MATH 110 Problem Set 4.3 Solutions

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1. (a) By the Fundamental Theorem of Calculus I,

$$g'(x) = \frac{d}{dx}\int_1^x (2+t^4)^5\,dt = (2+x^4)^5$$

(b) By the Fundamental Theorem of Calculus I,

$$h'(r) = \frac{d}{dr}\int_0^r \sqrt{x^2+4}\,dx = \sqrt{r^2+4}$$

(c) We can't apply the Fundamental Theorem of Calculus I directly to answer this question. We first have to modify G so it is in the form which allows us to apply FTC-I:

$$G(x) = \int_x^1 \cos\sqrt{t}\,dt = -\int_1^x \cos\sqrt{t}\,dt$$

by one of the properties of definite integrals. Then

$$G'(x) = -\frac{d}{dx}\int_1^x \cos\sqrt{t}\,dt = -\cos\sqrt{x}$$

2. (a) By the Fundamental Theorem of Calculus II we can write

$$\int_0^1 (3+x\sqrt{x})\,dx = \int (3+x\sqrt{x})\,dx\Big|_0^1$$

Evaluating the indefinite integral,

$$\int (3+x\sqrt{x})\,dx = \int (3+x^{3/2})\,dx = 3x+\frac{2}{5}x^{5/2}+C$$

So

$$\int (3+x\sqrt{x})\,dx\Big|_0^1 = 3x+\frac{2}{5}x^{5/2}\Big|_0^1 = \left(3(1)+\frac{2}{5}1^{5/2}\right)-\left(3(0)+\frac{2}{5}0^{5/2}\right) = 3+\frac{2}{5} = \frac{17}{5}$$

(b) We have

$$\int_0^{\pi/4} \sec\theta\tan\theta\,d\theta = \int \sec\theta\tan\theta\,d\theta\Big|_0^{\pi/4} = \sec\theta\Big|_0^{\pi/4} = \sec\frac{\pi}{4}-\sec 0 = \sqrt{2}-1$$

(c) We have

$$\int_1^2 \frac{s^4+1}{s^2}\,ds = \int \frac{s^4+1}{s^2}\,ds\Big|_1^2 = \int \left(s^2+s^{-2}\right)\,ds\Big|_1^2 = \frac{1}{3}s^3-s^{-1}\Big|_1^2 = \frac{1}{3}2^3-2^{-1}-\frac{1}{3}1^3+1^{-1} = \frac{17}{6}$$

3. The integrand is given in cases so we split the definite integral at the case boundaries:

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx = \int_{-2}^{0} 2\,dx + \int_{0}^{2} (4-x^2)\,dx$$

The first integral is

$$\int_{-2}^{0} 2\,dx = \int 2\,dx\Big|_{-2}^{0} = 2x\Big|_{-2}^{0} = 2(0) - 2(-2) = 4$$

The second integral is

$$\int_{0}^{2} (4-x^2)\,dx = \int (4-x^2)\,dx\Big|_{0}^{2} = 4x - \frac{1}{3}x^3\Big|_{0}^{2} = 4(2) - \frac{1}{3}2^3 = 8 - \frac{8}{3} = \frac{16}{3}$$

Altogether

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx = \int_{-2}^{0} 2\,dx + \int_{0}^{2} (4-x^2)\,dx = 4 + \frac{16}{3} = \frac{28}{3}$$

4. In both cases we are integrating over infinite discontinuities, which is not allowed for the Riemann integral.

   (a) The integrand $4/x^3$ has an infinite discontinuity at $x = 0$, so any integral of $4/x^3$ over an interval which includes $x = 0$, e.g., the interval $[-1, 2]$, is not well-defined, and the Fundamental Theorem of Calculus does not apply.

   (b) The integrand $\sec^2 x$ has an infinite discontinuity at $x = \pi/2$ because $\sec(\pi/2) = 1/\cos(\pi/2) = 1/0$. As in the previous question, the Riemann integral over the interval $[0, \pi]$ is not well-defined, and the Fundamental Theorem of Calculus does not apply.

   In general, before you can apply the Fundamental Theorem of Calculus, you should check that the integrand does not have an infinite discontinuity in the domain of integration. If it does, you can't apply the Fundamental Theorem of Calculus; you have to use more advanced alternative methods, which we will not cover in this course.

5. In this case $\Delta x = 1/n$, $x_i = i/n$, the interval is from 0 to 1, and the integrand is $\sqrt{x}$. Checking, the definition of the integral for $\sqrt{x}$ is

$$\int_{0}^{1} \sqrt{x}\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x = \lim_{n\to\infty} \left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdot + \sqrt{\frac{n}{n}} \right) \cdot \frac{1}{n}$$

   In this case the integral is easier to evaluate than the limit, so we conclude

$$\lim_{n\to\infty} \left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdot + \sqrt{\frac{n}{n}} \right) \cdot \frac{1}{n} = \int_{0}^{1} \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_{0}^{1} = \frac{2}{3}$$

   which is much easier than trying to evaluate the limit directly.