MATH 110-003 200730 Quiz 5 Solutions

Edward Doolittle

January 5, 2026

1. In Figure 1, ship A moves 'west' along the x-axis and ship B moves 'north' along the y axis. We have the Pythagorean relation among the variables x, y, and z:

$$x^2 + y^2 = z^2$$

Differentiating with respect to time,

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

From the given data, at 4:00 pm we have $dx/dt = -10$, $x = -35 - 4(10) = -75$, $dy/dt = 25$, $y = 0 + 4(25) = 100$. We can calculate z using the Pythagorean relation: $z^2 = x^2 + y^2 = 75^2 + 100^2 = 125^2$. Since the distance must be positive we have $z = 125$. Filling in that data we have

$$-75 \cdot -10 + 100 \cdot 25 = 125 \cdot \frac{dz}{dt} \implies \frac{dz}{dt} = 26$$

The ships are moving away from each other at a rate of 26 km/h at 4:00 pm.

2. **Solution 1:** The relations among the dimensions of the sphere are

$$C = 2\pi r \qquad A = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$$

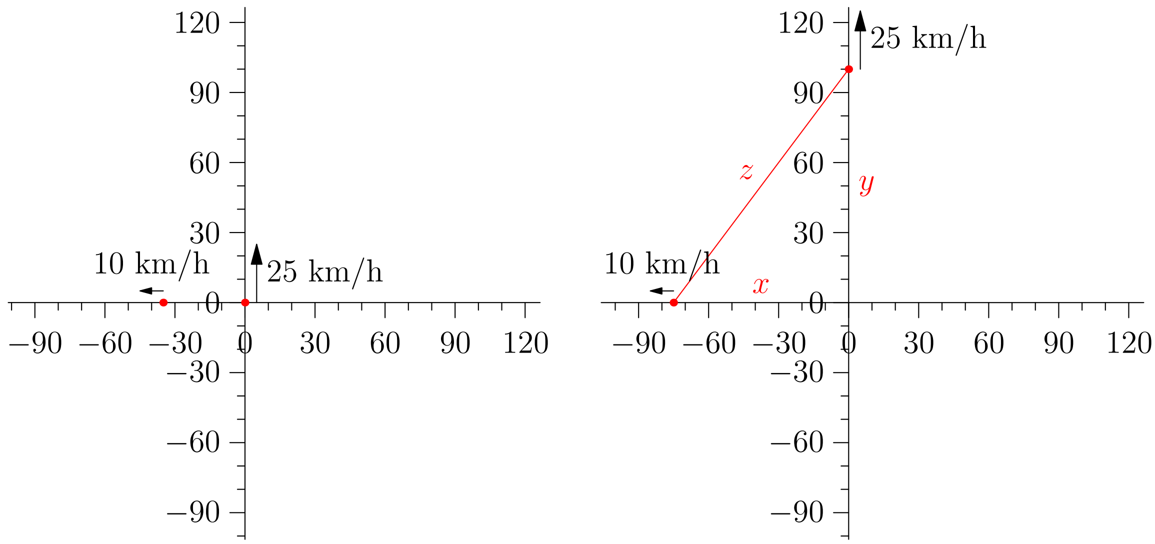
Differentiating,

$$dC = 2\pi dr \qquad dA = 8\pi r\, dr \qquad dV = 4\pi r^2\, dr$$

We are given $C = 84$ and $dC = 0.5$. Solving for r and dr using the first set of relations, $r = 84/2\pi \approx 13.4$, $dr = 0.5/2\pi \approx 0.080$, so

$$dA = 8\pi r\, dr \approx 26.9 \qquad dV = 4\pi r^2\, dr \approx 180$$

So the maximum error in the calculated value of the surface area is about 26.9 cm^2 and the maximum error in the calculated value of the volume is about 180 cm^3.



time t = noon   time t = 4:00 pm

Figure 1: Position of ships at noon and 4:00 pm

Since $A = 4\pi r^2 \approx 2260$ cm^2, the relative error in the value of A is about $dA/A = 0.012$ or about 1.2%. Since $V = (4/3)\pi r^3 \approx 10100$ cm^3, the relative error in V is about $dV/V \approx 180/10100 \approx 0.018$ or about 1.8%.

Solution 2: We can find formulas for A and V in terms of C without r. We have $r = C/2\pi$ so

$$A = 4\pi r^2 = 4\pi\left(\frac{C}{2\pi}\right) = 4\pi\frac{C^2}{4\pi^2} = \frac{C^2}{\pi} \qquad V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{C}{2\pi}\right)^3 = \frac{4}{3}\pi\frac{C^3}{8\pi^3} = \frac{C^3}{6\pi^2}$$

Differentiating,

$$dA = \frac{2}{\pi}C\,dC \qquad dV = \frac{1}{2\pi^2}C^2\,dC$$

Substituting $C = 84$ and $dC = 0.5$ we have

$$dA = \frac{2}{\pi}84 \times 0.5 \approx 26.7 \qquad dV = \frac{1}{2\pi^2}(84)^2 \times 0.5 \approx 179.$$

So the errors in the calculated values of the surface area and volume are about 26.7 cm^2 and 179 cm^3 respectively.

Since $A = C^2/\pi \approx 2250$ cm^2, the relative error in A is approximately $dA/A \approx 26.7/2250 \approx 0.012$ or 1.2%. Since $V = C^3/6\pi^2 \approx 10000$ cm^3, the relative error in V is approximately $dV/V \approx 0.018$ or about 1.8%.