Economics 1200 Spring 2012

Homework # 3

Write your answers to the following questions on separate sheets of paper. Your answers are due in class on **Thursday, February 23, 2012**. No late homeworks are accepted.

1. A public good is one for which those who do not pay for the good enjoy the same benefits as those who do pay for it. Think of publicly supported television or radio; paid members cannot exclude non-member "free-riders" from watching or listening. With this in mind, consider the following *three*-person game involving contributions to a public good.

   - Call the three players: Larry, Curly and Moe.
   - Each player has two action choices: contribute or not contribute one unit to the public good. Each player who chooses to contribute pays a cost of 1.5 units.
   - If a player contributes, his payoff is the total number of units contributed by all players, himself included (so a maximum of 3 units), less the cost of making his own contribution, 1.5 units.
   - If a player does not contribute, his payoff is the total number of units contributed by all other players.

   a) Write this three-player game down in normal (strategic) form.
   b) Find the Nash equilibrium of this game. Is this equilibrium the most efficient outcome? Explain why or why not.

2. Suppose you have a pair of dice, a red die and a white die and you roll them just once. Let A be the event that the sum of numbers from the two die add up to 7- "you roll a 7" and let B be the event that "the red die comes up 1".
   a) What is P(A)? P(B)?
   b) What is $P(A \cap B)$?

3. The king comes from a family of two children. What is the probability that the other child is his sister? Hint: write out the sample space of all possibilities. Then define the event A="one child is a girl" and the event B="one child is the king." Finally, find: P(A|B).

4. Blaise Pascal, a French mathematician and philosopher argued that we have to make a *wager* as to whether or not we choose to believe in God (assume that monotheism is the only possibility). This is known as *Pascal's wager*. Pascal's discussion suggests that we play a game with two strategies: "Believe in God" and "Do Not Believe in God," with payoffs as shown in the table below. Let us suppose that God Exists with some probability $p \geq 0$ and that God does not exist with probability 1-p. In this game, x is the payoff you get if God exists and you choose Believe in God; -y is the payoff to you of passing up on some worldly pleasures as a consequence of incorrectly believing in God (when God does not exist). If you choose Do Not Believe in God, but God exists (and God is a vengeful God), your payoff is –z; if you do not believe in God and God does not exist, your payoff is 0.

	God Exists with Probability p	God Does Not Exist with Probability 1-p
Believe in God	x	-y
Do Not Believe in God	-z	0

a) For what values of p should you believe in God? Should you Not Believe in God? Be indifferent between believing and not believing in God?
b) Pascal argued that the value of x is infinite and all other parameter values, y, z, in the table above are finite. Given Pascal's parameterization, what strategy did Pascal choose to play?
c) Suppose instead that it is y alone that is infinite. What strategy is the best response in that case?

5. Suppose you have an opportunity to invest in a company that is working on an important research project, e.g., a cure for cancer. If you invest, you have to put up \$1 million. If the project is successful, your \$1 million investment yields a \$6 million return, for a net return of \$5 million. If the project is unsuccessful, your return is 0, so you lose your \$1 million investment. The probability of success is 1/5.
   a) Draw the game in extensive form including Nature as a player.
   b) Compute the expected payoff in the event you choose to invest. Assuming you are risk neutral, do you choose to invest or not invest?
   c) Reconsider your answer to part b assuming you have the utility function $U(X) = \sqrt{X + 1{,}000{,}000}$, where X is your *net* return from the investment. Would you be willing to make the investment in this case? Why or why not? You might want to illustrate your answer with a graph.

6. In each of the three games shown below, let p be the probability that Player 1 plays Cooperate and let q be the probability that Player 2 plays Cooperate.

Prisoner's Dilemma

		Player 2 Cooperate	Defect
Player 1	Cooperate	70,70	10,80
	Defect	80,10	40,40

Stag Hunt

		Player 2 Cooperate	Defect
Player 1	Cooperate	70,70	5,40
	Defect	40,5	40,40

Chicken

		Player 2 Cooperate	Defect
Player 1	Cooperate	70,70	50,80
	Defect	80,50	40,40

a) For each game, draw a graph of Player 1's best response function (choice of p as a function of q), and a graph of Player 2's best response function (choice of q as a function of p). Combine the two graphs as was done in class (and as shown in your text, e.g., Figures 8,2,8.5), with p on the horizontal axis and q on the vertical axis. Using this combined graphs, find all the Nash equilibria for the game, both pure and mixed strategy Nash equilibria (if any). Label these equilibria on the combined graph.

b) In those games that have multiple pure strategy Nash equilibria, how do the expected payoffs from playing the mixed strategy Nash equilibrium compare with the payoffs from playing the pure strategy Nash equilibria? Which type of strategy (mixed or pure) would you prefer to play in these games and why?