MATH 110 Lecture 4.2

The Definite Integral

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Contents

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Definition of a Definite Integral

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- Such expressions show up in other contexts, e.g., calculating work done. They are so common that they have a special notation.
- **Definition:** Let f be a function on an interval $[a,b]$. Divide the interval into equal subintervals of length $\Delta x = (b-a)/n$ and let $x_0 = a, x_1 = a+(1/n)(b-a), \ldots, x_n = a+(n/n)(b-a)/n = b$ be the endpoints of those intervals. Let x_i^* be any **sample points** in those intervals. Then the **definite integral of f from a to b is**

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided that the limit exists. If it does exist, we say f is integrable.

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- In the notation $\int_a^b f(x)\,dx$, $f(x)$ is called the **integrand**.
- a and b are called the **limits of integration**; a is the **lower limit** and b is the **upper limit**.

Evaluating Integrals

- EJD

Properties of Integrals

- EJD

Examples and Exercises

Examples

1. 1.1 Evaluate the Riemann sum for $f(x) = x^2 - x$, $0 \leq x \leq 2$ with four subintervals, taking the sample points to be right endpoints. Explain, with the aid of a diagram, what the Riemann sum represents.

   1.2 Use the definition of a definite integral (with right enpoints) to calculate the value of the integral $\int_0^2 (x^2 - x)\, dx$.

2. Evaluate $\int_0^1 \left(x + \sqrt{1 - x^2}\right)\, dx$ by interpreting it in terms of areas.

3. If $\int_0^6 f(x)\, dx = 10$ and $\int_0^4 f(x)\, dx = 7$, find $\int_4^6 f(x)\, dx$.

Exercises

Now you should work on Problem Set 4.2. After you have finished it, you should try the following additional exercises from Section 4.2:

4.2 C-level: 1–8, 17–30, 33–51;
B-level: 52–64;
A-level: 65–75