Assignment 10

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1 Transaction Costs and the No Trade Region

Consider N stocks with $E[R_i] = \mu_i$ and $Var[R_i] = \sigma_i^2$ and correlation ρ_{ij}. In addition there is a risk-free rate R_f. Assume the investor starts with some initial dollar position vector X_0 and seeks the vector of terminal position X_1, so as to maximize the mean-variance objective function

$$\max_{X_1} R_f + X_1^\top(\mu - R_f) - \frac{\gamma}{2} X_1^\top \Sigma X_1 - |X_1 - X_0|^\top b$$

where b is a linear proportional transaction cost vector. We assume there are no transaction costs for trading the risk-free asset. The risk-free rate is equal to 2%. The asset-specific parameters are given in the table below.

	μ_i	σ_i	ρ_{ij}	b_i
Asset 1	5%	15%	50%	3%
Asset 2	15%	25%		3%

1. Solve for the optimal portfolio when there is one single risky asset and the risk-free asset. Derive an explicit solution for the no-trade region, that is two numbers $[X_L, X_H]$ such that when $X_L \le X_0 \le X_H$ it is optimal not to trade.

2. Now solve for the optimal portfolio in the case where there are two risky assets in addition to the risk-free asset. Characterize the no-trade region. Plot the optimal trading regions (no trade, buy 1/sell 2, buy 1/buy 2, ...) on a graph with x-axis X_{10} and y-axis X_{20}, that is the initial positions held in both assets.

3. How does the shape of the no-trade region change as you increase the correlation coefficient ρ between the two assets? As you make asset 2 riskier than asset 1?

2 Black Litterman

We will replicate the results of He-Litterman (1992) to better understand how to apply the Black-Litterman formula. We are considering the optimal asset allocation to seven country equity index returns with correlation matrix given on table 1 page 21 of the lecture notes (lecture 8) and with volatility and relative market capitalization weights given in table 2 of page 21 of the lecture notes (lecture 8).

1. Assume an investor has a risk-aversion coefficient $\gamma = 3.5$ and no uncertainty about his estimate of the mean vector μ_0. Compute the expected return vector μ_0 that would have him hold a portfolio equal to the market portfolio with weights w_{eq} given in table 2.

2. Assume another investor with risk-aversion $\gamma = 2$ views returns as $R = \mu + \epsilon$ where $\epsilon \sim N(0, \Sigma)$. He starts with a prior that $\mu \sim N(\mu_0, \tau\Sigma)$, where Σ is the empirical covariance matrix of returns. Suppose that $\tau = 0.03$. Derive his optimal portfolio w_0 and compare how it deviates from the equilibrium market weights w_{eq}.

3. Assume that same investor obtains two additional views on the relative performance of different country returns from two different analysts. The first analyst thinks that Germany will outperform a market value weighted basket of France and UK equities by 4.5%. The investor's confidence in this view is $\Omega_{11} = 0.025 \times \tau$. The second analyst thinks that the canadian equity market will outperform the US market by 2% on average. The investor's confidence in that view is $\Omega_{22} = 0.015 \times \tau$. He considers both signal to be independent as he obtained them from different analysts. Using the Black-Litterman formula, derive the posterior distribution of the mean return $\mu \sim N(\bar{\mu}, \bar{\Omega})$ as a function of the prior and the views. Verify numerically that the two sets of equations for $\bar{\mu}$ and $\bar{\Omega}$ on page 11 of the lecture notes indeed give the same answers.

4. Given his signals the investor sees returns as $R = \mu + \epsilon$ where $\epsilon \sim N(0, \Sigma)$ and $\mu \sim N(\bar{\mu}, \bar{\Omega})$. Derive his optimal unconstrained mean-variance portfolio w^*. Compare it to his prior portfolio w_0 and to the market weights w_{eq}.

5. Show that the optimal portfolio w^* can be decomposed into the prior portfolio and an 'overlay' of view portfolios. That is we can rewrite $w^* = w_0 + \lambda_1 P_1^\top + \lambda_2 P_2^\top$ where P_i denotes the i^th row of the view portfolio matrix P. Find the view-weights λ_1, λ_2.

6. In addition the investor has an absolute view that the Japanese stock market will outperform the equilibrium view. In particular he thinks that the Japanese market equity return will be 5.5%. His uncertainty about the view is $\Omega_{33}/\tau = 0.04$. Derive the new optimal portfolio and the weights on the three views $\lambda_1, \lambda_2, \lambda_3$. Discuss how the portfolio and the weights change as his uncertainty becomes smaller, e.g., $\Omega_{33}/\tau = 0.01$.