MATH 110 Problem Set 1.5b (one-sided and infinite limits)

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The following problems based on Section 1.5 of the textbook will help you study. *You do not need to hand in solutions to these problems.*

1. (Based on 1.5.2) Explain what it means to say that

$$\lim_{x\to 2^-} f(x) = 3 \text{ and } \lim_{x\to 2^+} f(x) = 5$$

   In this situation, is it possible for $\lim_{x\to 2} f(x)$ to exist? Explain.

2. (Based on 1.5.3) Explain what it means to say that

$$\lim_{x\to -1} f(x) = -\infty$$

3. (Based on 1.5.11) Sketch the graph of the function

$$f(x) = \begin{cases} 1-x & \text{if } x < -2 \\ 2x & \text{if } -2 \le x < 1 \\ 3-x^2 & \text{if } 1 \le x \end{cases}$$

   and use the graph to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

4. (Based on 1.5.15–18) Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$\lim_{x\to 0^-} f(x) = 1 \qquad \lim_{x\to 0^+} f(x) = -1 \qquad \lim_{x\to 2^-} f(x) = 0$$
$$\lim_{x\to 2^+} f(x) = 1 \qquad f(2) = 1 \qquad f(0) \text{ is undefined}$$

5. (Based on 1.5.19) Guess (to four decimal places) the value of the limit (if it exists)

$$\lim_{x\to 1} \frac{x^2 - x}{x^2 + x - 2}$$

   by evaluating the function at the numbers $x = 2$, 1.1, 1.01, 1.001 and at the numbers $x = 0$, 0.9, 0.99, 0.999.

6. (Based on 1.5.20) Guess (to four decimal places) the value of the limit (if it exists)

$$\lim_{x\to -2} \frac{x^2 - x}{x^2 + x - 2}$$

   by evaluating the function at the numbers $x = -3$, -2.1, -2.01, -2.001 and at the numbers $x = -1$, -1.9, -1.99, -1.999.

7. (Based on 1.5.29–39) Determine the following infinite limits.

(a) $\lim_{x\to -3^-} \dfrac{x+2}{x+3}$ (b) $\lim_{x\to 0} \dfrac{x-1}{x^2(x+2)}$ (c) $\lim_{x\to \pi^-} \cot x$ (d) $\lim_{x\to 2^-} \dfrac{x^2-2x}{x^2-4x+4}$

8. (Based on 1.5.40) Find the vertical asymptotes of the function $y = \dfrac{x^2+4}{2x-3x^2}$.

9. (Based on 1.5.47) Use a graph to estimate the equations of all the vertical asymptotes of the curve

$$y = \tan(2\cos x),\ -\pi \le x \le \pi$$

Then find the exact equations of the asymptotes using your knowledge of the trigonometric functions.

You may find the following additional exercises from Section 1.5 helpful.

1.5 C-level: 2–3, 4–5, 7, 8–9, 10, 11–12, 15–18, 24–25, 29–34, 38–39, 40;
B-level: 6, 11, 13–14, 35–37, 41, 46;
A-level: 47–48