Assignment 2

Computer Graphics

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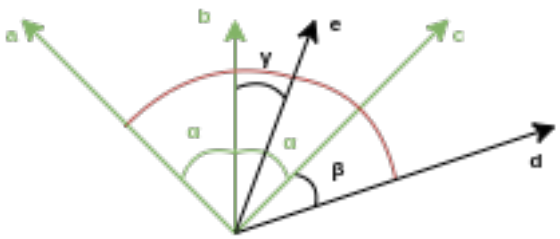
Exercise 2

In order to demonstrate that $\gamma = \frac{\beta}{2}$ we analyze three possible cases.

Case 1

In this case we have that d is after c, as shown in this image.

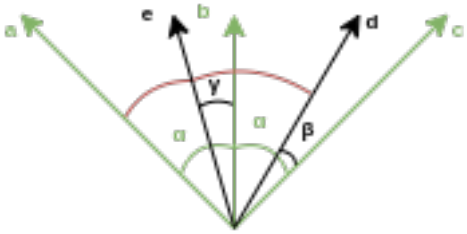


In this case we have that the two angles in red (the one between a and e, and the one between e and d) are the same. This is because e is the half-way vector between a and d. Now we can see that:

$$\begin{aligned} \alpha + \gamma &= \alpha + \beta - \gamma \\ \alpha - \alpha + \gamma + \gamma &= \beta \\ 2 \cdot \gamma &= \beta \\ \gamma &= \frac{\beta}{2} \end{aligned}$$

Case 2

In this case we have that d is between b and c, as shown in this image.

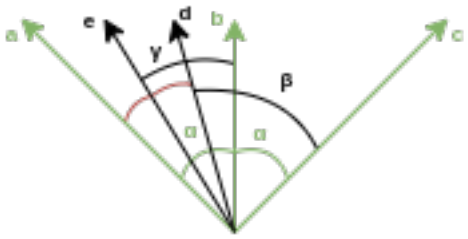


In this case we have that the two angles in red (the one between a and e, and the one between e and d) are the same. This is because e is the half-way vector between a and d. Now we can see that:

$$\begin{aligned} \alpha - \gamma &= \alpha - \beta + \gamma \\ \alpha - \alpha - \gamma - \gamma &= -\beta \\ -2 \cdot \gamma &= -\beta \\ 2 \cdot \gamma &= \beta \\ \gamma &= \frac{\beta}{2} \end{aligned}$$

Case 3

In this case we have that d is between a and b, as shown in this image.



In this case we have that the two angles in red (the one between a and e, and the one between e and d) are the same. This is because e is the half-way vector between a and d. Now we can see that:

$$\begin{aligned}
\alpha - \gamma &= \gamma - (\beta - \alpha) \\
\alpha - \gamma &= \gamma - \beta + \alpha \\
\alpha - \alpha - \gamma - \gamma &= -\beta \\
-2 \cdot \gamma &= -\beta \\
2 \cdot \gamma &= \beta \\
\gamma &= \frac{\beta}{2}
\end{aligned}$$