# More on Sequential and Simultaneous Move Games

- So far we have studied two types of games: 1) sequential move (extensive form) games where players *take turns* choosing actions and 2) strategic form (normal form) games where players *simultaneously* choose their actions.

- Of course it is possible to combine both game forms as, for example, happens in the game of football.

- The transformation between game forms may change the set of equilibria, as we shall see.

- We will also learn the concept of *subgame perfection*

Combining Sequential and Simultaneous Moves.

- Consider the following 2 player game, where Player 1 moves first:

Player 1

Stay Out   Enter

3,3   Player 1

Player 2

		A	B
	A	2,2	3,0
	B	0,3	4,4
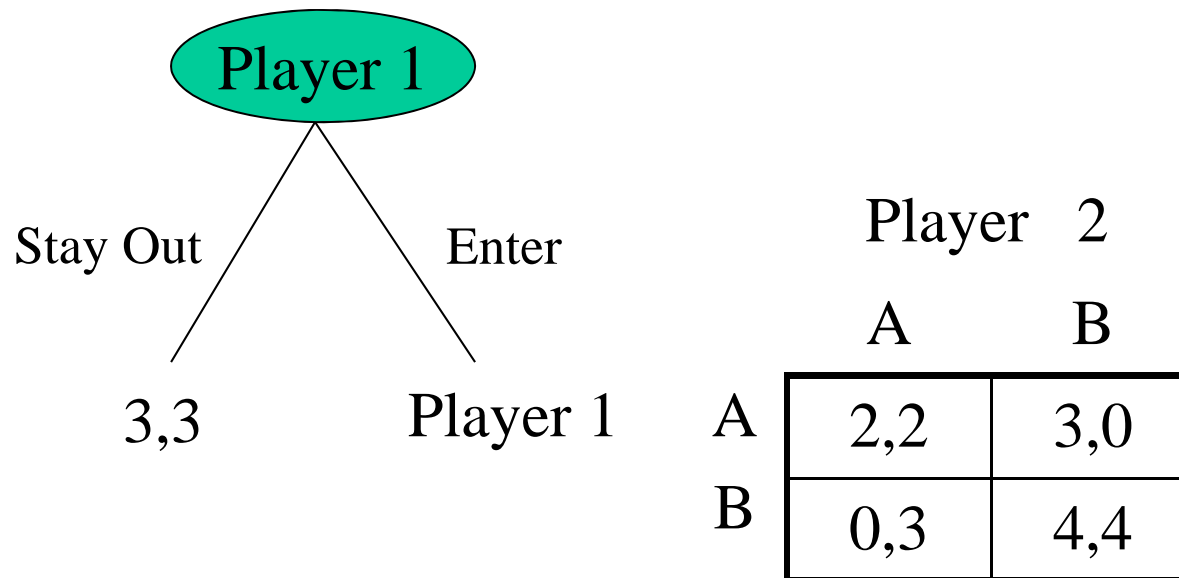
- If player 1 chooses to stay out, both he and player 2 earn a payoff of 3 each, but if player 1 chooses to enter, he plays a simultaneous move game with player 2.

Forward Induction

- The simultaneous move game has 3 equilibria: (A,A), (B,B) and a mixed strategy equilibrium where both players play A with probability 1/3 and earn expected payoff 8/3.

Player 1

Stay Out   Enter

3,3   Player 1

Player   2

	A	B
A	2,2	3,0
B	0,3	4,4
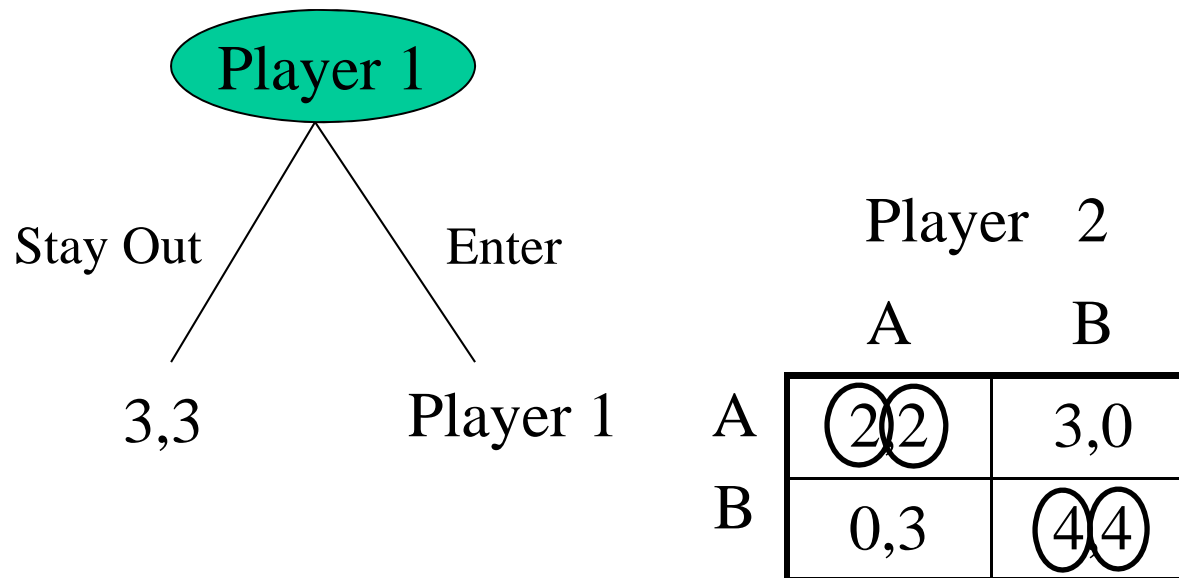
- If player 2 sees that player 1 has chosen to Enter, player 2 can use *forward induction* reasoning: since player 1 chose to forego a payoff of 3, it is likely that he will choose B, so I should also choose B.

- The likely equilibrium of the game is therefore: Enter, (B,B).

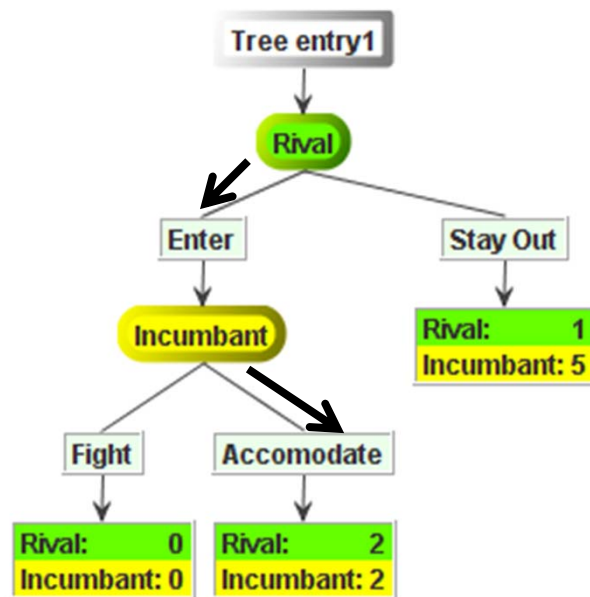# The Incumbent-Rival Game in Extensive and Strategic Form
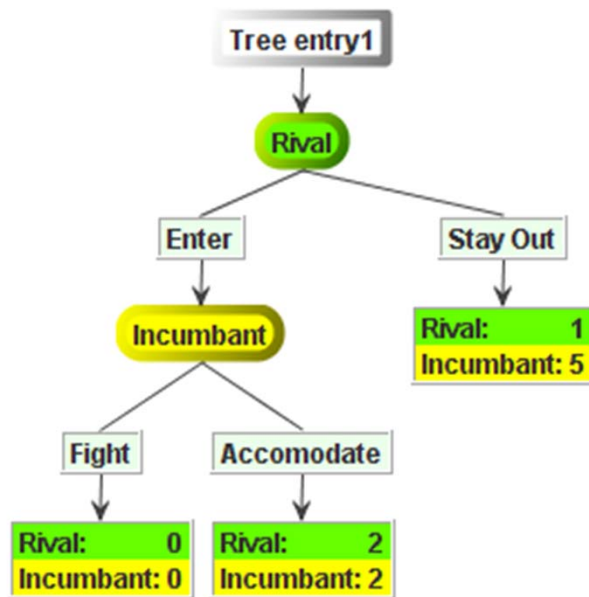
How many equilibria are there in the extensive form of this game?

How many equilibria are there in the strategic form of this game?

The Number of Equilibria Appears to be Different!

There appears to be just 1 equilibrium using rollback on the extensive form game.

There appears to be 2 equilibria using cell-by-cell inspection of the strategic form game

Subgame Perfection

- In the strategic form game, there is the additional equilibrium, Stay Out, Fight that is not an equilibrium using rollback in the extensive form game.

- Equilibria found by applying rollback to the extensive form game are referred to as **subgame perfect** equilibria: every player makes a perfect best response at every subgame of the tree.

  – Enter, Accommodate is a subgame perfect equilibrium.

  – Stay Out, Fight is not a subgame perfect equilibrium.

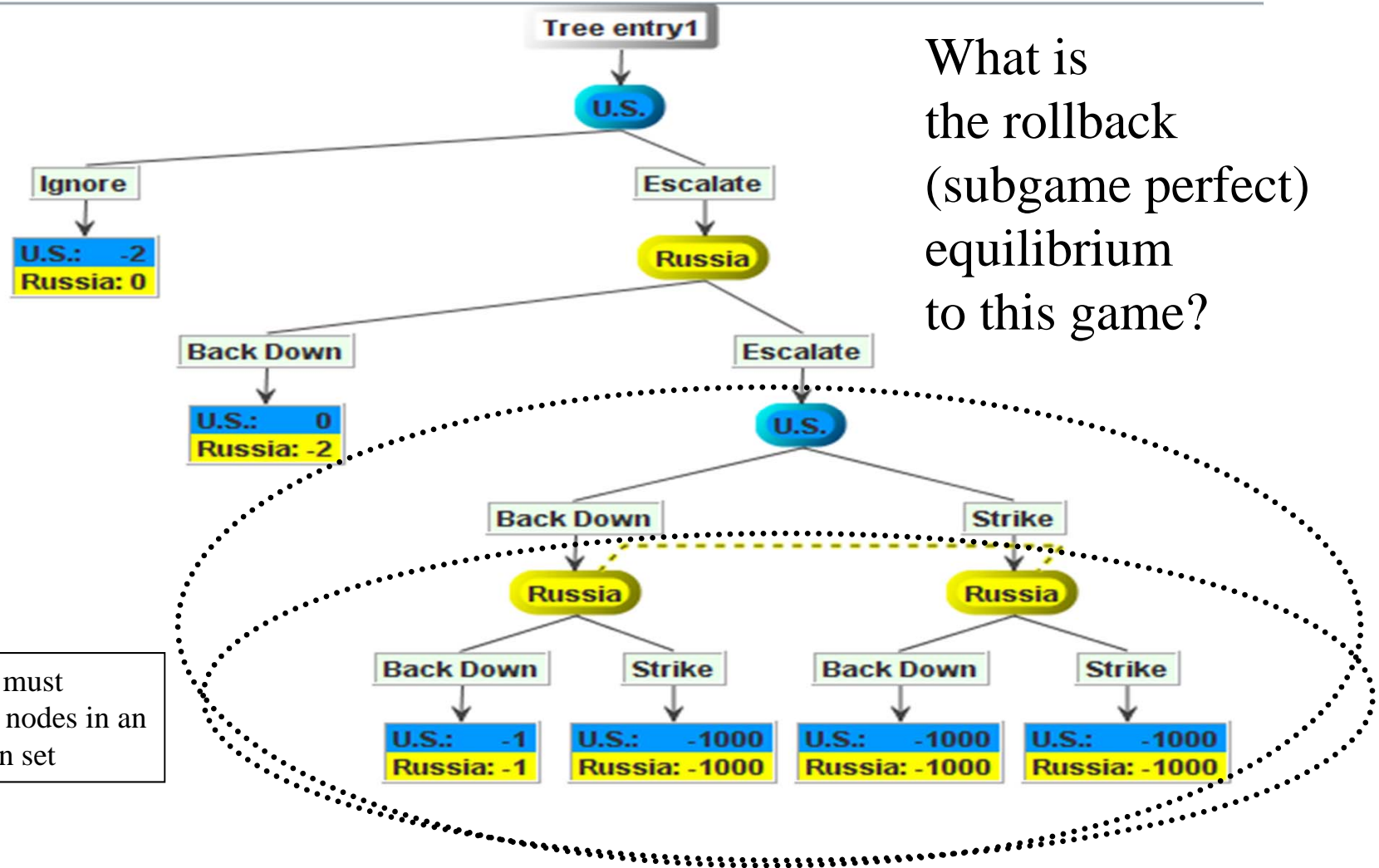- A subgame is the game that begins at any node of the decision tree.
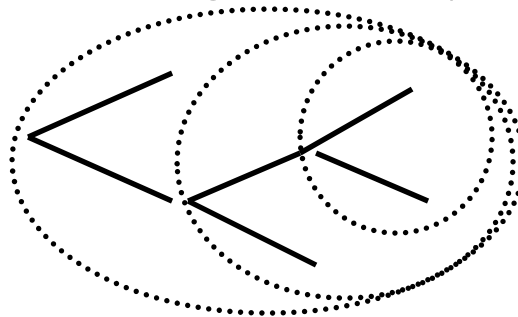
3 subgames (circled) are all the games beginning at all tree nodes including the root node (game itself)

Imperfect Strategies are Incredible

- Strategies and equilibria that fail the test of subgame perfection are called **imperfect**.

- The imperfection of a strategy that is part of an imperfect equilibrium is that at some point in the game it has an *unavoidable credibility problem*.

- Consider for example, the equilibrium where the incumbent promises to fight, so the rival chooses stay out.

- The incumbent's promise is *incredible*; the rival knows that if he enters, the incumbent is sure to accommodate, since if the incumbent adheres to his promise to fight, both earn zero, while if the incumbent accommodates, both earn a payoff of 2.

- Thus, Stay Out, Fight is a Nash equilibrium, but *it is not a subgame perfect Nash equilibrium.*

- Lesson: Every subgame perfect equilibrium is a Nash equilibrium but not every Nash equilbrium is a subgame perfect equilibrium!

Another Example:
Mutually Assured Destruction (MAD)

Tree entry1

U.S.

Ignore — U.S.: -2 / Russia: 0

Escalate — Russia

Back Down — U.S.: 0 / Russia: -2

Escalate — U.S.

Back Down — Russia

Strike — Russia

Back Down — U.S.: -1 / Russia: -1

Strike — U.S.: -1000 / Russia: -1000

Back Down — U.S.: -1000 / Russia: -1000

Strike — U.S.: -1000 / Russia: -1000

What is
the rollback
(subgame perfect)
equilibrium
to this game?

Subgames must
contain all nodes in an
information set

The Strategic Form Version of the Game Admits 3 Nash Equilibria

- Which Equilibrium is Subgame Perfect?

	Ignore	Escalate, Back Down	Escalate, Strike
Back Down	-2 / 0	0 / -2	0 / -2
Escalate, Back Down	-2 / 0	-1 / -1	-1000 / -1000
Escalate, Strike	-2 / 0	-1000 / -1000	-1000 / -1000

- Only the equilibrium where the strategies Escalate, Back Down are played by both the U.S. and Russia is *subgame perfect*. – Why?

From Simultaneous to Sequential Moves

- Conversion from simultaneous to sequential moves involves determining who moves first, which is not an issue in the simultaneous move game.
- In some games, where both players have dominant strategies, it does not matter who moves first.
  - For example the prisoner's dilemma game.
- When neither player has a dominant strategy, the subgame perfect equilibrium will depend on the order in which players move.
  - For example, the Senate Race Game, the Pittsburgh Left-Turn Game.

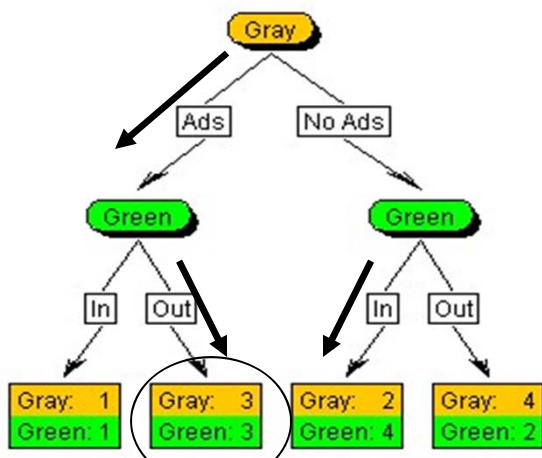# The Equilibrium in Prisoner's Dilemma is the Same *Regardless of Who Moves First*
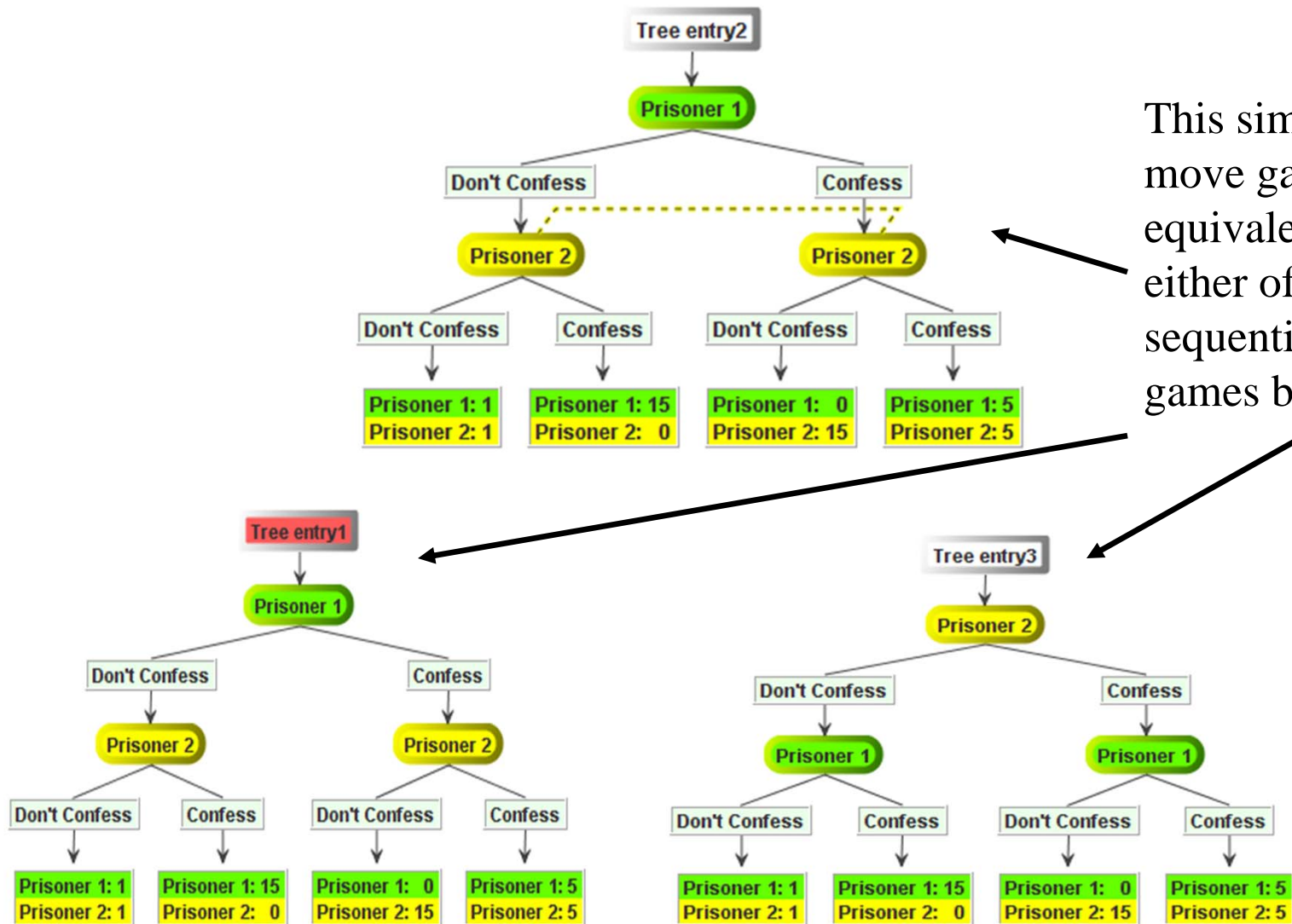
This simultaneous move game is equivalent to either of the 2 sequential move games below it.
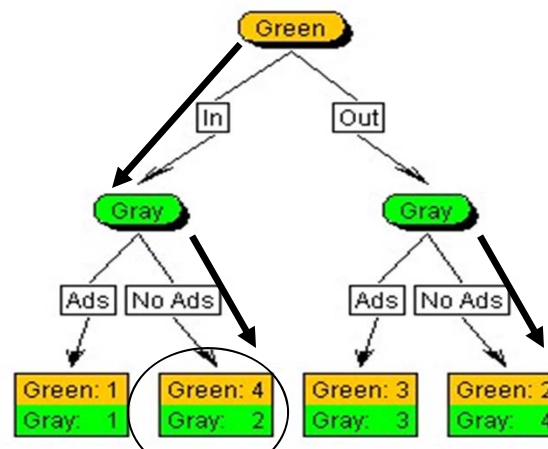
The Senate Race Game has a Different Subgame Perfect Equilibrium Depending on Who moves first.

In the simultaneous move game, there is only one Nash equilibrium.
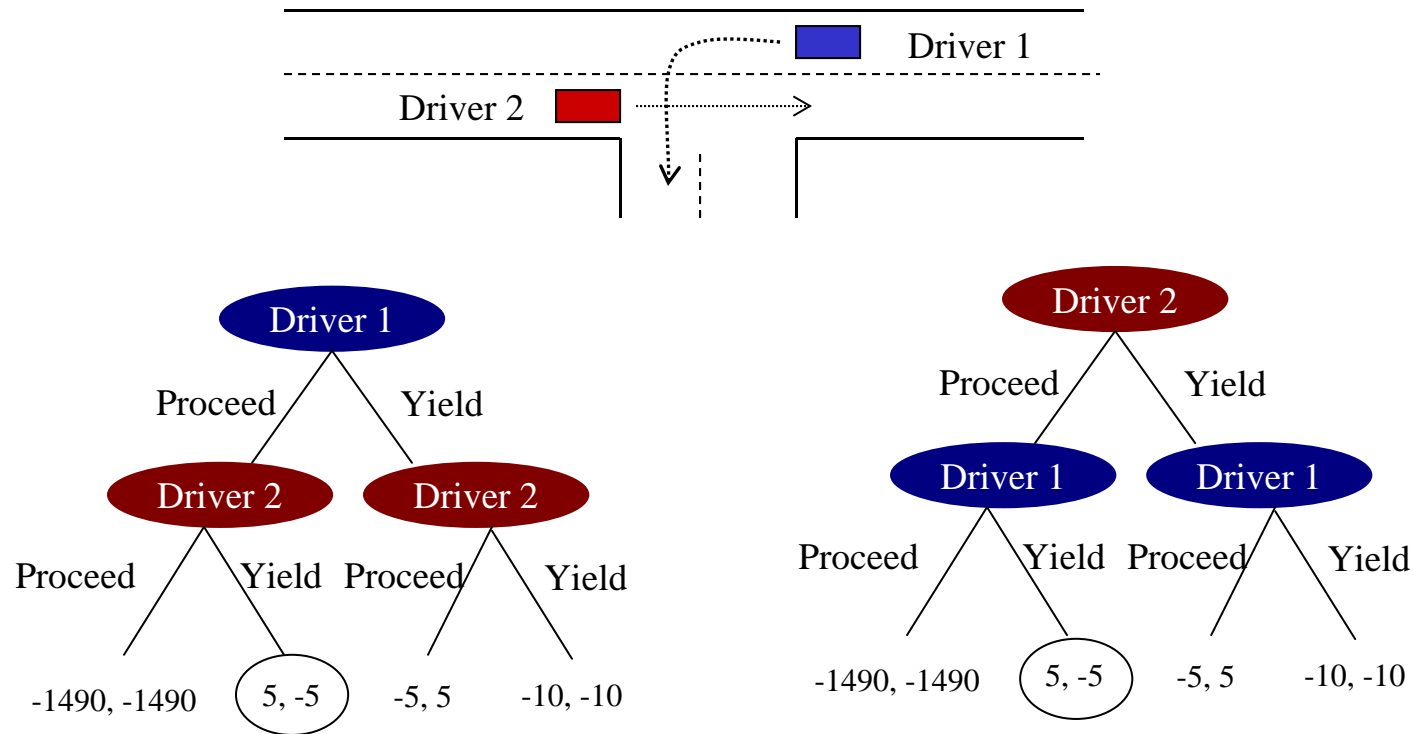
subgame perfect eq,

subgame perfect eq,

# Similarly, in the Pittsburgh Left-Turn Game

These subgame perfect equilibria look the same, but if Driver 1 moves first he gets a payoff of 5, while if Driver 2 moves first Driver 1 gets a payoff of –5, and vice versa for Driver 2.

Going from a Simultaneous Move to a Sequential Move Game may eliminate the play of a mixed strategy equilibrium

- This is true in games with a unique mixed strategy Nash equilibrium.
    - Example: The Tennis Game

		Venus Williams	
		DL	CC
Serena Williams	DL	50, 50	80, 20
	CC	90, 10	20, 80

The Pure Strategy Equilibrium is Different, Depending on Who Moves First.
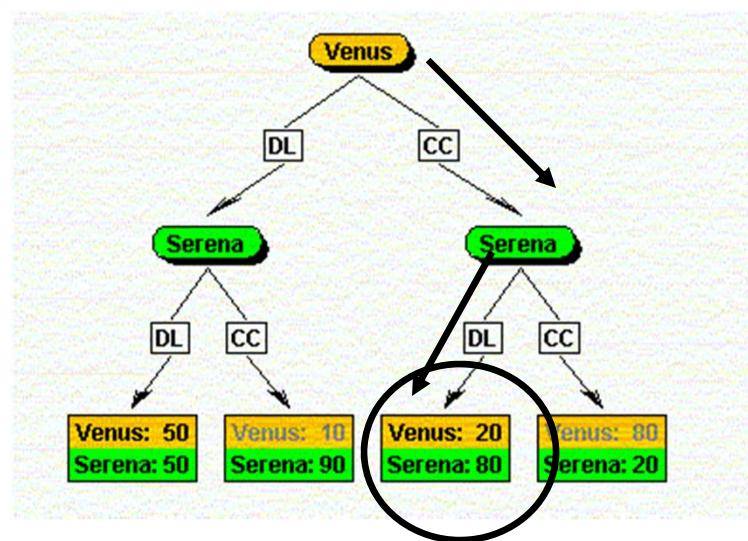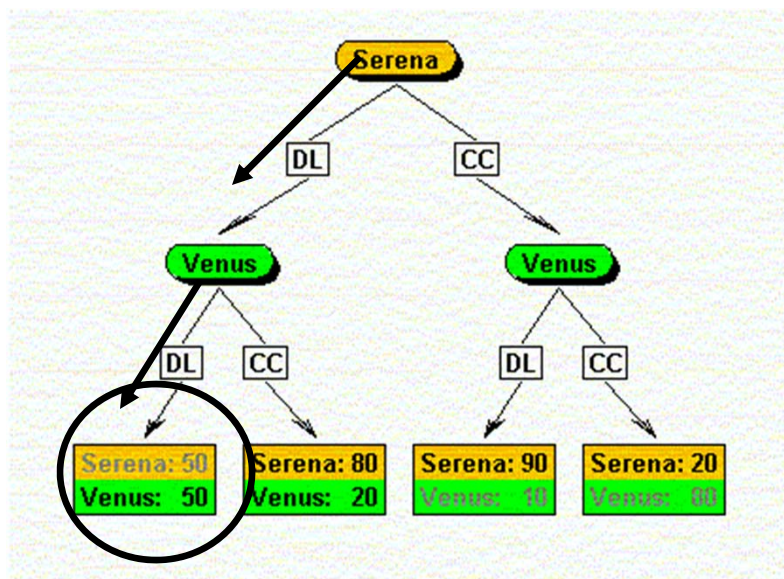
There is no possibility of mixing in a sequential move game without any information sets.

Empirical Plausibility
of Subgame Perfection?

- Consider the "Centipede Game":

The original game has 100 decision nodes hence, the name. For our purposes, four nodes will suffice.
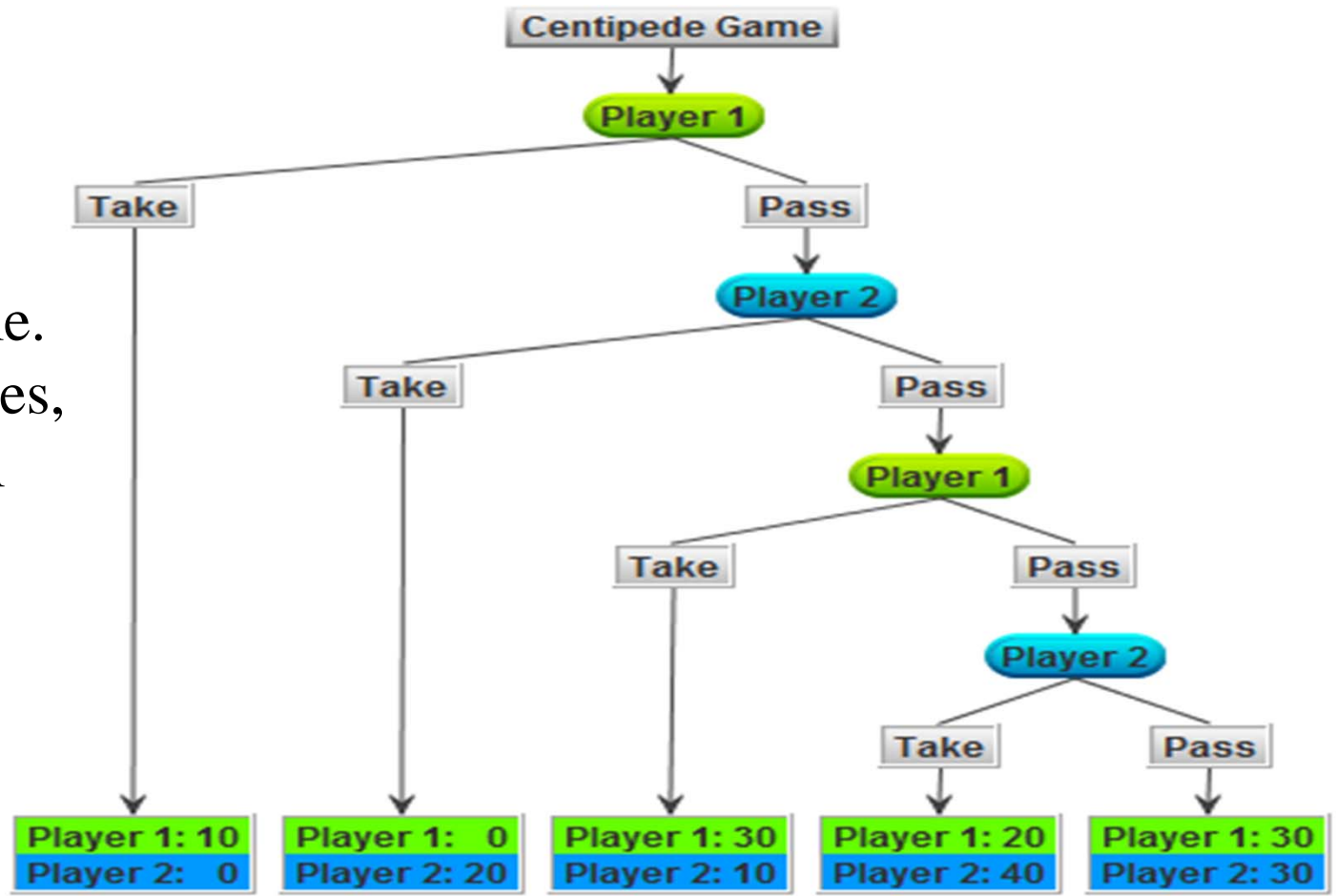
Unique Subgame Perfect Equilibrium

- Player 1 chooses Take at the First Opportunity.

- Empirically it takes time for players to figure this equilibrium prediction out.

- Players are boundedly rational

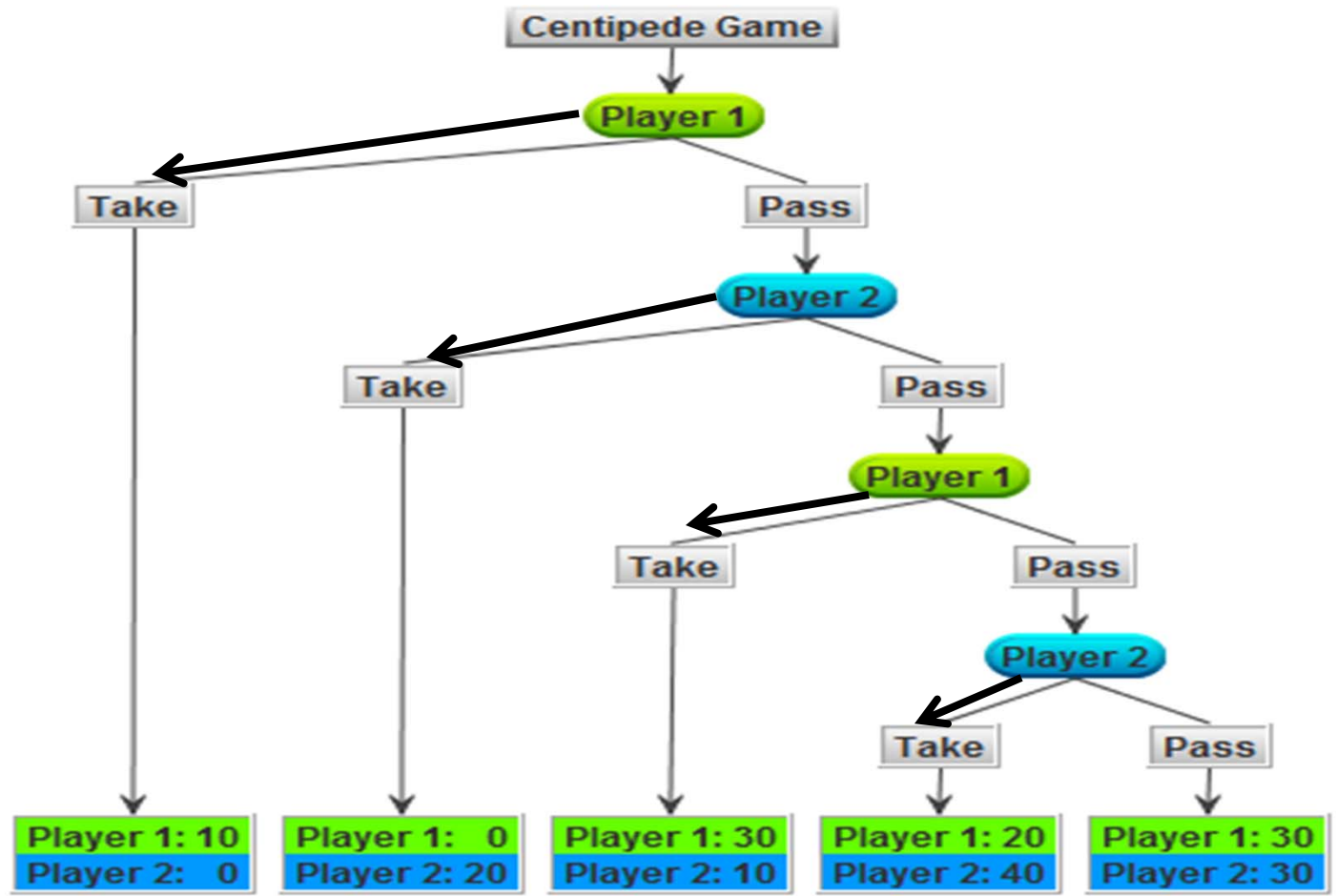# Strategic Moves

- So far we have supposed that the rules of the game were fixed, e.g., who moves first, the timing of decisions, payoffs, etc.

- In real strategic situations, players will have incentives to attempt to *manipulate the rules of the game/action choices/payoffs available for their own benefit.*

  – E.g. who moves first, what choices remain, etc.

- A **strategic move** is an action taken outside the rules of the game effectively transforming the original game, into a two-stage game—such moves are sometimes called "*game-changers.*"

- Some kind of strategic move is made in stage one and some version of the original game, possibly with altered payoffs, is then played in stage 2.

- For strategic moves to work, they must (1) be *observable* to the other players and (2) *irreversible* (to the extent this is credible) so that they alter other player's expectations and make the outcome more favorable to the player making the strategic move.

Kinds of Strategic Moves

- **Commitments**:
  - Irreversibly limit your choice of action thereby forcing the other player to choose his/her best response to your preferred action.
  - E.g., in the Pittsburgh left-turn game, I hit the gas pedal hard and jump the light early to make my left turn, so your best response is to yield.
- **Threats**: If you do not choose an action I prefer, I will respond in a manner that will be bad for you (in the second stage).
  - E.g., if you do not support my bill in Congress, I will raise money for your opponent this November.
- **Promises**: If you choose an action I prefer, I will respond in a manner that will be good for you.
  - E.g., if you vote for my bill in Congress, I will send a check for your re-election campaign.
- Note that both threats and promises are costly if they have to be carried out. However, if a threat works to alter the target player's behavior, there is no cost to the player of issuing the threat, while if a promise works, there is a cost.

Why Strategic Moves?

- Freedom of choice / a larger action space can be a bad thing.
  - For example, allowing students to make up a midterm exam or perform extra credit is costly – a new exam, extra assignments have to be written/graded.
  - By announcing and adhering to a no make-up, no extra credit, no exceptions policy and announcing it on day 1 of class, writing it into the syllabus, repeating it over and over, etc. your professor is made strictly better off.

Credibility of Strategic Moves

- The problem with strategic moves, especially threats and promises is that they may not be credible.

  - Ex-post, you may not want to carry out a costly threat or follow through with a promised reward.

- Strategic moves that are not credible will be ignored.

- The way to make strategic moves credible is to either take options off the table completely by making a truly irreversible move, or make it costly for the strategic mover to deviate from his strategic move, i.e., change the payoffs of the game so that following through with the strategic move is a best response in the second stage.

# Illustration of Credibility Issue: Nuisance Lawsuits

- Consider a game between a Plaintiff and a Defendant.

- Plaintiff moves first, deciding whether to file a lawsuit against Defendant at cost to the Plaintiff of k>0. If a lawsuit is filed, the Plaintiff makes a take-it-or-leave it settlement offer of s>0.

- Defendant accepts or rejects. If Defendant rejects, Plaintiff has to decide whether or not to go to trial at cost c>0 to the Plaintiff and at cost d>0 to the Defendant.

- If the case goes to trial, Plaintiff wins the amount w with probability p and loses (payoff=0) with probability 1-p.

- Assume that pw < c, and this fact is common knowledge-a critical assumption.
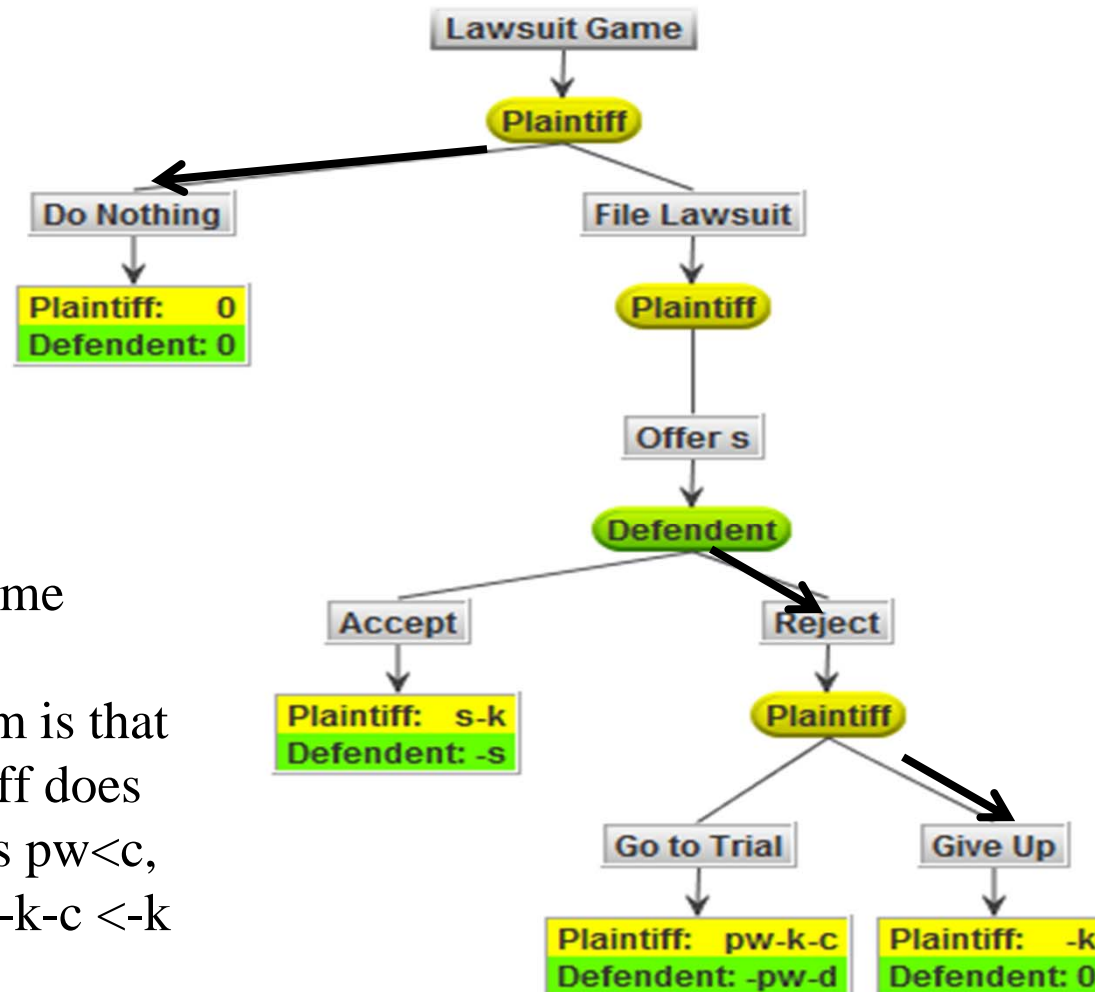
The Game in Extensive Form
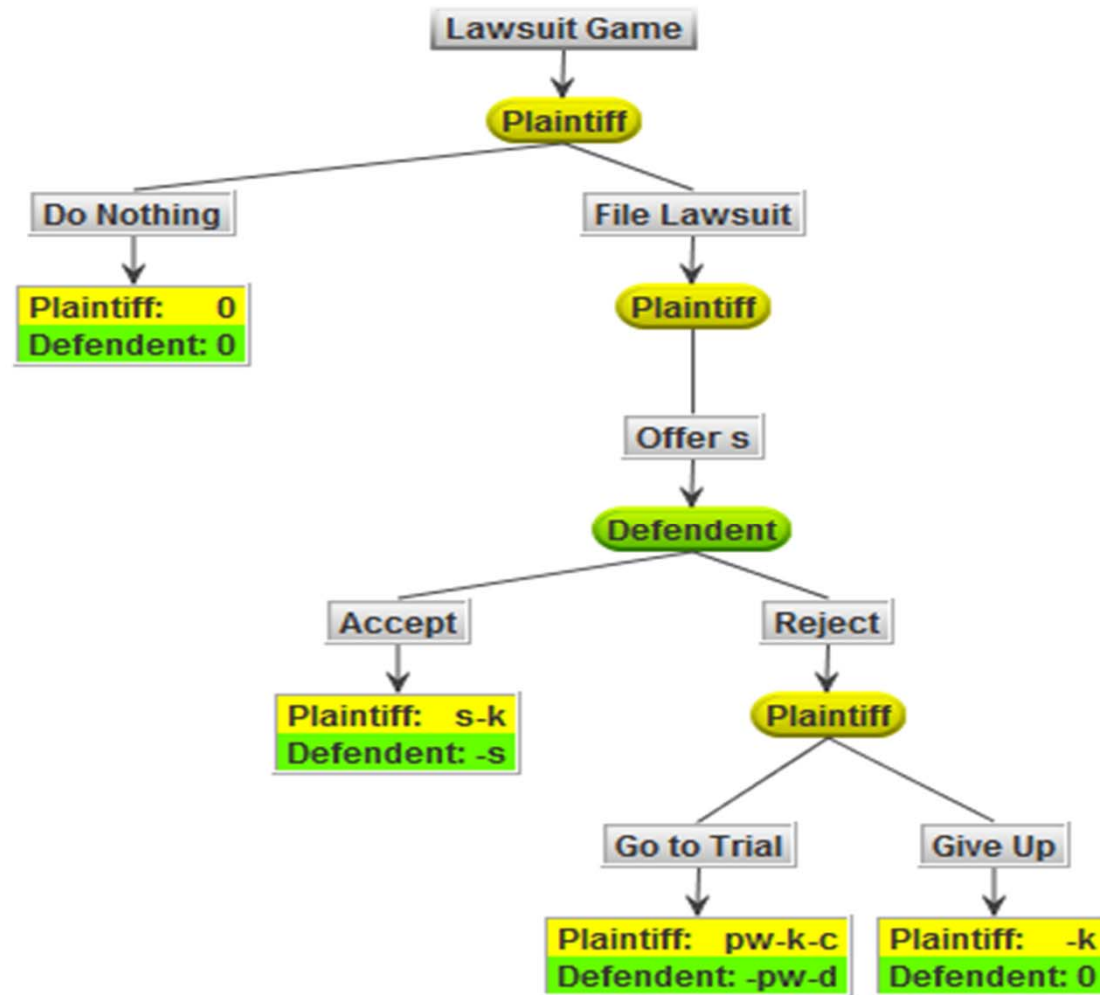
A Specific Parameterization of the Game to Play

- k, cost of filing a lawsuit = 10
- c=d=cost of a trial =30
- Settlement offer =50
- Winnings, w=100
- Probability the Plaintiff wins,p=.10
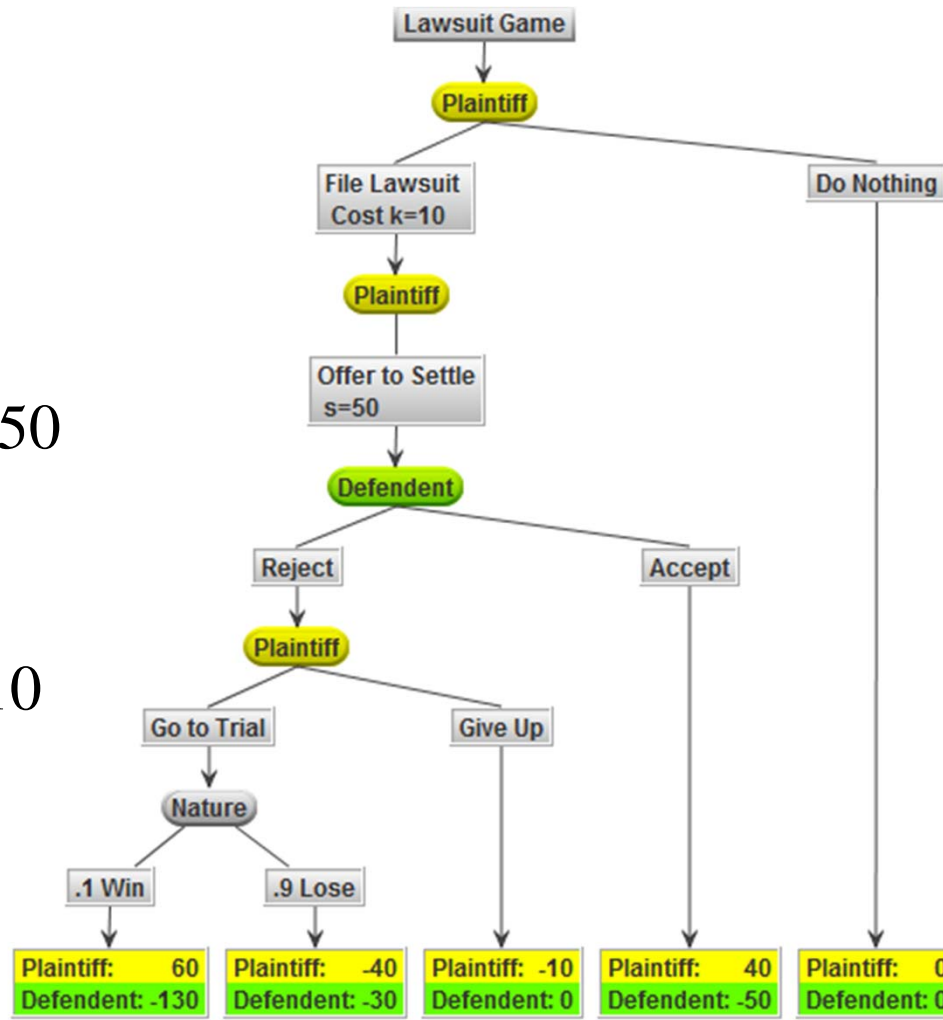
The Lawsuit is Not Credible

- The subgame perfect equilibrium is that the Plaintiff does nothing, as pw<c, so that pw-k-c <-k

How to Make the Nuisance Lawsuit Credible?

- Consider the strategic move by which Plaintiff pre-pays his lawyer the costs of a trial, c, up-front (Plaintiff puts money in a non-refundable retainer account guaranteeing the attorney's future availability for trial).

- In this case the payoff to Give Up changes from –k to –k-c.

- Plaintiff now goes to trial so long as pw>0, i.e. if there is any positive probability p of winning the amount w.

- This further implies that the Defendant prefers to go to trial only if s>pw+d.

- Defendant is now willing to settle for any amount s<pw+d.

- So the settlement range is [pw, pw+d]. If the Plaintiff can make a take-it-or-leave-it offer, what is the settlement amount s?

- Outcome is Plaintiff sues, offers to settle, and a settlement is reached.

Further Considerations

- If Plaintiff settles, her payoff is s-k-c, where k is the cost of filing the lawsuit (threat), c is the retainer cost.

- If Plaintiff can make a take-it-or-leave-it offer of s=pw+d, then the question remains as to whether the nuisance lawsuit should have been brought in the first place.

- Since doing nothing yields a payoff of 0 (this *is* a *nuisance* lawsuit, no real harm was done), then we must have that: s-k-c>0 and if s=pw+d, then we require that pw+d-k-c>0.

- One can further incorporate *malice*, whereby d enters positively in the plaintiff's payoff from choosing to go to trial.

Another Example: Criminal Law

- Accused criminal can plead guilty or not guilty.

- If he pleads not guilty, the prosecutor can offer a full sentence, valued at s or a reduced sentence valued at r. For the prosecutor, s>r, but for the accused, the payoffs are losses: -s, -r, so -r>-s; the accused strictly prefers a reduced sentence.

- Accused can accept prosecutor's offer or reject it and go to trial.

- Probability of being convicted, p, is assumed known.

- Cost to the prosecutor of taking the case to trial=c.

- Cost to the accused of defending himself at trial=d.

- Suppose d/s > 1-p, probability of an acquittal. In this case, in equilibrium the prosecutor offers no reduction, and the accused pleads guilty and gets sentence s.

- **Strategic move**: Accused requests a public defender (free lawyer), so that d=0. In response, the prosecutor offers a reduced sentence, r=ps-c<s; Accused accepts.

- Logic: By eliminating the cost to the accused of going to trial, the accused will *always go to trial* rather than plead guilty since ps<s if p<1. Knowing this, prosecutor will cut a deal offering a reduced sentence of r–ps-c, greater than or equal to the prosecutor's expected payoff from going to trial. The accused will only accept the lowest possible reduced sentence, r=ps-c. Prosecutor agrees to this deal as he is made indifferent between accepting it and going to trial.

Other Means of Acquiring Credibility

- Delegation of your decision to a disinterested /non-emotional third party.
  - If you do not pay by this date, a repo man is scheduled to take your car.
- Legally binding contracts.
  - If you violate parole, you will go back to prison.
- Reputation may be a substitute for commitment in a repeated game setting –more on this next week.
  - You have a reputation of getting angry if you don't get your way. So you get your way.
- Rational irrationality.
  - You know I'm so crazy that I will follow through with the incredible threat.

Ways to Counter Strategic Moves

- Counter sunk costs with sunk costs.
  - In the nuisance lawsuit game the defendant can pay *his* lawyer in advance for the cost of a trial. Better yet, hire "*in house*" lawyers to deter nuisance suits in the first place.
- Eliminate communication/observation – if a strategic move is not communicated/observed, it is useless.
  - For example, I'm not going to look at your sign that reads "homeless, please help."
- Undermine an opponent's need to carry out threats.
  - I won't tell if you make an exception for me; it can be "our little secret."
- Salami tactics: Reduce an opponent's threat by complying to a very small degree, especially repeatedly over time, like cutting off and offering thin slices of salami. In this way, the threat is never triggered, though you may fail to completely comply.
  - "Sorry for the delay. Times are tough for me right now. I can repay you this amount now and will send the rest of what I owe you shortly."