Leiden University
Medical Center

Confounding adjustment and estimating treatment effects

Without models

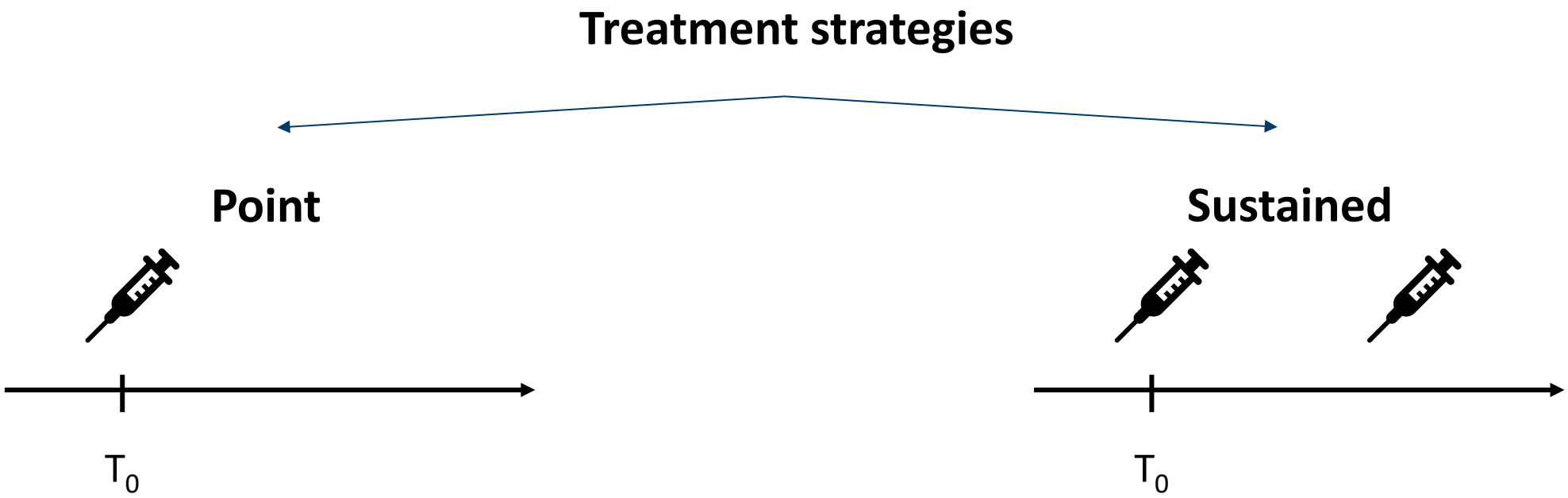
Edouard Fu, PhD

Department of Clinical Epidemiology, LUMC

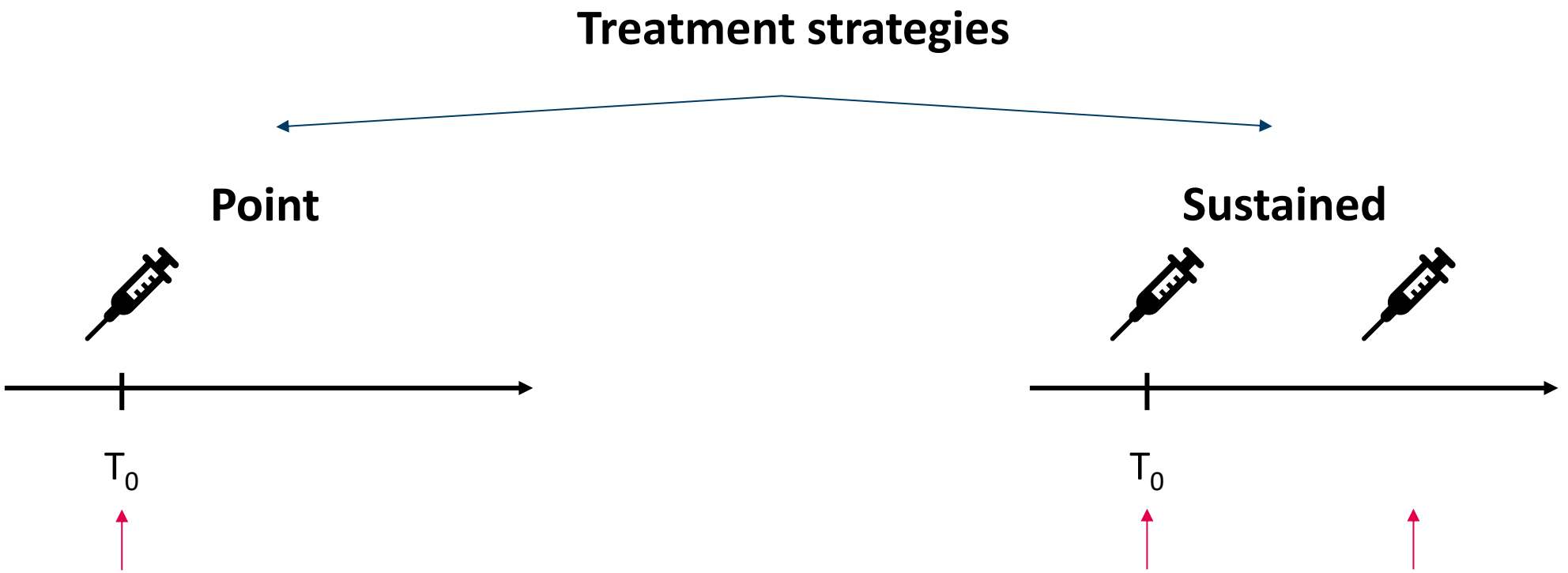
- This is going to be an interactive lecture

- Go to **classpoint.app** and fill in the classcode at the top right corner of this slide

Classification of treatment strategies



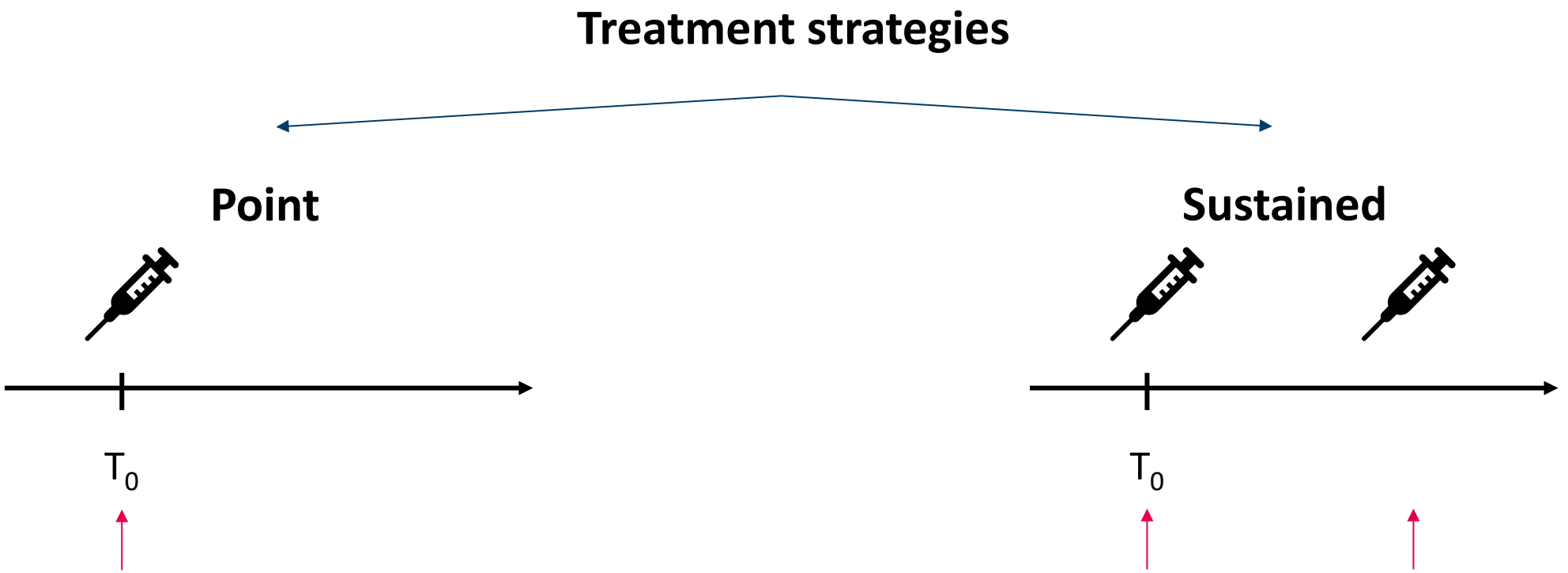
Baseline vs. time-varying confounding



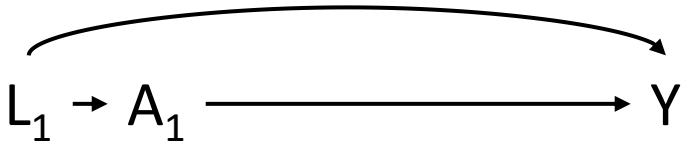
- Groups need to be similar at time zero
- Only baseline confounding

- Groups need to be similar at time zero & during follow-up
- Baseline & time-varying confounding

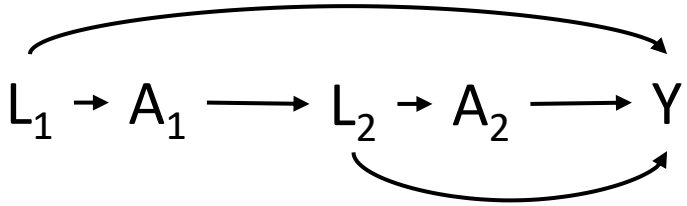
Baseline vs. time-varying confounding



- Groups need to be similar at time zero
- Only baseline confounding



- Groups need to be similar at time zero & during follow-up
- Baseline & time-varying confounding



Let’s practice with classifying treatment strategies

Go to classpoint.app

Point strategy or sustained treatment strategy?

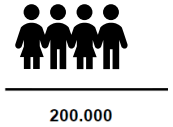
1. Receive bariatric surgery
2. Receive Pfizer first dose now, and second dose 3 weeks later
3. Start SGLT-2i within 3 months from now
4. Never start SGLT-2i
5. Start GLP-1RA when a cardiovascular event develops

A: point strategy

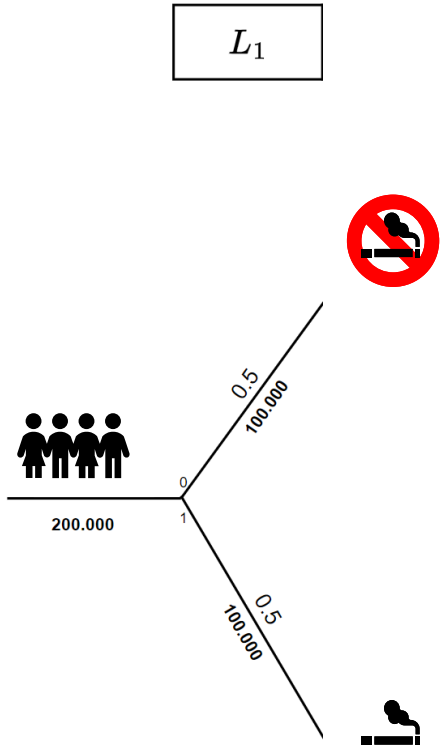
B: sustained strategy

Multiple Choice

Visualizing the history of a population in a tree graph

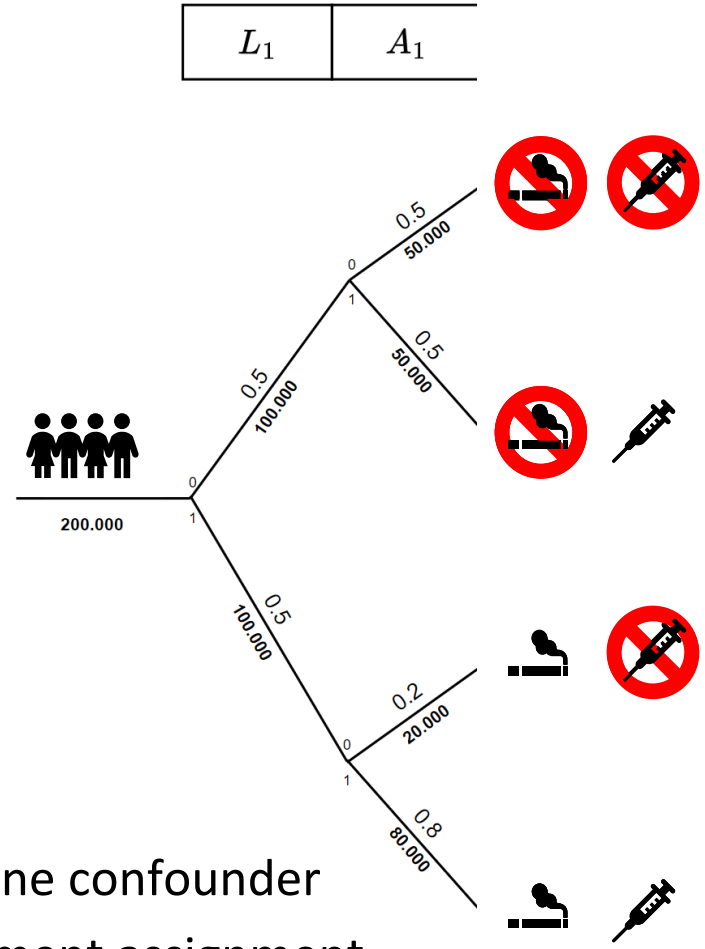


Visualizing the history of a population in a tree graph



L_1: Baseline confounder

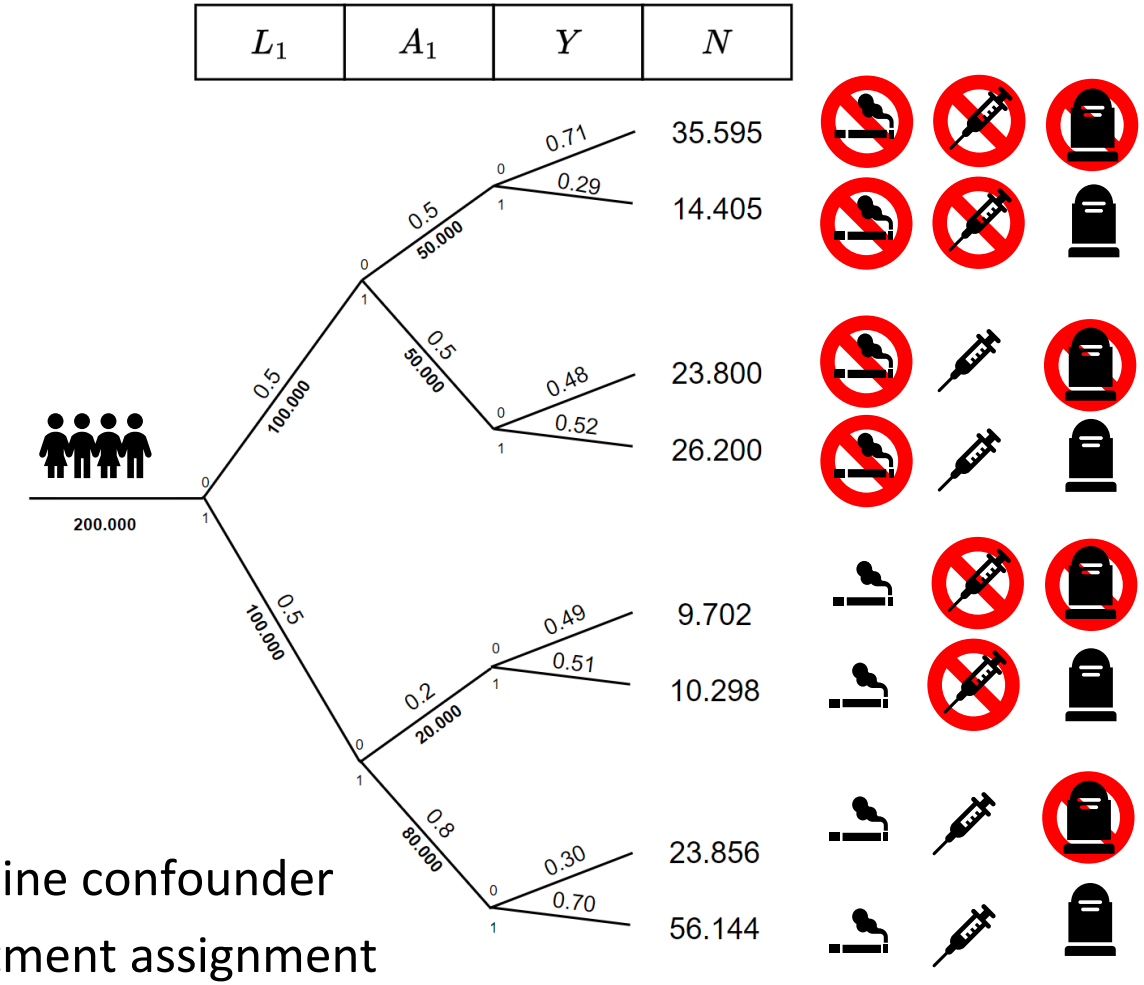
Visualizing the history of a population in a tree graph



L_1: Baseline confounder

A_1: Treatment assignment

Visualizing the history of a population in a tree graph



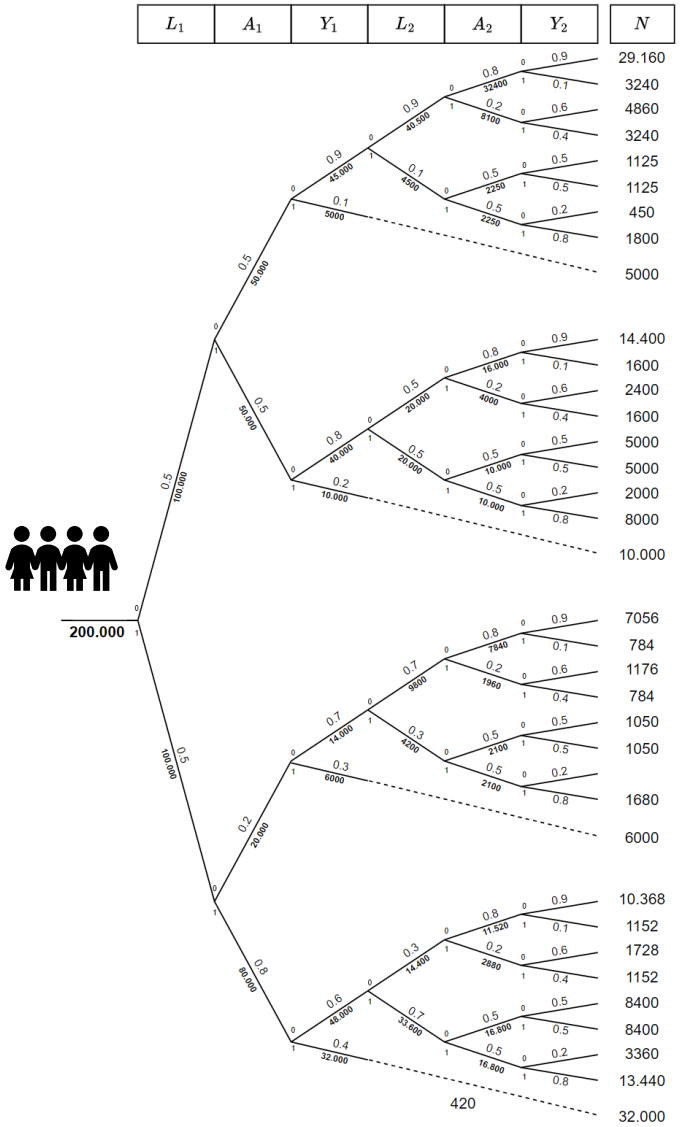
L_1: Baseline confounder

A_1: Treatment assignment

Y: Outcome

This is the whole tree of a **point intervention** because we only have treatment at single point in time!

Visualizing the history of a population as a tree



Quickly becomes more complex for **sustained strategies** because of multiple A_t

L_1: Baseline confounder

A_1: Treatment at time t=1

Y_1: Outcome at time t=1

L_2: Time-varying confounder

A_2: Treatment at time t=2

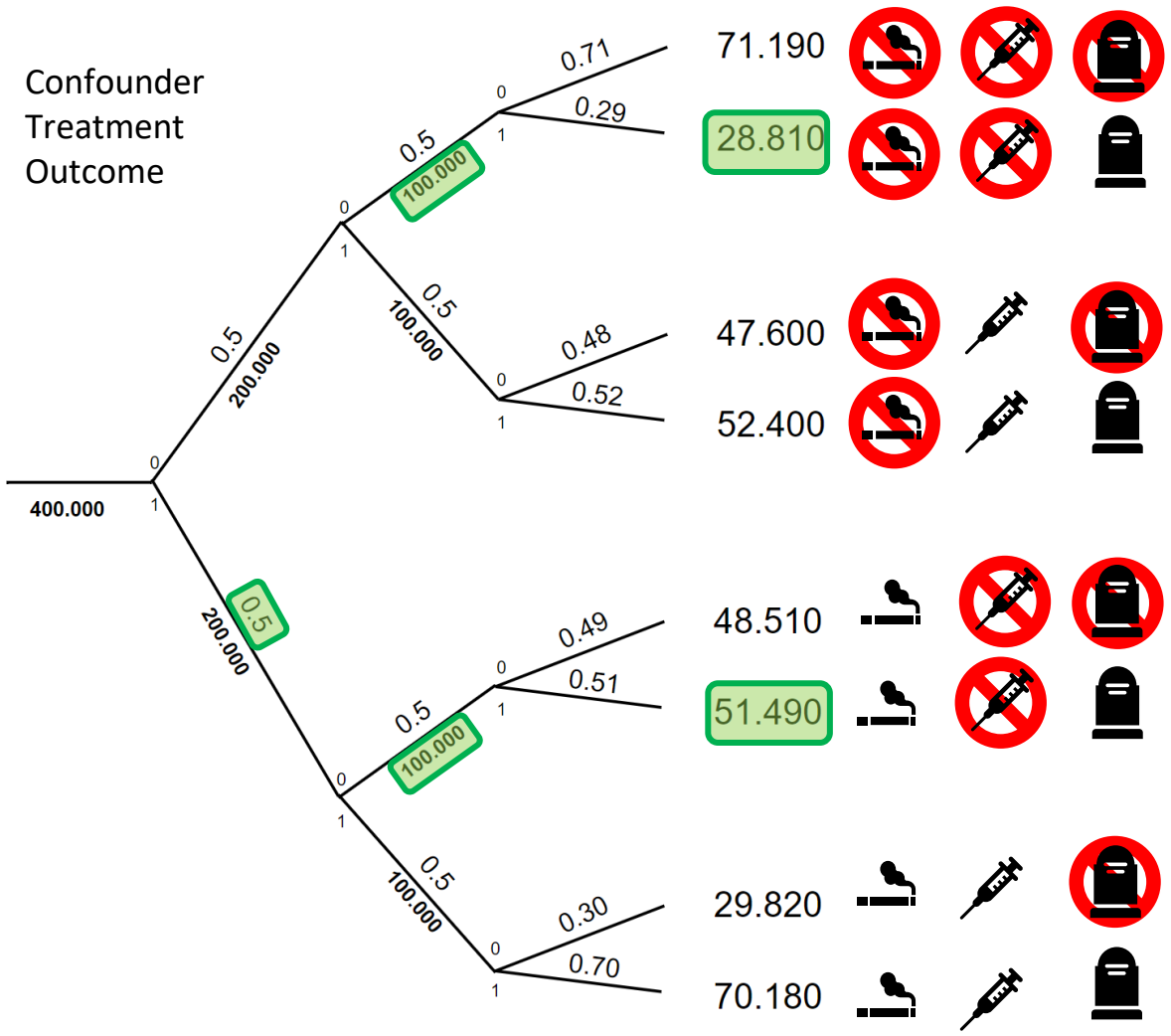
Y_2: Outcome at time t=2

Some exercises

Go to classpoint.app

L_1	A_1	Y	N

L: Confounder
A: Treatment
Y: Outcome



Instructions on reading the tree

1 binary confounder L (smoking)

1 binary treatment A (medication)

1 binary outcome Y (death)

Number above the lines represent proportions

Number below the lines represent number of patients

Short Answer

Question 1:

What is the probability that L_1=1? **0.5**

Question 2:

How many are **untreated**? **100.000 + 100.000 = 200.000**

Question 3:

How many die among **untreated**? **28.810 + 51.490 = 80.300**

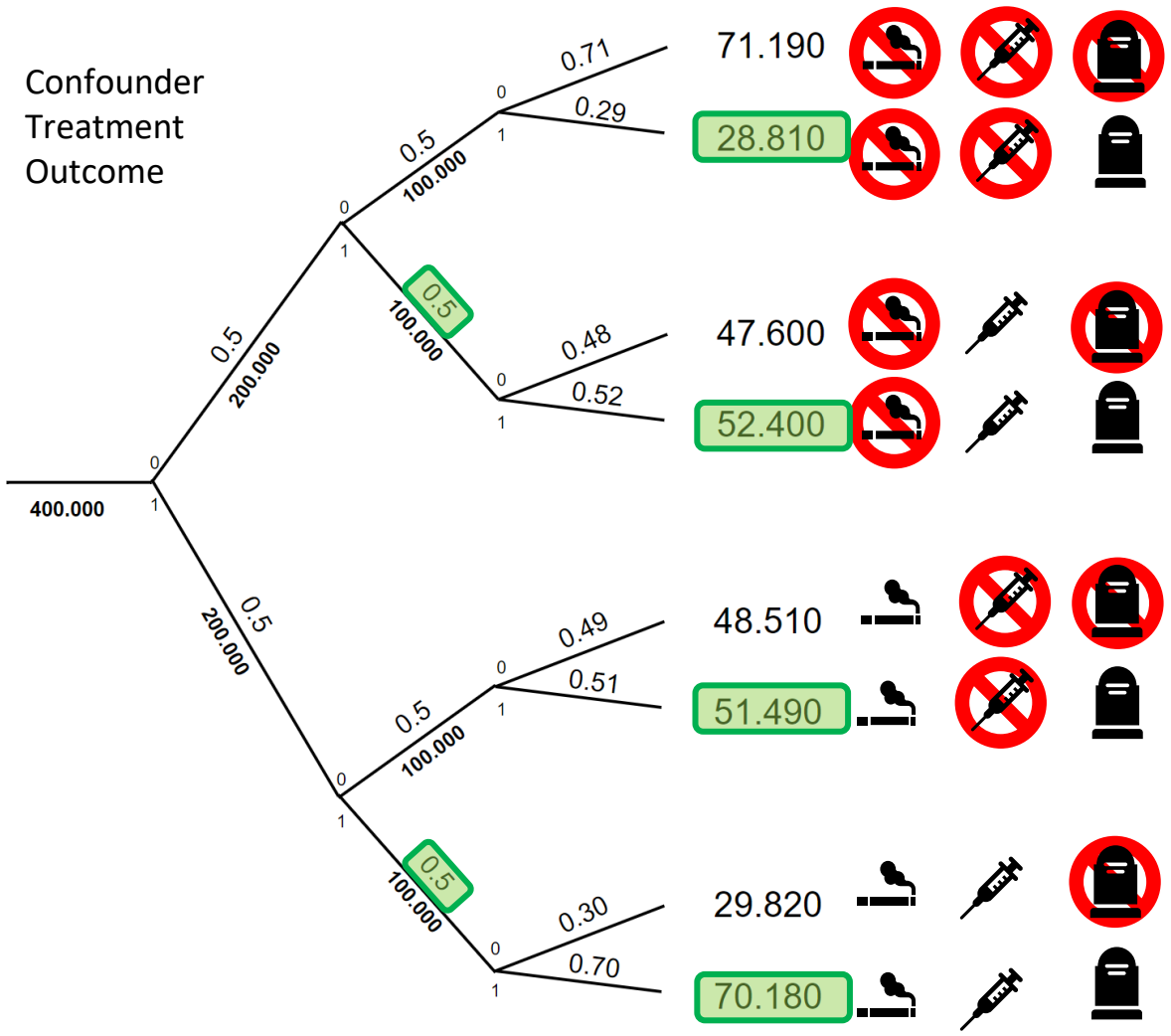
Question 4:

What is risk of death among **untreated**? **80.300/200.000 = 0.402**

Some exercises

L_1	A_1	Y	N

L: Confounder
A: Treatment
Y: Outcome



Instructions on reading the tree

1 binary confounder L (smoking)

1 binary treatment A (medication)

1 binary outcome Y (death)

Number above the lines represent proportions

Number below the lines represent number of patients

Question 5:
Does L_1 predict A_1?

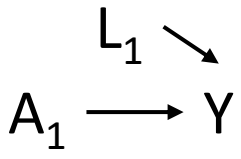
No
$Pr[A_1 = 1 | L_1 = 1] = 0.5$
$Pr[A_1 = 1 | L_1 = 0] = 0.5$

Question 6:
Does L_1 predict Y?

Yes:
$Pr[Y = 1 | L_1 = 1] = (51.490+70.180)/200.000 = 0.61$
$Pr[Y = 1 | L_1 = 0] = (28.810+52.400)/200.000 = 0.41$

Question 7:
Is L_1 a confounder?

No

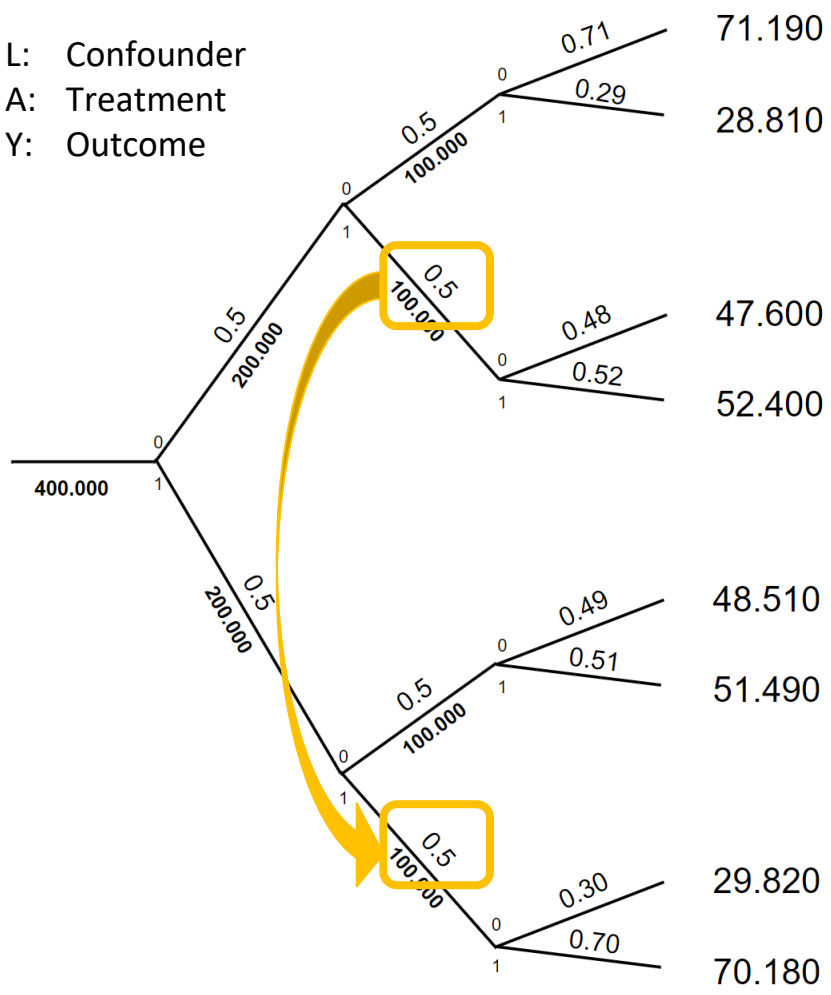


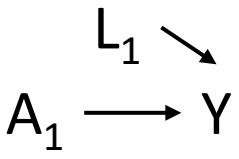
Baseline confounding

Let's check that these data indeed come from a randomized trial

L_1	A_1	Y	N

L: Confounder
A: Treatment
Y: Outcome





In a randomized trial

- Prognostic factor does not determine whether someone receives treatment or not

- Association is causation in randomized trial

Step 3: Effect estimation

Risk among untreated
(28.810+51.490)/(100.000+100.000) = 0.40

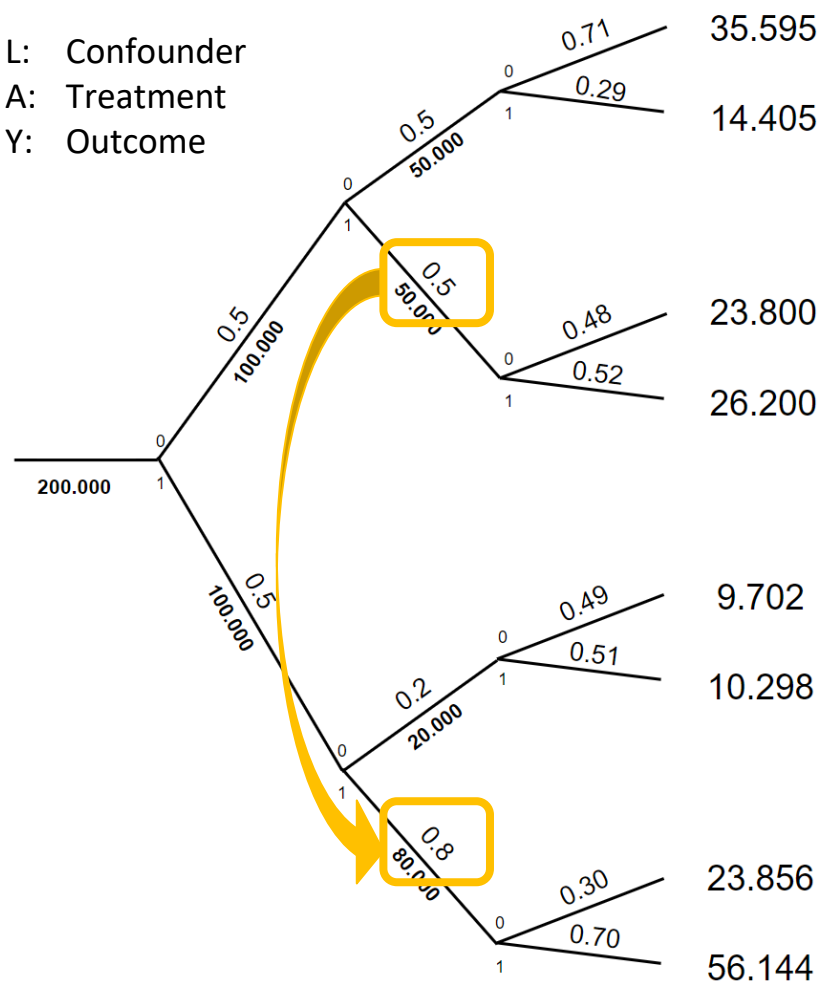
Risk among treated
(52.400+70.180)/(100.000+100.000) = 0.61

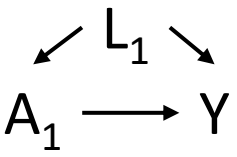
Causal risk difference: 0.61-0.40 = 0.21 (= 21%)
Causal risk ratio: 0.61/0.40 = 1.52

New tree graph. Do these new data come from a randomized trial?

L_1	A_1	Y	N

L: Confounder
A: Treatment
Y: Outcome





In observational studies

- Prognostic factor determines whether someone receives treatment or not (L_1 = confounder)
- Association is NOT causation

Step 3: Effect estimation without adjustment for baseline confounding

Risk among untreated
(14.405+10.298)/(50.000+20.000) = 0.35 ≠ 0.40

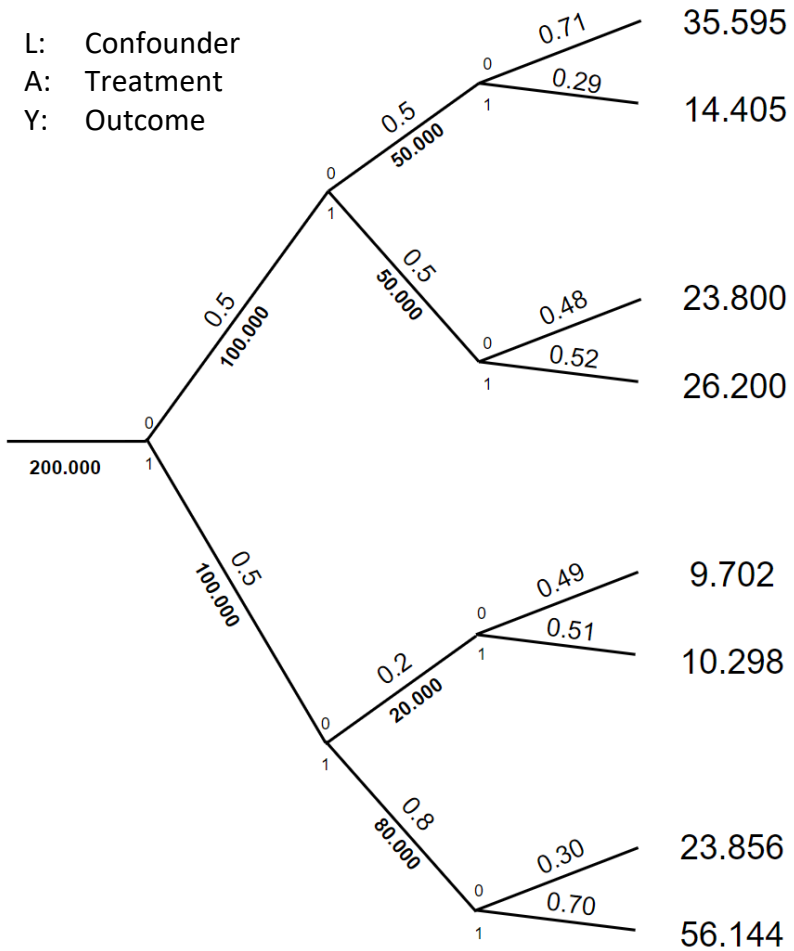
Risk among treated
(26.200+56.144)/(50.000+80.000) = 0.63 ≠ 0.61

Confounded risk difference: 0.63-0.35 = 0.28 (= 28%) ≠ 0.21
Confounded risk ratio: 0.63/0.35 = 1.80 ≠ 1.52

Adjusting for baseline confounding with weighting (IPTW)

L_1	A_1	Y	N

L: Confounder
A: Treatment
Y: Outcome



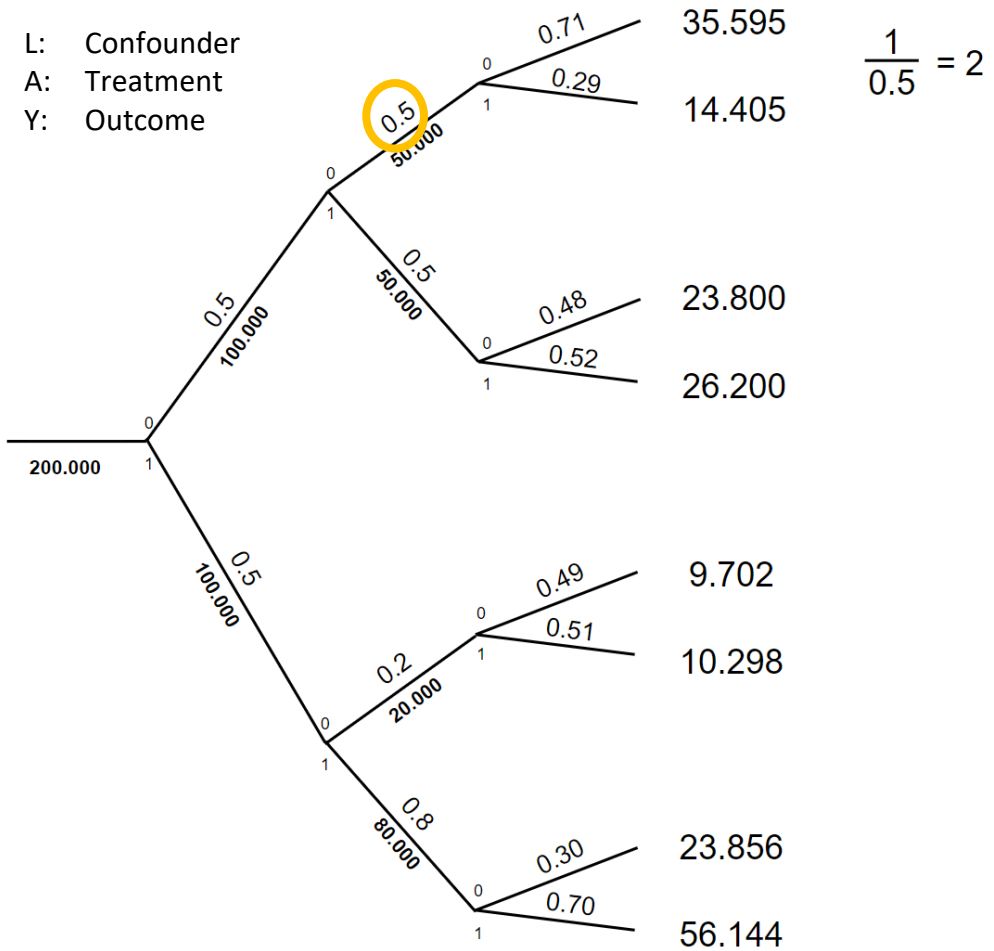
Inverse Probability of Treatment Weights are the inverse of the probability of having received your treatment history given confounders

Here: $w_t = \frac{1}{\Pr[A_1|L_1]}$

Adjusting for baseline confounding with weighting (IPTW)

L_1	A_1	Y	N	w_t	N_w

L: Confounder
A: Treatment
Y: Outcome



$$\frac{1}{0.5} = 2$$

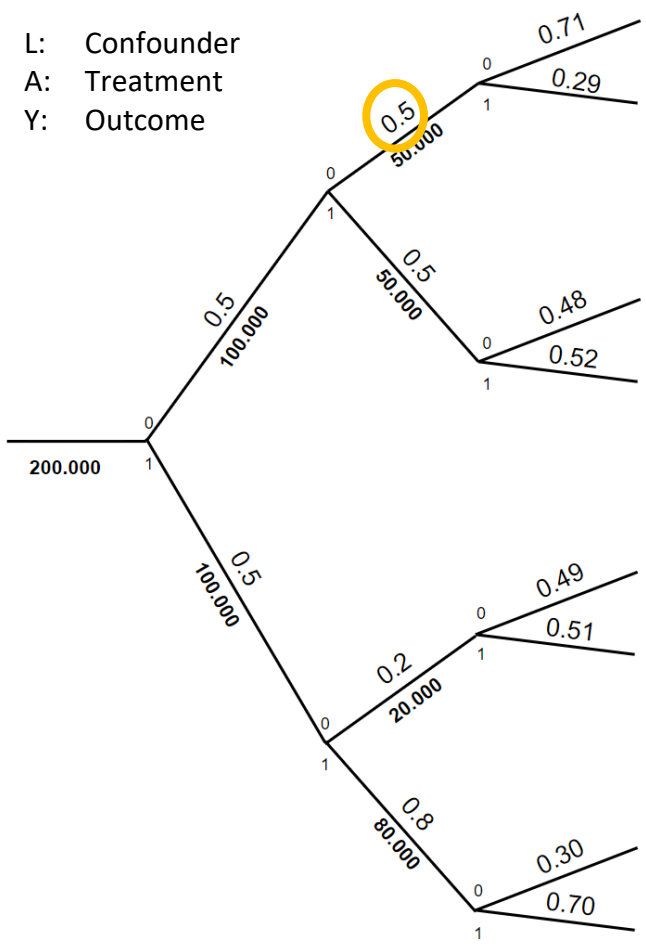
Inverse Probability of Treatment Weights are the inverse of the probability of having received your treatment history given confounders

Here: $w_t = \frac{1}{\Pr[A_1|L_1]}$

Adjusting for baseline confounding with weighting (IPTW)

L_1	A_1	Y	N	w_t	N_w
			35.595	$\frac{1}{0.5} = 2$	71.190
			14.405		28.810
			23.800	?	
			26.200		
			9.702	?	
			10.298		
			23.856	?	
			56.144		

L: Confounder
A: Treatment
Y: Outcome



Inverse Probability of Treatment Weights are the inverse of the probability of having received your treatment history given confounders

Here: $w_t = \frac{1}{\Pr[A_1|L_1]}$

Adjusting for baseline confounding with weighting (IPTW)

L_1	A_1	Y	N	w_t	N_w
		0.71	35.595	$\frac{1}{0.5} = 2$	71.190
	0.5 50.000	0.29	14.405		28.810
0.5 100.000		0.48	23.800	$\frac{1}{0.5} = 2$	47.600
	0.5 50.000	0.52	26.200		52.400
200.000		0.49	9.702	$\frac{1}{0.2} = 5$	48.510
	0.2 20.000	0.51	10.298		51.490
0.5 100.000		0.30	23.856	$\frac{1}{0.8} = 1.25$	29.820
	0.8 80.000	0.70	56.144		70.180

L: Confounder
A: Treatment
Y: Outcome

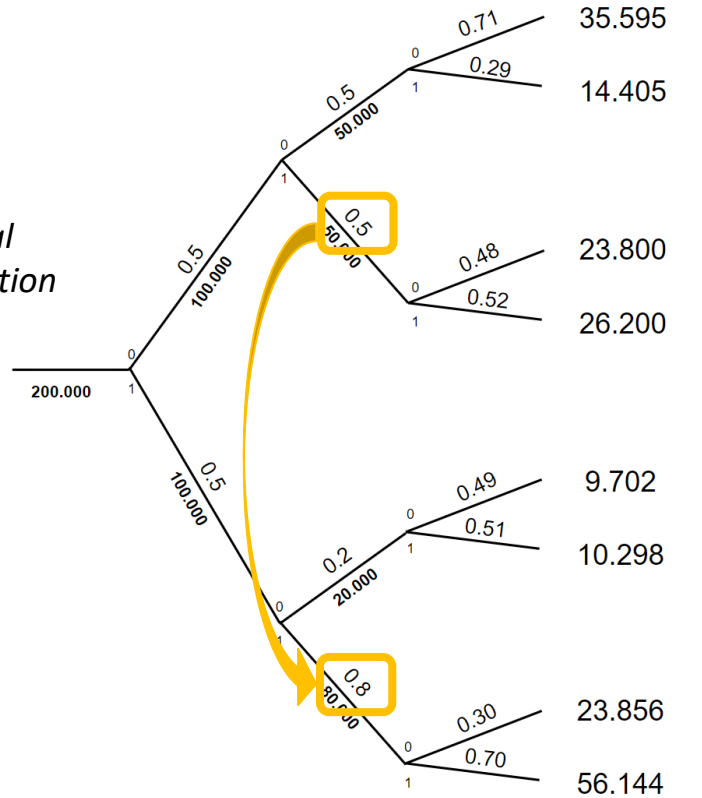
Inverse Probability of Treatment Weights are the inverse of the probability of having received your treatment history given confounders

Here: $w_t = \frac{1}{\Pr[A_1|L_1]}$

Turning our observational study into a randomized trial

Original population

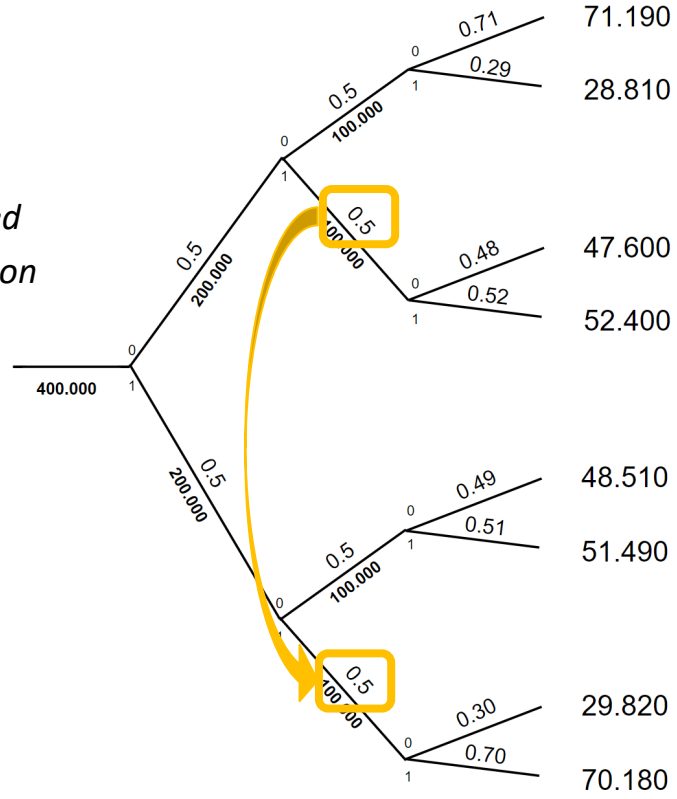
L_1	A_1	Y	N



In the original population, treatment is predicted by confounder L_1

Weighted population

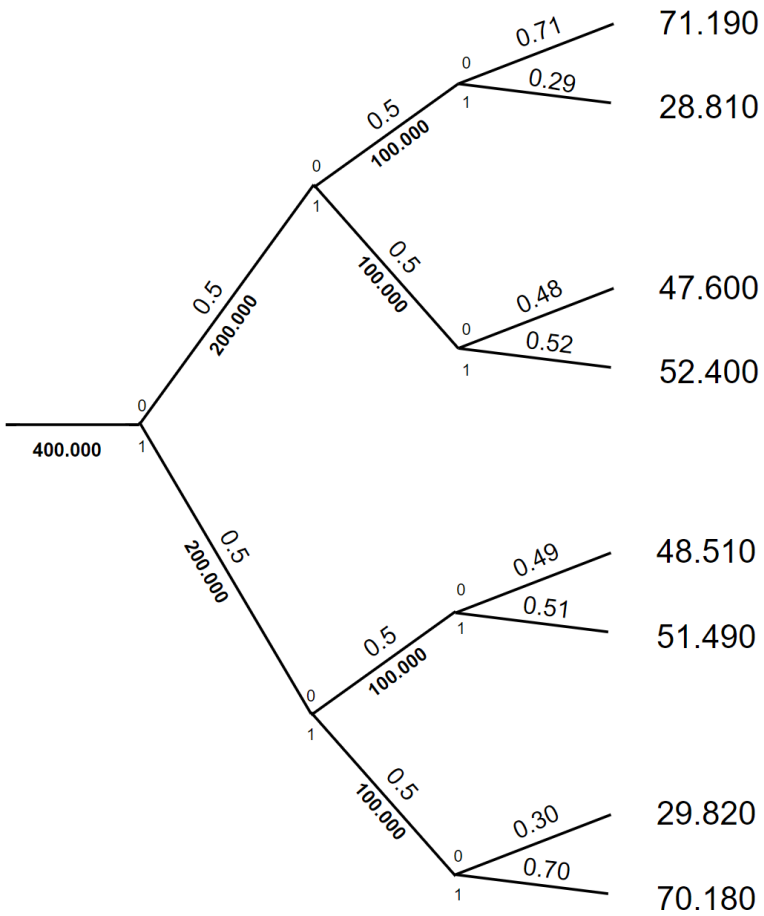
L_1	A_1	Y	N_w

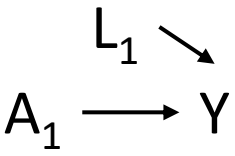


In the weighted population, treatment is no longer predicted by confounder L_1

Treatment effect estimation in the weighted pseudopopulation

L_1	A_1	Y	N_w





In weighted pseudopopulation

- Confounder no longer determines whether someone receives treatment or not

- Association is causation in the weighted pseudopopulation

Effect estimation

Risk among untreated
(28.810+51.490)/(100.000+100.000) = 0.40 ✓

Risk among treated
(52.400+70.180)/(100.000+100.000) = 0.61 ✓

Causal risk difference: 0.61-0.40 = 0.21 (= 21%) ✓
Causal risk ratio: 0.61/0.40 = 1.52

Some comments on weighting

- Note that we only assumed 1 binary confounder – So we could calculate the weights nonparametrically (i.e., without models)

- In practice, there may be many confounders, which may be categorical and continuous → need to **fit models** to estimate the weights (e.g. logistic regression model)

- Note that if there are unmeasured confounders (e.g. if we had not measured L_1), we cannot use them to estimate our inverse probability of treatment weights, and our resulting treatment effects will be biased (then we have not turned our observational study into a randomized trial)

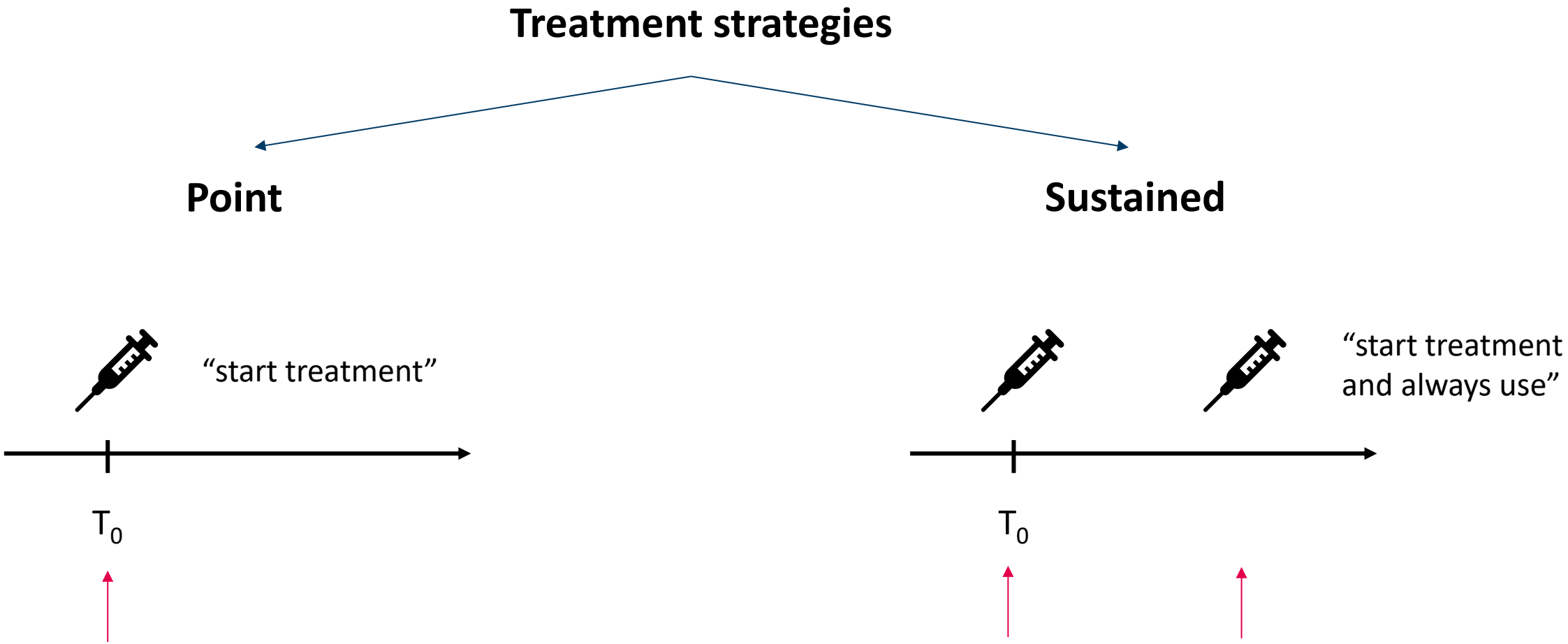
Some comments on outcome model

- In practice, we also fit a model for the outcome (e.g. a *weighted* Cox regression) since survival times are not observed for everyone (there is censoring)

- To obtain correct confidence intervals we need to take into account the weighting, e.g. with robust standard error or bootstrapping

Time-varying confounding

Recap baseline vs. time-varying confounding



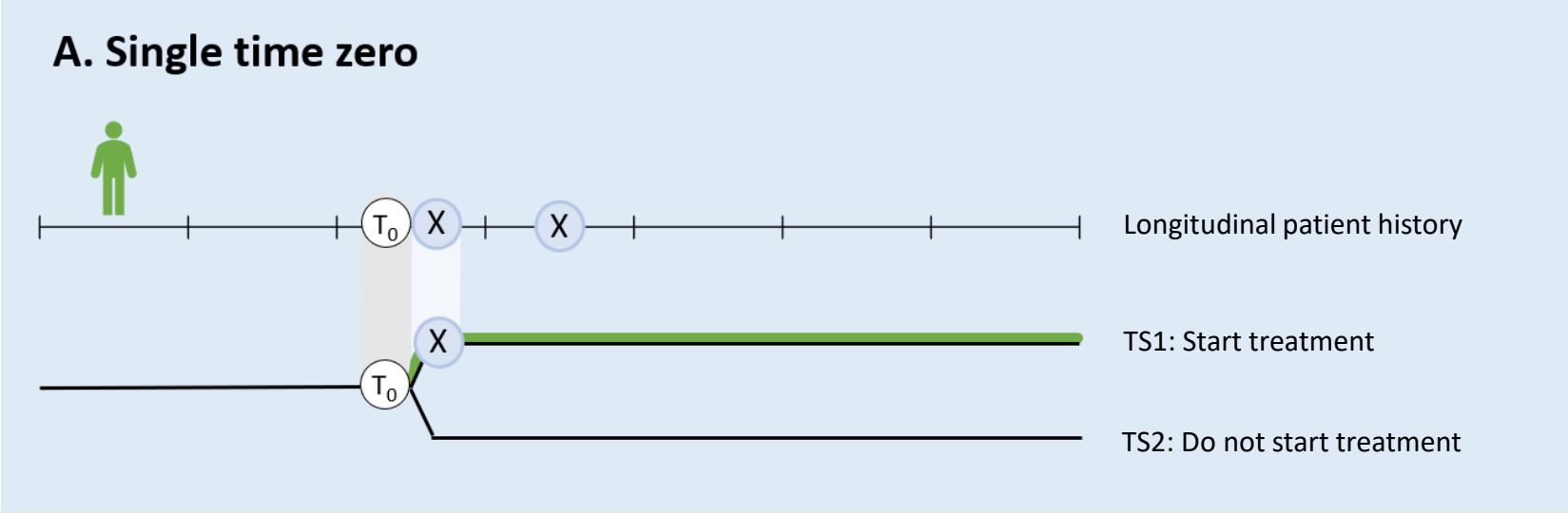
- Groups need to be similar at time zero
- Only baseline confounding

- Groups need to be similar at time zero & during follow-up
- Baseline & **time-varying confounding**

Why the effects of sustained strategies are more interesting

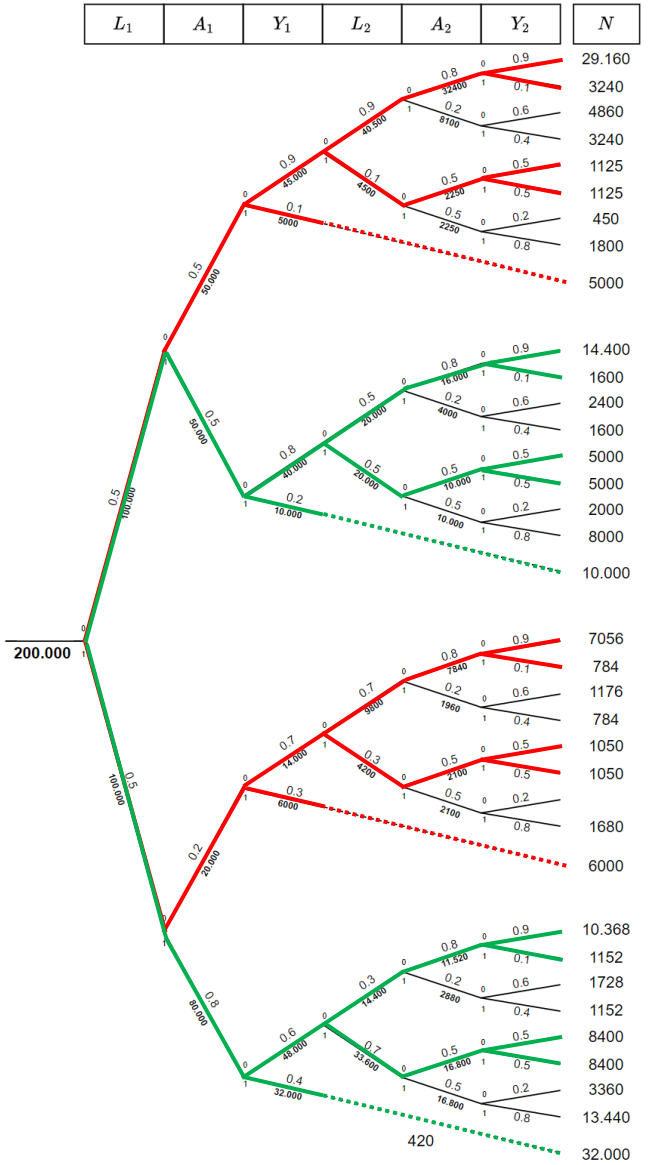
If we compare the point strategies “start treatment” vs. “do not start treatment”, what problems arise?

- Many people in “start treatment” group may stop treatment during follow-up
- Conversely, many people in “do not start treatment” group may start it during follow-up
- We may then find a hazard ratio of 1.0 even for a treatment known to have benefits



Sustained strategies: tree graph with 2+ timepoints

Go to classpoint.app



Let's say we are interested in the sustained strategies:

- "always treat"
- "never treat"

Multiple Choice

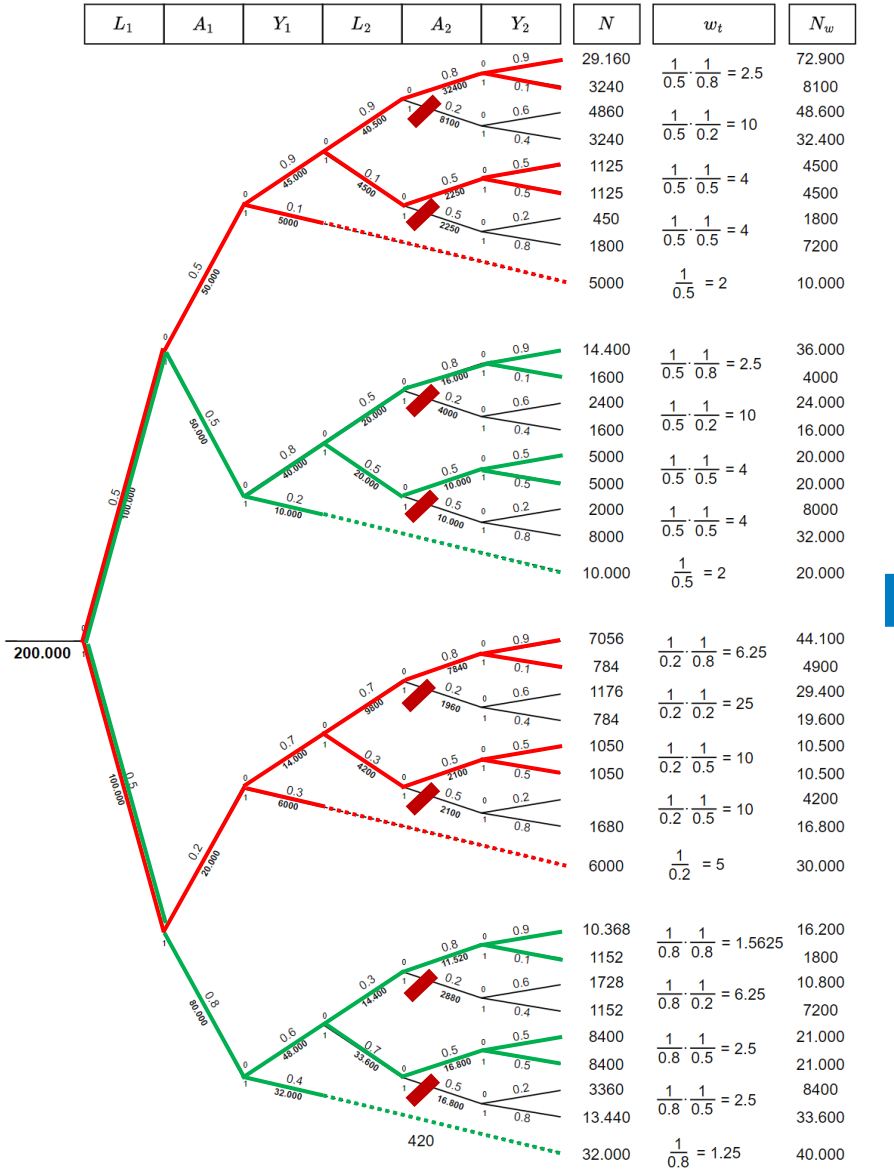
Which strategy is highlighted in the tree?

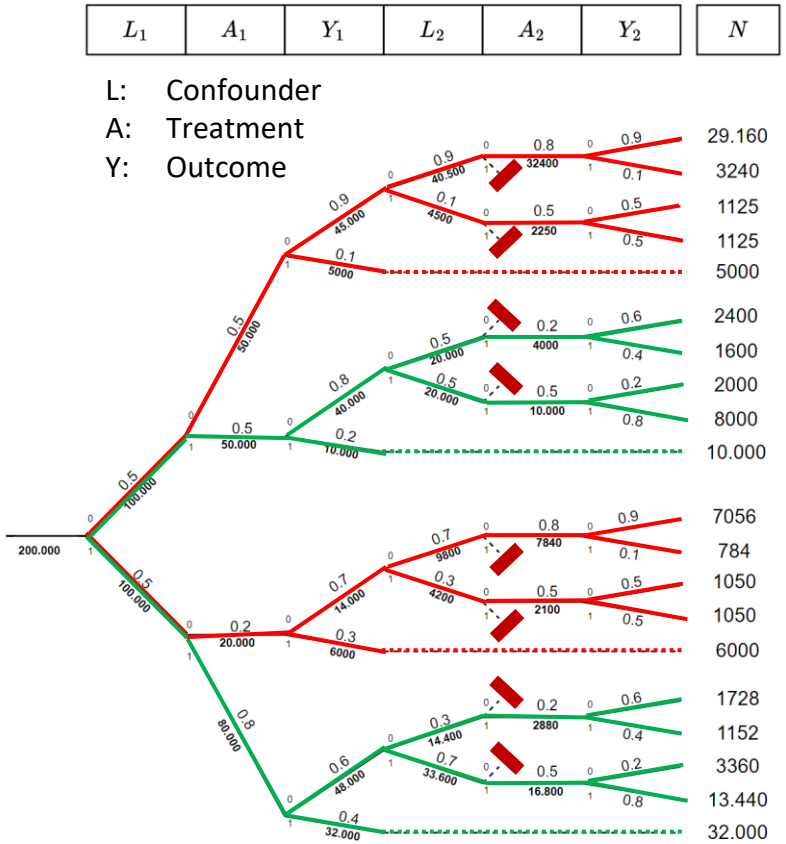
A: Always treat

B: Never treat

C: Neither

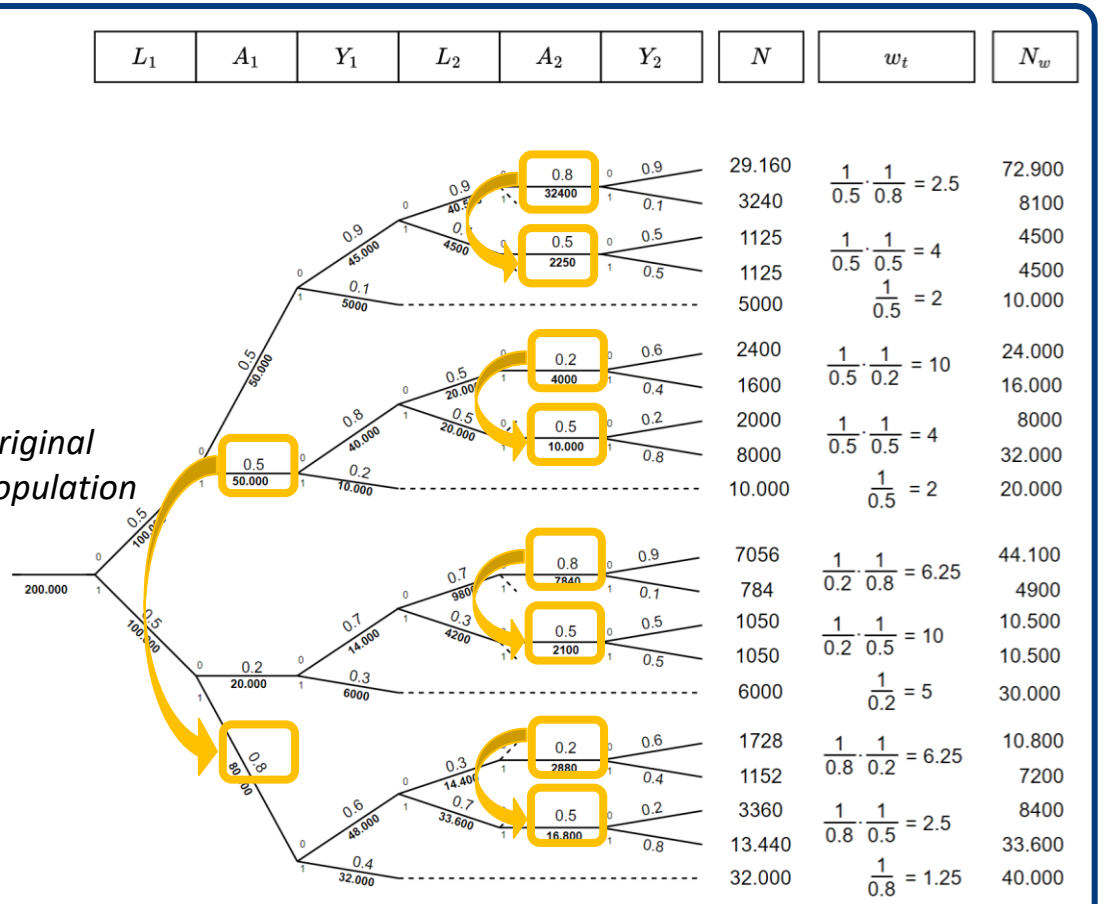
Censoring: focus only on branches of interest





Censor patients who deviate from the strategies of interest

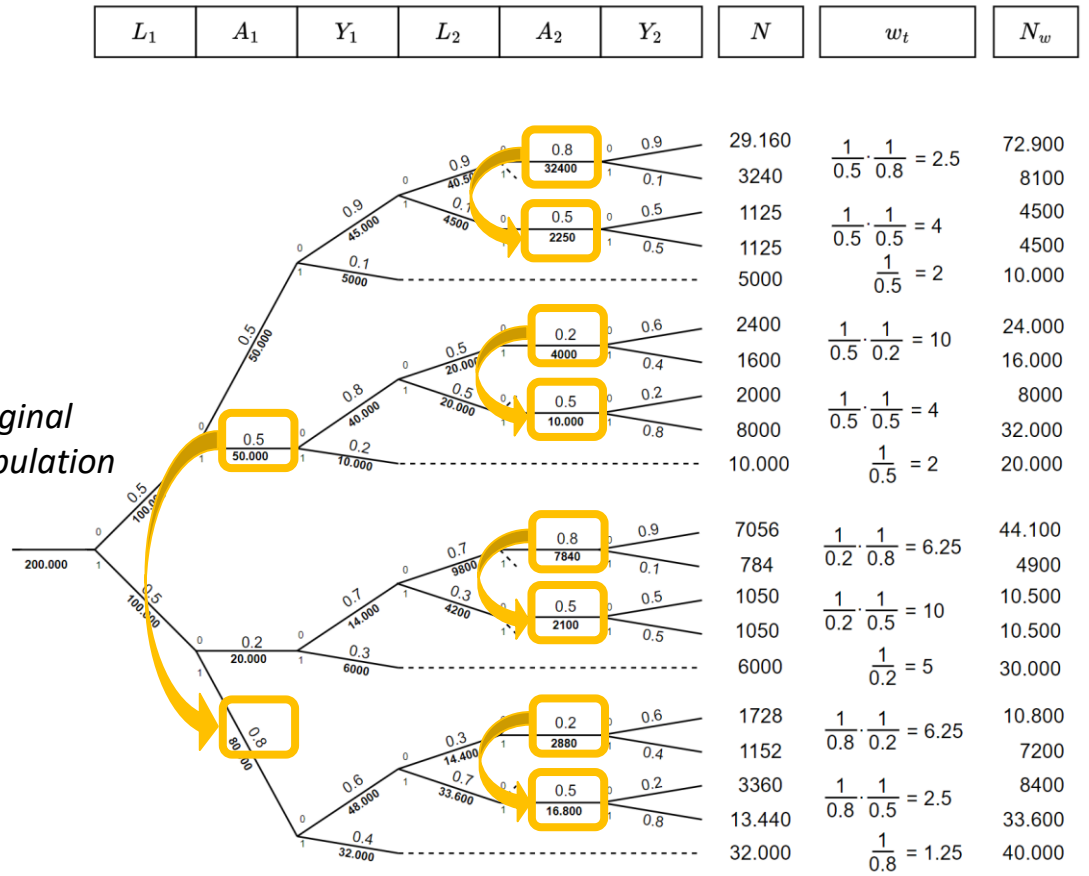
Turning our observational study into a sequentially randomized trial



In the original population, treatment A_k is predicted by confounder L_k

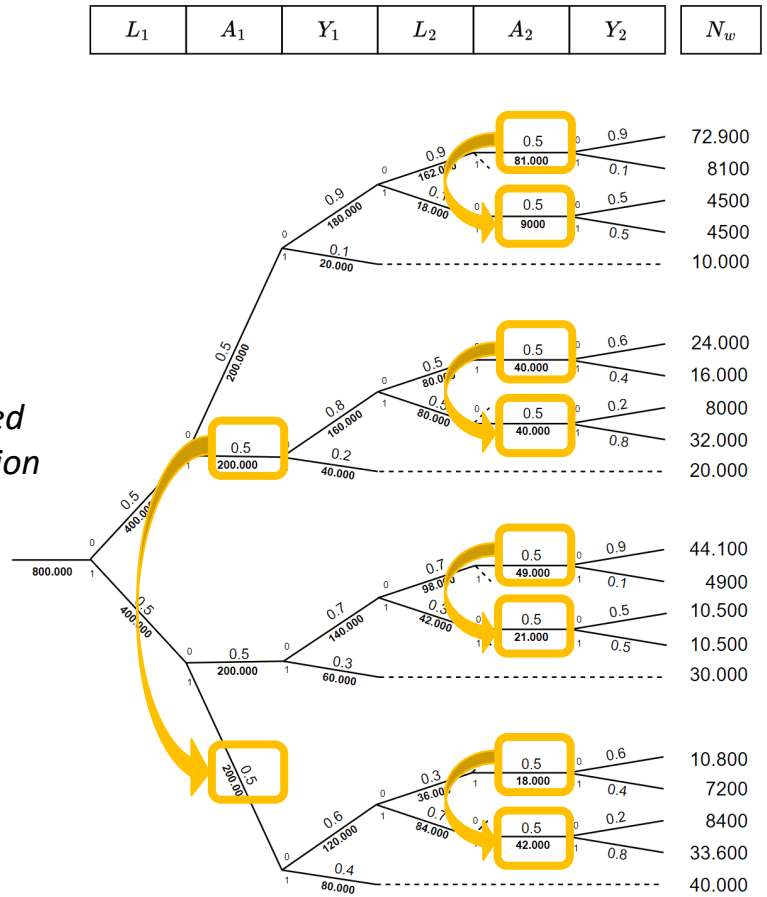
Turning our observational study into a sequentially randomized trial

Original population



In the original population, treatment A_k is predicted by confounder L_k

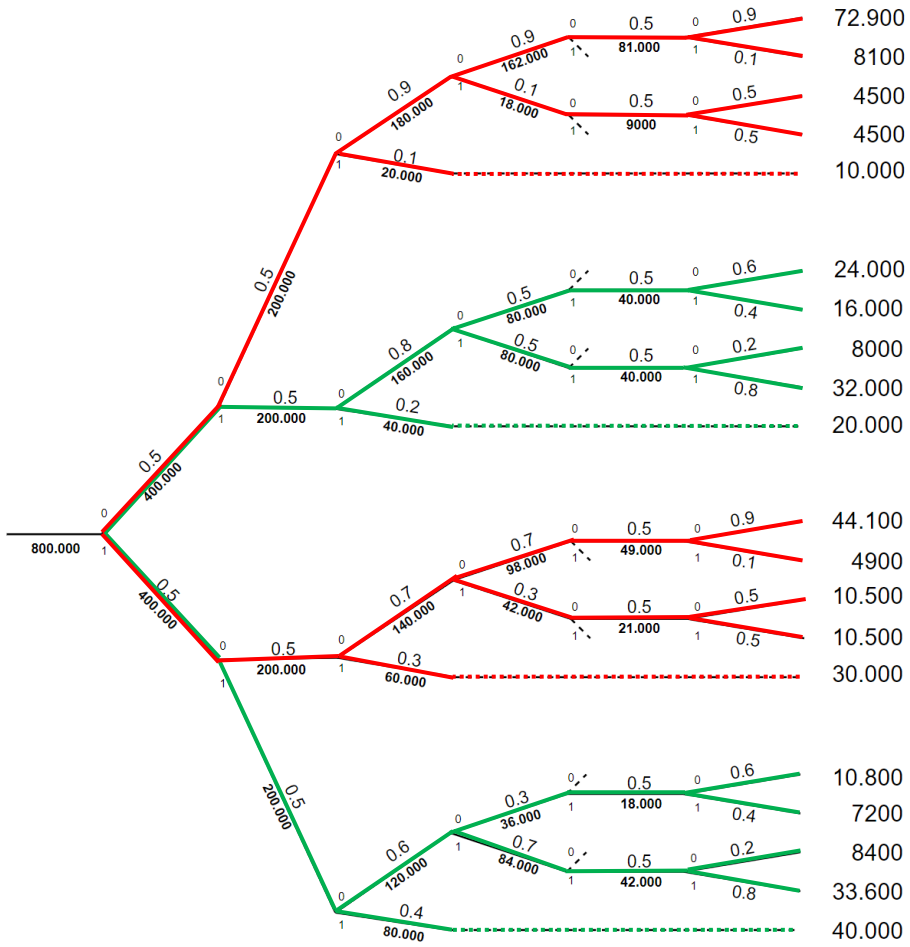
Weighted population



In the weighted population, treatment A_k is no longer predicted by confounder L_k

Treatment effect estimation in the weighted pseudopopulation

L_1	A_1	Y_1	L_2	A_2	Y_2	N_w



Effect estimation sustained strategies

Risk among never treated
(8100+4500+10.000+4900+10.500+30.000)/(200.000) = 0.34

Risk among always treated
(16.000+32.000+20.000+7200+33.600+40.000)/(200.000) = 0.74

Causal risk difference: 0.74-0.34 = 0.40 (= 40%)
Causal risk ratio: 0.74/0.34 = 2.19

Effect estimation point strategies

Risk among untreated
(28.810+51.490)/(100.000+100.000) = 0.40

Risk among treated
(52.400+70.180)/(100.000+100.000) = 0.61

Causal risk difference: 0.61-0.40 = 0.21 (= 21%)
Causal risk ratio: 0.61/0.40 = 1.52

Conclusions

1. Important distinction between point vs. sustained strategies

2. Always need to adjust for baseline confounding

3. If interested in sustained strategies, also need to adjust for time-varying confounding

4. We showed how weighting can be used to turn the observational data into a randomized or sequentially randomized trial

5. Results are biased if there are unmeasured confounders

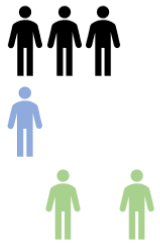
Leiden University
Medical Center

Questions

e.l.fu@lumc.nl

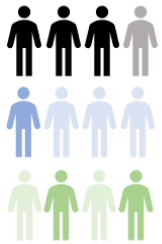
Censoring & weighting on a group-level

Artificial censoring



Censored during follow-up if not following strategy of interest

Weighting



Uncensored replicates (dark color) are upweighted to account for censored replicates (light color) with similar characteristics

Standardization (time-fixed exposure)

Standardization – Identification proof

$$E[Y^a] = \sum_l \mathrm{E}[Y^a|L = l] * \Pr[L = l]$$

By the law of total expectation

$$= \sum_l \mathrm{E}[Y^a|A = a, L = l] * \Pr[L = l]$$

By exchangeability assumption

$$= \sum_l \mathrm{E}[Y|A = a, L = l] * \Pr[L = l]$$

By consistency assumption

We have now expressed a counterfactual quantity as something that does not contain counterfactuals and that we can estimate from our data

Standardization on the tree graph

Standardization as simulation

Equivalence of standardization and IPW

Standardization for ATT

IPW for ATT

G-formula (aka standardization in the time-varying setting)

G-formula – Identification proof

G-formula as simulation

Equivalence of G-formula and IPW

Probability review (taken from Robins' lecture)