UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 110–003–004 200730

Midterm Test 1 (A version)

Time: 50 minutes
Instructors:
Dr. Edward Doolittle

Name: _____
Student #: _____
Section: _____

You have 50 minutes to do each of the following questions. The test is worth a total of 40 marks.
(marks) Please justify your conclusions and show all your work. A non-programmable calculator of approved type is permitted. No other aids are permitted. Use the backs of the pages for rough work.

1. Find the following derivatives. Do not simpify!

(4) (a) y' where $y = \tan^2(3x)$

(4) (b) g'' where $g(t) = \sqrt[3]{t^2+1}$

(4) 2. Find the differential dy of $y = \dfrac{s}{1+2s}$

(4) 3. Find $\lim\limits_{x\to\infty}\left(\sqrt{4x^2+2x}-2x\right)$

(6) 4. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of the base must be twice the width. Find the dimensions of the container that minimize the amount of material used.

(a) Sketch a diagram illustrating the situation, labelling important quantities with letters.

(b) Write formulas and for the objective function to be minimized and for any constraints.

(c) Find the minimum value of the objective function. Use the First Derivative Test to ensure that you have a minimum.

(d) State the solution to the problem in words.

(5) 5. Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

(5) 6. Consider the function f, given below along with its derivatives.

$$f(x) = \frac{2x^2-4x+1}{x^2-2x+4} \qquad f'(x) = \frac{14(x-1)}{(x^2-2x+4)^2} \qquad f''(x) = \frac{-42x(x-2)}{(x^2-2x+4)^3}$$

(Note that the denominators in the above three expressions are always positive.) Find each of the following. **You do not need to sketch the graph!**

(a) The interval(s) on which f is increasing and concave up

(b) The interval(s) on which f is increasing and concave down

(c) The interval(s) on which f is decreasing and concave up

(d) The interval(s) on which f is decreasing and concave down

(e) The horizontal asymptote(s) of f.

(4) 7. Find $\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\tan\theta}$. Do not use L'Hôpital's Rule!

(4) 8. Show that $2\sqrt{x} > 3 - \dfrac{1}{x}$ for all $x > 1$.