MATH 110 Quiz 1 Solutions

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1. (a) The table is as follows. The $9 - x$ column is trivial, but you might want to double check with a calculator just in case. be powers of 10).

x	$9 - x$	$\sqrt{x} - 3$	$f(x)$
10.00	-1.0000	+0.1623	-6.1623
9.10	-0.1000	+0.01622	-6.0166
9.01	-0.01000	+0.001666	-6.0017
8.99	+0.01000	-0.001667	-5.9983
8.90	+0.1000	-0.01671	-5.9833
8.00	+1.0000	-0.1716	-5.8284

Note that to have four decimal points of accuracy in the final column, I had to keep more than four decimal points in the intermediate calculations; I had to keep four significant figures (five would have been better).

   (b) Based on the above table, a reasonable guess for the limit would be a number about halfway between -6.0017 and -5.9983, i.e., about -6.0000 to four decimal places. (Using limit theorems, we can now show that that is the exact answer.)

2. Candidates for vertical asymptotes of rational functions are vertical lines over x values at which the denominator goes to 0. Factoring the denominator, we have $y = \dfrac{x+3}{(x+3)(x-2)}$ so our candidates for asymptotes are the lines $x = -3$ and $x = 2$. The rational function may have a removable discontinuity at those x values rather than an infinite discontinuity, however, so we must check (one-sided) limits as x approaches those candidate values.

   For $\lim\limits_{x\to -3^-}$ we have $(x+3)/(x+3) = 1$ and $x-2$ non-zero (in fact, close to -5), so $\lim\limits_{x\to -3^-} \dfrac{x+3}{(x+3)(x-2)}$ is not infinite (in fact, close to $-1/5$).

   For $\lim\limits_{x\to -3^+}$, againg $(x+3)/(x+3)$ and $x-2$ is non-zero (in fact, close to -5), so $\lim\limits_{x\to -3^+} \dfrac{x+1}{(x+3)(x-2)}$ is not infinite (in fact, close to $-1/5$).

   For $\lim\limits_{x\to 2^-}$, $(x+3)/(x+3) = 1$ and $x-2$ is negative and close to zero, so we have 1 divided by a number close to 0 but negative, and overall we have a limit of $-\infty$. For $\lim\limits_{x\to 2^+}$ we have a limit of $+\infty$.

   In summary, there is only one vertical asymptote, at $x = 2$. See the graph in Figure 1. Note that there is a hole in the graph at $x = -3$ but no asymptote there.

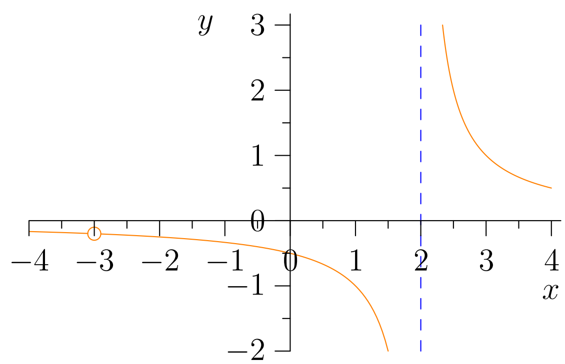

Figure 1: Graph of $y = \dfrac{x+3}{x^2+x-6}$