Assignment 4

Computer Graphics

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1 Exercise 1

1.1 Task 1

The two matrices R_90 and T in homogeneous coordinates are as follows:

$$R_{90} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1.2 Task 2

The following are the computations for the rotated and translated points and vector:

$$\begin{aligned} p_1' &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \cdot 1 + (-1) \cdot 1 + 0 \cdot 1 \\ 1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 \\ 0 \cdot 1 + 0 \cdot 1 + 1 \cdot 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-1) + 0 \cdot 1 + 1 \cdot 1 \\ 0 \cdot (-1) + 1 \cdot 1 + (-2) \cdot 1 \\ 0 \cdot (-1) + 0 \cdot 1 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} p_2' &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \cdot 3 + (-1) \cdot 2 + 0 \cdot 1 \\ 1 \cdot 3 + 0 \cdot 2 + 0 \cdot 1 \\ 0 \cdot 3 + 0 \cdot 2 + 1 \cdot 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-2) + 0 \cdot 3 + 1 \cdot 1 \\ 0 \cdot (-2) + 1 \cdot 3 + (-2) \cdot 1 \\ 0 \cdot (-2) + 0 \cdot 3 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} -1 \\ 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} p_2' &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \cdot 1 + (-1) \cdot 4 + 0 \cdot 1 \\ 1 \cdot 1 + 0 \cdot 4 + 0 \cdot 1 \\ 0 \cdot 1 + 0 \cdot 4 + 1 \cdot 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-4) + 0 \cdot 1 + 1 \cdot 1 \\ 0 \cdot (-4) + 1 \cdot 1 + (-2) \cdot 1 \\ 0 \cdot (-4) + 0 \cdot 1 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} -3 \\ -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} p' &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.5 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \cdot 1.5 + (-1) \cdot 2.5 + 0 \cdot 1 \\ 1 \cdot 1.5 + 0 \cdot 2.5 + 0 \cdot 1 \\ 0 \cdot 1.5 + 0 \cdot 2.5 + 1 \cdot 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2.5 \\ 1.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-2.5) + 0 \cdot 1.5 + 1 \cdot 1 \\ 0 \cdot (-2.5) + 1 \cdot 1.5 + (-2) \cdot 1 \\ 0 \cdot (-2.5) + 0 \cdot 1.5 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix} \end{aligned}$$

$$u' = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1.5 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \cdot 0.5 + (-1) \cdot 1.5 + 0 \cdot 0 \\ 1 \cdot 0.5 + 0 \cdot 1.5 + 0 \cdot 0 \\ 0 \cdot 0.5 + 0 \cdot 1.5 + 1 \cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-1.5) + 0 \cdot 0.5 + 1 \cdot 0 \\ 0 \cdot (-1.5) + 1 \cdot 0.5 + (-2) \cdot 0 \\ 0 \cdot (-1.5) + 0 \cdot 0.5 + 1 \cdot 0 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} -1.5 \\ 0.5 \end{bmatrix}$$

Now we can see that $u' = p' - p'_1$, as shown below:

$$\begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix}$$

The translation only works with points, while the rotation works with both points and vectors. For this reason, the vector u' is only affected by the rotation matrix, and not by the translation matrix.

Finally, here is a visual representation of all the points and vector before and after the transformations.