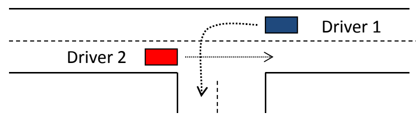
Coordination Problems and Other Topics in Simultaneous Move Games

More Complicated Simultaneous Move Games

Coordination Problems

- Any time we have multiple Nash equilibrium, we loose predictive power; we cannot say which equilibrium will be chosen.
- Games with multiple Nash equilibria are known as coordination games.
- Coordination games can be strategic settings where all players want to do the same thing, e.g. convict the guilty acquit the innocent.
- Coordination games can also be strategic settings where players want to do different things, e.g. drive through an intersection / yield to the other driver.

The Pittsburgh Left-Turn Game



Payoffs are in terms of seconds gained or lost (with minus sign)

		Driver 2 (Plan: proceed through intersection)	
		Proceed with Plan	**Yield** to Other Driver
Driver 1 (Plan: make left turn)	**Proceed** with Plan	-1490, -1490	5, -5
	Yield to Other Driver	-5, 5	-10, -10

Equilibria in the Pittsburgh Left-Turn Game

- Both equilibria have one driver proceeding, while the other yields.
- **Both equilibria are efficient**.
- In such cases, governments often step in to impose one equilibrium *by law.*

This game is a particular version of the game of "Chicken"

		Driver 2 (Plan: proceed through intersection)	
		Proceed with Plan	**Yield** to Other Driver
Driver 1 (Plan: make left turn)	**Proceed** with Plan	-1490, -1490	5, -5
	Yield to Other Driver	-5, 5	-10, -10

"Anti-Coordination" Coordination Games

- Not the same dress! (Fashion)
- Not the same words/ideas (Writing)
- LUPI, a Swedish lottery game designed in 2007:
  - Choose a positive integer from 1 to 99 inclusive.
  - The winner is the one person who chooses the lowest unique positive integer (LUPI).
  - If there is no unique integer choice, there is no winner.

"Coordination" Coordination Games

- Suppose you and a partner are asked to choose one element from the following sets of choices. If you both make the same choice, you earn $1, otherwise nothing.
  - {Red, Green, Blue}
  - {Heads, Tails}
  - {7, 100, 13, 261, 555}
- Write down an answer to the following questions. If your partner writes the same answer you win $1, otherwise nothing.
  - A positive number
  - A month of the year
  - A woman's name

Another Example of Multiple Equilibria in Pure Strategies: The Alpha-Beta game.

	L	R
U	1 5	0 4
M	0 6	1 3
D	4 6	4 4

- Strategies for Alpha (Row) are Up (U), Middle (M) or Down (D). Strategies for Beta (Column) are Left (L) or Right (R).
- What are the multiple equilibria in this game? Why?

Finding Equilibria by Eliminating Dominated Strategies....

- Strategies U and M are weakly dominated for Player Alpha by strategy D.
- Suppose we eliminate strategy U first. The resulting game is:

		Beta	
		L	R
Alpha	M	0 6	1 3
	D	4 6	4 4

- It now appears as though there is a unique equilibrium at D,R.

...Can Lead to the Wrong Conclusion.

- Suppose instead we eliminated Alpha's M strategy first. The resulting game is:

		Beta	
		L	R
Alpha	U	1 5	0 4
	D	4 6	4 4

- Now it appears as though D,L is the unique equilibrium.
- Lesson: If there are weakly dominated strategies, consider all possible orders for removing these strategies when searching for the Nash equilibria of the game.

Finding Equilibria via Best Response Analysis *Always* Works:

		Beta	
		L	R
Alpha	U	1 5	0 4
	M	0 6	1 3
	D	4 6	4 4

- There are two mutual best responses: (D,L) and (D,R).These are the two Nash equilibria of the game.

Multiple Equilibria: Fact of Life or Problem to be Resolved?

- Suppose we have multiple Nash equilibria –the Alpha-Beta game is an example. What can we say about the behavior of players in such games?
- One answer is *we can say nothing*: both equilibria are mutual best responses , so what we have is a coordination problem as to which equilibrium players will select. Such coordination problems seem endemic to lots of interesting strategic environments.
  - For instance there are two ways to drive, on the right and on the left and no amount of theorizing has led to the conclusion that one way to drive is better than another. The U.S. and France drive on the right; the U.K and Japan drive on the left.
- A second answer is that *if economics strives to be a predictive science, then multiplicity of equilibria is a problem that has to be dealt with*. For those with this view, the solution is to adopt some *additional criteria, beyond mutual best response,* for selecting from among multiple equilibria.
- Some criteria used are:
  - Focalness/salience (we have already seen this in the coordination games).
  - Fairness /envy-freeness (we have already seen this in the alpha-beta game).
  - Efficiency /Payoff dominance
  - Risk dominance

Efficiency

- In selecting from among multiple equilibria, economists often make use of efficiency considerations: which equilibrium is most efficient?
- An *equilibrium* is **efficient** if there exists no other equilibrium in which *at least* one player earns a higher payoff and no player earns a lower payoff.
- Efficiency considerations require that all players know all payoffs and believe that all other player value efficiency as a selection criterion.
  - This may be unrealistic: Consider the Alpha-Beta game, for example: many pairs coordinate on D,R over D,L.
- Efficiency may be more relevant as a selection criterion if agents can communicate/collude with one another. Why?

Line Formation as an Example

- There are two ways for a firm to line customers up to conduct business with spatially separated agents of the firm:
  1. There is a line for every agent of the firm, and each customer chooses which line to get in (as at the grocery store or a toll plaza).

     or
  2. There is one "snake line," and customers at the head of the line go to the first agent who becomes available (as at a bank or airport check-in counter).
- *Efficiency considerations* govern the choice between these two line conventions. Can you explain why?

Efficiency Considerations Cannot Always be Used to Select an Equilibrium

- There can be multiple *efficient* equilibria.
- Example 1: Driving on the left (U.K.) or right hand side (U.S.) of the road.
  - Coexistence of both suggest there is not efficiency difference.
- Example 2: The Pittsburgh Left-Turn Game.
  - What is more efficient letting the left turner move or making the left turner wait?
  - While the first left turner moving might be efficiency enchancing, e.g., if there are no left turn signals, if more than one left turning vehicle exploits the Pittsburgh left turn it might be less efficient.
  - Bottom line: It's not clear whether the Pittsburgh left-turn is efficiency enhancing or not.

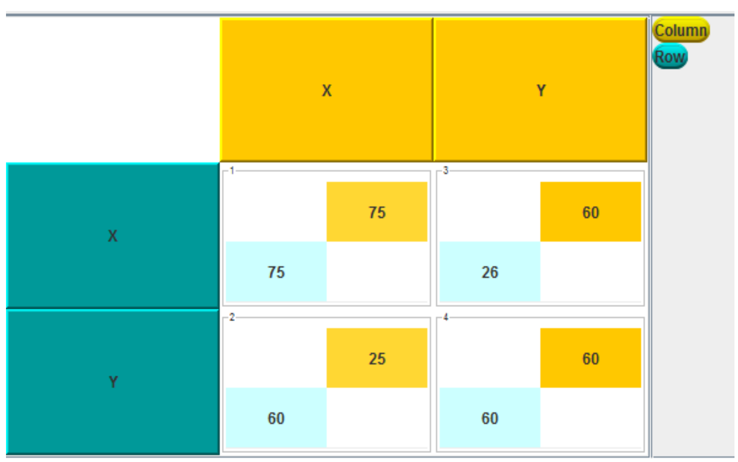
Risk Considerations and Equilibrium Selection.

- Consider the following game

	X	Y
X	75, 75	25, 60
Y	60, 25	60, 60

- What are the Nash equilibria?
- Which equilibrium do you think is the most likely one to be chosen?

Computer Screen View



Risk Dominance

- The two equilibria are X,X and Y,Y.
- X,X is efficient (also known as *payoff dominant*) but Y,Y is less risky, or what we call the *risk dominant* equilibrium – why?

	X	Y
X	**75, 75**	**25, 60**
Y	**60, 25**	**60, 60**

- In choosing Y, each player insures himself a payoff of 60 regardless of what the other player does. So, Y is a risk-free or "safe" strategy.
- How can we evaluate the expected payoff from playing X?

The Principle of *Insufficient Reason*

- If you are completely ignorant of which of n possible outcomes will occur, assign probability 1/n that each outcome will occur.

	X	Y
X	**75, 75**	**25, 60**
Y	**60, 25**	**60, 60**

- Applying this principle to this two state game, we assign probability ½ to our opponent choosing X and probability ½ to our opponent choosing Y.
- The *expected payoff* from playing X is: ½(75)+½(25)=50.
- The *expected payoff* from playing Y is: ½(60)+½(60)=60.
- Since the expected payoff from playing Y, 60 is greater than the expected payoff from playing X, 50, Y is the preferred choice. If both players reason this way, they end up at Y,Y.

Maximin/Minimax Method

- For zero, or constant-sum games only, so this is not so general…
- Each player reasons that what's good for me is bad for my opponent.
- Suppose payoffs are written for row only (Network game is constant-sum).
- Row looks only at the lowest payoff in each row and chooses the row with the highest of these lowest payoffs (maximizes the minimum Row gets -maximin)
  - Network 1 chooses sitcom because 52% > 45%.
- Column looks only at the highest payoffs in each column and chooses the column with the lowest of these highest payoffs (minimizes the maximum Row gets minimax).
  - Network 2 chooses game show because 52%<55%.

Network 2

		Sitcom	GameShow	Row Min
Network 1	Sitcom	55%	52%	52%
	Game Show	50%	45%	45%
	Column Max	55%	52%	

Adding More Strategies

- Suppose we add a third choice of a "talent show" to *Battle of the Networks*.

Network 2

		Sitcom	Game Show	Talent Show
	Sitcom	55%, 45%	52%, 48%	51%, 49%
Network 1	Game Show	50%, 50%	45%, 55%	46%, 54%
	Talent Show	52%, 48%	49%, 51%	48%, 52%

- What is the Nash equilibrium in this case? First ask: are there any dominated strategies? If so, eliminate them from consideration.

Eliminating the Dominated Strategies Reduces the Set of Strategies that May Comprise Nash Equilibria.

		Network 2		
		~~Sitcom~~	Game Show	Talent Show
	Sitcom	~~55%, 45%~~	52%, 48%	51%, 49%
Network 1	~~Game Show~~	~~50%, 50%~~	~~45%, 55%~~	~~46%, 54%~~
	Talent Show	~~52%, 48%~~	49%, 51%	48%, 52%

- A game show is a dominated strategy for Network 1.
- A sitcom is a dominated strategy for Network 2.

Continuing the search for dominated strategies among the remaining choices…

		Network 2		Nash Equilibrium
		~~Sitcom~~	~~Game Show~~	Talent Show
	Sitcom	~~55%, 45%~~	~~52%, 48%~~	51%, 49%
Network 1	~~Game Show~~	~~50%, 50%~~	~~45%, 55%~~	~~46%, 54%~~
	~~Talent Show~~	~~52%, 48%~~	~~49%, 51%~~	~~48%, 52%~~

- Talent show is now a dominated strategy for Network 1
- Game show is now a dominated strategy for Network 2

Best Response Analysis Also Works

Network 2

Nash Equilibrium

Network 1		Sitcom	Game Show	Talent Show
	Sitcom	55%, 45%	52%, 48%	51%, 49%
	Game Show	50%, 50%	45%, 55%	46%, 54%
	Talent Show	52%, 48%	49%, 51%	48%, 52%

Adding a Third Player

- Consider again the case of two strategies, sitcom and game show and suppose there is a third player, Network 3.
- The normal-form representation of this three-player game is:

Network 3

Network 3: Sitcom

Network 2

Network 1		Sitcom	Game Show
	Sitcom	34%,25%,41%	32%,32%,36%
	Game Show	32%,30%,38%	33%,31%,36%

Network 3: Game Show

Network 2

Network 1		Sitcom	Game Show
	Sitcom	34%,29%,37%	38%,32%,30%
	Game Show	35%,38%,27%	36%,39%,25%

- Network 3's payoff is now the third percentage given.
- What is the Nash equilibrium of this game?

Computer Screen View

Use Best Response Analysis

		Network 3: Sitcom		Network 3: Game Show	
		Network 2: Sitcom	Network 2: Game Show	Network 2: Sitcom	Network 2: Game Show
Network 1	Sitcom	34%,25%,41%	32%,32%,36%	34%,29%,37%	38%,32%,30%
Network 1	Game Show	32%,30%,38%	33%,31%,36%	35%,38%,27%	36%,39%,25%

- Pure strategy Nash equilibrium to this game is for Network 1 to run a game show, Network 2 to run a game show and Network 3 to run a sitcom.

Non-Constant-Sum Games

- The Network Game is an example of a constant sum game.
- The payoffs to both players always add up to the constant sum of 100%.
- We could make that game zero sum by redefining payoffs relative to a 50%-50% share for each network.
- Nash equilibria also exist in non-constant sum or **variable sum games, where players may have some *common interest.***
- For example, prisoner's dilemma type games. Payoffs are "profits", so more is better:

		Burger King	
		Value Meals	No Value Meals
McDonald's	Value Meals	2, 2	4, 1
	No Value Meals	1, 4	3, 3

Asymmetry In Payoffs

- Payoffs need not be symmetric in equilibrium.
- Consider this version of Prisoner's Dilemma:

		Burger King	
		Value Meals	No Value Meals
McDonald's	Value Meals	3, 1	5, 0
	No Value Meals	1,6	4, 5

- Both players still have a dominant strategy of Value Meals: Game is Still a PD, as the mutually efficient (most jointly profitable) outcome is not a Nash equilibrium

Rationalizability

- An alternative solution concept that is a generalization of Nash equilibrium.
- Identify strategies that are *never a best response* for a player given any beliefs about the play of his opponent. Strictly dominated strategies are never a best response, but there can be strategies that are not strictly dominated but which are *never a best response*.
- The set of strategies that survive elimination on the grounds of never being a best response are the set of "rationalizable equilibria" they can be rationalized via certain beliefs.
- Every Nash equilibrium is a rationalizable equilibriuim, but not every rationalizable equilibrium is a Nash equilibrium.
- In this sense, rationalizability is a more general solution concept.

Rationalizability Example

		Column		
		X	Y	Z
Row	A	3,2	0,3	2,0
	B	1,3	2,0	1,2
	C	2,1	4,3	0,2

- Note that neither player has any dominated strategies.
- Nash equilibrium is: CY
- For Row, Strategy B is never a best response.
- For Column, Strategy Z is never a best response.
- Eliminating these, we see that X is never a best response for Column.
- It further follows that A is not a best response for Row, leaving only CY.
- More generally, sequential elimination of never best responses need not lead to Nash equilbrium

Elimination of Never Best Responses Need Not Select a Solution

- Consider this example:

		Column		
		X	Y	Z
Row	A	1,5	2,4	5,1
	B	4,2	3,3	4,2
	C	5,1	2,4	1,5

- For Row, A is rationalizable if Row thinks Column will play Z; B is rationalizable if Row thinks Column will play Y; C is rationalizable if Row thinks Column will play X.
- For Column, X is rationalizable if Column thinks Row will play A; Y is rationalizable if Column thinks Row will play B, Z is rationalizable if Column thinks Row will play C.
- Thus, all strategies are rationalizable in this example.
- However, the Nash equilibrium, using best response analysis is BY

Some Games Have No Equilibrium in Pure Strategies

- Some games have no pure strategy equilibria.
- Consider, for example, the following "tennis game" between the Williams sisters, Serena and Venus.
- Suppose Serena is in a position to return the ball and and can choose between a down-the-line (DL) passing shot or a cross-court (CC) diagonal shot.
- Venus (on defense) has to guess what Serena will do, and position herself accordingly. DL positions Serena for the DL shot, and CC for the CC shot.
- Payoffs are the fraction (%) of times that each player wins the point.

		Venus Williams	
		DL	CC
Serena Williams	DL	50, 50	80, 20
	CC	90, 10	20, 80

In such cases, playing a pure strategy is usually not a winning strategy

- Using best response/cell-by-cell inspection, we see that there is no pure strategy Nash equilibrium; Serena's best response arrows do not point to the same cell as Venus's best response arrows.
- In such cases, it is better to behave unpredictably using a *mixed strategy*.

		Venus Williams	
		DL	CC
Serena Williams	DL	50, 50	80, 20
	CC	90, 10	20, 80