MATH110–S01–S02 200930 Quiz 3 Solutions

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1. To find the equation of the tangent line in point-slope form we need to find a point (x_0, y_0) through which the tangent line passes and we need to find the slope of the tangent line. The point is given: $(4, 10)$ (though it's a good idea to check that that point really is on the curve). So we now need to find the slope of the tangent line. The slope of the tangent line is the derivative y' evaluated at $x = 4$. By the product rule we have

$$y' = \frac{d}{dx}\left(x^{1/2}(x+1)\right) = \left(\frac{d}{dx}x^{1/2}\right)(x+1) + x^{1/2}\left(\frac{d}{dx}(x+1)\right) = \frac{1}{2}x^{-1/2}(x+1) + x^{1/2} \cdot 1$$

so

$$m = y'(4) = \frac{1}{2}4^{-1/2}(4+1) + 4^{1/2} \cdot 1 = \frac{1}{2}\frac{1}{\sqrt{4}}(5) + \sqrt{4} = \frac{5}{4} + 2 = \frac{13}{4}$$

Assembling all the information, the equation of the tangent line is

$$y - 10 = \frac{13}{4}(x-4)$$

2. The first derivative of f is

$$f'(x) = \frac{\left(\frac{d}{dx}1\right)(2-x) - 1\frac{d}{dx}(2-x)}{(2-x)^2} = \frac{1}{(2-x)^2}$$

so the second derivative of f is

$$f''(x) = \frac{d}{dx}\frac{1}{(2-x)^2} = \frac{\left(\frac{d}{dx}1\right)(2-x)^2 - 1\frac{d}{dx}(2-x)^2}{(2-x)^4}$$

$$= \frac{-2(2-x)\frac{d}{dx}(2-x)}{(2-x)^4} = \frac{2(2-x)}{(2-x)^4} = \frac{2}{(2-x)^3}$$