Assignment 6

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Problem 1: Mean-variance portfolio choice and leverage constraints

Consider an economy with $N = 3$ risky assets R_1, R_2, R_3 and one risk-free asset R_0. The expected return vector is $\mu = [0.08; 0.14; 0.16]$ and standard deviation $\sigma = [0.10; 0.30; 0.35]$. The pair-wise correlation between any two returns is 0.2 . There is a risk-free rate $R_0 = 0.05$. We want to solve the problem of a mean-variance investor who faces leverage constraints and cannot borrow more than 30% of his wealth. The investor seeks the portfolio R_P such that $\max E\left[R_P\right] - \frac{a}{2} V\left[R_p\right]$ subject to $w'\mathbf{1} \leq m$ where $m = 1.3$ and w is the vector of weights invested in the risky assets.

1. Determine the tangency portfolio w_t, its mean, variance and Sharpe ratio.
2. Determine the portfolio w_z which is a combination of the tangency portfolio and the global minimum variance portfolio ($w_z = x w_t + (1-x) w_{min}$ for some x you should determine), which has zero correlation with the tangency portfolio. Compute its mean, variance and Sharpe ratio.
3. Prove that the investor will optimally choose to invest in a combination of a risky-asset-only mean-variance efficient portfolio and the risk-free rate. Prove further that this implies that we can restrict his optimal portfolio choice to portfolios with returns of the form $R_P = R_0 + x_t\left(R_t - R_0\right) + x_z\left(R_z - R_0\right)$. Set up the Lagrangian of the agent's problem, derive the first-order condition, and compute the optimal portfolio in terms of x_t, x_z the holdings of tangency and zero beta portfolio.

4. Prove that there exists a risk-aversion level a^* so that if $a > a^*$ then the agent is unconstrained and does not hold the zero-beta portfolio. Instead, if $a < a^*$ then the agent will also invest in the zero-beta portfolio.
5. Plot the Sharpe ratio on the optimal portfolio as a function of the risk-aversion level. What happens to the Sharpe ratio of the optimal portfolio as a falls below a^* ? Interpret the findings.

Problem 2: Betting against Beta and information ratio

Suppose you estimate the following regression $R_i - R_0 = \alpha_i + \beta_i (R_M - R_0) + \epsilon_i$ for $i = H, L$ two portfolios of High and low beta firms respectively. You find the following $\alpha_L = 2.36\%, \alpha_H = -4.36\%, \beta_L = 0.55, \beta_H = 1.78$. The risk-free rate is 0.5%. In addition, $\sigma_L = \sigma_H = 12\%$ and the residuals are uncorrelated.

1. Construct a zero cost portfolio, using H and L, with a positive alpha and zero beta. What is the alpha of that portfolio? what is its information ratio?
2. Given $\sigma_M = 14\%, \mu_M - R_0 = 7.5\%$ what is the optimal position in the BAB portfolio that you should take? What is the optimal position in the high-beta portfolio and the low-beta portfolio?
3. What is the Sharpe ratio of your optimal portfolio? How does it compare to that of the market?

Problem 3: APT

Assume that the following two-factor model describes returns

$$R_j = \alpha_j + \beta_{j1} F_1 + \beta_{j2} F_2 + \varepsilon_j \tag{1}$$

Assume also that the following three portfolio characteristics are:

1. Find the equation that describes expected returns in equilibrium.
2. Suppose the risk premium on the market portfolio is given by $\mu_M - R_0 = 0.04$. Consider a portfolio 4, that has $\beta_{41} = 1$ and $\beta_{42} = 0$, and a beta with respect to the market

Portfolio	μ_j	β_{j1}	β_{j2}
1	0.12	1	0.5
2	0.134	3	0.2
3	0.12	3	-0.5

portfolio denoted by β_{4M}. Similarly, consider a portfolio 5 , that has $\beta_{51} = 0$ and $\beta_{52} = 1$, and a beta with respect to the market portfolio denoted by β_{5M}. Determine R_0, β_{4M}, and β_{5M} such that expected returns are consistent with the CAPM.

Problem 4: CAPM: True or False?

1. The CAPM implies that stocks with the same expected return cannot have the same beta.

2. The CAPM implies that two securities with different levels of idiosyncratic risk must have different expected returns, otherwise, no agent would choose to hold the security with higher idiosyncratic risk.

3. According to the CAPM standard deviation is the right measure of risk for all assets? for some assets?

4. According to the CAPM, beta is the right measure of risk for all assets.

5. Suppose an asset has a positive alpha (i.e., it is above the security market line). Is this asset undervalued? Should you invest all your wealth in this asset if you are a mean-variance investor?