MATH 110 Lecture 2.8

Related Rates

Edward Doolittle

Thursday, February 26, 2026

Department of Indigenous Knowledge and Science
First Nations University of Canada

Contents

Related Rates

Related Rates

- Related rates problems are word problems which can be solved by taking first derivatives.

Related Rates

- Related rates problems are word problems which can be solved by taking first derivatives.
- Related rates problems have a reputation of being difficult.

Related Rates

- Related rates problems are word problems which can be solved by taking first derivatives.
- Related rates problems have a reputation of being difficult.
- Until now, the skills you have been learning are mostly technical.

Related Rates

- Related rates problems are word problems which can be solved by taking first derivatives.
- Related rates problems have a reputation of being difficult.
- Until now, the skills you have been learning are mostly technical.
- Related rates problems require more conceptual understanding, including the ability to develop your own formulas, but are technically easy.

Related Rates

- Related rates problems are word problems which can be solved by taking first derivatives.
- Related rates problems have a reputation of being difficult.
- Until now, the skills you have been learning are mostly technical.
- Related rates problems require more conceptual understanding, including the ability to develop your own formulas, but are technically easy.
- With practice, related rates problems will become easier.

Colony of Bacteria Problem

- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.

Colony of Bacteria Problem

- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.
- We make periodic measurements of the diameter of the colony and find that the radius is increasing at a constant rate of 0.2 cm per hour.

Colony of Bacteria Problem

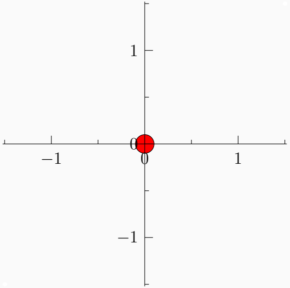
- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.
- We make periodic measurements of the diameter of the colony and find that the radius is increasing at a constant rate of 0.2 cm per hour.
- The real question we have, though, is "How fast is the area of the colony is increasing after 5 hours?"

Colony of Bacteria Problem

- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.
- We make periodic measurements of the diameter of the colony and find that the radius is increasing at a constant rate of 0.2 cm per hour.
- The real question we have, though, is "How fast is the area of the colony is increasing after 5 hours?"
- It's not true that the area is increasing at a constant rate!

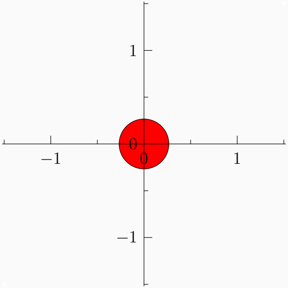
Colony of Bacteria Problem

- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.
- We make periodic measurements of the diameter of the colony and find that the radius is increasing at a constant rate of 0.2 cm per hour.
- The real question we have, though, is "How fast is the area of the colony is increasing after 5 hours?"
- It's not true that the area is increasing at a constant rate!



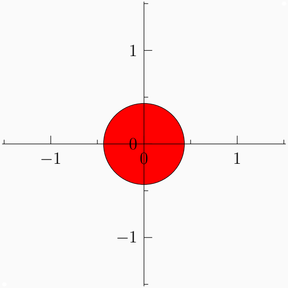
Colony of Bacteria Problem

- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.
- We make periodic measurements of the diameter of the colony and find that the radius is increasing at a constant rate of 0.2 cm per hour.
- The real question we have, though, is "How fast is the area of the colony is increasing after 5 hours?"
- It's not true that the area is increasing at a constant rate!



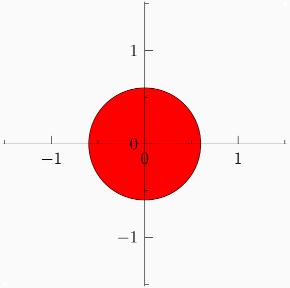
Colony of Bacteria Problem

- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.
- We make periodic measurements of the diameter of the colony and find that the radius is increasing at a constant rate of 0.2 cm per hour.
- The real question we have, though, is "How fast is the area of the colony is increasing after 5 hours?"
- It's not true that the area is increasing at a constant rate!



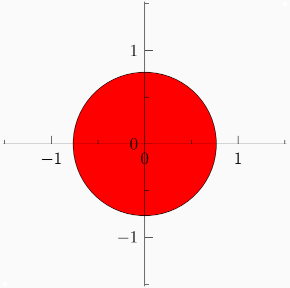
Colony of Bacteria Problem

- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.
- We make periodic measurements of the diameter of the colony and find that the radius is increasing at a constant rate of 0.2 cm per hour.
- The real question we have, though, is "How fast is the area of the colony is increasing after 5 hours?"
- It's not true that the area is increasing at a constant rate!



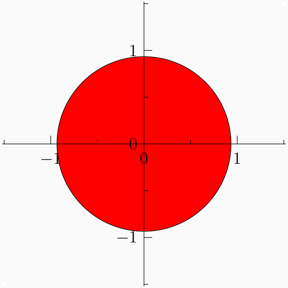
Colony of Bacteria Problem

- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.
- We make periodic measurements of the diameter of the colony and find that the radius is increasing at a constant rate of 0.2 cm per hour.
- The real question we have, though, is "How fast is the area of the colony is increasing after 5 hours?"
- It's not true that the area is increasing at a constant rate!



Colony of Bacteria Problem

- As a simple example, consider a roughly circular shaped colony of bacteria growing in a petri dish. The colony has initial radius close to 0 cm.
- We make periodic measurements of the diameter of the colony and find that the radius is increasing at a constant rate of 0.2 cm per hour.
- The real question we have, though, is "How fast is the area of the colony is increasing after 5 hours?"
- It's not true that the area is increasing at a constant rate!



Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
    1. Identify the variables in the problem, including time.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
  1. Identify the variables in the problem, including time.
  2. Find a **relation** between the non-time variables.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
  1. Identify the variables in the problem, including time.
  2. Find a **relation** between the non-time variables.
  3. Differentiate the relation.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
    1. Identify the variables in the problem, including time.
    2. Find a **relation** between the non-time variables.
    3. Differentiate the relation.
    4. Fill in all the numbers we know.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
    1. Identify the variables in the problem, including time.
    2. Find a **relation** between the non-time variables.
    3. Differentiate the relation.
    4. Fill in all the numbers we know.
    5. Solve for the quantity we don't know.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
    1. Identify the variables in the problem, including time.
    2. Find a **relation** between the non-time variables.
    3. Differentiate the relation.
    4. Fill in all the numbers we know.
    5. Solve for the quantity we don’t know.
- We must take these steps in the correct order, or we will get the wrong answer.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
    1. Identify the variables in the problem, including time.
    2. Find a **relation** between the non-time variables.
    3. Differentiate the relation.
    4. Fill in all the numbers we know.
    5. Solve for the quantity we don't know.
- We must take these steps in the correct order, or we will get the wrong answer.

- In the colony of bacteria problem, the steps are as follows:

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
  1. Identify the variables in the problem, including time.
  2. Find a **relation** between the non-time variables.
  3. Differentiate the relation.
  4. Fill in all the numbers we know.
  5. Solve for the quantity we don't know.
- We must take these steps in the correct order, or we will get the wrong answer.

- In the colony of bacteria problem, the steps are as follows:
  1. The variables are time t, radius $r(t)$, and area $A(t)$.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
  1. Identify the variables in the problem, including time.
  2. Find a **relation** between the non-time variables.
  3. Differentiate the relation.
  4. Fill in all the numbers we know.
  5. Solve for the quantity we don't know.
- We must take these steps in the correct order, or we will get the wrong answer.

- In the colony of bacteria problem, the steps are as follows:
  1. The variables are time t, radius $r(t)$, and area $A(t)$.
  2. The relation between r and A is $A(t) = \pi (r(t))^2$.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
  1. Identify the variables in the problem, including time.
  2. Find a **relation** between the non-time variables.
  3. Differentiate the relation.
  4. Fill in all the numbers we know.
  5. Solve for the quantity we don't know.
- We must take these steps in the correct order, or we will get the wrong answer.

- In the colony of bacteria problem, the steps are as follows:
  1. The variables are time t, radius $r(t)$, and area $A(t)$.
  2. The relation between r and A is $A(t) = \pi\left(r(t)\right)^2$.
  3. $dA/dt = 2\pi r\, dr/dt$

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
  1. Identify the variables in the problem, including time.
  2. Find a **relation** between the non-time variables.
  3. Differentiate the relation.
  4. Fill in all the numbers we know.
  5. Solve for the quantity we don't know.
- We must take these steps in the correct order, or we will get the wrong answer.

- In the colony of bacteria problem, the steps are as follows:
  1. The variables are time t, radius $r(t)$, and area $A(t)$.
  2. The relation between r and A is $A(t) = \pi\,(r(t))^2$.
  3. $dA/dt = 2\pi r\, dr/dt$
  4. At $t = 5$, $dr/dt = 0.2$,

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
  1. Identify the variables in the problem, including time.
  2. Find a **relation** between the non-time variables.
  3. Differentiate the relation.
  4. Fill in all the numbers we know.
  5. Solve for the quantity we don't know.
- We must take these steps in the correct order, or we will get the wrong answer.

- In the colony of bacteria problem, the steps are as follows:
  1. The variables are time t, radius $r(t)$, and area $A(t)$.
  2. The relation between r and A is $A(t) = \pi\,(r(t))^2$.
  3. $dA/dt = 2\pi r\, dr/dt$
  4. At $t = 5$, $dr/dt = 0.2$, and $r = 0.2 \times 5 = 1$.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
  1. Identify the variables in the problem, including time.
  2. Find a **relation** between the non-time variables.
  3. Differentiate the relation.
  4. Fill in all the numbers we know.
  5. Solve for the quantity we don't know.
- We must take these steps in the correct order, or we will get the wrong answer.

- In the colony of bacteria problem, the steps are as follows:
  1. The variables are time t, radius $r(t)$, and area $A(t)$.
  2. The relation between r and A is $A(t) = \pi\left(r(t)\right)^2$.
  3. $dA/dt = 2\pi r\, dr/dt$
  4. At $t = 5$, $dr/dt = 0.2$, and $r = 0.2 \times 5 = 1$.
  5. $dA/dt = 2\pi \cdot 1 \cdot 0.2 \approx 1.26$ cm^2/hr.

Colony of Bacteria Solution

- In order to solve this problem, we must take each of the following steps.
  1. Identify the variables in the problem, including time.
  2. Find a **relation** between the non-time variables.
  3. Differentiate the relation.
  4. Fill in all the numbers we know.
  5. Solve for the quantity we don't know.
- We must take these steps in the correct order, or we will get the wrong answer.

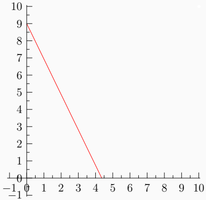
- In the colony of bacteria problem, the steps are as follows:
  1. The variables are time t, radius $r(t)$, and area $A(t)$.
  2. The relation between r and A is $A(t) = \pi\left(r(t)\right)^2$.
  3. $dA/dt = 2\pi r\,dr/dt$
  4. At $t = 5$, $dr/dt = 0.2$, and $r = 0.2 \times 5 = 1$.
  5. $dA/dt = 2\pi \cdot 1 \cdot 0.2 \approx 1.26$ cm^2/hr.
- Note that we differentiated *before* we substituted any numbers.

Relations in Implicit Form

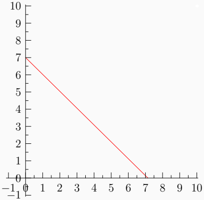
- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?

Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?

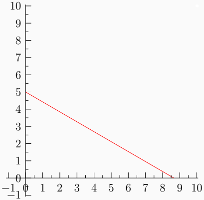


- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?



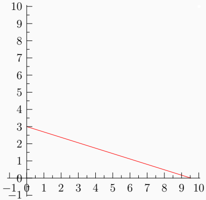
Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?



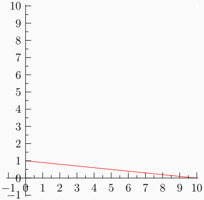
Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?



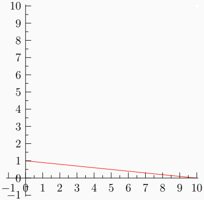
Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?



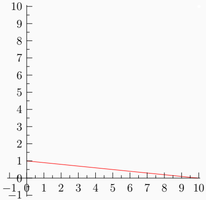
Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?
- The variables are time, x, and y.



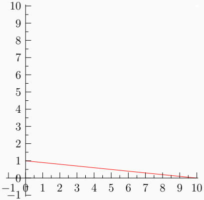
Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?
- The variables are time, x, and y.
- The relation is $x^2 + y^2 = 10^2$.



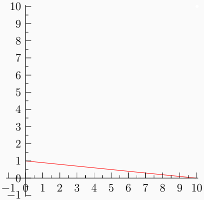
Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?
- The variables are time, x, and y.
- The relation is $x^2 + y^2 = 10^2$.
- Differentiating, $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$.



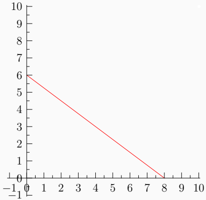
Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?
- The variables are time, x, and y.
- The relation is $x^2 + y^2 = 10^2$.
- Differentiating, $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$.
- Filling in what we know, $\dfrac{dy}{dt} = -2$, $x = 8$.



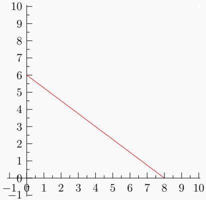
Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?
- The variables are time, x, and y.
- The relation is $x^2 + y^2 = 10^2$.
- Differentiating, $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$.
- Filling in what we know, $\dfrac{dy}{dt} = -2$, $x = 8$.
- We are not given y but we can figure it out: $x^2 + y^2 = 10^2$ and $x = 8$ give $y = 6$.



Relations in Implicit Form

- Consider the following problem: A 10 ft ladder, propped against a wall, slides down at a rate of 2 ft/s. How fast is the bottom of the ladder moving when the bottom is 8 ft from the wall?
- The variables are time, x, and y.
- The relation is $x^2 + y^2 = 10^2$.
- Differentiating, $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$.
- Filling in what we know, $\dfrac{dy}{dt} = -2$, $x = 8$.
- We are not given y but we can figure it out: $x^2 + y^2 = 10^2$ and $x = 8$ give $y = 6$.
- So $16\dfrac{dx}{dt} + 12 \cdot -2 = 0$, i.e., $\dfrac{dx}{dt} = 3/2$ ft/s.



Water Level Problem

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?

Water Level Problem

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- The variables are time, the height h of the water, the radius r of the surface of the water, and V the volume of the water.

Water Level Problem

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- The variables are time, the height h of the water, the radius r of the surface of the water, and V the volume of the water.
- The relation is $V = \dfrac{1}{3}\pi r^2 h$.

Water Level Problem

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- The variables are time, the height h of the water, the radius r of the surface of the water, and V the volume of the water.
- The relation is $V = \dfrac{1}{3}\pi r^2 h$.
- Note that $V(t)$, $r(t)$, and $h(t)$ are all dependent variables.

Eliminating a non-Time Variable

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?

Eliminating a non-Time Variable

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- We need to reduce the number of variables. We can either try to reduce the number of variables before or after we differentiate.

Eliminating a non-Time Variable

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- We need to reduce the number of variables. We can either try to reduce the number of variables before or after we differentiate.
- Let's try to reduce the number of variables before we differentiate.

Eliminating a non-Time Variable

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- We need to reduce the number of variables. We can either try to reduce the number of variables before or after we differentiate.
- Let's try to reduce the number of variables before we differentiate.
- We need *another relation*.

Eliminating a non-Time Variable

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- We need to reduce the number of variables. We can either try to reduce the number of variables before or after we differentiate.
- Let's try to reduce the number of variables before we differentiate.
- We need *another relation*.
- Similar triangles give $r/3 = h/10$.

Eliminating a non-Time Variable

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- We need to reduce the number of variables. We can either try to reduce the number of variables before or after we differentiate.
- Let's try to reduce the number of variables before we differentiate.
- We need *another relation*.
- Similar triangles give $r/3 = h/10$.
- Then $r^2 = 9h/100$ so $V = \dfrac{3\pi}{100}h^3$.

Solving the Water Level Problem

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?

Solving the Water Level Problem

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- Now $\dfrac{dV}{dt} = \dfrac{9\pi}{100} h^2 \dfrac{dh}{dt}$.

Solving the Water Level Problem

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- Now $\dfrac{dV}{dt} = \dfrac{9\pi}{100} h^2 \dfrac{dh}{dt}$.
- We know $\dfrac{dV}{dt} = 2$ and $h = 5$.

Solving the Water Level Problem

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- Now $\dfrac{dV}{dt} = \dfrac{9\pi}{100} h^2 \dfrac{dh}{dt}$.
- We know $\dfrac{dV}{dt} = 2$ and $h = 5$.
- Substituting the values we know, we have $2 = \dfrac{9\pi}{100} 5^2 \dfrac{dh}{dt}$.

Solving the Water Level Problem

- A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?
- Now $\dfrac{dV}{dt} = \dfrac{9\pi}{100} h^2 \dfrac{dh}{dt}$.
- We know $\dfrac{dV}{dt} = 2$ and $h = 5$.
- Substituting the values we know, we have $2 = \dfrac{9\pi}{100} 5^2 \dfrac{dh}{dt}$.
- Solving, $\dfrac{dh}{dt} = \dfrac{8}{9\pi}$.

Two Moving Objects

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

Two Moving Objects

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- Let O be the point on the road directly under the balloon.

Two Moving Objects

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- Let O be the point on the road directly under the balloon.
- The variables are t, $x(t)$, $y(t)$, and $z(t)$.

Two Moving Objects

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- Let O be the point on the road directly under the balloon.
- The variables are t, $x(t)$, $y(t)$, and $z(t)$.
- The relation is $x^2 + y^2 = z^2$.

Two Moving Objects

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- Let O be the point on the road directly under the balloon.
- The variables are t, $x(t)$, $y(t)$, and $z(t)$.
- The relation is $x^2 + y^2 = z^2$.
- We could try to eliminate variables using our knowledge of the motion, but let's try another approach.

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- The relation is $x^2 + y^2 = z^2$.

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- The relation is $x^2 + y^2 = z^2$.
- Differentiate: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$.

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- The relation is $x^2 + y^2 = z^2$.
- Differentiate: $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 2z\dfrac{dz}{dt}$.
- One equation, six variables; we need values for five variables before we can find $\dfrac{dz}{dt}$.

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- The relation is $x^2 + y^2 = z^2$.
- Differentiate: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$.
- One equation, six variables; we need values for five variables before we can find $\frac{dz}{dt}$.
- We are given $\frac{dx}{dt} = 15$ and $\frac{dy}{dt} = 5$. We still need x, y, and z.

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- $x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$, $\frac{dx}{dt} = 15$, and $\frac{dy}{dt} = 5$

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- $x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$, $\frac{dx}{dt} = 15$, and $\frac{dy}{dt} = 5$
- After 3 s the boy has traveled $x = 3 \cdot 15 = 45$ ft.

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- $x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$, $\frac{dx}{dt} = 15$, and $\frac{dy}{dt} = 5$
- After 3 s the boy has traveled $x = 3 \cdot 15 = 45$ ft.
- After 3 s the balloon has risen $3 \times 5 = 15ft$, total height $y = 45 + 15 = 60$ ft.

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- $x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$, $\frac{dx}{dt} = 15$, and $\frac{dy}{dt} = 5$
- After 3 s the boy has traveled $x = 3 \cdot 15 = 45$ ft.
- After 3 s the balloon has risen $3 \times 5 = 15ft$, total height $y = 45 + 15 = 60$ ft.
- z satisfies $z^2 = 45^2 + 60^2 = 5625$ so $z = 75$.

Retaining All non-Time Variables

- A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- $x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}$, $\frac{dx}{dt}=15$, and $\frac{dy}{dt}=5$
- After 3 s the boy has traveled $x=3\cdot 15=45$ ft.
- After 3 s the balloon has risen $3\times 5=15ft$, total height $y=45+15=60$ ft.
- z satisfies $z^2=45^2+60^2=5625$ so $z=75$.
- Solving, $\frac{dz}{dt}=\frac{1}{75}(45\cdot 15+60\cdot 5)=13$ ft/s.

Examples and Exercises

Examples

1. The volume of a cube is increasing at a rate of $10\ \text{cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 30 cm?
2. A waterskier skis over a ramp in the shape of a right-angled wedge 15 ft long, 4 ft above the water at the end. How fast is she rising when she leaves the ramp?
3. The angle of elevation of the sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400 ft tall building increasing when the angle of elevation of the sun is $\pi/6$?

Exercises

Now you should work on Problem Set 2.8. After you have finished it, you should try the following additional exercises from Section 2.8:

2.8 C-level: 1–35;
B-level: 36–40;
A-level: 41–50