Bargaining Games

An Application of Sequential Move Games

The Bargaining Problem

- The "Bargaining Problem" arises in economic situations where there are **gains from trade,** for example, when a buyer values an item more than a seller.
- The problem is *how to divide the gains, for example, what price should* be charged?
- Bargaining problems arise when the size of the market is *small and there are no obvious price standards* because the good is *unique, e.g., a house at a particular location. A custom contract to develop a web page, etc.*
- We can describe **bargaining games (in extensive form)** that allow us to better understand the bargaining problem in various economic settings.

Bargaining Games

- A bargaining game is one in which two (or more) players bargain over how to divide the gains from trade.
- The gains from trade are represented by a sum of money, *M, that is "on the table."*
- Players move sequentially, making alternating offers.
- Examples:
  - A Seller and a Buyer bargain over the price of a house.
  - A Labor Union and Firm bargain over wages & benefits.
  - Two countries, e.g., the U.S. and Canada bargain over the terms of a trade agreement.

The *Disagreement Value*

- If both players in a 2-player bargaining game disagree as to how to divide the sum of money M, (and walk away from the game) then each receives their **disagreement value.**
- Let a=the disagreement value to the first player and let b=the disagreement value to the second player.
- In many cases, a=b=0, e.g., if a movie star and film company cannot come to terms, the movie star doesn't get the work and the film company doesn't get the movie star.
- The disagreement value is know by some other terms, e.g., the best alternative to negotiated agreement "BATNA."
- By **gains from trade we mean that M>a+b.**

Take it or Leave it Bargaining Games

- “Take-it-or-leave-it” is the simplest sequential move bargaining game between two players; each player makes one move.
- Player 1 moves first and proposes a division of M.
  - For example, x for player 1 and M-x for player 2.
- Player 2 moves second and must decide whether to accept or reject Player 1’s proposal.
- If Player 2 accepts, the proposal is implemented: Player 1 gets x and Player 2 gets M-x.
- If Player 2 rejects, then both players receive their disagreement values, a for Player 1 and b for Player 2.
- This game has a simple “rollback” equilibrium:
  - Player 2 accepts if M-x – b, her disagreement value.
- Often we can give an even more precise solution.

Used Car Example

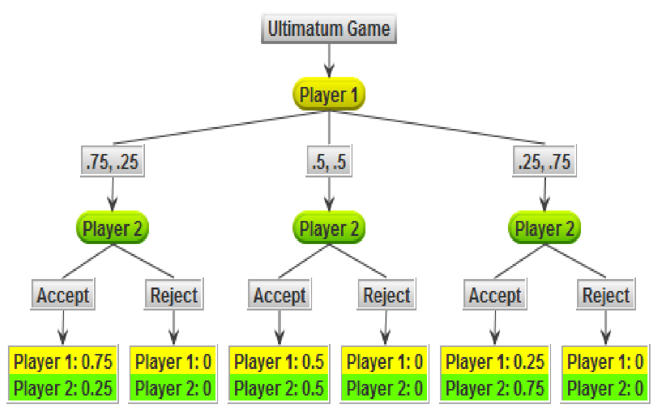
- Buyer is willing to pay a maximum price of $8,500.
- Seller will not sell for a price less than $8,000.
- M=$8,500-$8000=$500, a=b=0.
- Suppose the seller moves first and there is *perfect information*: the seller knows the maximum value the buyer attaches to the car. Then the seller knows the buyer will reject any price p>$8,500, and will accept any price p ˛ $8,500.
- The seller maximizes his profits by proposing p=$8,500, or x=$500. The buyer accepts, since M-x – b.
- The seller gets the entire amount, M=$500.
- What happens if the buyer moves first?

"Ultimatum Game"

Discrete Version of Take it or Leave it Bargaining

- Player 1 moves first and proposes a division of $1.00. Suppose there are just 3 possible discrete divisions, limited to $0.25 increments:
  - Player 1 can propose x=$0.25, x=$0.50, or x=$0.75 for himself, with the remainder, 1-x going to Player 2.
- Player 2 moves second and can either *accept* or *reject* Player 1's proposal.
- If Player 2 *accepts*, the proposal is implemented.
- If Player 2 ***rejects***, both players get $0 each. The $1.00 gains from trade *vanish*.

Computer Screen View

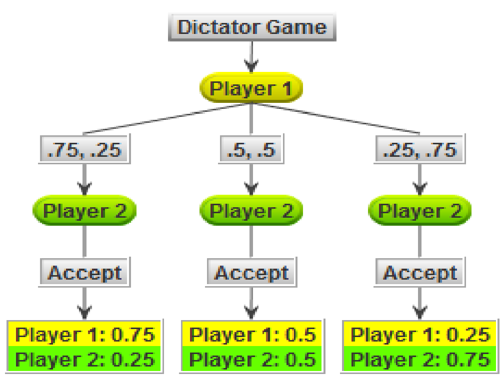


Problems with Take-it-or-Leave-it

- Take-it-or-leave-it games are too trivial; there is no back-and-forth bargaining.
- Another problem is the credibility of take-it-or leave-it proposals.
  - If player 2 rejects player 1's offer, *is it really believable* that both players walk away even though there are potential gains from trade?
  - Or do they continue bargaining? Recall that M>a+b.
- What about fairness? Is it really likely that Player 1 will keep as much of M as possible for himself?

The Dictator Game

- Are Player 1's concerned about *fairness, or are* they concerned that Player 2's will reject their proposals? The Dictator Game gets at this issue.



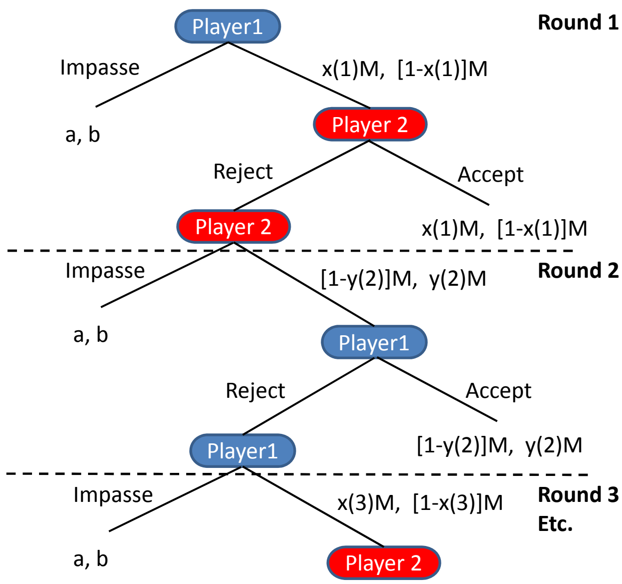
The Alternating Offers Model of Bargaining

- A sequential move game where players have perfect information at each move.
- Players take turns making alternating offers, with one offer per round, i.e., this is real back-and-forth bargaining.
- Round numbers t = 1,2,3,…
- Let $0 \le x(t) \le 1$ be the fraction of M that player 1 asks for in bargaining round t, and let $0 \le y(t) \le 1$ be the fraction of M that player 2 asks for in bargaining round t.

Alternating Offer Rules

- Player 1 begins in the first round by proposing to keep x(1)M for himself and giving Player 2 [1-x(1)]M.
- If Player 2 accepts, the deal is struck. If Player 2 rejects, another bargaining round may be played. In round 2, player 2 proposes to keep y(2)M for herself, and giving [1-y(2)]M to player 1.
- If Player 1 accepts, the deal is struck, otherwise, it is round 3 and Player 1 gets to make another proposal.
- Bargaining continues in this manner until a deal is struck or no agreement is reached (an *impasse* is declared by one player – a "holdout").
- If no agreement is reached, Player 1 earns a, and Player 2 earns b (the disagreement values).

Alternating Offers in Extensive Form



When Does it End??

- Alternating offer bargaining games could continue indefinitely. In reality they do not.
- Why not?
  - Both sides have agreed to a deadline in advance (or M=0 at a certain date).
  - The gains from trade, M, *diminish* in value over time, and may fall below a+b.
  - The players are *impatient* (time is money!).
- Take-it-or-leave-it has 1 round deadline. Let’s focus on the last two possibilities.

Decreasing Gains from Trade

- Suppose there are 2 rounds, Player 1 proposes a division of M first, player 2 accepts/rejects.
- If accepted the proposal is implemented.
- If rejected, player 2 gets to make a counter-proposal for how to split a reduced amount of money ○M, where 0<○<1 is the rate of decay.
- Player 1 can then accept or reject this final proposal. The bargaining game is then over with certainty.
- Example: Suppose that M=$12 ○=1/3.

Shrinking Gains Equilibrium

- What is the equilibrium?
- Note first that M=$12 in round 1, but only $4 in round 2.
- Working backward, player 2 knows s/he can get $3.99 in round 2 as the first mover since player 1 will strictly prefer $0.01 to nothing in round 2, and will accept that division (this follows from our assumption that players are forward looking, rational, payoff maximizers).
- Knowing this, player 1 must offer player 2 $4.00>$3.99 in round 1 keeping $12-4 = 8 for himself. As player 2 recognizes this as a better payoff than can be had by waiting, she accept player 1's first round proposal.
- Player 1's first stage offer equals the amount at stake at the start of the final round, ○M. This generalizes to any finite n-round bargaining game, where 2 < n < ∞.

Impatience as a Reason for Ending Bargaining: The Period Discount Factor, g

- The period discount factor, *0 < g < 1, provides a means of* evaluating future money amounts in terms of current equivalent money amounts.
- Suppose a player values a \$1 offer now as equivalent to \$1(1+*r) one period later. The discount factor in this* case is: g*=1/(1+r), since* g×*\$1=\$1/(1+r) now = \$1* later.
- If r is high, g is low: players discount future money amounts heavily, and are therefore very impatient.
- If r is low, g is high; players regard future money almost the same as current amounts of money and are more patient (less impatient).

Example: Bargaining over a House

- Suppose the minimum price a seller will sell her house for is \$150,000, and the maximum price the buyer will pay for the house is \$160,000. Therefore, M=\$10,000.
- Suppose both players have the exact same discount factor, g*=.80. (This implies that r=.25).*
- Suppose that there are just two rounds of bargaining. Why? The Seller has to sell by a certain date (buying another house or the Buyer has to start a new job and needs a house.
- Suppose the buyer makes a proposal in the first round, and the seller makes a proposal in the second round.
- Work backward starting in the second (last) round of bargaining and apply backward induction.

Two period bargaining over a house, continued

- Work backward. From the perspective of today, the value of the gain from trade in the second and final round is δM. In that round, the seller has to make a counterproposal.
- In that second and final round, the seller's proposal will be to keep δM for herself and the buyer must accept or reject. Since he is indifferent between accepting and rejecting (he gets 0 in either case), let's suppose he will accept the proposal.
- Knowing this, the buyer must offer the seller δM in the first period and, since in this case the seller is made indifferent between waiting and accepting, the seller accepts immediately.
- In our example, where δ=.8 and M=$10,000, the buyer offers .8M to the seller, or $8,000, keeping $2,000 for himself. The sale price of the home is thus $150,000+$8,000=$158,000.
- So, the same logic as in the decreasing gain case.

Infinitely Repeated Analysis

- Now suppose there is no end to the number of bargaining rounds; bargaining can go on forever (an infinitely repeated game)
- If the Buyer's moves first, the amount he proposes to keep for himself, *x(1)M, must leave the Seller an amount that is* equivalent to that which the Seller can get in the next round, *2*, by rejecting and proposing *y(2)M for herself next round.* The equivalent amount now, in period *1*, has value to the seller of $\delta y(2)M$, where δ is the *period discount factor*.
- Dropping the time indexes, the Buyer offers $(1-x)M=\delta yM$ *to the Seller,* $\Rightarrow \quad x=1-\delta y$
- By a similar argument, the Seller must offer $(1-y)M=\delta xM$ *to* the buyer, $\Rightarrow \quad y=1-\delta x$.
- $x=1-\delta(1-\delta x)$, $x=(1-\delta)/(1-\delta^2)=1/(1+\delta)$.
- $y=1-\delta(1-\delta y)$, $y=(1-\delta)/(1-\delta^2)=1/(1+\delta)$.

Infinitely Repeated Analysis (Continued)

- $x=y=1/(1+\delta)$. (Note that $x+y > 1$!) What is x and y?
- x is the amount the Buyer gets if he makes the first proposal in the very first round; the Seller would get $(1-x)=\delta/(1+\delta)$, because the shares must add to 100%
- $y=1/(1+\delta)$ is the amount the Seller gets if she makes the first proposal in the very first round; Buyer would get $(1-y)= \delta/(1+\delta)$ in this case.
- If the Buyer is the first proposer, he gets *xM, and the Seller* gets *(1-x)M. Price is $150,000+(1-x)M.*
- If the Seller is the first proposer, she gets *yM and the Buyer* gets *(1-y)M. Price is $150,000+yM.*
- In our example, the Buyer was the first proposer: *x=1/(1+.8)=1/1.8=.556M. The Seller gets (1-x)M=(1-* .556)M=.444M. Since M=10,000, the price of the house is $154,440. ($150,000+.444*10,000).

First Mover Advantage and Limiting Outcomes

- Summarizing, the first mover gets $1/(1+\delta)$ and the second mover gets $\delta/(1+\delta)$.
- Note that so long as $0<\delta<1$ there is a first mover advantage, as $1/(1+\delta)> \delta/(1+\delta)$.
- What happens to the first mover advantage as $\delta\rightarrow 1$? We have x=1/2, 1-x=1/2; the first mover advantage disappears!
- As $\delta\rightarrow 0$? We get x=1, 1-x=0; strong first mover advantage.
- The extent of the first mover advantage depends on the discount factor.

Differing Discount Factors

- Suppose the two players have different discount factors, for example the buyer's discount factor δ_b *is less than the seller's* discount factor δ_s.
  - Buyer is less patient than the seller. Who gets more in this case?
- When Buyer is the first mover, he now offers *(1-x)M=* δ_s *yM to* the Seller, and when Seller is the first mover she offers *(1-y)M=* δ_b *xM to the Buyer.*
- $x=1-\delta_s y$ *and* $y=1-\delta_b x$. $x^{new}=(1-\delta_s)/(1-\delta_s\delta_b)$.
- It is easy to show that $x^{new} < (1-\delta)/(1-\delta^2)=x$ *when both buyer* and seller had the same discount factors.
- Example: δ_b =.5, δ_s =.8; $x=(1-\delta_s)/(1-\delta_s\delta_b)=.2/.6=.333$. Recall that when $\delta_b = \delta_s = \delta$=.8 that x $=(1-\delta)/(1-\delta^2)=.2/.36=.556$.

Practical Lessons

- In reality, bargainers do not know one another's discount factors, δ *(or their relative levels of* patience), but may try to guess these values.
- Signal that you are patient, even if you are not. For example, do not respond with counteroffers right away. Act unconcerned that time is passing- have a "poker face."
- Remember that our bargaining model indicates that the more patient player gets the higher fraction of the amount M that is on the table.