, 9)$, so the differences are $x_2 - x_1 = 5 - 2 = 3$, $y_2 - y_1 = 9 - 4 = 5$ and the slope is
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5}{3}$$
   (b) Again as before, we should be careful with the negatives, and also with the order of the terms in the subtraction: always $x_2 - x_1$ in the denominator, and always $y_2 - y_1$ in the numerator. The slope is
$$m = \frac{-6 - 5}{2 - (-3)} = \frac{-11}{5}$$
   (c) The slope is
$$m = \frac{-7 - 0}{-3 - (-1)} = \frac{-7}{-2} = \frac{7}{2}$$

3. (a) This is a vertical line consisting of all points with $x = 2$, so it passes through the points $(2, 3)$ and $(2, -4)$ for example. (Any other points of the form $(2, y)$ would do.) We graph those two points on the plane, see Figure 1(a), and then draw a straight line through them, see Figure 1(b). (Of course, you don't have to first find two points if you're using grid paper and you recognize that the line is a vertical grid line.)
   (b) This is similar to the previous problem. Choose two points on the horizontal line $y = -3$, say $(-4, -3)$ and $(5, -3)$, then draw the line connecting them. See Figure 2.



(a) Two points on the line (b) Completed graph

Figure 1: Graph of $x = 2$



(a) Two points on the line (b) Completed graph

Figure 2: Graph of $y = -3$

4. (a) Since we have a point and a slope, we use point-slope form:

$$y - y_1 = m(x - x_1) \implies y - 1 = -\frac{5}{3}(x - (-3)) \implies y - 1 = -\frac{5}{3}(x + 3)$$

The final simplification is optional.

(b) There are several ways to handle this problem, but I recommend first finding the slope of the line between the points:

$$m = \frac{-2 - (-3)}{-4 - 1} = \frac{-2 + 3}{-4 - 1} = \frac{1}{-5} = -\frac{1}{5}$$

Now using the point $P(1, -3)$ as our point and $m = -1/5$ as our slope, we can use point-slope form:

$$y - y_1 = m(x - x_1) \implies y - (-3) = -\frac{1}{5}(x - 1) \implies y + 3 = -\frac{1}{5}(x - 1)$$

The latter simplification is optional in this context.

(c) We use the slope-intercept form with $m = 2$ and $b = 5$:

$$y = mx + b \implies y = 2x + 5$$

(d) If the x-intercept is 4, that means the line passes through the point $P(4, 0)$. Similarly, if the y-intercept is -3, that means the line passes through the point $Q(0, -3)$. Next we find the slope:

$$m = \frac{-3 - 0}{0 - 4} = \frac{-3}{-4} = \frac{3}{4}$$

Now we can use the point-slope form:

$$y - y_1 = m(x - x_1) \implies y - 0 = \frac{3}{4}(x - 4) \implies y = \frac{3}{4}(x - 4)$$

(e) You could use point-slope form, recalling that any line parallel to the x-axis has slope 0:

$$y - y_1 = m(x - x_1) \implies y - (-4) = 0(x - 1) \implies y + 4 = 0 \implies y = -4$$

Either of the last two equations is acceptable as an answer.

(f) The best way to answer this question is just to remember that all lines parallel to the y-axis have equation of the form $x = a$. Since the line passes through the point $(x, y) = (1, -4)$, we must have $a = 1$. So the equation of the line is $x = 1$.

(g) The slope of the given line can be found by putting it into slope-intercept form (solving for y). We have

$$4x - 5y = 7 \implies 5y = 4x - 7 \implies y = \frac{4}{5}x - \frac{7}{5}$$

It follows that the slope of the given line is $4/5$. Since parallel lines have the same slope, it follows that the slope of the unknown line is also $4/5$. Now we have a point on the unknown line, $P(2, -3)$, and its slope, $m = 4/5$, so we can use point-slope form to write an equation for the line:

$$y - y_1 = m(x - x_1) \implies y - (-3) = \frac{4}{5}(x - 2) \implies y + 3 = \frac{4}{5}(x - 2)$$

where the latter simplification is optional.

(h) Again, we first find the slope of the given line by solving for y:

$$3x + 7y = 2 \implies 7y = -3x + 2 \implies y = -\frac{3}{7}x + \frac{2}{7}$$

so the slope of the given line is $-3/7$. The slope of any perpendicular line is the negative reciprocal of that number so we can calculate

$$m = -\frac{1}{-3/7} = -1 \times -\frac{7}{3} = \frac{7}{3}$$

(The end result can be obtained quickly by changing the sign (negative) and flipping the fraction over (reciprocal).) Now we have a point on the unknown line, $P(-1/2, 5/3)$, and its slope, $7/3$, so we can write an equation for the line in point-slope form:

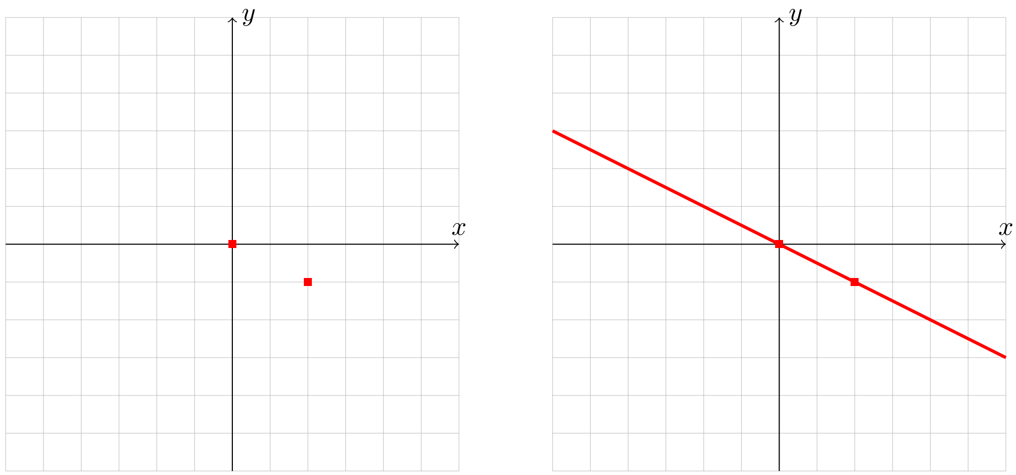
$$y - y_1 = m(x - x_1) \implies y - \frac{5}{3} = \frac{7}{3}\left(x - \left(-\frac{1}{2}\right)\right) \implies y - \frac{5}{3} = \frac{7}{3}\left(x + \frac{1}{2}\right)$$

where the latter simplification is optional. (Other optional simplifications may be helpful: you may find it easier to work with such an equation, for example, if you "clear fractions" by multiplying every term by 6.)

5. (a) To find the slope and y-intercept, we put the equation of the line into slope-intercept form by solving for y:

$$x + 2y = 0 \implies y = -\frac{1}{2}x + 0$$

so the slope is $m = -1/2$ and the y-intercept is $(0, b) = (0, 0)$. A second point could be found by substituting (say) $x = 2$ into the equation of the line to obtain $y = (-1/2)(2) + 0 = -1$ so $(2, -1)$ is a point on the line. Plot the points $(0, 0)$ and $(2, -1)$ and draw a line through them. See Figurefig:x+2y=0.



(a) Two points on the line

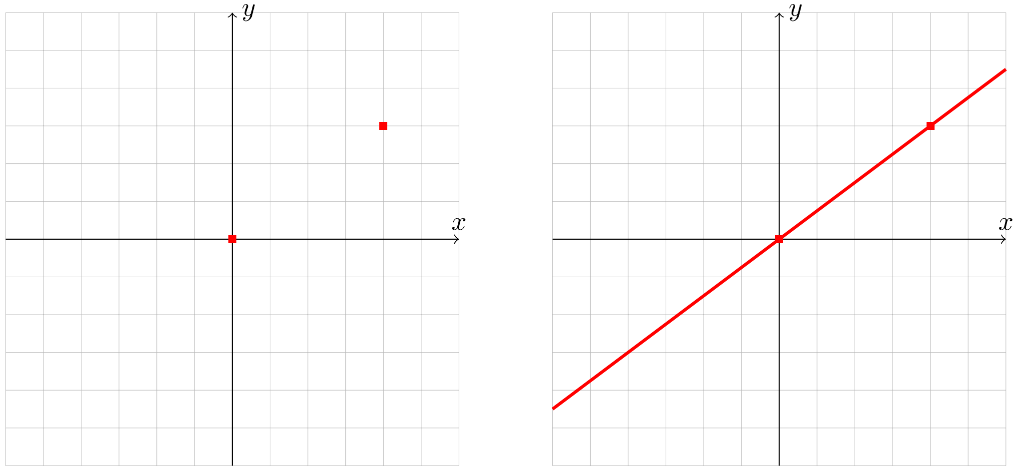
(b) Completed graph

Figure 3: Graph of $x + 2y = 0$

(b) Putting the equation into slope-intercept form we have

$$3x - 4y = 0 \implies 4y = 3x \implies y = \frac{3}{4}x + 0$$

so the slope is $m = 3/4$ and the y-intercept is $(0, 0)$. We need a second point on the graph to determine the line. We could solve as we solved the previous, or we could reason like this: since the slope is $3/4$, there is a rise of 3 units for every run of 4 units. Starting at $(0, 0)$, we rise by 3 and run by 4 to get to $(4, 3)$, which is therefore a second point on the graph. We plot those to points and draw a line through them. See Figure 4.



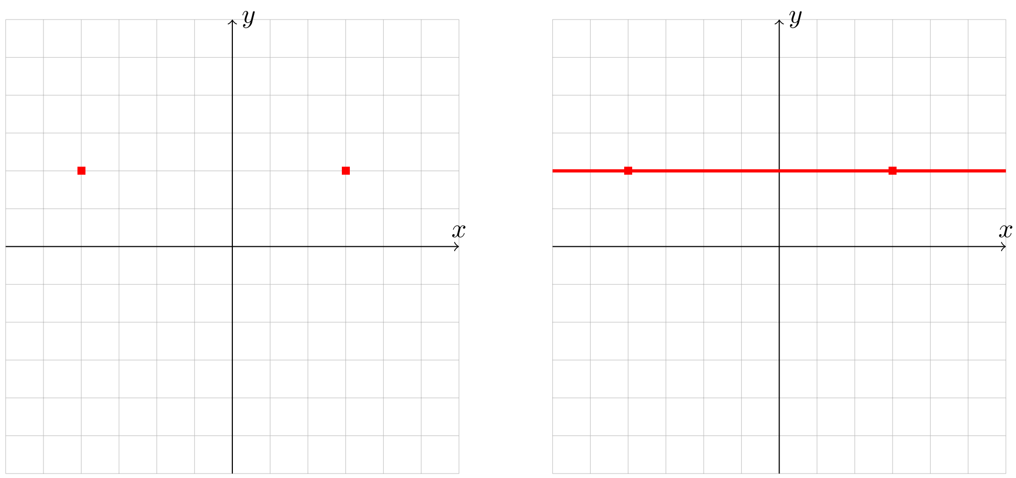
(a) Two points on the line (b) Completed graph

Figure 4: Graph of $3x - 4y = 0$

(c) Writing in slope-intercept form, we have

$$y = 2 \implies y = 0x + 2$$

so its slope is 0 and y-intercept is $(0, 2)$. This is a line in horizontal form, so it's easy to graph on grid paper (it is the grid line parallel to the x-axis and passing through $(0, 2)$). Or you could pick any two points of the form $(x, 2)$, e.g., $(-4, 2)$, $(3, 2)$, and draw the line through them. See Figure 5.



(a) Two points on the line (b) Completed graph

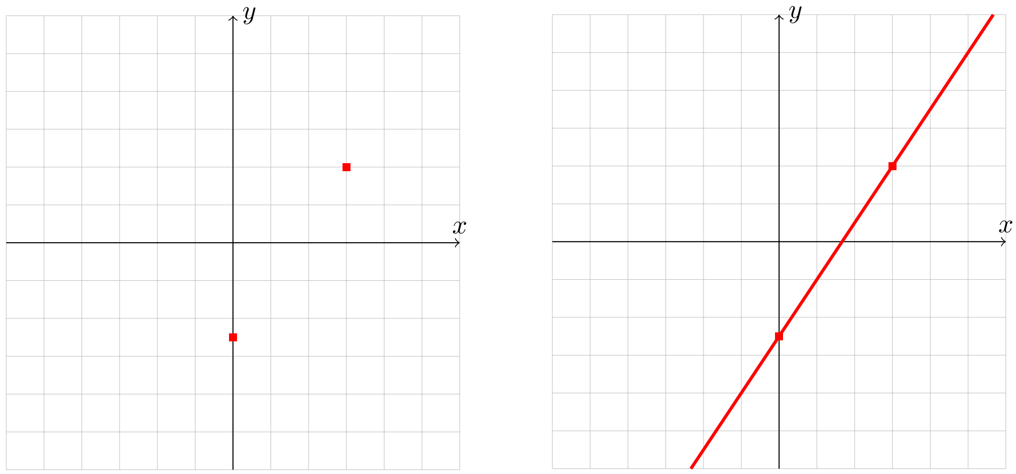
Figure 5: Graph of $y = 2$

(d) The equation is given in general form. Writing in slope-intercept form by solving for y,

$$3x - 2y - 5 = 0 \implies 2y = 3x - 5 \implies y = \frac{3}{2}x - \frac{5}{2}$$

so the slope is $m = 3/2$ and the y-intercept is $(0, b) = (0, -5/2)$. We can find a second point on the line by substituting $x = 3$ (say) to obtain $y = (3/2)(3) - (5/2) = 9/2 - 5/2 = 4/2 = 2$, so

$(3, 2)$ is a second point on the line. We plot those points and draw the line through them. See Figure 6

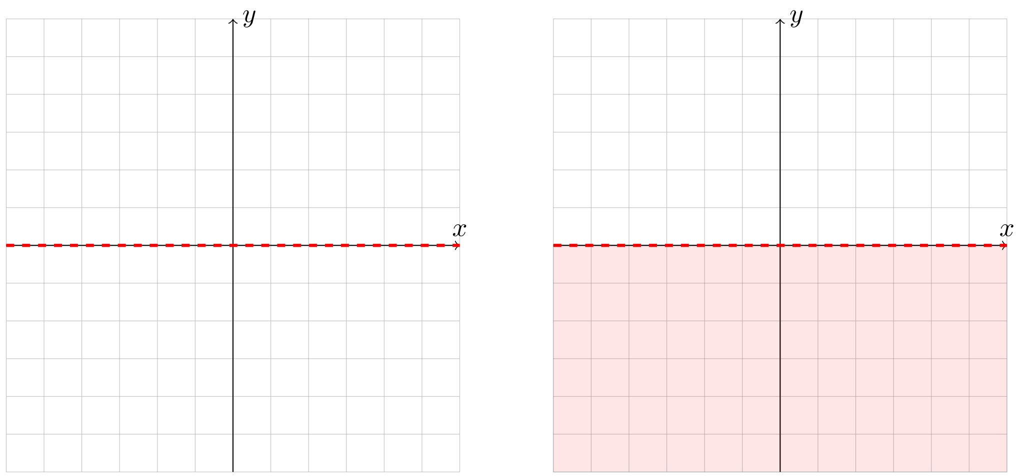


(a) Two points on the line

(b) Completed graph

Figure 6: Graph of $3x - 2y - 5 = 0$

6. (a) We graph the line $y = 0$ with a dashed line (because the intequality is strict, i.e., $y < 0$ not $y \leq 0$) and since y is less than the expression that defines the line we shade all the points below the line. See Figure 7.

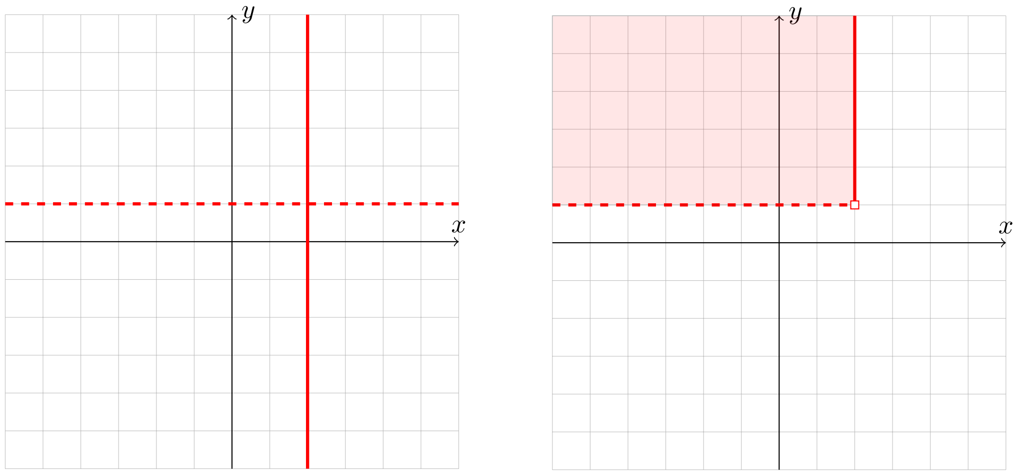


(a) Boundary $y = 0$ of the region

(b) Completed graph

Figure 7: Graph of $y < 0$

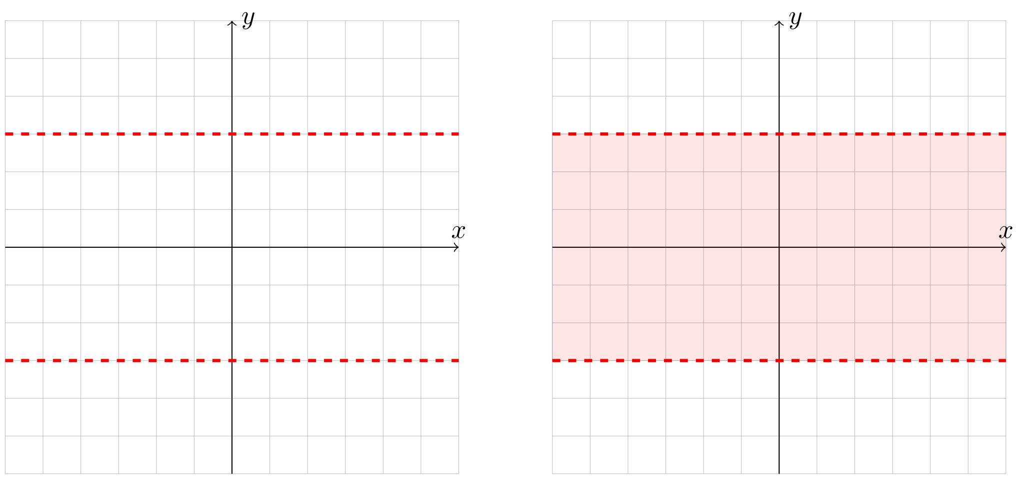
(b) First we graph the vertical line $x = 2$ with a solid line. Then we shade all points to the left of the line, which gives us $x \leq 2$. Then we graph the horizontal line $y = 1$ with a dashed line. Then we shade all points to the abve the line, which gives us $y > 1$. All points which are "double shaded" are part of the final answer. Note that the corner $(2, 1)$ is not part of the final answer (why not?), so we draw it with an open circle for emphasis. See Figure 8.



(a) Boundary of the region (b) Completed graph

Figure 8: Graph of $x \leq 2$ and $y > 1$

(c) The condition without absolute values translates to "$-3 < y$ and $y < 3$". We graph each of those regions by the method of the previous problems and combine them to obtain the final result. See Figure 9.



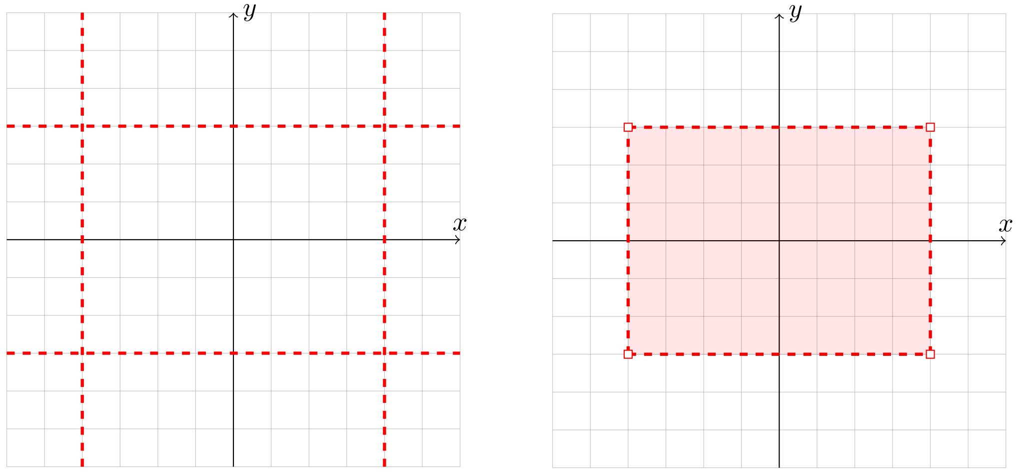
(a) Boundary of the region (b) Completed graph

Figure 9: Graph of $|y| < 3$

(d) The first condition translates to "$-4 < x$ and $x < 4$", while the second condition translates to "$-3 < y$ and $y < 3$". We graph each of those regions, and the answer is the resulting "quadruple-shaded" region, also known as an "open box" (*open* because it does not include its boundary).

7. The distances are the same because

$$|AC| = \sqrt{(-3-1)^2 + (1-0)^2} = \sqrt{16+1} = \sqrt{17}$$
$$|BC| = \sqrt{(-3-(-2))^2 + (1-(-3))^2} = \sqrt{1+16} = \sqrt{17}$$



(a) Boundary of the region

(b) Completed graph

Figure 10: Graph of $|x| < 4$ and $|y| < 3$

8. (a) We have

$$
\begin{aligned}
|AB| &= \sqrt{(0-(-4))^2 + (-5-7)^2} = \sqrt{16+144} = \sqrt{160} = \sqrt{16}\sqrt{10} = 4\sqrt{10} \\
|BC| &= \sqrt{(2-0)^2 + (-11-(-5))^2} = \sqrt{4+36} = \sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10} \\
|AC| &= \sqrt{(2-(-4))^2 + (-11-7)^2} = \sqrt{36+324} = \sqrt{360} = \sqrt{36}\sqrt{10} = 6\sqrt{10}
\end{aligned}
$$

So we see that $|AB| + |BC| = 4\sqrt{10} + 2\sqrt{10} = 6\sqrt{10} = |AC|$. If B were not directly between A and C, the trip from A through B to C would be longer than the trip directly from A to C.

(b) Use slopes to show that A, B, and C are collinear. The slopes of the lines are

$$
\begin{aligned}
\text{slope}(AB) &= \frac{-5-7}{0-(-4)} = \frac{-12}{4} = -3 \\
\text{slope}(BC) &= \frac{-11-(-5)}{2-0} = \frac{-6}{2} = -3
\end{aligned}
$$

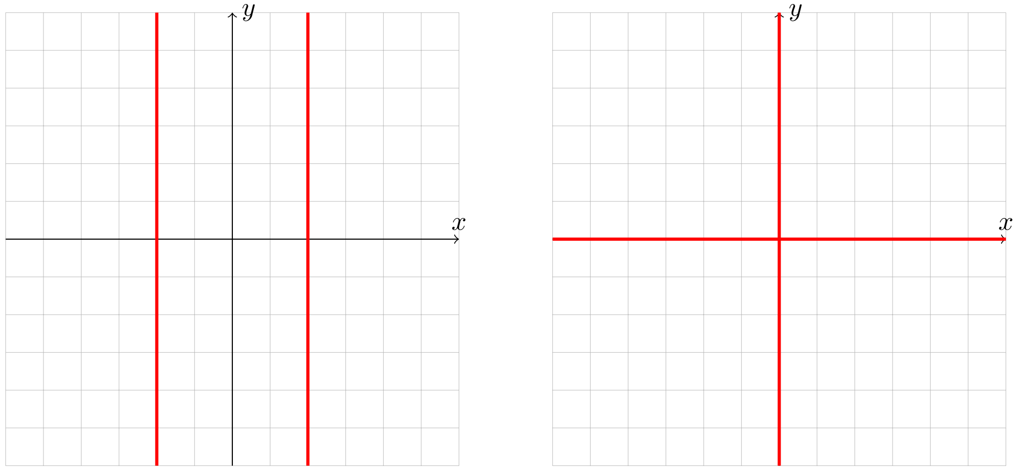
If B did not lie on the line betwen A and C, there would be a "kink" on the path from A to B to C and the slopes would be different. Since the slopes are the same, B does lie on the line from A to C.

9. (a) This is similar to question 6. The condition translates to "$x = -2$ or $x = 2$", so we graph those two vertical lines. See Figure 11(a).

(b) Factoring, the condition $xy = 0$ reduces to "$x = 0$ or $y = 0$". The line $x = 0$ is the y-axis, and the line $y = 0$ is the x-axis. See Figure 11(b).

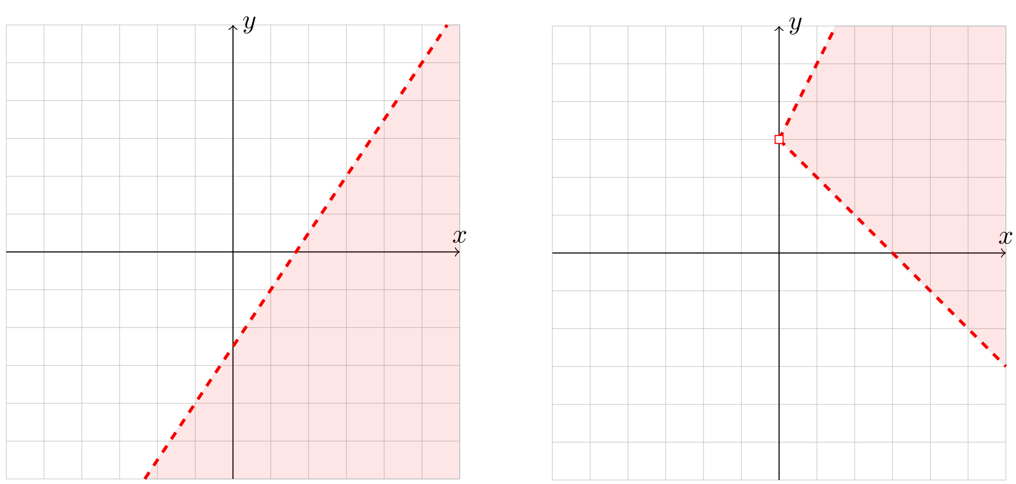
10. (a) Solving for y, the condition is $y < (3/2)x - (5/2)$. We graphed this line in question 5d, so we can just re-draw the graph (but with a dotted line). Then we shade the region below the line. See Figure 12(a).

(b) The two conditions are $y > 3 - x$ and $y < 2x + 3$. We graph each of those lines with dashed lines, shade the region above the first and below the second, and the final result is the region that is double-shaded. See Figure 12(b). Note that I have also marked the intersection of the two lines, which has coordinate $(0, 3)$. (How can you find that intersection?)



(a) Graph of $|x| = 2$ (b) Graph of $xy = 0$

Figure 11: Two graphs



Graph of $2y < 3x - 5$ Graph of $3 - x < y < 3 + 2x$

Figure 12: Two graphs

11. Points on the y-axis are of the form $(0, y)$. The distance from such a point to $(4, -3)$ is

$$\sqrt{(-3-y)^2+(4-0)^2} = \sqrt{(y+3)^2+16} = \sqrt{y^2+6y+25}$$

The distance from such a point to $(2, 5)$ is

$$\sqrt{(5-y)^2+(2-0)^2} = \sqrt{y^2-10y+29}$$

If the point $(0, y)$ is equidistance from the two given points, we must have

$$\sqrt{y^2+6y+25} = \sqrt{y^2-10y+29}$$

Squaring both sides, we get

$$y^2+6y+25 = y^2-10y+29 \implies 16y = 4 \implies y = \frac{1}{4}$$

So the point $(0, 1/4)$ satisfies the conditions of the problem. (Check!)

If you are familiar with geometry, another (better) way to solve this problem would be to find the intersection of two lines, the y-axis and the perpendicular bisector of the segment connecting the two points.