Economics 1200 Spring 2012

Homework # 4

Write your answers to the following questions on separate sheets of paper. Your answers are due in class on **Tuesday, March 27**. No late homeworks are accepted.

1. An employer and its unionized workforce are having difficulty agreeing on a new contract and so there is talk of a possible strike by the workers. Before the vote to strike or not, the union can either build up its strike fund, or not, and the employer can either build up its inventories to continue service to its customers or not. The decisions to build up or not are made simultaneously. Thereafter, the union decides whether to strike or not. The game can be represented in normal form as follows (ignoring the sequence of moves).

		UNION			
		Build up fund		Don’t build up fund	
		strike	no strike	strike	no strike
EMPLOYER	Build up Inventories	-5, -5	-2, -2	-2, -10	-2, 0
	Don’t Build up Inventories	-10, 10	0, -2	-5, 5	0, 0

   a) Using the payoff table, are there any Nash equilibria in pure strategies? If so, what are they? Show your work.
   b) Write the game down in extensive form, being careful about payoff assignments and including any information sets. Then, identify all subgames.
   c) Solve for the unique, subgame perfect equilibrium of the game and address the question: is a strike threat credible? Show your work. How/does your answer to this part differ from your answer to part a?

2. Consider the game between a parent and a child. The child can choose to be good (G) or bad (B). The parent can punish the child (P) or not (N). The child gets enjoyment worth 1 from bad behavior and hurt worth -2 from punishment. The payoff to a child who behaves well and is not punished is 0; one who is bad and is not punished gets 1, one who is bad and gets punished earns 1-2=-1 and one who is good but is punished gets -2. The parent gets -2 from the child’s bad behavior and 0 for good behavior. The parent also gets -1 if s/he inflicts punishment; thus if the child is good and the parent punishes the parent gets 0-1=-1, if bad and no punishment, the parent gets -2 and if bad and punishment the parent gets -3.
   a) Set this up as a simultaneous move game and find all the Nash equilibria.
   b) Next suppose the child chooses G or B first. After observing the child’s behavior the parent chooses P or N. Draw the game tree and find the subgame perfect equilibrium.
   c) Now suppose that before the child acts in the sequential move version of the game, the parent can decide whether or not to threaten to punish the child for

bad behavior: Threat P if B (with the implicit promise of no punishment for good behavior) or No Threat. Suppose that, if made, the threat is perfectly credible. Illustrate the game in this case, find the subgame perfect equilibria and explain how and why your answers to parts b and c differ.

3. Consider the following 2 player, 3 move stage game:

		Player 2		
		L	M	R
	L	1,1	5,0	0,0
Player 1	M	0,5	4,4	0,0
	R	0,0	0,0	3,3

a) Find two pure strategy Nash equilibria of the stage game.
b) Suppose the stage game is repeated exactly twice, and this fact is known. Show that a subgame perfect equilibrium of the twice-repeated game is (M, M) in period 1 and (R, R) in period 2, i.e. write down strategies that support this as an equilibrium outcome.

4. Consider the following sequential move "investment" game. Investor moves first and must decide whether or not to invest his endowment of \$4 with a Trustee. If the Investor does not invest, his payoff is \$4 and the Trustee's payoff is \$0. If the Investor invests the \$4, the amount is tripled to \$12 and the Trustee can then decide whether to keep the \$12 for himself giving the Investor \$0, or return half of the \$12 to the Investor in which case the payoff to both players is \$6 each.

a) What is the rollback equilibrium in the one-shot version of this game?
b) Now consider the case where the sequential move investment game described above is played infinitely many times. Under what conditions on the discount factor δ can "trust and reciprocity" (i.e., the Investor invests and the Trustee returns half of \$12 in every round) be sustained as an equilibrium using a grim trigger strategy. Show your work.

5. Consider a two-player game between Child's Play and Kid's Korner, each of which produces and sells wooden swing sets for children. Each player can set either a high or a low price for a standard two-swing, one slide set. If they both set a high price, each receives profits of \$64,000 per year. If one sets a low price and the other sets a high price, the low-price firm earns profits of \$72,000 per year while the high price firm earns profits of \$20,000 per year. If they both set a low price, each earns profits of \$57,000 per year.
a) What are the Nash equilibrium strategies and payoffs in the simultaneous-play version of this game if the two players make price decisions only once?
b) If the two firms decide to play this game for a fixed number of periods, say 4 years, what are each firm's total profits at the end of the four years? (don't discount). Explain your reasoning.
c) Suppose the two firms play this game repeatedly forever. Let each of them use the grim strategy in which they both price high (cooperate) unless one of them

"defects" in which case they price low for the duration of the game. What is the one-time gain from defecting against an opponent playing this strategy? How much does each firm lose in each future period if it defects once? If δ=.8 (r=.25), will it be worthwhile for them to cooperate? Find the range of δ (or r) values for which the grim strategy is able to sustain cooperation (mutual high prices) between the two firms.

d) Suppose the firms play this game repeatedly year after year neither expecting any change in their interaction. If the world were to end after 4 years without either firm having anticipated this event, what are each firm's total profits (not discounted) over the four year period? Compare your answer with part b. Is there a difference? Explain why or why not.

e) Suppose now that the firms know there is a 10% probability that one of them may go bankrupt in any given year. If bankruptcy occurs, the repeated game between the two firms ends. Will this knowledge change the firms' actions when δ=.8 (r=.25)? Explain why or why not. What if the probability of a bankruptcy increases to 35% in any year? Again, explain why or why not.