Simultaneous Move Games

Decision making without knowledge of the strategy choice of opponents

Simultaneous Moves

- Arise when players have to make their strategy choices *simultaneously, without knowing the strategies that* have been chosen by the other player(s).
  - Student studies for a test; the teacher writes questions.
  - Two firms independently decide whether or not to develop and market a new product.
- While there is no information about what other players will actually choose, we assume that the strategic choices available to each player are known by all players.
- Players must think not only about their own best strategic choice but also the best strategic choice of the other player(s).
- We will consider both discrete and continuous strategy spaces.

Normal or Strategic Form

- A simultaneous move game is depicted in "Normal" or "Strategic" form using a **game table that relates the** strategic choices of the players to their payoffs.
- The convention is that the row player's payoff is listed *first* and the column player's payoff is listed *second.*

		Column Player	
		Strategy C1	Strategy C2
Row Player	Strategy R1	a, b	c, d
	Strategy R2	e, f	g, h

- For example, if Row player chooses R2 and Column player chooses C1, the Row player's payoff is e and the Column player's payoff is f.

Special Zero-Sum Form

- For zero (or constant sum games), knowing the payoffs sum to zero (or some other constant) allows us to write a simultaneous move game in normal form more simply:

		Warden	
		Guard Wall	Inspect Cells
Prisoner	Climb Wall	-1	1
	Dig Tunnel	1	-1

- Payoffs are shown only for the Prisoner; the Warden's payoffs are the negative of the prisoner's payoff

The Role of Beliefs

- When players move simultaneously, what does it mean to say that in equilibrium strategies are a mutual best response?
  - One cannot see what the other is doing and condition your behavior on their move.
- In simultaneous move games, rational players consider all of the strategies their opponents *may* take and they form beliefs (subjective probabilities) about the likelihood of each strategy their opponent(s) could take.
- After forming these beliefs, rational players maximize their expected payoff by choosing the strategy that is a best response to their beliefs about the play of their opponent(s). The same is true of the opponent(s).

Coordination Game Example

How would you play this game?

	X	Y
X	1: 0, 0	3: 1, 1
Y	2: 1, 1	4: 0, 0

Example of the Role of Beliefs

- Consider the pure coordination game.

			Column Player	
			X	Y
Row Player	X		0, 0	1, 1
	Y		1, 1	0, 0

- Suppose Row player assigns probability p>.5 to column player playing Y. Then Row's best response to this belief is to play X:

  Row's expected payoff from playing X is 0(1-p)+1(p)=p, while Row's expected payoff from playing Y is 1(1-p) + 0(p)=1-p. Since we assumed p>.5, the expected payoff to Row from playing X, p, is greater than the expected payoff to Y, 1-p.

How Might Such Beliefs be Formed?

- Players' subjective beliefs about the play of an opponent in a simultaneous move game may be formed in one of several ways:
  - *Introspection*: given my knowledge of the opponent's payoffs ***what would I do if I were the other player***?
  - *History* (repeated games only): what strategy has the same opponent played in the past.
  - *Imitation/learning from others*: what strategies have players (other than my current opponent) chosen in this type of strategic setting?
  - *Pre-play communication*.
  - Other type of *signaling*.
- We focus for now on the first, introspective method.

Pure vs. Mixed Strategies

- A player pursues a **pure strategy if she always** chooses the same strategic action out of all the strategic action choices available to her in every round.
  - e.g. Always refuse to clean the apartment you share with your roommate.
- A player pursues a **mixed strategy if she** randomizes in some manner among the strategic action choices available to her in every round.
  - e.g. Sometimes pitch a curveball, sometimes a slider (“mix it up,” “keep them guessing”).
- We focus for now on pure strategies only.

Example: *Battle of the Networks*

- Suppose there are just two television networks. Both are battling for shares of viewers (0-100%). Higher shares are preferred (= higher advertising revenues).
- Network 1 has an advantage in sitcoms. If it runs a sitcom, it always gets a higher share than if it runs a game show.
- Network 2 has an advantage in game shows. If it runs a game show it always gets a higher share than if it runs a sitcom.

		Network 2	
		Sitcom	Game Show
Network 1	Sitcom	55%, 45%	52%, 48%
	Game Show	50%, 50%	45%, 55%

Computer Screen View

	Sitcom	Game Show
Sitcom	1: 55, 45	3: 52, 48
Game Show	2: 50, 50	4: 45, 55

Nash Equilibrium

- We cannot use rollback in a simultaneous move game, so how do we find a solution?
- We determine the “best response” of each player to a particular choice of strategy by the other player.
- We do this for both players. Note that in thinking of an opponent’s best response, we are using introspection to form beliefs about what the (rational) opponent will do.
- If each player’s strategy choice is a best response to the strategy choice of the other player, then we have found a solution or *equilibrium to the game.*
- This solution concept is know as a **Nash equilibrium, after John Nash who first proposed it.**
- A game may have 0, 1 or more Nash equilibria in pure strategies.

Best Response Analysis

- Best response analysis (a.k.a. cell-by-cell inspection) is the most reliable for method for finding Nash equilibria. First Find Network 1's best response to Network 2's possible strategies:
  - If Network 2 runs a sitcom, Network 1's *best response is* to run a sitcom. Circle Network 1's payoff in this case, 55%
  - If Network 2 runs a game show, Network 1's *best response* is to run a sitcom. Circle Network 1's payoff in this case, 52%

		Network 2	
		Sitcom	Game Show
Network 1	Sitcom	55%, 45%	52%, 48%
	Game Show	50%, 50%	45%, 55%

Best Response Analysis, Continued

- Next, we find Network 2's best response.
  - If Network 1 runs a sitcom, Network 2's *best response is* to run a game show. Circle Network 2's payoff in this case, 48%
  - If Network 1 runs a game show, Network 2's *best response* is to run a game show. Circle Network 2's payoff in this case, 55%
- The unique Nash equilibrium is for Network 1 to run a sitcom and Network 2 to run a game show.
- This is found by the cell with the two circled payoffs. This is the method of best response analysis for locating Nash equilibria.

		Network 2	
		Sitcom	Game Show
Network 1	Sitcom	55%, 45%	52%, 48%
	Game Show	50%, 50%	45%, 55%

Dominant Strategies

- A player has a **dominant strategy if it outperforms** (has higher payoff than) all other strategies regardless of the strategies chosen by the opposing player(s).
- For example, in the battle of the networks game, Network 1 has a dominant strategy of always choosing to run a sitcom. Network 2 has a dominant strategy of always choosing to run a game show.
- Why?
- Successive elimination of non-dominant or "**dominated"** strategies can help us to find a Nash equilibrium.

Successive Elimination of Dominated Strategies

- Another way to find Nash equilibria
- Draw lines through (successively eliminate) each player's dominated strategy(s).
- If successive elimination of dominated strategies results in a *unique* outcome, that outcome is the Nash equilibrium of the game.
- We call such games **dominance solvable.**
- But, not all games have unique equilibria/are dominance solvable, so this method will not work as generally as best response analysis.

		Network 2	
		~~Sitcom~~	Game Show
Network 1	Sitcom	~~55%, 45%~~	52%, 48%
	~~Game Show~~	~~50%, 50%~~	~~45%, 55%~~

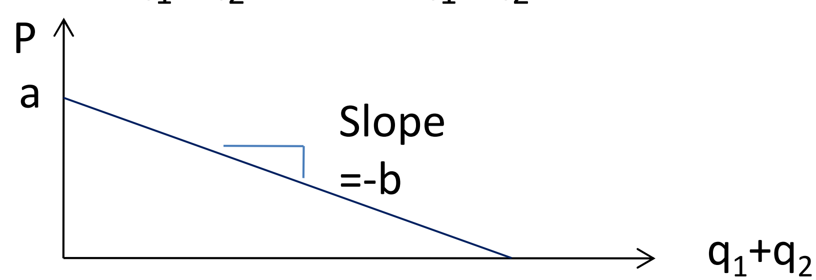
Oligopolies and Game Theory

- One of the important applications of game theory to economics is to understand the pricing and quantity decisions of firms in markets where there are not too many firms.
- We call such markets oligopoly markets; they lie in between monopoly (one firm) and perfect competition (many firms).
- With small numbers of firms, one firm's quantity or price decisions can depend on or matter for the quantity or price decisions of the other firms in that market.
- Thus firm decisions involve strategic interactions and are the domain of game theory.

Cournot Competition

- A game where two firms (duopoly) compete in terms of the quantity sold (market share) of a homogeneous good is referred to as a Cournot game after the French economist who first studied it.
- Let q_1 and q_2 be the number of units of the good that are brought to market by firm 1 and firm 2.
- Assume the market price, P, is determined by **market demand:**

$$P=a-b(q_1+q_2) \text{ if } a>b(q_1+q_2),\ \ P=0 \text{ otherwise.}$$



- Firm 1's profits are $(P-c)q_1$ and firm 2's profits are $(P-c)q_2$, where *c is the marginal cost of producing each unit of the good.*
- Assume both firms seek to maximize profits.

Monopoly or Duopoly?

- If the firms could *collude*, they could effectively act as a single firm, a monopoly instead of a duopoly and split the monopoly profits.
- The monopolist would choose the monopoly quantity, $q_{m,}$ where marginal revenue equaled marginal cost, and given that quantity the monopolist would choose the price to charge based on the market demand curve, $P(q_{m,})$.
- However, generally, collusion is *not* allowed (e.g., due to anti-trust laws), so the two firms cannot act as a single firm, but rather must make decisions simultaneously and independently of one another.
- That is, firm 1(2) must guess what quantity firm 2(1) will choose and choose its best response in terms of its own quantity to bring to the market. The result is a non-cooperative Nash equilibrium (mutual best response) that is generally less profitable than the Monopoly (non-strategic) equilibrium outcome where $q_1=q_2=q_m/2$.

Numerical Example, Discrete Choices

- Suppose $P = 130-(q_1+q_2)$, so $a=130$, $b=1$
- Suppose the marginal cost per unit, $c=\$10$ for both firms.
- Suppose there are just three possible quantities that each firm $i=1,2$ can choose $q_i = 30, 40$ or 60.
- There are thus 3x3=9 possible market prices and profit outcomes for the two firms.
- For example, if firm 1 chooses $q_1=30$, and firm 2 chooses $q_2=60$, then Market $P=130-(30+60)=\$40$.
- Firm 1's profit is then $(P-c)q_1=(\$40-\$10)30=\$900$.
- Firm 2's profit is then $(P-c)q_2=(\$40-\$10)60=\$1800$.

Cournot Game Payoff Matrix

		Firm 2		
		q_2=30	q_2=40	q_2=60
	q_1=30	1800,1800	1500, 2000	900, 1800
Firm 1	q_1=40	2000, 1500	1600, 1600	800, 1200
	q_1=60	1800, 900	1200, 800	0, 0

- Depicts all 9 possible profit outcomes for each firm.

Find the Nash Equilibrium

Firm 2

		q_2=30	q_2=40	q_2=60
	q_1=30	1800,1800	1500, 2000	900, 1800
Firm 1	q_1=40	2000, 1500	1600, 1600	800, 1200
	q_1=60	1800, 900	1200, 800	0, 0

Nash Equilibrium

- q=60 is *weakly dominated for both firms; use cell-by-cell* inspection to complete the search for the equilibrium.

Profit Maximization with Continuous Strategies

- Firm 1's profit π_1= $(P-c)q_1=(a-b(q_1+q_2)-c)\ q_1=(a-bq_2-c)\ q_1-b(q_1)^2$.
- Firm 2's profit π_2= $(P-c)q_2=(a-b(q_1+q_2)-c)\ q_2=(a-bq_1-c)\ q_2-b(q_2)^2$.
- Both firms seek to maximize profits. We find the profit maximizing quantity using calculus:
  - Firm 1: $d\pi_1/dq_1=a-bq_2-c-2bq_1$. At a maximum, $d\pi_1/dq_1=0$,
    - $q_1=(a-bq_2-c)/2b$. This is firm 1's **best response function.**
  - Firm 2: $d\pi_2/dq_2=a-bq_1-c-2bq_2$. At a maximum, $d\pi_2/dq_2=0$,
    - $q_2=(a-bq_1-c)/2b$. This is firm 2's **best response function.**
- In our numerical example, firm 1's best response function is :

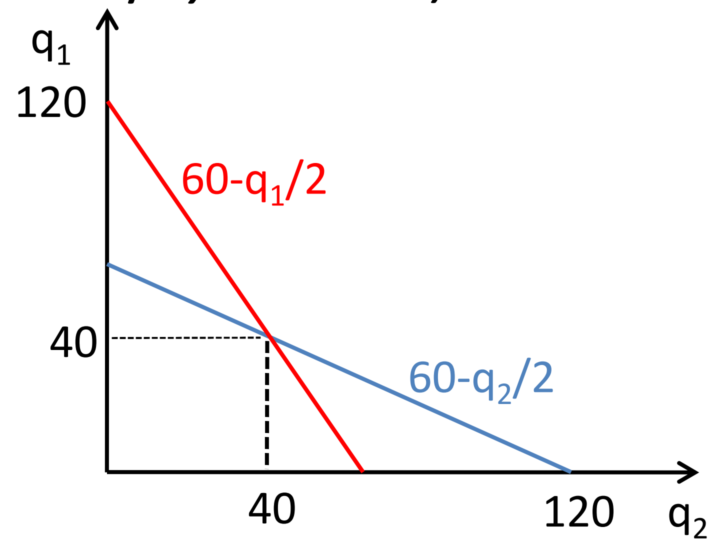
$$q_1=(a-bq_2-c)/2b = (130-q_2-10)/2=60-q_2/2.$$

- Similarly, firm 2's best response function in our example is:

$$q_2=(a-bq_1-c)/2b = (130-q_1-10)/2=60-q_1/2.$$

Equilibrium with Continuous Strategies

- Equilibrium can be found *algebraically or graphically.*
- Algebraically we have two reaction functions and two unknown quantities: $q_1=60-q_2/2$ and $q_2=60-q_1/2$,
- Substitute out using one of these equations: $q_1=60-(60-q_1/2)/2$ $=60-30+q_1/4$, so $q_1 (1-1/4)=30$, $q_1=30/.75=40$.
- Using the equation for $q_{2,}$ and q_1 =40, we fined that q_2=40 as well (the problem is perfectly *symmetric).*
- Graphically:



Reaction Functions More Generally

- The reaction function approach can be applied in any case where the strategy is chosen over some *continuous* interval.
- For example, the army size game.
- Two hostile countries red and blue have to decide on the size of their armies.
- Red country reasons that it needs an army of 250,000+1/2 the size the blue army.
- Suppose that blue country reasons that it needs an army of 300,000 + 1/3 the size of the red army.

Equilibrium Army Sizes

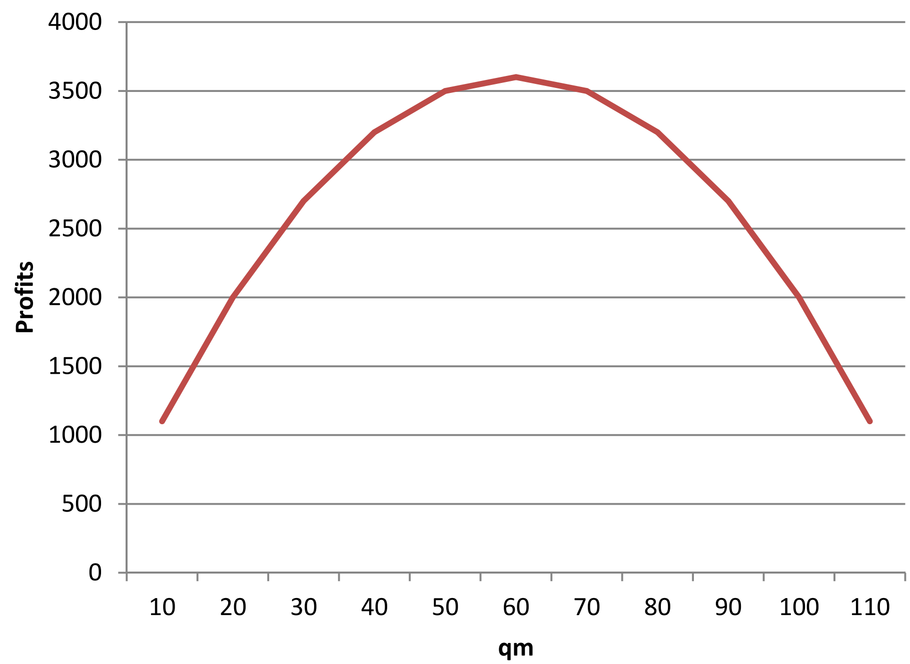
- Red's reaction function $q_r = 250{,}000 + 1/2\ q_b$
- Blue's reaction function $q_b = 300{,}000 + 1/3\ q_r$.
- This is just two equations in two unknowns.
- Solution: $q_r = 250{,}000 + 1/2(300{,}000 + 1/3\ q_r)$
  - $q_r = 250{,}000 + 150{,}000 + 1/6 q_r$
  - $5/6\ q_r = 400{,}000$, $q_r = 6*(400{,}000)/5 = 480{,}000$.
  - $q_b = 300{,}000 + 1/3(480{,}000) = 460{,}000$.

Cournot vs. Monopoly Outcome

- As mentioned, the oligopoly outcome is thought to differ from the monopoly outcome because the two firms cannot collude and so cannot act as a single supplier of the good to the market.
- But what is the difference between the oligopoly and the monopoly outcome?
- To figure this out, we need to find the monopoly quantity. Let q_m denote the monopolist's quantity. The monopolist seeks to maximize:

$$P(q_m)q_m - c\, q_m = (130 - q_m)q_m - 10q_m = 120q_m - (q_m)^2.$$

Monopoly Profit Function $120q_m-(q_m)^2$ Graphed



- It is easy to see that profits are maximized at q_m=60.
- By contrast, in the Cournot duopoly model, both firms chose a quantity q_i= 40, so total quantity q_1+q_2=80.
- Thus quantity is *lower* under monopoly (and prices are *higher*) than under Cournot competition.

Cournot vs. Perfect Competition Equilibrium Outcome

- Under perfect competition, firms make zero profits. This implies that each of firm in the market is choosing a quantity such that price = marginal cost, c=10. $P=10 \Rightarrow 130-q=10$, or q=60 for *each* firm.
- Note that if each of the two firms in this example market chooses q=60, then $q_1+q_2=120$ and the price, P=130-120=10; Profits, $\pi_i= (P-c)q_i$ are 0, since P-c=0 by definition.

Summary

- The monopoly outcome corresponds to the case where the two firms act as one and produce qi=30 each, or a total of $q_1+q_2=60$. The price charged P=130-60=**70**, and profits are (70-10)*60=3,600 (or **1,800** per firm).
- The Cournot-Nash equilibrium outcome corresponds to the case where the two firms solve their reaction functions, that specify what quantity is a best response to the quantity choice of the other firm. In this case we found that each firm should produce $q_i=40$ units. The price charged is P=130-(40+40)=**50** and profits are (50-10)*40=**1,600** per firm.
- Finally, under perfect competition, the price charged is such that neither firm makes any profits, P=c=**10**, and profits are **0** per firm.

Profits Summary

Monopoly
Cournot Oligopoly
Perfect Competition

		Firm 2		
		q_2=30	q_2=40	q_2=60
	q_1=30	1800,1800	1500, 2000	900, 1800
Firm 1	q_1=40	2000, 1500	1600, 1600	800, 1200
	q_1=60	1800, 900	1200, 800	0, 0