

# MATH 110 Lecture 3.5

## Summary of Curve Sketching

---

Edward Doolittle

Tuesday, March 17, 2026

Department of Indigenous Knowledge and Science  
First Nations University of Canada

Summary of Curve Sketching

Guidelines for Sketching a Curve

Slant Asymptotes

Examples and Exercises

# Summary of Curve Sketching

---

# Guidelines for Sketching a Curve

A Domain

# Guidelines for Sketching a Curve

A Domain

B Intercepts

# Guidelines for Sketching a Curve

A Domain

B Intercepts

C Symmetry

# Guidelines for Sketching a Curve

A Domain

B Intercepts

C Symmetry

D Asymptotes

# Guidelines for Sketching a Curve

A Domain

B Intercepts

C Symmetry

D Asymptotes

E Intervals of Increase/Decrease



# Guidelines for Sketching a Curve

- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of Increase/Decrease
- F Local Max/Min

# Guidelines for Sketching a Curve

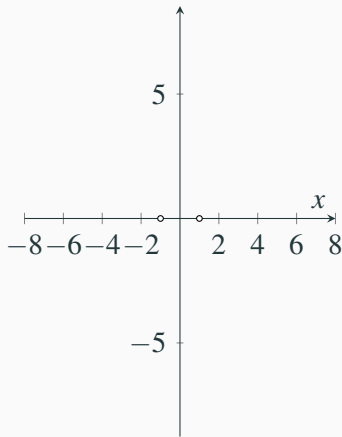
- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of Increase/Decrease
- F Local Max/Min
- G Concavity and Inflection Points

# Guidelines for Sketching a Curve

- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of Increase/Decrease
- F Local Max/Min
- G Concavity and Inflection Points
- H Sketch the Curve

**Sketch**  $y = \frac{2x^2}{x^2-1}$ ,  $y = \frac{2}{1-1/x^2}$ ,  $y' = \frac{-4x}{(x^2-1)^2}$ ,  $y'' = \frac{12x^2+4}{(x^2-1)^3}$

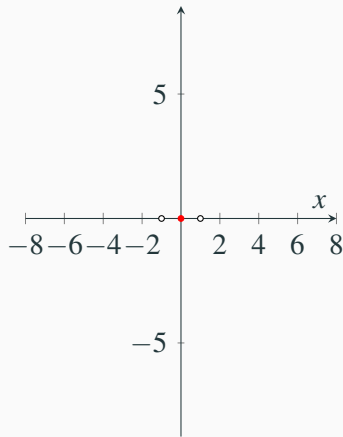
A Domain



**Sketch**  $y = \frac{2x^2}{x^2-1}$ ,  $y = \frac{2}{1-1/x^2}$ ,  $y' = \frac{-4x}{(x^2-1)^2}$ ,  $y'' = \frac{12x^2+4}{(x^2-1)^3}$

A Domain

B Intercepts

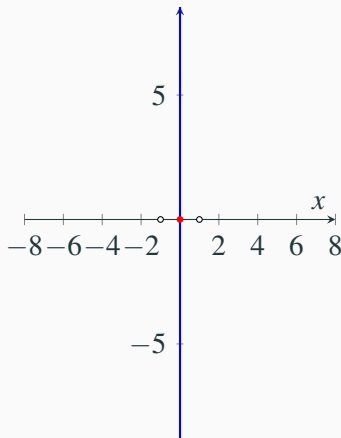


**Sketch**  $y = \frac{2x^2}{x^2-1}$ ,  $y = \frac{2}{1-1/x^2}$ ,  $y' = \frac{-4x}{(x^2-1)^2}$ ,  $y'' = \frac{12x^2+4}{(x^2-1)^3}$

A Domain

B Intercepts

C Symmetry



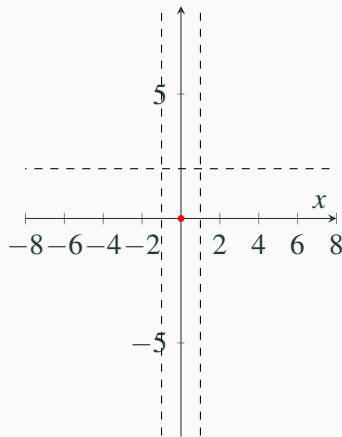
**Sketch**  $y = \frac{2x^2}{x^2-1}$ ,  $y = \frac{2}{1-1/x^2}$ ,  $y' = \frac{-4x}{(x^2-1)^2}$ ,  $y'' = \frac{12x^2+4}{(x^2-1)^3}$

A Domain

B Intercepts

C Symmetry

D Asymptotes



**Sketch**  $y = \frac{2x^2}{x^2-1}$ ,  $y = \frac{2}{1-1/x^2}$ ,  $y' = \frac{-4x}{(x^2-1)^2}$ ,  $y'' = \frac{12x^2+4}{(x^2-1)^3}$

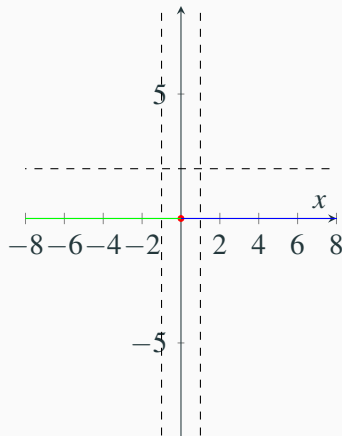
A Domain

B Intercepts

C Symmetry

D Asymptotes

E Intervals of  
Increase/Decrease





**Sketch**  $y = \frac{2x^2}{x^2-1}$ ,  $y = \frac{2}{1-1/x^2}$ ,  $y' = \frac{-4x}{(x^2-1)^2}$ ,  $y'' = \frac{12x^2+4}{(x^2-1)^3}$

A Domain

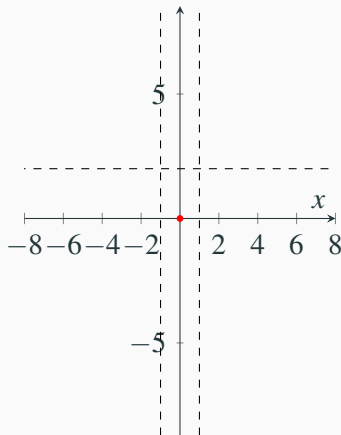
B Intercepts

C Symmetry

D Asymptotes

E Intervals of  
Increase/Decrease

F Local Max/Min



**Sketch**  $y = \frac{2x^2}{x^2-1}$ ,  $y = \frac{2}{1-1/x^2}$ ,  $y' = \frac{-4x}{(x^2-1)^2}$ ,  $y'' = \frac{12x^2+4}{(x^2-1)^3}$

A Domain

B Intercepts

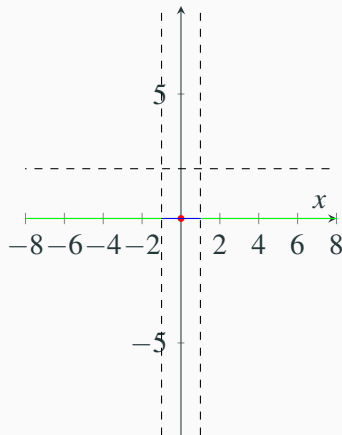
C Symmetry

D Asymptotes

E Intervals of  
Increase/Decrease

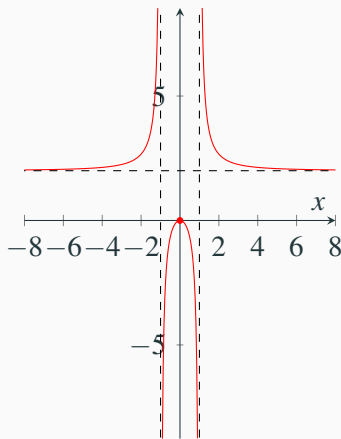
F Local Max/Min

G Concavity and Inflection  
Points



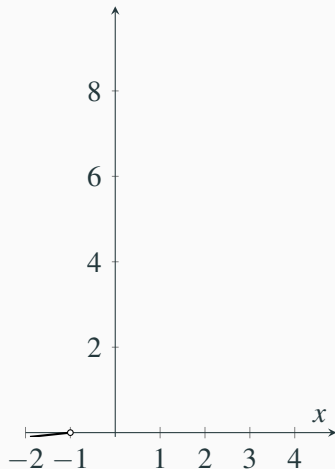
**Sketch**  $y = \frac{2x^2}{x^2-1}$ ,  $y = \frac{2}{1-1/x^2}$ ,  $y' = \frac{-4x}{(x^2-1)^2}$ ,  $y'' = \frac{12x^2+4}{(x^2-1)^3}$

- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of  
Increase/Decrease
- F Local Max/Min
- G Concavity and Inflection  
Points
- H Sketch the Curve



**Sketch**  $f(x) = \frac{x^2}{\sqrt{x+1}}, f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}}, f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$

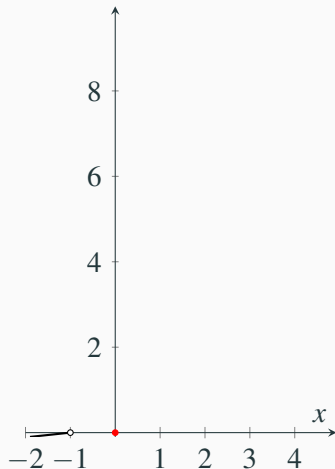
A Domain



**Sketch**  $f(x) = \frac{x^2}{\sqrt{x+1}}, f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}}, f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$

A Domain

B Intercepts

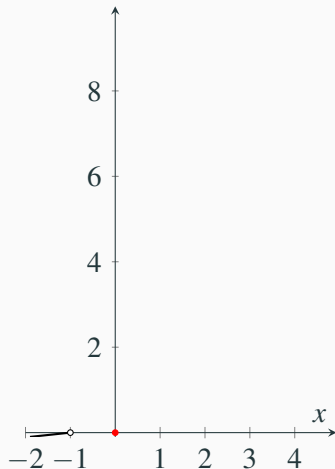


**Sketch**  $f(x) = \frac{x^2}{\sqrt{x+1}}$ ,  $f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}}$ ,  $f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$

A Domain

B Intercepts

C Symmetry



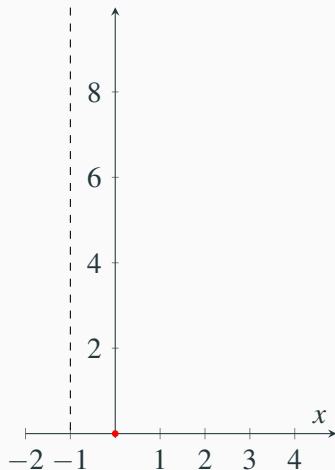
**Sketch**  $f(x) = \frac{x^2}{\sqrt{x+1}}$ ,  $f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}}$ ,  $f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$

A Domain

B Intercepts

C Symmetry

D Asymptotes



**Sketch**  $f(x) = \frac{x^2}{\sqrt{x+1}}, f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}}, f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$

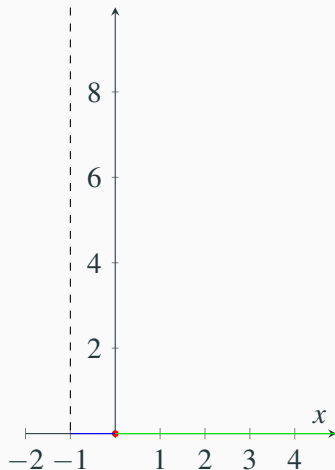
A Domain

B Intercepts

C Symmetry

D Asymptotes

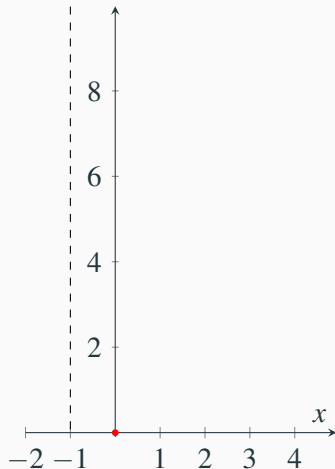
E Intervals of  
Increase/Decrease





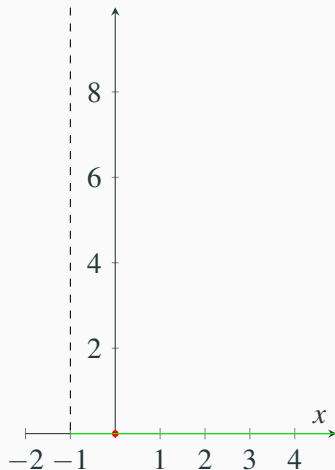
**Sketch**  $f(x) = \frac{x^2}{\sqrt{x+1}}, f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}}, f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$

- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of  
Increase/Decrease
- F Local Max/Min



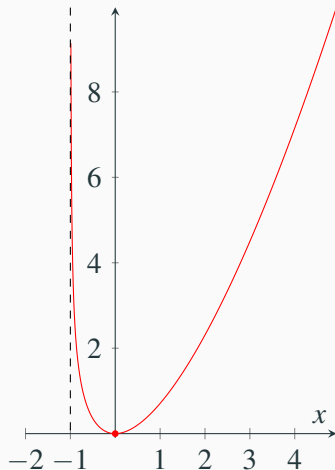
**Sketch**  $f(x) = \frac{x^2}{\sqrt{x+1}}, f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}}, f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$

- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of  
Increase/Decrease
- F Local Max/Min
- G Concavity and Inflection  
Points



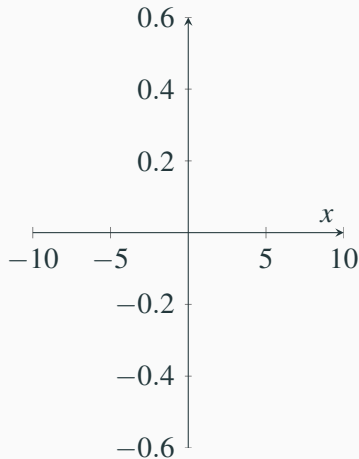
**Sketch**  $f(x) = \frac{x^2}{\sqrt{x+1}}, f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}}, f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$

- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of  
Increase/Decrease
- F Local Max/Min
- G Concavity and Inflection  
Points
- H Sketch the Curve



**Sketch**  $y = \frac{\cos x}{2+\sin x}$ ,  $y' = -\frac{2\sin x+1}{(2+\sin x)^2}$ ,  $y'' = \frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

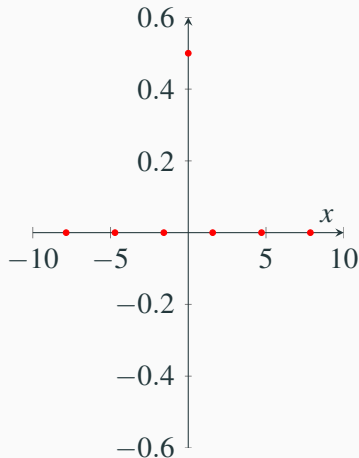
A Domain



**Sketch**  $y = \frac{\cos x}{2+\sin x}$ ,  $y' = -\frac{2\sin x+1}{(2+\sin x)^2}$ ,  $y'' = \frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

A Domain

B Intercepts

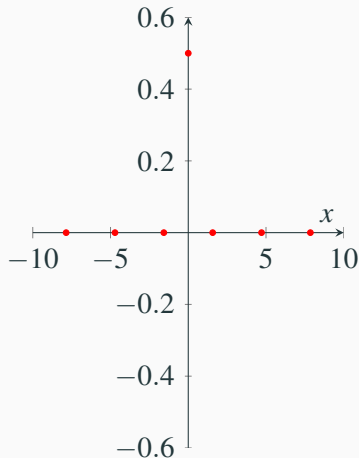


**Sketch**  $y = \frac{\cos x}{2+\sin x}$ ,  $y' = -\frac{2\sin x+1}{(2+\sin x)^2}$ ,  $y'' = \frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

A Domain

B Intercepts

C Symmetry



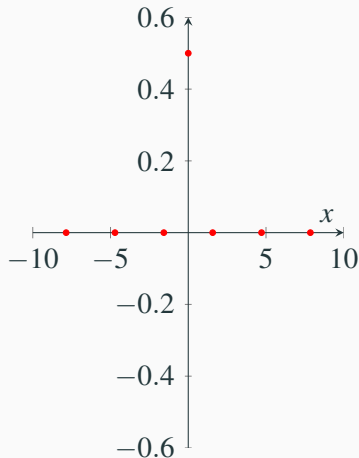
**Sketch**  $y = \frac{\cos x}{2+\sin x}$ ,  $y' = -\frac{2\sin x+1}{(2+\sin x)^2}$ ,  $y'' = \frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

A Domain

B Intercepts

C Symmetry

D Asymptotes



**Sketch**  $y = \frac{\cos x}{2+\sin x}$ ,  $y' = -\frac{2\sin x+1}{(2+\sin x)^2}$ ,  $y'' = \frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

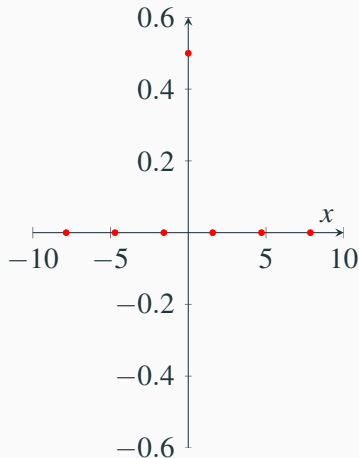
A Domain

B Intercepts

C Symmetry

D Asymptotes

E Intervals of  
Increase/Decrease





**Sketch**  $y = \frac{\cos x}{2+\sin x}$ ,  $y' = -\frac{2\sin x+1}{(2+\sin x)^2}$ ,  $y'' = \frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

A Domain

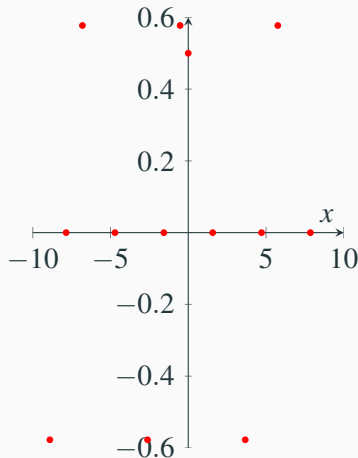
B Intercepts

C Symmetry

D Asymptotes

E Intervals of  
Increase/Decrease

F Local Max/Min



**Sketch**  $y = \frac{\cos x}{2+\sin x}$ ,  $y' = -\frac{2\sin x+1}{(2+\sin x)^2}$ ,  $y'' = \frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

A Domain

B Intercepts

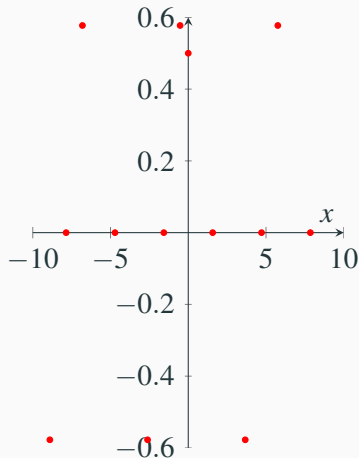
C Symmetry

D Asymptotes

E Intervals of  
Increase/Decrease

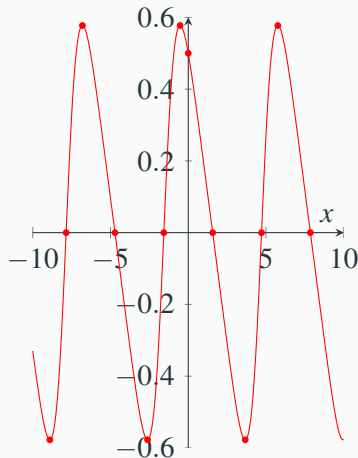
F Local Max/Min

G Concavity and Inflection  
Points



**Sketch**  $y = \frac{\cos x}{2+\sin x}$ ,  $y' = -\frac{2\sin x+1}{(2+\sin x)^2}$ ,  $y'' = \frac{2\cos x(1-\sin x)}{(2+\sin x)^3}$

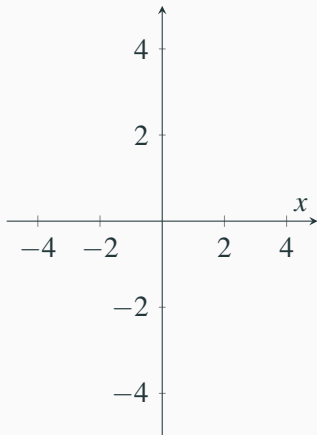
- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of  
Increase/Decrease
- F Local Max/Min
- G Concavity and Inflection  
Points
- H Sketch the Curve



# Slant Asymptotes

**Sketch**  $y = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}, y' = \frac{x^2(x^2+3)}{(x^2+1)^2} y'' = \frac{2x(3-x^2)}{(x^2+1)^3}$

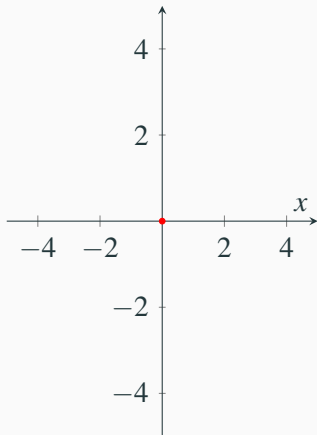
A Domain



**Sketch**  $y = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}, y' = \frac{x^2(x^2+3)}{(x^2+1)^2} y'' = \frac{2x(3-x^2)}{(x^2+1)^3}$

A Domain

B Intercepts

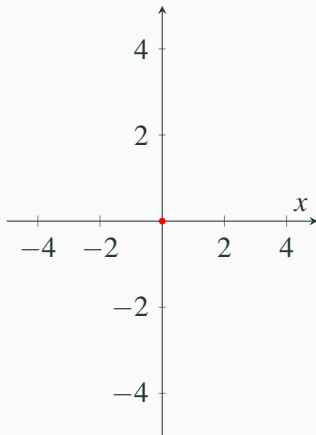


**Sketch**  $y = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}, y' = \frac{x^2(x^2+3)}{(x^2+1)^2} y'' = \frac{2x(3-x^2)}{(x^2+1)^3}$

A Domain

B Intercepts

C Symmetry



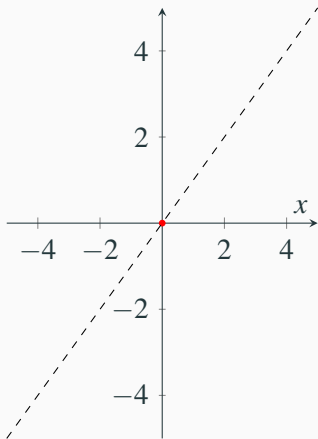
**Sketch**  $y = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}, y' = \frac{x^2(x^2+3)}{(x^2+1)^2} y'' = \frac{2x(3-x^2)}{(x^2+1)^3}$

A Domain

B Intercepts

C Symmetry

D Asymptotes





**Sketch**  $y = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}, y' = \frac{x^2(x^2+3)}{(x^2+1)^2} \quad y'' = \frac{2x(3-x^2)}{(x^2+1)^3}$

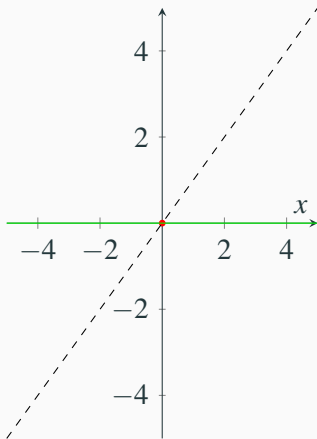
A Domain

B Intercepts

C Symmetry

D Asymptotes

E Intervals of  
Increase/Decrease



**Sketch**  $y = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}, y' = \frac{x^2(x^2+3)}{(x^2+1)^2} y'' = \frac{2x(3-x^2)}{(x^2+1)^3}$

A Domain

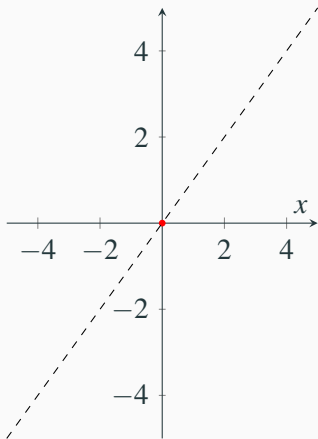
B Intercepts

C Symmetry

D Asymptotes

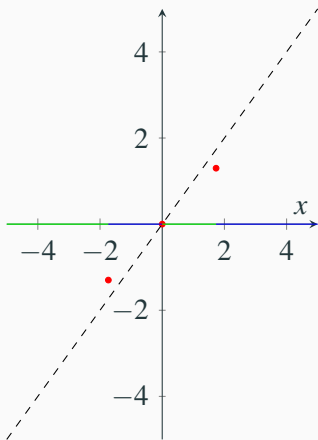
E Intervals of  
Increase/Decrease

F Local Max/Min



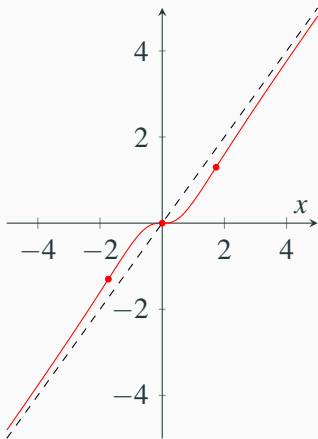
**Sketch**  $y = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}, y' = \frac{x^2(x^2+3)}{(x^2+1)^2} \quad y'' = \frac{2x(3-x^2)}{(x^2+1)^3}$

- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of  
Increase/Decrease
- F Local Max/Min
- G Concavity and Inflection  
Points



**Sketch**  $y = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}, y' = \frac{x^2(x^2+3)}{(x^2+1)^2} y'' = \frac{2x(3-x^2)}{(x^2+1)^3}$

- A Domain
- B Intercepts
- C Symmetry
- D Asymptotes
- E Intervals of  
Increase/Decrease
- F Local Max/Min
- G Concavity and Inflection  
Points
- H Sketch the Curve



# Examples and Exercises

---

Now you should work on Problem Set 3.5. After you have finished it, you should try the following additional exercises from Section 3.5:

3.5 C-level: 1–20, 45–48, 49–54;

B-level: 21–40;

A-level: 41–44, 55–60