Assignment 1

Computer Graphics

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Exercise 1

Exercise 1.1

In order to compute the cosine of the angle α between the two vectors, we need to use the dot product equation, namely:

$$< a, b > \;=\; ||a|| \cdot ||b|| \cdot \cos(\alpha) \tag{1}$$

Thus by moving the elements of the equation around, we obtain that:

$$\begin{aligned}
\cos(\alpha) &= \frac{< a, b >}{||a|| \cdot ||b||} \\
\cos(\alpha) &= \frac{< a, b >}{\sqrt{2 + 1^2 + 0^2} \cdot \sqrt{1^2 + 1^2 + 1^2}} \\
\cos(\alpha) &= \frac{< a, b >}{\sqrt{3} \cdot \sqrt{3}} \\
\cos(\alpha) &= \frac{\sqrt{2} + 1}{3} \\
\cos(\alpha) &= 0.9107
\end{aligned}$$

Exercise 1.2

In order to see whether the vector is perpendicular to both the given vectors, we need to use the cross product. The x, y and z elements of vector z are:

$$\begin{aligned}
z_x &= (1 \cdot 1) - (0 \cdot 1) = 1 \\
z_y &= (0 \cdot 1) - (\sqrt{2} \cdot 1) = -\sqrt{2} \\
z_z &= (\sqrt{2} \cdot 1) - (1 \cdot 1) = \sqrt{2} - 1
\end{aligned}$$

Thus the vector is defined as:

$$z = (1, -\sqrt{2}, \sqrt{2} - 1)^T$$

Exercise 1.3

To calculate u we need to do a multiplication between a matrix and a vector. In our case:

$$\begin{aligned}
\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & -3 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -\sqrt{2} \\ \sqrt{2} - 1 \end{pmatrix} &= \begin{pmatrix} 1 \cdot 1 + 1 \cdot \left(-\sqrt{2}\right) + 1 \cdot \left(\sqrt{2} - 1\right) \\ 2 \cdot 1 + 2\left(-\sqrt{2}\right) + 1 \cdot \left(\sqrt{2} - 1\right) \\ (-1) \cdot 1 + (-3)\left(-\sqrt{2}\right) + (-3)\left(\sqrt{2} - 1\right) \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 1 - \sqrt{2} \\ 2 \end{pmatrix}
\end{aligned}$$