

? ? ? Signaling Games ? ? ?

- In incomplete information games, one player knows more information than the other player.
- So far, we have focused on the case where the “type” of the more informed player was known to that player but unknown to the less informed player.
- Signaling games are incomplete information games where the more informed player has to decide whether to *signal* in some way their true type, and the less informed player has to decide how to respond to both the uncertainty about his opponent’s type *and* the signal his opponent has sent, recognizing that signals may be *strategically chosen*.

What are Signals?

- **Signals are *actions* that more informed players use to convey information to less informed players about the unobservable type of the more informed player.**
 - Example: A player who wants the trust of less informed player may signal past instances of trust, may provide verbal assurances of trustworthiness, the names of character references/former employees on a resume, discuss church attendance, charity work, etc.
- **Signals may or may not be credible: Why? Because individuals will use signals *strategically* when it suits them. Less qualified applicants may “pad” their resumes, lie about their past work history/qualifications, embezzle from their church/charity.**
 - Talk is cheap: “Yeah, right”; “whatever”; “I could care less” are common.
 - The more credible signals involve costly actions, e.g. a college diploma, an artistic portfolio, a published book, a successful business.

Examples of Strategic Signaling

- Insurance contracts: Accident prone types will want greater coverage, lower deductibles, while those unlikely to be in accidents will require minimal coverage, higher deductibles. Insurance companies respond to these signals by charging higher prices for greater coverage/lower deductible.
- Used cars: The dealer has to decide whether to offer a warranty on a used car or offer the car “as is.”
- Pittsburgh left-turn game: The left-turner can attempt to signal whether he is a Pittsburgher or an Out-of-Towner.
- Letter grade or pass/fail grades: Letter grade signals more commitment, risk-taking; pass grade signals lowest possible passing letter grade, C-.

Example 1: Prisoner's Dilemma Again.

- Recall the Prisoner's Dilemma game from last week, where player 1's preferences depend on whether player 2 is nice or selfish.

		Player 2	
		C	D
1	C	4,4	0,6
	D	6,0	2,2

Player 2 selfish

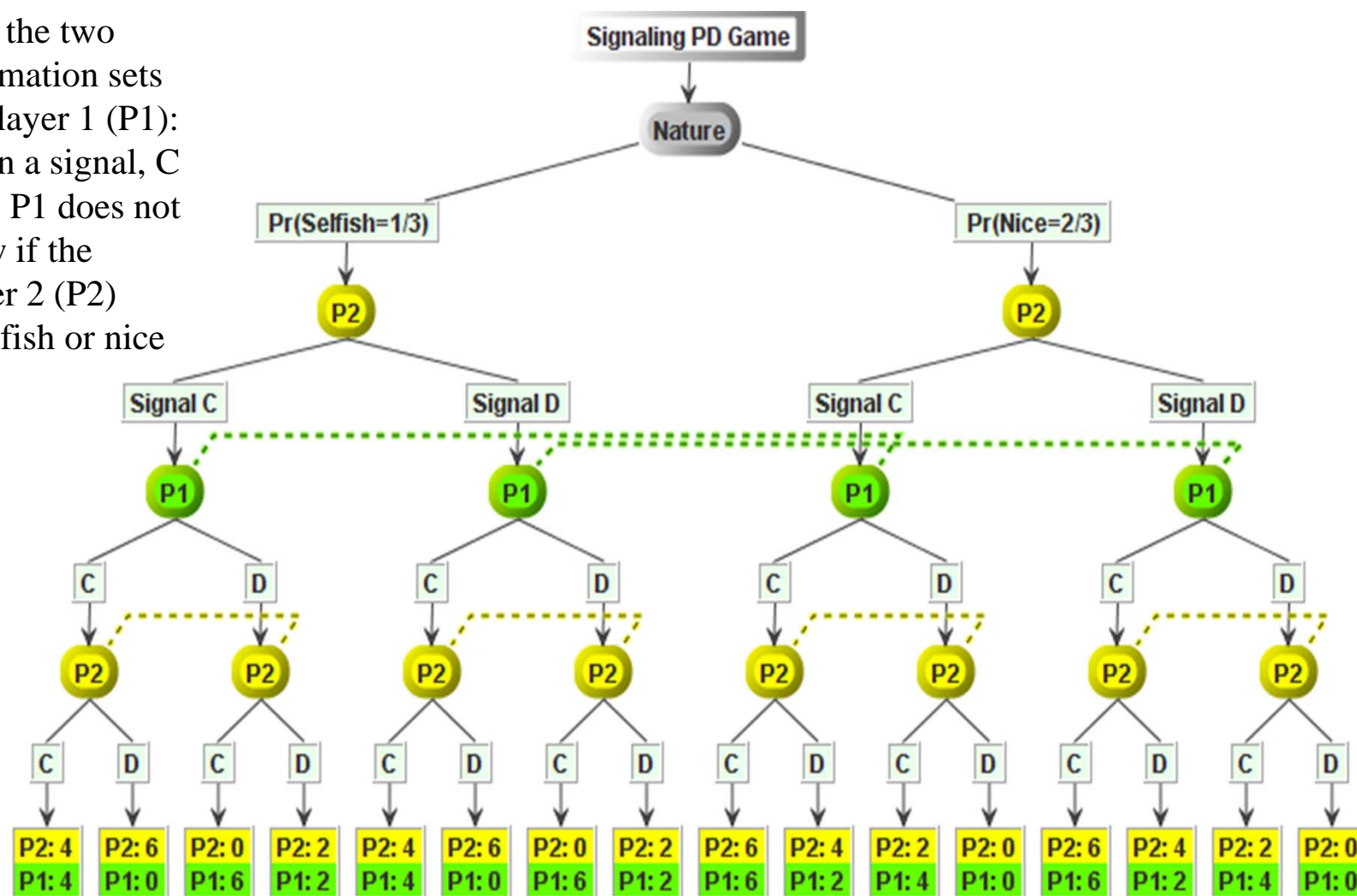
		Player 2	
		C	D
1	C	6,6	2,4
	D	4,2	0,0

Player 2 nice

- Suppose the player 2 can costlessly signal to player 1 her action choice before Player 1 gets to choose. The signal is **nonbinding**, “**cheap talk**.” Player 1 observes this signal before making his own move, but still does not know what type of player 2 he is facing, selfish or nice.
- For example, if player 2 signals C, player 1 wants to play C if player 2 is nice, but D if player 2 is selfish.

Example 1 in Extensive Form

Note the two information sets for player 1 (P1):
Given a signal, C or D, P1 does not know if the player 2 (P2) is selfish or nice



Analysis of Example 1

- The signal is player 2's cheap talk message of C or D.
 - “I intend to play C” or “I intend to play D”
- Both player 2 types have an incentive to signal C. A selfish player 2 wants player 1 to play C so she can play D and get the highest possible payoff for herself. A nice player 2 wants player 1 to play C so she can play C and get the highest possible payoff for herself.
- If the two types sent different signals, player 1 would be able to differentiate between the two types of player 2, and the game would then be like a game of complete information.
- Therefore, both player 2 types signal C: the signal is perfectly uninformative; this is called a *pooling equilibrium* outcome.
- Player 1 will play C if the prior probability that player 2 is selfish $p < \frac{1}{2}$ and will play D if $p > \frac{1}{2}$.
 - In this example, since $p = \frac{1}{3} < \frac{1}{2}$, player 1 should play C.

Example 2: Signaling in Coordination Games (1)

- Consider the battle of the sexes game:

		Kaylee	
		Basketball Game	Party
Luke	Basketball Game	2,1	0,0
	Party	0,0	1,2

- Now suppose that, prior to play of the game, Kaylee says: “Luke, I’m going to the party.”
- Kaylee’s message is *self-committing*; if she thinks Luke believes her message that she will go to the party, then her best response is to go to the party.
- It is also *self-signaling*: Kaylee wants to say she is going to the party if and only if that is where she is planning to go.
- Messages that are both self-committing and self-signaling are highly credible.

Signaling in Coordination Games

(2)

- Consider this Stag Hunt game: Amy and Pete are working on their Econ 1200 term project. Either can choose to supply high or low effort:

		Pete	
		High effort	Low effort
Amy	High effort	5,5	1,3
	Low effort	3,1	2,2

- Suppose prior to the game, Amy says: “Pete, I plan to put in high effort on the term project.” Then, the game is played.
- Amy’s message is self-committing if she thinks Pete will believe her message, then it is a best response for her to supply high effort.
- But Amy’s message is *not* self-signaling – Amy would like Pete to believe she will supply high effort even if she plans to supply low effort. So perhaps the message is less credible in this case?

Credibility of Signaling Continued

- Note that in a standard PD, where it is known that there are *no* type distinctions, i.e., *both players are known to be selfish*, then a message by one player that he intends to cooperate (not confess) is neither self-committing nor self-signaling! In other words, it is *incredible*.
- What about two-way communication?
 - Generally, this would lead to some kind of negotiation game.
 - Alternatively, if communication is expensive, time consuming, one can appeal to social norms of behavior, e.g., the bride's family pays for the wedding.

Example 3: Market Entry Game with Signaling

- Two firms, incumbent is Oldstar, the new firm is Nova.
- Oldstar is a known, set-in-its-ways company, but Nova is a relatively unknown startup. Oldstar reasons that Nova is one of two types: “strong” or “weak.” Nova knows its type.
- In a fight, Oldstar can beat a weak Nova, but a strong Nova can beat Oldstar. The winner has the market all to itself.
- If Oldstar has the market to itself, it makes a profit of 3, and if Nova has the market to itself it makes a profit of 4.
- The cost of a fight is -2 to both firms.

These facts are reflected in the payoff matrix given to the right

	Oldstar	
	Fight	Retreat
Nova	Strong ¹	3
	<div>2</div> <div>-2</div>	<div>4</div> <div>0</div>
Weak ²	4	
	<div>-2</div> <div>1</div>	<div>2</div> <div>0</div>

The Equilibrium Without Signaling

- Let w be the probability that Nova is weak, and so $1-w$ is the probability that Nova is strong.
- In the absence of any signals from Nova, Oldstar will calculate the expected payoff from fighting, which is

$$(w)1 + (1-w)(-2) = 3w - 2,$$

and compare it with the payoff from retreating which is 0.

- If $3w - 2 > 0$, Oldstar's best response is to fight, or in other words, Oldstar fights if:

$$3w > 2, \text{ or } w > 2/3.$$

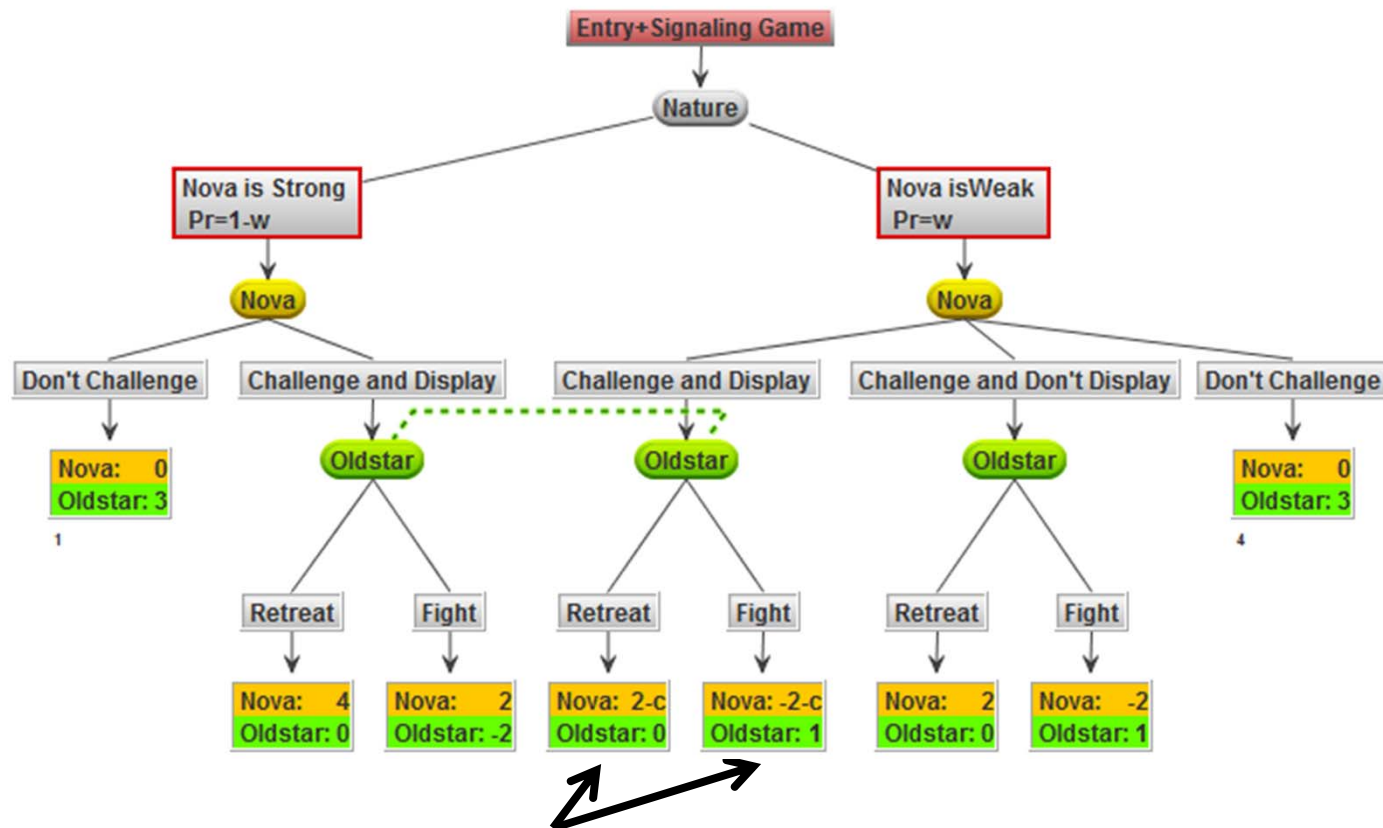
\Rightarrow Oldstar fights only if its prior belief is that Nova is very likely to be weak, (chance is 2 out of 3).

Signaling in Example 3

- Suppose Nova can provide a signal of its type by presenting some evidence that it is strong, in particular, by displaying prototypes of its new advanced products before it has the ability to produce and distribute a large quantity of these products.
- If it is unable to produce/distribute enough to meet market demand --if it is “weak”--, Oldstar may be able to copy the new products and quickly flood the market. But if Nova is “strong” and is ready to produce/distribute enough to meet market demand, it will squeeze Oldstar out of the market.
- Nova’s signal choice is therefore to **display** the new products, or **not display** the new products.
- Suppose it is costly for a weak Nova to imitate a strong Nova. The cost for a weak Nova to display, c , is *common knowledge* (along with w). The cost for a strong Nova to display is 0.

The Game in Extensive Form

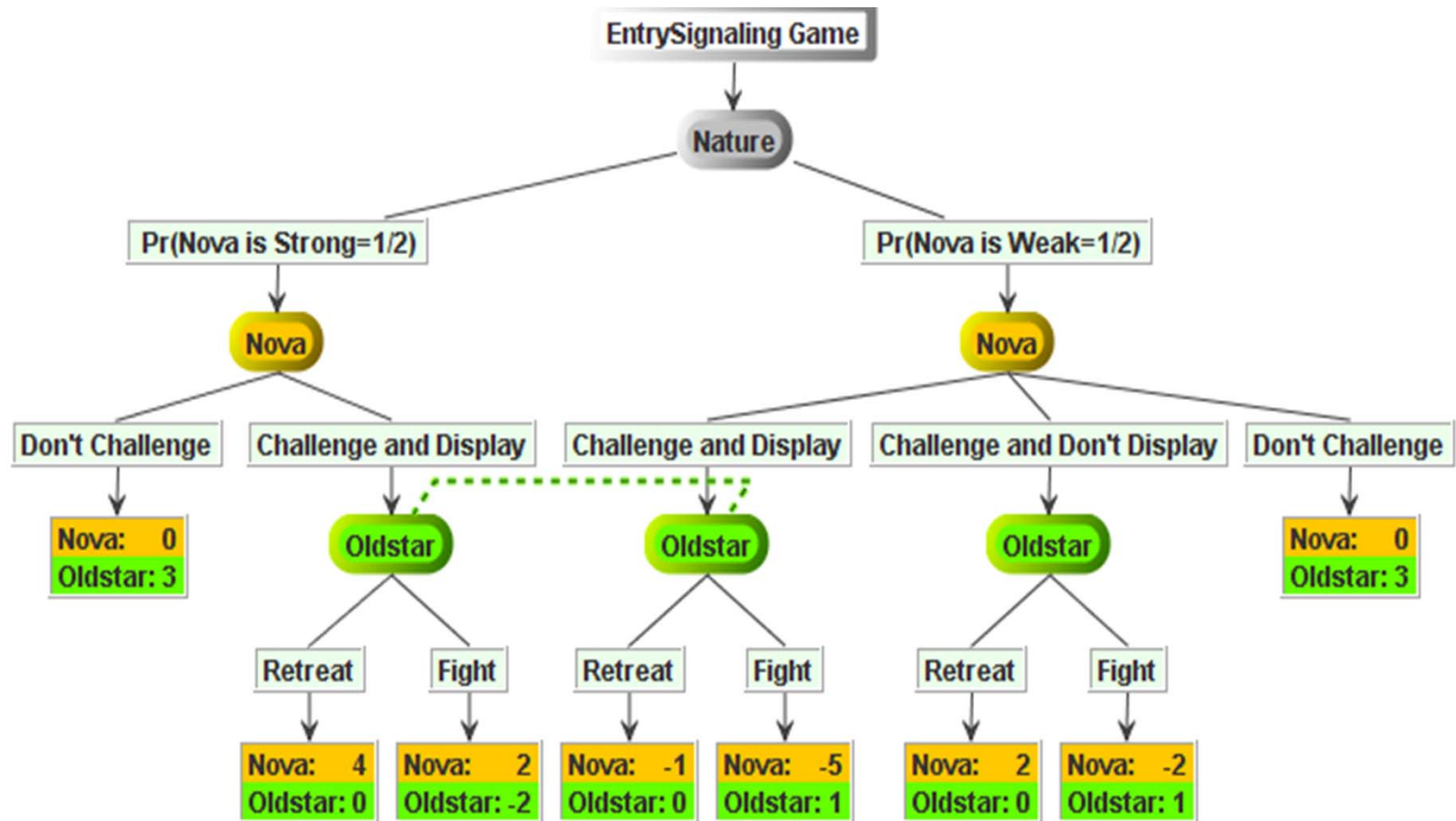
- Suppose w , the probability that Nova is weak is $\frac{1}{2}$



The cost c of displaying only applies to a weak Nova who chooses to display.

Case 1

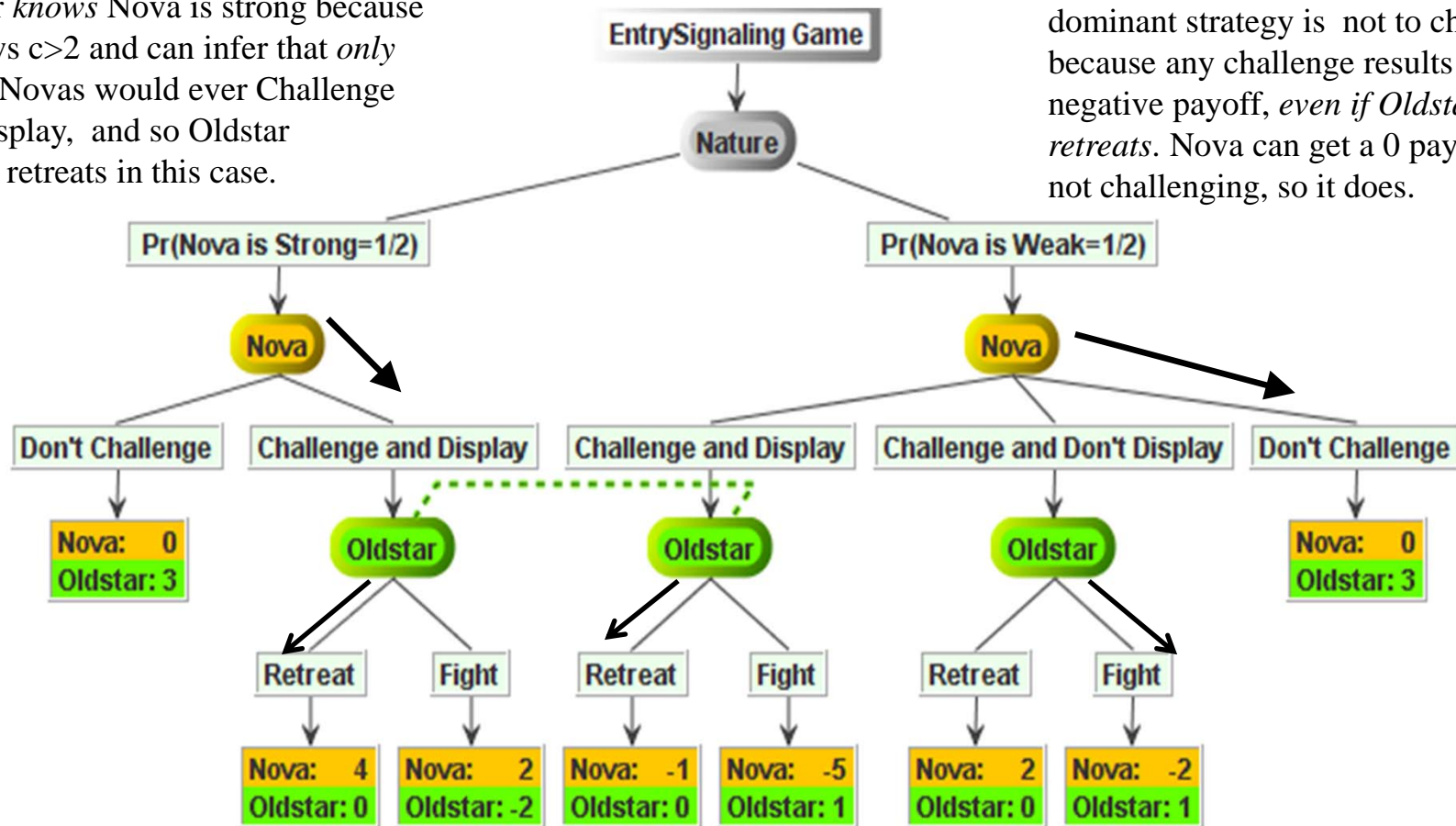
- Suppose $c > 2$, for example, $c=3$.



Strong Novas Challenge and Display, Weak Don't Challenge: There is Perfect Separation

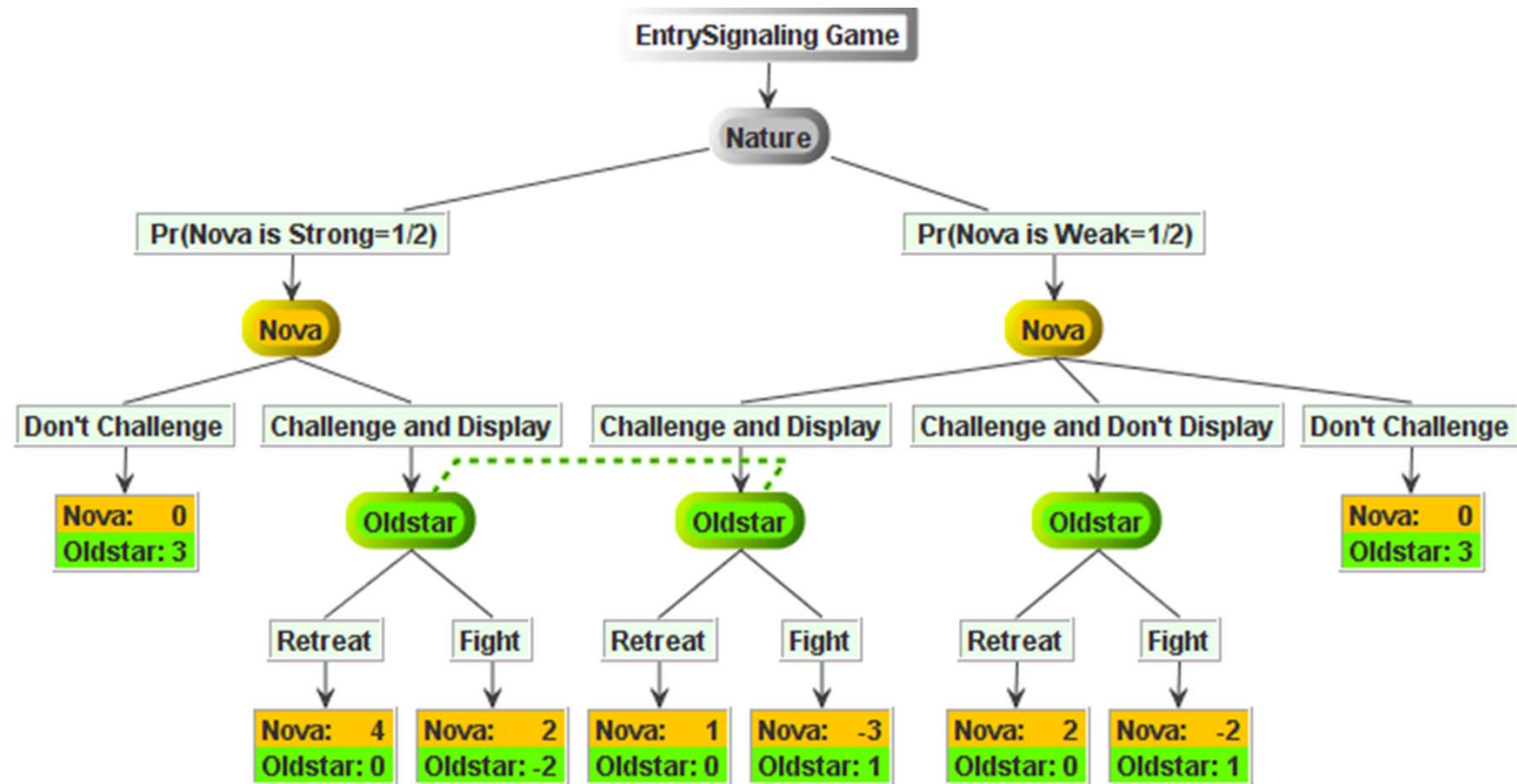
If Nova Challenges and Displays, Oldstar *knows* Nova is strong because it knows $c > 2$ and can infer that *only strong* Novas would ever Challenge and Display, and so Oldstar always retreats in this case.

If Nova is weak, and $c > 2$, Nova's dominant strategy is not to challenge, because any challenge results in a negative payoff, *even if Oldstar retreats*. Nova can get a 0 payoff from not challenging, so it does.



Case 2

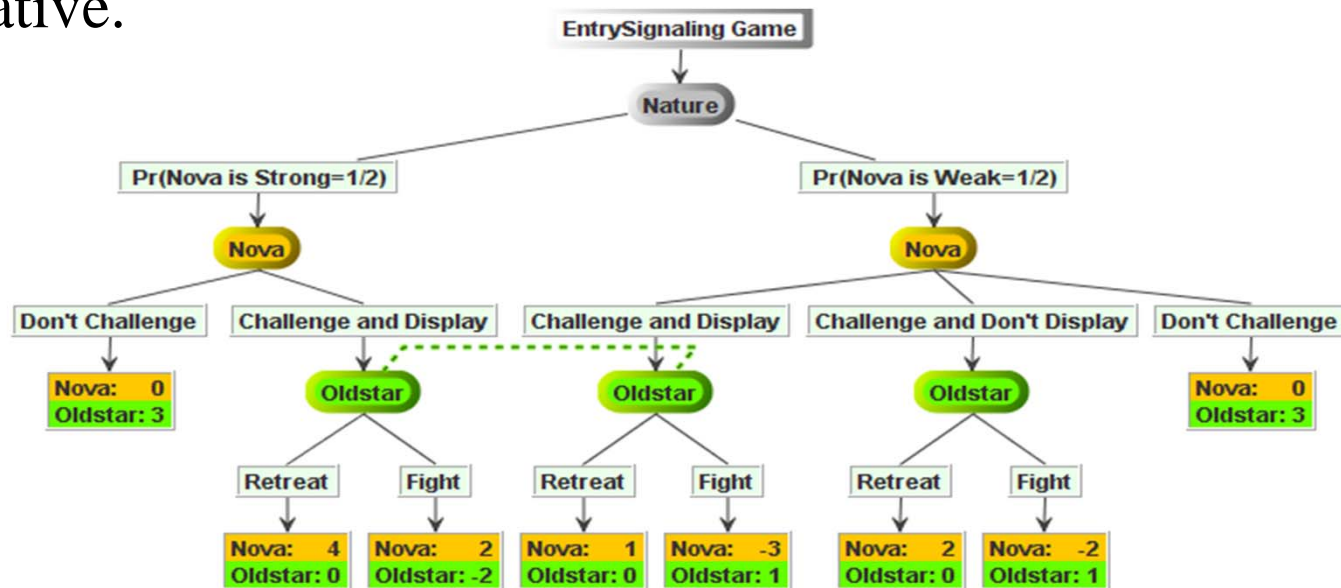
- Suppose $w < 2/3$ and $c < 2$, for example, $w=1/2$ and $c=1$.



Suppose Oldstar's strategy is to retreat if Nova Challenges and Displays. If $c < 2$, even weak Novas get a positive payoff from challenging with a Display

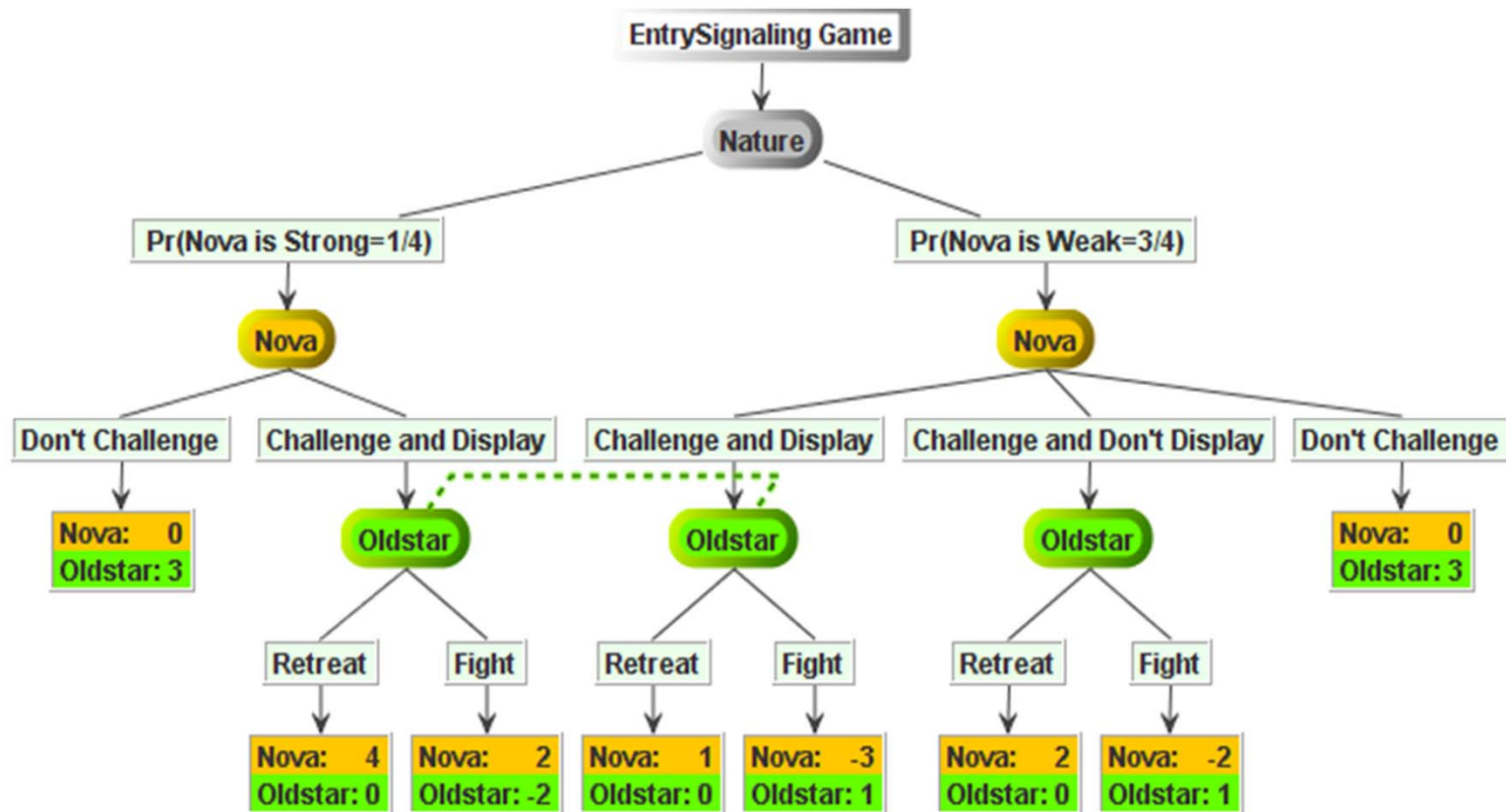
The Equilibrium involves Pooling by Both Types

- If $w < 2/3$ Oldstar retreats if it sees a Display.
- If $c < 2$, Both types of Novas find it profitable to Challenge and Display because Oldstar will retreat— a pooling equilibrium.
- If Oldstar fights, it gets $w(1)+(1-w)(-2)=3w-2$.
- If $w < 2/3$, Oldstar's expected payoff from fighting is negative.



Case 3

- Suppose $c < 2$ and $w > 2/3$, for example, $c=1$ and $w=3/4$.
- Neither a separating nor a pooling equilibrium is possible.
 - Weak Novas challenge with some probability p .



How Does Oldstar React?

- Oldstar draws inferences *conditional* on whether or not Nova displays. It does this according to Bayes Rule.
- Oldstar responds to a display by fighting with probability q .

		Display		Sum of Row
		Yes	No	
Nova's True Type	Strong	$1-w$	0	$1-w$
	Weak	wp	$w(1-p)$	w
Sum of Column		$1-w+wp$	$w(1-p)$	

- If Oldstar sees a display, with probability $wp/(1-w+wp)$ Nova is weak, and with probability $(1-w)/(1-w+wp)$ Nova is strong.

Semi-Separation Involves a Mixed Strategy

- The prior probabilities of a weak/strong Nova get updated following observation of “Challenge and Display”.
- Oldstar’s expected payoff from fighting conditional on observing a display is now:

$$1(wp/(1-w+wp) + (-2)[(1-w)/(1-w+wp)]) \\ = [wp-2(1-w)]/(1-w+wp)$$

- Oldstar’s (expected) payoff from retreating is remains 0.
- So, in mixing, Nova chooses a p so as to keep Oldstar perfectly indifferent between fighting and retreating:

$$[wp-2(1-w)]/(1-w+wp)=0$$

$$\text{or } [wp-2(1-w)]=0$$

$$p=2(1-w)/w.$$

Mixed Strategy, Continued

- Given Oldstar's strategy of fighting when it sees a display with probability q , a weak Nova's expected payoff from challenging with a display is:

$$q(-2-c)+(1-q)(2-c)=2-c-4q$$

- A weak Nova's (expected) payoff from not challenging is always 0.
- So, Oldstar chooses q to keep a weak Nova perfectly indifferent between challenging with display and not challenging

$$2-c-4q=0$$

$$q=(2-c)/4.$$

- Summary:** Mixed strategy, semi-separating equilibrium is for weak Nova, to display with probability $p=2(1-w)/w$, and for Oldstar to challenge with probability $q=(2-c)/4$.

Summary:

3 Equilibrium Types Depend on c and w

		Probability Nova is Weak, w	
		$w < 2/3$	$w > 2/3$
Cost of Display, c	$c < 2$	Pooling	Semi-Separating
	$c > 2$	Separating	