MATH110–S01–S02 200930 Quiz 4 Solutions

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1. By the quotient rule,

$$f'(x) = \frac{\left(\frac{d}{dx}\cos x\right)(\tan x + \sin x) - \cos x\left(\frac{d}{dx}(\tan x + \sin x)\right)}{(\tan x + \sin x)^2}$$

   By the derivative of cos, the sum rule, and the derivatives of tan and sin, we have

$$f'(x) = \frac{-\sin x(\tan x + \sin x) - \cos x(\sec^2 x + \cos x)}{(\tan x + \sin x)^2}$$

   Various simplifications are possible here, but are not necessary. For example, multiplying the numerator out and applying the Pythagorean identity,

$$f'(x) = \frac{-\sin x\tan x - \sin^2 x - \sec x - \cos^2 x}{(\tan x + \sin x)^2} = \frac{-\sin x\tan x - \sec x - 1}{(\tan x + \sin x)^2}$$

   Further simplification might be possible by multiplying the numerator and denominator through by $\cos^2 x$, etc.

2. Call the limit in the question L. By the chain rule for limits (or the product rule for limits), we have

$$L = \lim_{t\to 0}\left(\frac{\sin 3t}{t}\right)^2 = \left(\lim_{t\to 0}\frac{\sin 3t}{t}\right)^2$$

   provided the limit inside the bracket exists. But it is just a variation on the basic trig limit we studied:

$$\lim_{t\to 0}\frac{\sin 3t}{t} = \lim_{t\to 0} 3\frac{\sin 3t}{3t} = 3\lim_{t\to 0}\frac{\sin 3t}{3t} = 3\cdot 1$$

   Putting it all together, $L = 9$.