MATH 110-004 200730 Quiz 2 Solutions

Edward Doolittle

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1. The function under the limit can be rewritten as
$$\frac{(5+h)^{-1}-5^{-1}}{h} = \frac{1}{h}\left(\frac{1}{5+h}-\frac{1}{5}\right) = \frac{1}{h}\frac{5-(5+h)}{(5+h)(5)} = \frac{1}{h}\frac{-h}{5(5+h)}$$
(5 marks). Therefore the limit can be evaluated as follows:
$$\lim_{h\to 0}\frac{(5+h)^{-1}-5^{-1}}{h} = \lim_{h\to 0}\frac{1}{h}\frac{-h}{5(5+h)} = \lim_{h\to 0}\frac{-1}{5(5+h)} = -\frac{1}{25}$$
(5 marks)

2. The x values of interest are the points where the denominator of $\dfrac{x-3}{2x^2-5x-3}$ is zero, and the points where the definition of the function changes. At all other points the function is guaranteed to be continuous by the theorem on the continuity of rational functions.

   The denominator can be factored as $2x^2-5x-3=(2x+1)(x-3)$, so the values of interest are $x=-1/2$ and $x=3$ (the latter for two reasons). To determine whether f is continuous at those values we have to take limits and. if necessary, compare with the values of the function. (2 marks)

   For $x=-1/2$, we have
$$\lim_{x\to -1/2^-} f(x) = \lim_{x\to -1/2^-}\frac{x-3}{(2x+1)(x-3)} = \lim_{x\to -1/2^-}\frac{1}{2x+1} = -\infty$$
since the denominator is a negative number close to zero. Therefore $\lim_{x\to -1/2} f(x)$ does not exist and f is not continuous at $x=-1/2$. The type of discontinuity is infinite. (We don't have to check the limit $x\to -1/2^+$, but you can if you want; you should get $+\infty$.) (4 marks)

   For $x=3$ we have
$$\lim_{x\to 3} f(x) = \lim_{x\to 3}\frac{x-3}{(2x+1)(x-3)} = \lim_{x\to 3}\frac{1}{2x+1} = \frac{1}{7}$$
and $f(3)=1/7$ by the definition of the function. Since the value of the function agrees with the limit at 3, f is continuous at 3. (4 marks)

   In summary, the only discontinuity of f is at $-1/2$, and the type of that discontinuity is infinite.