

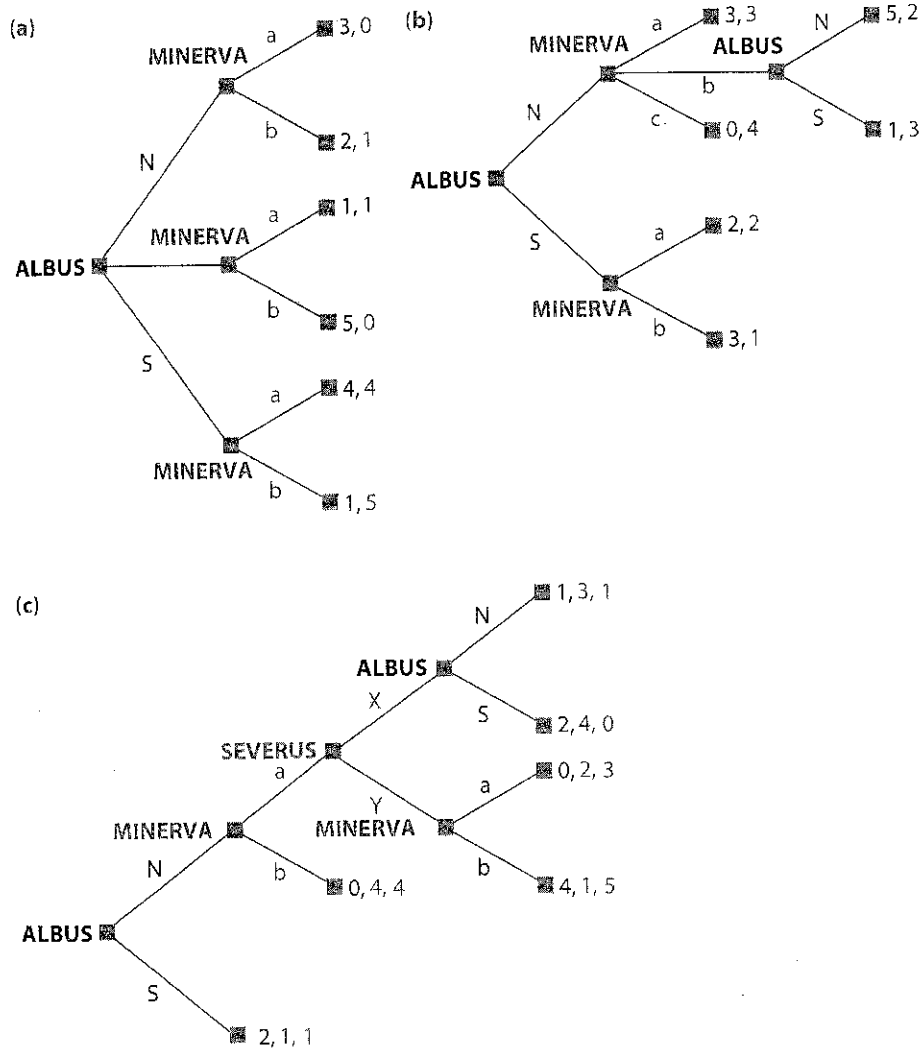
## Homework # 1

Write your answers to the following questions on separate sheets of paper. Your answers are due in class on **Tuesday, January 24**. Late homeworks are not accepted.

1. A prisoner is trying to escape from prison. He can attempt to climb over the prison wall or dig a tunnel under the prison wall. The warden can prevent the prisoner from climbing over the wall by posting guards at the wall and he can prevent the prisoner from tunneling under the wall by having regular inspections of cells, but he has only enough guards to do one or the other and not both.
  - a. Choose (and justify) some simple numerical payoffs for this game, and then write the game in *normal form*. Be sure to label the strategies and players in your game matrix. Is this game constant sum or non-constant sum? Explain/justify your answer.
  - b. Now express this game in *extensive form*, assuming the prisoner and warden make their decision at the same time. Use a dashed line to indicate the information set for the second mover in the game tree.
  - c. What is the mutual best response (Nash equilibrium) for the simultaneous move version of this game, as you have depicted in parts a or b? Give your reasoning.
  - d. Now express the same game in extensive form assuming that the warden makes his decision first and the prisoner, *after seeing the warden's strategy*, moves second. Use rollback to find the Nash equilibrium in the sequential move version of this game. Does it differ from your answer to c? Why or why not.
2. Bruster's and Rita's both sell equally delicious ice cream, and compete for the same customers. Each can offer customers a rewards card (offering free ice cream after a certain number of purchases), or not. Suppose that profits at each firm are greater if neither offers a rewards card than if both do. If one firm offers a rewards card and the other doesn't, the one offering the rewards card earns higher profits than in the case where neither firm offers a reward card, while the firm that does not offer a rewards card earns lower profits than in the case where both firms offer a rewards card.
  - a. Choose some profit numbers (payoffs) for this game that are consistent with the description above and write the game in normal form. Be sure to label the strategies and players in your game matrix.
  - b. Does either player have a *dominant strategy* in this game? If so what is it?
  - c. If Bruster's Sweet Rewards card offers 1 free ice cream for every 10 purchased, what would you expect Rita's Cool Card to offer in equilibrium and why?

3. Dixit, Skeath and Reiley, Chapter 3, exercises U2 and U3, p. 85.
4. Dixit, Skeath and Reiley, Chapter 3, exercise U4, pp. 85-6.
5. Consider the game of Marienbad which, like the game of Nim, is zero-sum (there is a winner and a loser). As in Nim, there are two piles of match sticks and two players, player 1 and player 2. Let  $m$  be the number of sticks in the first pile and let  $n$  be the number of sticks in the second pile. Player 1 moves first and thereafter the players take turns. At each turn, a player can pick up any number of matches (up to the maximum) from one of the two piles. The player who removes the last match in Marienbad loses the game (this is just the opposite of the rule for winning Nim). Prove/explain the following claims:
  - a. If  $m=n=1$ , then player 1 has a winning strategy.
  - b. If  $m=n>1$  (e.g.  $m=5, n=5$ ), the player 2 has a winning strategy.
  - c. If  $m \neq n$ , then player 1 has a winning strategy.
6. Suppose an investor is thinking of opening a restaurant. She has \$100,000 to invest. If she opens the restaurant, the probability is .35 that the restaurant succeeds and she earns a gross of \$300,000 (including the initial 100,000 investment). With probability .65, the restaurant fails and the investor loses her investment, that is, she has \$0 left. If the investor doesn't open the restaurant, she keeps her \$100,000 investment.
  - a. Draw the game in extensive form assuming that the investor's decision to open or not open a restaurant is made first, and that "nature" moves second. Be sure to label the two players, their action choices, and include the (expected) payoffs at the terminal nodes of the tree.
  - b. What action would a risk neutral investor choose? Would a risk averse investor necessarily follow the same choice?
  - c. Now suppose that the order of moves is reversed, so that nature moves first, but the investor does not know the move that nature has made in advance of making the decision to open or not open the firm. Draw the game tree in this case, using a dashed line on your game tree to indicate that the investor does not know the outcome of the chance move made by "nature". Is your answer to part b any different? Is this version of the game a sequential or a simultaneous move game?

U2. In each of the following games, how many pure strategies (complete plans of action) are available to each player? List all of the pure strategies for each player.



U3. For each of the games illustrated in Exercise U2, identify the rollback equilibrium outcome and the complete equilibrium strategy for each player.

U4. Two distinct proposals, A and B, are being debated in Washington. The Congress likes proposal A, and the president likes proposal B. The proposals are not mutually exclusive; either or both or neither may become law. Thus

there are four possible outcomes, and the rankings of the two sides are as follows, where a larger number represents a more favored outcome.

Outcome	Congress	President
A becomes law	4	1
B becomes law	1	4
Both A and B become law	3	3
Neither (status quo prevails)	2	2

- (a) The moves in the game are as follows. First, the Congress decides whether to pass a bill and whether it is to contain A or B or both. Then the president decides whether to sign or veto the bill. Congress does not have enough votes to override a veto. Draw a tree for this game and find the rollback equilibrium.
  - (b) Now suppose the rules of the game are changed in only one respect: the president is given the extra power of a line-item veto. Thus, if the Congress passes a bill containing both A and B, the president may choose not only to sign or veto the bill as a whole, but also to veto just one of the two items. Show the new tree and find the rollback equilibrium.
  - (c) Explain intuitively why the difference between the two equilibria arises.
- U5. Two players, Amy and Beth, play the following game with a jar containing 100 pennies. The players take turns; Amy goes first. Each time it is a player's turn, she takes between one and 10 pennies out of the jar. The player whose move empties the jar wins.
- (a) If both players play optimally, who will win the game? Does this game have a first-mover advantage? Explain your reasoning.
  - (b) What are the optimal strategies (complete plans of action) for each player?
  - (c) Now suppose we change the rules so that the player whose move empties the jar loses. Does this game have a first-mover advantage? Explain your reasoning.
  - (d) In this second variant, what are the optimal strategies for each player?
- U6. Now Amy and Beth play a game with two jars, each containing 100 pennies. The players take turns; Amy goes first. Each time it is a player's turn, she chooses one of the jars and removes anywhere from one to 10 pennies from it. The player whose move leaves both jars empty wins. (Note that when a player empties the second jar, the first jar must already have been emptied in some previous move by one of the players.)
- (a) Does this game have a first-mover advantage or a second-mover advantage? Explain which player can guarantee victory, and how she can do it.