Homework # 5

Write your answers to the following questions on separate sheets of paper. Your answers are due in class on **Tuesday April 10**. <u>No late homeworks are accepted</u>.

1. Consider a simple version of the two-firm, Cournot duopoly game in which there is incomplete information. In particular, firm 1 does not know firm 2's type - whether firm 2 has high or low costs. As a result, it is uncertain about firm 2's payoffs. Suppose firm 1 believes that firm 2 has high costs with probability 1/3 and low costs with probability 2/3. Firm 2 knows its type and knows firm 1's prior belief about whether it has high or low costs. As usual, the two firms have to decide between producing high or low output levels, with the payoffs (profits) to each firm given as follows:

		Firm 2 has high costs	
		High Output	Low Output
Firm 1	High Output	1,1	2,3
	Low Output	0,2	1,2

		Firm 2 has low costs	
		High Output	Low Output
Firm 1	High Output	1,1	0,0
	Low Output	2,0	1,-1

a. If firm 2 is a high cost firm, what is its best strategy? If it is a low cost firm, what is its best strategy?

b. If firm 1 chooses the strategy "high output" what is its expected profit? If it chooses the strategy "low output" what is its expected profit? Which strategy should it choose?

c. Suppose more generally that the probability that firm 2 has high costs is denoted by p, where $0 < p < 1$. What is the Bayes-Nash equilibrium of this game?

2. Consider a variant of the incomplete information prisoner's dilemma game discussed in class. In this variant, only the selfish type 2 player has a dominant strategy.

		2	
		C	D
1	C	4,4	0,6
	D	6,0	2,2

Player 2 selfish

		2	
		C	D
1	C	6,6	2,4
	D	4,0	0,2

Player 2 nice

a. Explain why the selfish type 2 player will always play D, while the nice type 2 player might play C or D.

b. Let p be the probability that a player 1 faces a selfish player 2. Draw the incomplete information game in extensive form.
c. Suppose that player 1 is expected to play D. What is a nice player 2's best response?
d. For what values of p (if any) is it a Bayes-Nash equilibrium for player 1 to play D in response to D by both types of player 2s? Explain.
e. For what values of p (if any) is it a Bayes-Nash equilibrium for player 1 to play C in response to D by a selfish player 2 and C by a nice player 2? Explain.

3. Consider a world in which there are two rival species – A and B. Some proportion of species A is weak; the rest are strong. There is some feature that all strong A's naturally possess. By contrast, a weak A can also display this feature, but at some cost to himself. Each member of A chooses whether to challenge a member of B for valuable territory. Strong As always challenge. If the B player fights back, a strong A wins the fight and a weak A (whether displaying or not) loses. A B player cannot tell the difference between a strong A and a displaying weak A merely from observation. A weak A in this scenario has to make a decision [back down or (display and) challenge], and a B player that is challenged also has to make a decision [fight or retreat]. Two critical factors influence the behavior of weak As and of Bs --- the cost to a weak A of displaying and the proportion of As who are weak. Depending on these factors, a weak A will always, sometimes or never (display and) challenge, and a B will always, sometimes, or never fight.

Consider the following table representing the combinations of the two critical factors:

	High cost of displaying	Low cost of displaying
Small proportion of weak As	1	2
Large proportion of weak As	3	4

Which of the four cells, 1,2,3, or 4 provides the conditions under which a pooling equilibrium exists where weak As always (display and) challenge and Bs always retreat? Carefully explain and defend your cell choice (all credit is given to your explanation and not to your choice of the cell number).

4. Consider the following 2-player, seller-buyer signaling game. The game starts with a random move by nature to determine whether the item the seller has is good or bad (think used car). The seller knows whether his item is good or bad. All that the buyer knows is there is some probability q that the item is good, and so 1-q is the probability that the item is bad. After being informed of the quality of the good, the seller has to decide whether to offer it for sale or withhold it from the market. If the seller withholds the item, the game ends with the status quo payoff of 0 for both seller and buyer. If the seller offers the item for sale and it is good, it can be sold as is, but if it is bad and the seller offers it for sale, the item has to be cleaned up at a cost c. After cleanup, a bad item looks just like a good item and the buyer can't tell the difference (the buyer does not see whether or not a cleanup has occurred). Moreover, the price of the item if sold is p, regardless of whether it is good or bad, so the price does not reveal the quality either. If the item is offered for sale, the buyer has to decide whether to buy it or not, but he does not know whether the item is good or bad. The buyer can say either yes or no to the seller's item at price p. If he says no, there is no deal and he receives a payoff of 0. The seller receives a

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payoff of 0 if the item is good and -c if the item is bad. If the buyer says yes, the item is exchanged for price p. The value a buyer receives from a good item is V and from a bad item is W. His payoff if V-p if the item is good and W-p if the item is bad. Assume that V > p > W > 0, so the buyer has a positive payoff only if he buys the good item. The seller's payoff is the price p if the item is good and p-c if the item is bad.

a. Draw the signaling game in extensive form, including the payoffs at the end of the tree.
b. Suppose p-c >0. Under what conditions will the buyer buy an item that the seller has brought to market? Will the buyer always get a good quality item? Why or why not (what type of equilibrium is this, pooling or separating?)
c. Suppose now that c>p. Under what conditions will the buyer buy an item that the seller has been brought to market? Will the buyer always get a good quality item? Why or why not (what type of equilibrium is this, pooling or separating?)