MATH 110-003 200730 Quiz 4 Solutions

Edward Doolittle

January 5, 2026

1. (a) The velocity of the stone after t seconds is $h'(t) = 10 - 1.66t$, so the velocity after 3 seconds is $h'(3) = 10 - 1.66(3) \approx 5$ m/s.

   (b) The stone reaches its maximum height when $h'(t) = 0$, i.e., when $10 - 1.66t = 0$, i.e., $t \approx 10/(5/3) = 6$ s. The maximum height attained at $t = 6$ is $h(6) = 10(6) - 0.83(6)^2 \approx 60 - (5/6)(6)^2 = 30$ m.

2. Differentiating implicitly, we have

$$\begin{aligned}\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(2x^2+2y^2-x)^2 &\implies 2x + 2y\frac{dy}{dx} = 2(2x^2+2y^2-x)\cdot\frac{d}{dx}(2x^2+2y^2-x)\\ &\implies 2x + 2y\frac{dy}{dx} = 2(2x^2+2y^2-x)\cdot\left(4x+4y\frac{dy}{dx}-1\right)\end{aligned}$$

Since we are only evaluating this when $(x,y) = (0,1/2)$ we can save some effort by postponing the algebra until we have substituted those values into the above expression. We have

$$\begin{aligned}2(0)+2\left(\frac{1}{2}\right)\frac{dy}{dx} &= 2(2(0)^2+2\left(\frac{1}{2}\right)^2-0)\cdot\left(4(0)+4\frac{1}{2}\frac{dy}{dx}-1\right)\\ \frac{dy}{dx} &= 2\left(\frac{1}{2}\right)\cdot\left(4\frac{1}{2}\frac{dy}{dx}-1\right)\\ \frac{dy}{dx} &= 2\frac{dy}{dx}-1\end{aligned}$$

which implies $dy/dx = 1$. The equation of the tangent line in point-slope form is $y - (1/2) = 1(x-0)$.