# MATH 110 Problem Set 2.4 Solutions

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1. (a) We apply the sum rule for derivatives:

$$\begin{aligned} f'(x) &= \frac{d}{dx}(2x^3 + \cos x) \\ &= \frac{d}{dx}2x^3 + \frac{d}{dx}\cos x \\ &= 2\frac{d}{dx}x^3 + \frac{d}{dx}\cos x \\ &= 2 \cdot 3x^2 - \sin x \\ &= 6x^2 - \sin x \end{aligned}$$

using the constant multiple rule, power rule, and rule for $\cos x$.

(b) Using the product rule,

$$\begin{aligned} G'(t) &= \frac{d}{dt}(t^2 \cos t) \\ &= \left(\frac{d}{dt}t^2\right)\cos t + t^2\frac{d}{dt}\cos t \\ &= 2t\cos t + t^2(-\sin t) \\ &= 2t\cos t - t^2 \sin t \end{aligned}$$

where we used the power rule and the rule for $\cos t$ on the second line.

(c) By the quotient rule

$$\frac{d}{d\theta}\frac{1 - \sin\theta}{\tan\theta + \cos\theta} = \frac{(\tan\theta + \cos\theta)(-\cos\theta) - (1 - \sin\theta)(\sec^2\theta - \sin\theta)}{(\tan\theta + \cos\theta)^2}$$

You can do lots of algebra and apply various trig identities to simplify the expression slightly, but my advice is to leave it alone, unless you need to do something else with it that requires simplification.

2. To find a tangent line we need to find its slope, which is a derivative.

$$\begin{aligned} y' &= \frac{d}{dx}x\cos x \\ &= \frac{dx}{dx}\cos x + x\frac{d}{dx}\cos x \\ &= \cos x + x(-\sin x) \\ &= \cos x - x\sin x \end{aligned}$$

where we applied the product rule. We evaluate the derivative at the base point $x = \pi$ to find the slope:

$$y'(\pi) = \cos\pi - \pi\sin\pi = -1 - \pi(0) = -1$$

So the equation of the tangent line is

$$y - (-\pi) = -1(x - \pi) \implies y + \pi = \pi - x \implies y = -x$$

The negative reciprocal of $-1$ is $-1/-1 = 1$, which is the slope of the normal line, so the equation of the normal line is

$$y - (-\pi) = 1(x - \pi) \implies y + \pi = x - \pi \implies y = x - 2\pi$$

3. We calculated the first derivative in the previous problem. We take the derivative of that to obtain the second derivative:

$$\begin{aligned}
\frac{d^2y}{dx^2} &= \frac{d}{dx}\frac{dy}{dx} \\
&= \frac{d}{dx}(\cos x - x\sin x) \\
&= \frac{d}{dx}\cos x - \frac{d}{dx}x\sin x \\
&= -\sin x - \sin x - x\cos x \\
&= -2\sin x - x\cos x
\end{aligned}$$

4. The function has a horizontal tangent where its derivative is 0. So we need to calculate the derivative:

$$f'(x) = 1 - \sin x$$

We need to solve the equation

$$f'(x) = 0 \implies 1 - \sin x = 0 \implies \sin x = 1$$

The $\sin^{-1}$ button on the calculator can give one solution: $x = \sin^{-1}(1) = \pi/2$. You should look at a graph of $\sin x$ to find all the other solutions: $x = \pi/2 \pm 2k\pi$, $k = 0, 1, 2, \ldots$.

5. (a) Multiply the numerator and denominator by 2 to obtain

$$\lim_{x\to 0}\frac{\sin 2x}{x} = \lim_{x\to 0} 2\frac{\sin 2x}{2x} = 2$$

(b) Multiply the numerator and denominator by $2x$, and then by $3x$, to obtain

$$\lim_{x\to 0}\frac{\sin 2x}{\sin 3x} = \lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{2x}{3x}\cdot\frac{3x}{\sin 3x} = 1\cdot\frac{2}{3}\cdot 1 = \frac{2}{3}$$

(c) Call the limit in the question $L$. By the product rule for limits we have

$$L = \lim_{t\to 0}\left(\frac{\sin 2t}{t}\right)^3 = \left(\lim_{t\to 0}\frac{\sin 2t}{t}\right)^3$$

provided the limit inside the bracket exists. But it is just a variation on the basic trig limit we studied:

$$\lim_{t\to 0}\frac{\sin 2t}{t} = \lim_{t\to 0} 2\frac{\sin 2t}{2t} = 2\lim_{t\to 0}\frac{\sin 2t}{2t} = 2\cdot 1 = 2$$

Putting it all together, $L = 2^3 = 8$.

(d) **Solution 1:** Call the limit $L$. Write $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$:

$$L = \lim_{\theta\to 0} \frac{\cos\theta - 1}{\tan\theta} = \lim_{\theta\to 0} \frac{\cos\theta - 1}{\frac{\sin\theta}{\cos\theta}} = \lim_{\theta\to 0} \frac{(\cos\theta - 1)\cos\theta}{\sin\theta}$$

Divide both numerator and denominator by $\theta$:

$$L = \lim_{\theta\to 0} \frac{\frac{\cos\theta - 1}{\theta}\cos\theta}{\frac{\sin\theta}{\theta}}$$

By the quotient rule for limits we have

$$L = \frac{\lim_{\theta\to 0}\frac{\cos\theta-1}{\theta}\lim_{\theta\to 0}\cos\theta}{\lim_{\theta\to 0}\frac{\sin\theta}{\theta}} = \frac{0\cdot 1}{1} = 0$$

**Solution 2:** Multiply numerator and denominator by the "conjugate" $\cos\theta + 1$ to obtain

$$L = \lim_{\theta\to 0} \frac{\cos\theta - 1}{\sin\theta} = \lim_{\theta\to 0} \frac{\cos\theta - 1}{\tan\theta} \cdot \frac{\cos\theta + 1}{\cos\theta + 1} = \lim_{\theta\to 0} \frac{\cos^2\theta - 1}{\tan\theta(\cos\theta + 1)}$$

Using the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$,

$$\begin{aligned} L &= \lim_{\theta\to 0} \frac{\cos^2\theta - 1}{\tan\theta(\cos\theta+1)} = \lim_{\theta\to 0} \frac{-\sin^2\theta}{(\sin\theta/\cos\theta)(\cos\theta+1)} \\ &= \lim_{\theta\to 0} \frac{-\sin\theta}{(\cos\theta+1)/\cos\theta} = \frac{-\sin 0}{(\cos 0 + 1)/\cos 0} = \frac{-0}{(1+1)/1} = \frac{0}{2} \\ &= 0 \end{aligned}$$

6. This is a question from the lecture. Using the definition of sec in terms of cos and the quotient rule,

$$\begin{aligned} \frac{d}{dx}\sec x &= \frac{d}{dx}\frac{1}{\cos x} \\ &= \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x} \\ &= \frac{\sin x}{\cos^2 x} \\ &= \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} \\ &= \sec x \tan x \end{aligned}$$

7. This is a question from the lecture. The solution is essentially parallel to the calculation of the derivative for sin. Using the definition of derivative,

$$\frac{d}{dx}\cos x = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$

Now we need an angle addition formula for cos, which you can look up in the trig identity tables in the textbook:

$$\cos(x+h) = \cos x \cos h - \sin x \sin h$$

Continuing with our calculation,

$$\begin{aligned}
\frac{d}{dx}\cos x &= \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h} \\
&= \cos x \lim_{h\to 0}\frac{\cos h - 1}{h} - \sin x \lim_{h\to 0}\frac{\sin h}{h} \\
&= \cos x(0) - \sin x(1) \\
&= -\sin x
\end{aligned}$$

by the basic trig limits.

8. (a) Note that $\mu$ and $W$ are constants in this problem, so we treat them as constants when we apply the differentiation rules. By the quotient rule,

$$\begin{aligned}
\frac{dF}{d\theta} &= \frac{(\mu\sin\theta + \cos\theta)(0) - \mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2} \\
&= -\frac{\mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2}
\end{aligned}$$

(b) By the previous, $dF/d\theta = 0$ implies that

$$\mu\cos\theta - \sin\theta = 0$$

(Note: we are assuming that $\mu$ and $W$ are not 0. What happens if either number is 0? What would that mean physically?)
To solve that equation, we move $\sin\theta$ to the right-hand side and then divide by $\cos\theta$:

$$\begin{aligned}
\mu\cos\theta &= \sin\theta \\
\mu &= \frac{\sin\theta}{\cos\theta} \\
\mu &= \tan\theta
\end{aligned}$$

You can solve that equation with a calculator using the $\tan^{-1}$ button: $\theta = \tan^{-1}\mu$ gives the solution as an angle in radians if your calculator is set to radian mode, or in degrees if your calculator is set to degree mode. For example, if $\mu = 1$ your calculator should give the answer $\pi/4$ radians, or $45°$.