MATH 110 Lecture 3.7

Optimization Problems

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Department of Indigenous Knowledge and Science
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  5. **Find formulas for constraints.** Use them to reduce Q to a function of one variable.
  6. **Use the methods of 3.1, 3.3, 3.4 to find the *absolute* maximum or minimum of Q.** Check end behaviour!

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- The problems we will look at divide into categories (dealing with numbers, geometry with rectangles, geometry with circles and rectangles, three dimensional geometry with boxes, trigonometry, etc.)
- Once you have done a few problems in each category, you will be able to solve others in that category more quickly.
- Start with easier problems to build skill and then work through as many of the harder problems as you can.

Sum is Constraint, Product is Objective

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- We use the constraint to eliminate one of the variables, e.g., $y=23-x$ so we can write $Q(x)=xy=x(23-x)$.
- To maximize Q, we consider $Q'(x)=23-2x$. The max will occur where $Q'(x)=0$, or where $Q'(x)$ doesn't exist (but it always exists), or at an endpoint of the domain (there is no endpoint). Solving $Q'(x)=0$ we have $23-2x=0$ or $x=23/2$.

Sum is Constraint, Product is Objective, ctd.

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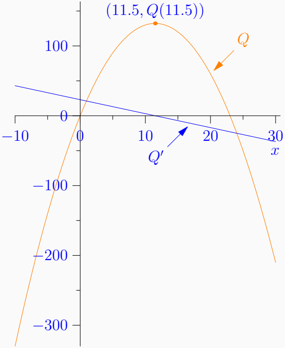
- We have found $Q(x) = xy = x(23 - x)$ has a critical point at $x = 23/2$.
- We need to check that $Q(x)$ has a **maximum** at $x = 23/2$.



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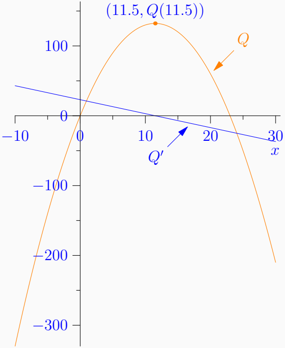
- We have found $Q(x) = xy = x(23 - x)$ has a critical point at $x = 23/2$.
- We need to check that $Q(x)$ has a **maximum** at $x = 23/2$.
- $Q'(x) > 0$ for $x < 23/2$, and $Q'(x) < 0$ for $x > 23/2$, so $Q(x)$ increases to $x = 23/2$ then decreases.



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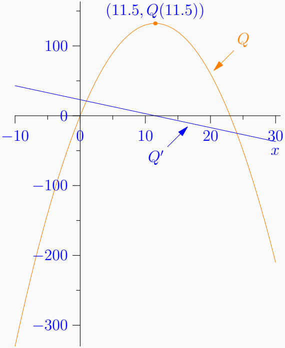
- We need to check that $Q(x)$ has a **maximum** at $x = 23/2$.
- $Q'(x) > 0$ for $x < 23/2$, and $Q'(x) < 0$ for $x > 23/2$, so $Q(x)$ increases to $x = 23/2$ then decreases.
- By the first derivative test, Q has an absolute maximum at $x = 23/2$.



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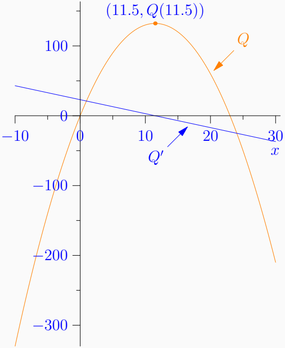
- By the first derivative test, Q has an absolute maximum at $x = 23/2$.
- Now we can answer the problem: The two numbers which add to 23 and have the largest possible product are $x = 23/2$ and $y = 23 - x = 23/2$.



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- We also have the constraint that x and y must be positive; in symbols, $x > 0$ and $y > 0$.

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- Eliminating the variable $y = 100/x$, we reduce the problem to minimizing $Q(x) = x + y = x + 100/x$, $x > 0$. Note that the domain of Q is not the whole real line, only the half line $(0, \infty)$.

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- The derivative is $Q'(x) = 1 - 100/x^2$, with critical points where $Q'(x) = 0$, $1 - 100/x^2 = 0$, $x = \pm 10$.
- We throw away $x = -10$ because it is outside of the domain of Q.

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- Once again, we need to check that we have an absolute minimum at $x = 10$.

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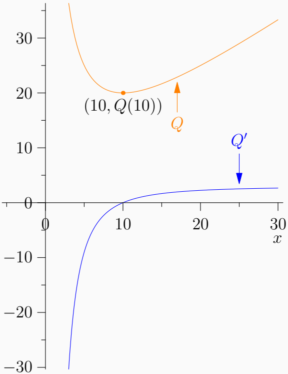
- We have $Q'(x) = 1 - 100/x^2 < 0$ for $x < 10$, and $Q'(x) > 0$ for $x > 10$.



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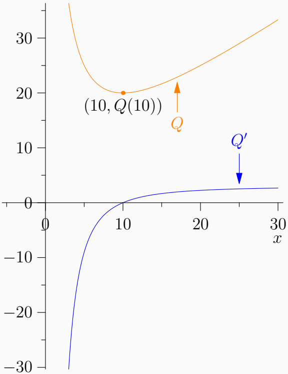
- Once again, we need to check that we have an absolute minimum at $x = 10$.
- We have $Q'(x) = 1 - 100/x^2 < 0$ for $x < 10$, and $Q'(x) > 0$ for $x > 10$.
- So $Q(x)$ decreases to $x = 10$ and then increases.



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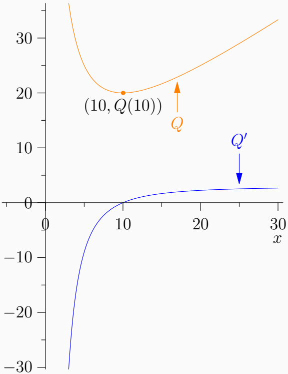

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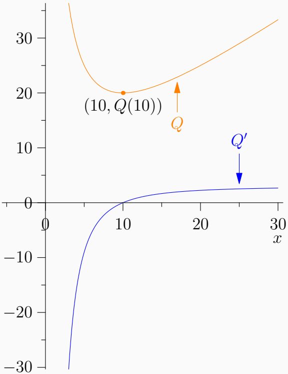
- So $Q(x)$ decreases to $x = 10$ and then increases.
- By the First Derivative Test, Q has an absolute minimum at $x = 10$.
- The answer to the question is: the two numbers, the product of which is 100 and the sum of which is a minimum, are $x = 10$ and $y = 100/x = 10$.



Plane Geometry

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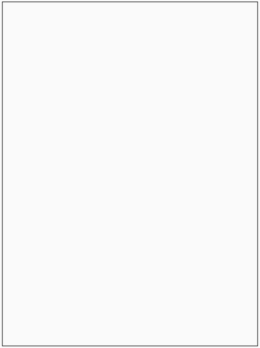

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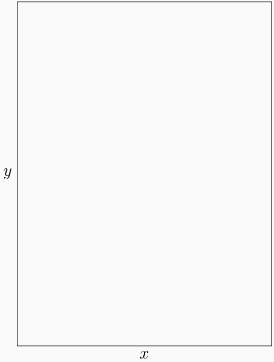

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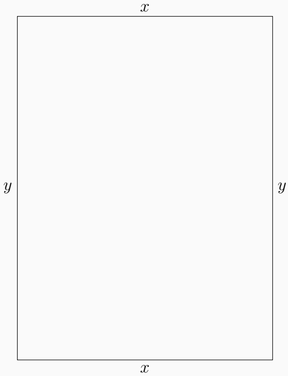

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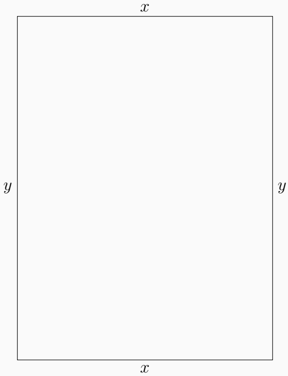

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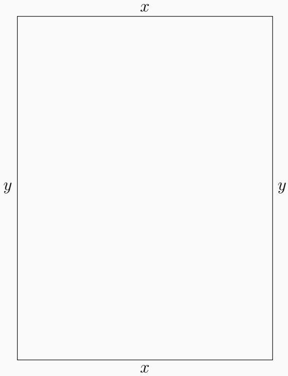
- Our constraint is $P = 100$, i.e., $x + y = 50$.



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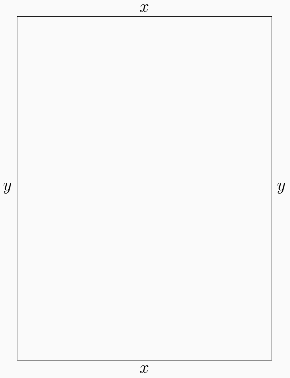
- Our objective is to maximize $A = xy$.



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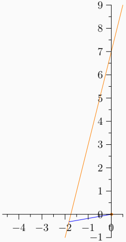

Analytic Geometry

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- Again, we draw a diagram, including both the line and the origin.
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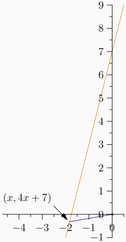

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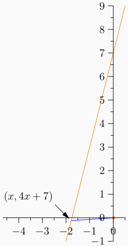
- We introduce a variable x to describe a point on the line. Then the coordinates of the point on the line are $(x, 4x+7)$.
- We try to imagine how we might find the point that is closest to the origin as x moves. We need a formula for the length of the line segment connecting $(0,0)$ and $(x, 4x+7)$.



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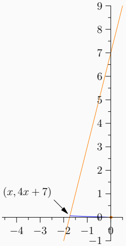

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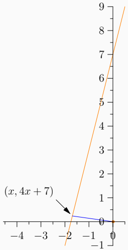

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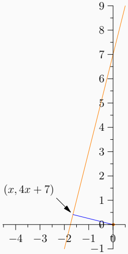

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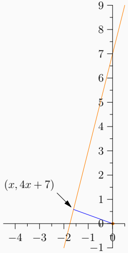

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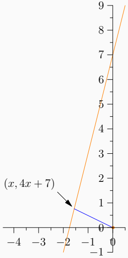

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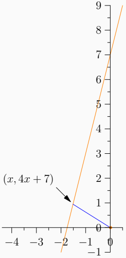

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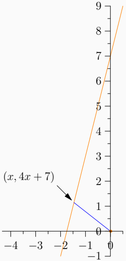

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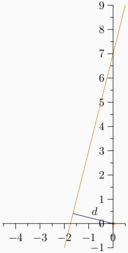

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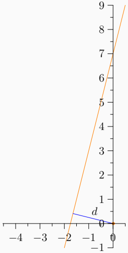

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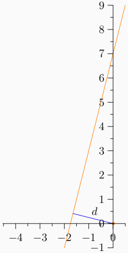

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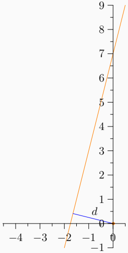

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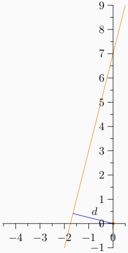
- $D'(x) = 0$ implies $x = -56/34 = -28/17$.



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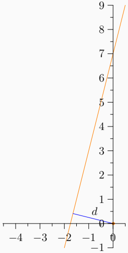

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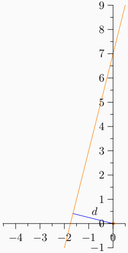
- $D'(x) = 0$ implies $x = -56/34 = -28/17$.
- The first derivative test tells us there's an absolute min at $x = -28/17$.
- The point on the line that is closest to the origin is $(x, y) = (-28/17, 4(-28/17) + 7)$.



Solid Geometry

- If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Physics

- A boat leaves a dock at 2 pm and travels south at a speed of 20 km/h. Another boat has been heading east at 15km/h and reaches the dock at 3pm. At what time were the boats closest together?

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    1. Find the demand function, assuming that it is linear.
    2. How should ticket prices be set to maximize revenue?

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    1. Find the demand function.
    2. How large a rebate should the company offer to maximize its revenue?
    3. If its weekly cost function is $C(x) = 68,000 + 150x$, how should the manufacturer set the size of the rebate in order to maximize its profit?

Examples and Exercises

Examples

1. A farmer wants to fence an area 1.5 million square feet in rectangular field and then divide it in half with a fence parallel to one of the sides. How can he do this so as to minimize the cost of the fence?
2. Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(0, 1)$.
3. A right circular cylinder is inscribed in a sphere of radius r. Find the largest possible volume of such a cylinder.
4. The illumination of an object by a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. If two light sources, one three times as strong as the other, are placed 10 ft apart, where should the object be placed on the line between the sources so as to receive the least illumination?

Exercises

Now you should work on Problem Set 3.7. After you have finished it, you should try the following additional exercises from Section 3.7:

3.7 C-level: 1–64;
B-level: 65–67, 69–70;
A-level: 71–80