Applied Time Series

Course timetable:

- Thursday 2 October 2025 - 8:30 - 11:45 - P508
- Thursday 9 October 2025 - 8:30 - 11:45 - C131
- Thursday 16 October 2025 - 8:30 - 11:45 - C131
- Thursday 23 October 2025 - 8:30 - 11:45 - C131
- Thursday 23 October 2025 - 12:00 - 13:30 - C131
- Thursday 6 November 2025 - 8:30 - 11:45 - Online
- Thursday 13 November 2025 - 8:30 - 11:45 - P205
- Thursday 20 November 2025 - 8:30 - 11:45 - P205
- Thursday 27 November 2025 - 8:30 - 10:00 - P205

Assessment:

- Assignment (30%, due date: December 29, 2025, at midnight)
- Final Exam (70%, December 11, 2025 - Salle A Bis - from 10:15 AM to 12:15 PM)

References:

1. Brooks, C. (2008), Introductory Econometrics for Finance, Cambridge University Press.
2. Enders, W. (2014), Applied Econometric Time Series, 4th Edition, Wiley.
3. Hamilton, J. D. (1994), Time Series Analysis, Princeton University Press.
4. Hilpisch, Y. (2015), Python for Finance: Analyze Big Financial Data, O'Reilly Publishing.
5. Sargent, T. J. and Stachurski (2018), Lectures in Quantitative Economics.
6. Sheppard, K. (2018), Introduction to Python for Econometrics, Statistics and Numerical Analysis.
7. Tsay, R. S. (2010), Analysis of Financial Time Series, 3rd Edition, Wiley.

Assignment 2025

Part 1 (10 points): Theoretical exercises.

Exercise 1 (3 points): Stationarity

Let $\{\epsilon_t\}_{t\in\mathbb{Z}}$ be a weak white noise process with variance σ^2, i.e., a random process such that:

$$\forall t, \quad \mathbb{E}[\epsilon_t] = 0, \quad \mathrm{Var}(\epsilon_t) = \mathbb{E}[\epsilon_t^2] = \sigma^2 < \infty, \quad \text{and} \quad \mathrm{Cov}(\epsilon_t, \epsilon_{t-h}) = \mathbb{E}[\epsilon_t \epsilon_{t-h}] = 0 \text{ for } h \neq 0.$$

Are the following processes second-order (weak) stationary?

1. $Y_t = \epsilon_t - \epsilon_{t-1}$
2. $Z_t = a\epsilon_t + b\epsilon_{t-1}$
3. $W_0 = 0$, and $W_t = W_{t-1} + \epsilon_t$, for all $t > 0$

Exercise 2 (2 points): Transformations of a stationary process

Let ϵ_t be a weak white noise with variance σ^2. Fix $\mu \in \mathbb{R}$, and a sequence (θ_k) such that $\sum_{k=0}^{\infty} \theta_k^{\infty} < \infty$. Let U_t be the process defined by:

$$U_t = \mu + \sum_{k=0}^{\infty} \theta_k \epsilon_{t-k},$$

Compute the expectation, the variance and the covariance ($\gamma_h = \mathrm{cov}\left(U_t, U_{t-h}\right)$), for any $h > 0$.

Exercise 3 (4 points): Forecast errors

Consider the following stochastic process: $X_t = 0.5 + 0.8X_{t-1} + \epsilon_t$

1. Determine the best linear forecast ($\mathbb{E}[X_{t+h}|\Omega_t] = X_t^*(h)$) for $h = 1, 2, 3, \ldots, h, \ldots, \infty$.
2. Determine the forecast error ($e_t(h) = X_{t+h} - X_t^*(h)$.) for $h = 1, 2, 3, \ldots, h$.
3. Determine the variance of the forecast error ($\mathbb{V}[e_t(h)] = \sigma_\epsilon^2(h)$) for $h = 1, 2, 3, \ldots, h, \ldots, \infty$
4. Determine the prediction interval for h and when $h \to \infty$). Under the normality assumption of the errors, the h-step-ahead $(1-\alpha)$-level prediction interval is constructed as:

$$CI^{\alpha}(X_t^*(h)) = X_t^*(h) \pm z_{\alpha/2}\sqrt{\sigma_\epsilon^2}$$

where $z_{\alpha/2}$ is the ($\alpha/2$)-level quantile of the $\mathcal{N}(0, 1)$ distribution.

Exercise 4 (1 point): The MA(1)-GARCH(1) model

Consider the model $y_t = \theta_0 + \epsilon_t + \theta_1 \epsilon_{t-1}$, where the conditional variance of ϵ_t is $h_t = \sigma_t^2 = \mathrm{V}_{t-1}(\epsilon_t) = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1}$ with $\alpha_0 > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$ and $\alpha_1 + \alpha_2 < 1$.

1. Find the conditional and unconditional mean of y_t.

2. Find the conditional and unconditional variance of y_t.

Part 2 (10 points): Empirical exercises.

Exercise 4 (4 points): SARIMA-GARCH model (A or B)

Guided exercise A. Retrieve the monthly macro-variable of your choice from DBnomics) or Bloomberg and fit the appropriate SARIMA-GARCH models to this time series. Your in-sample period starts from the first observation of your time-series until December 2019, whereas the out-of-sample periods starts in January 2020 and ends in September 2025. Fit and describe the estimated model of the in-sample period. Please use a recursive window to re-estimate your model every months from January 1, 2020 to September 30, 2025. Use your fitted models to produce 1-step ahead forecasts on the out-of-sample period and compare these forecasts to the true realisations.[1] Describe precisely in a Jupyter Notebook the different steps you follow (stationarity, seasonality, Box-Jenkins approach, test-statistics, forecast evaluation.). Bonus question: Are you able to apply Darts or Google TimesFM to do these forecasts on your time series? Use the RMSE to select the best model.

Guided exercise B. Retrieve the daily asset price of your choice from Yahoo Finance or Bloomberg and fit the appropriate SARIMA-GARCH models to this time series. Your in-sample period starts from the first observation of your time-series until December 31, 2024, whereas the out-of-sample periods starts in January 1, 2025 and ends in April 30, 2025. Fit and describe the estimated model on the in-sample period. Please use a rolling window of 4 years to re-estimate your model and then produce a daily forecast on the out-of-sample period. Indeed, you use these estimated models to produce 1-step ahead forecasts on the out-of-sample period (from January 1, 2025 to April 30, 2025).[2] For both periods, compute the VaR and the ES of your choice and run some backtesting tests for the

[1] 69 forecasts have to be done with this monthly variable.
[2] Approximately 70 forecasts have to be done with this daily variable.

VaR. Describe precisely in a Jupyter Notebook the different steps you follow (stationarity, seasonality, Box-Jenkins approach, test-statistics, forecast evaluation, VaR/ES computation and backtesting.).

Exercise 5 (4 points): Markov Switching Autoregression Models

Do It Yourself. Go to Bloomberg or Yahoo Finance to download the daily commodity price of your choice from January 1, 2020 until September 30, 2025. Compute its continous daily, monthly and quarterly return. Specify a markov switching autoregression model on these 3 time series from the beginning of your sample till December 31, 2022. Next, do some forecasts over the next 3 quarters, 9 months and the subsequent days (for this daily prediction, please predict 20-step ahead forecasts and next re-estimaste your model by including the true realizations - recursive window - and predict again 20-step ahead forecasts, do it till the end of the sample). You may use statmodels to do it. Be careful, you have to figure out which SARIMA-GARCH model you should apply first on the time series before adding a Markov Switching process. Describe your models and comment your results in a Jupyter Notebook.

Exercise 6 (2 points): Volatility is (Mostly) Path Dependent

Replication exercise. Recent empirical studies, extending the ARCH literature, have shown that the volatility of financial markets is mostly path-dependent, i.e., is very well explained by the history of past asset returns. Please do the following replication exercises on the stock prices of your choice:

- Perform the fitting of one model on historical price (see empirical_study.ipynb) and compare the predicted volatility to a SkewStudent GJR-GARCH model.
- Compute its vanilla call price using our 4-factor markovian PDV model (see option_pricing_4fmpdv.ipynb).

Describe precisely in a Jupyter Notebook the different steps you follow and comment your results.