Cornell Engineering
Sibley School of Mechanical and Aerospace Engineering

Optimal Cislunar Trajectories with Continuous, High-Thrust Nuclear-Thermal Propulsion

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Outline

- Background & Motivation
- Research Methods
- Case Study Results
- Conclusion

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- Background & Motivation
  - Propulsion and Trajectories
  - Minimum Time Transfers
  - NTP Overview
- Research Methods
- Case Study Results
- Conclusion

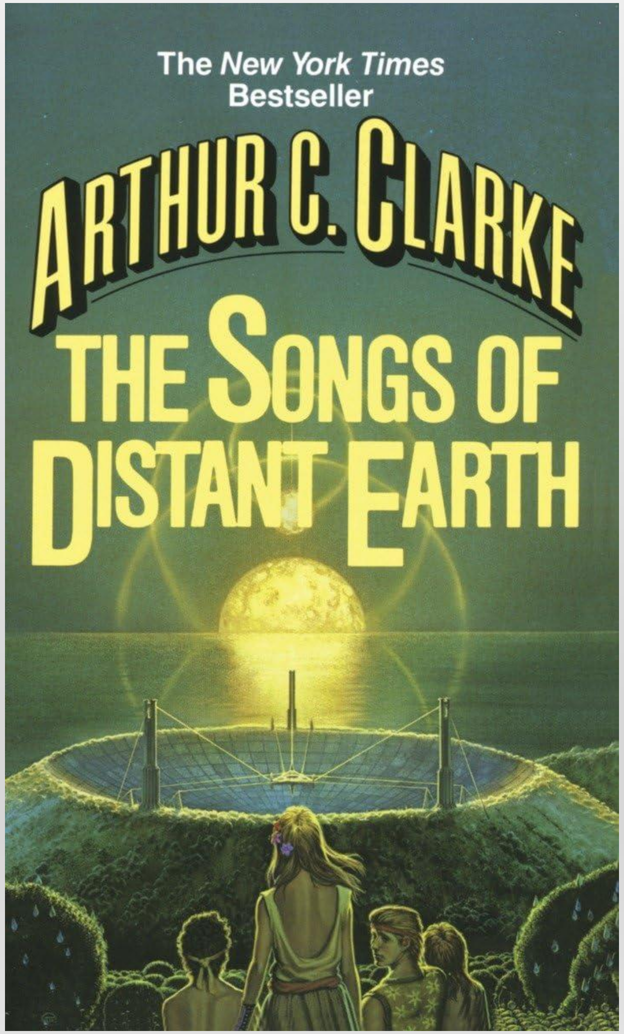
Background

- Propulsion Systems
  - Chemical Propulsion: High Thrust, Low I_{sp}
  - Electric Propulsion: Low Thrust, High I_{sp}

- Cis-lunar Trajectories
  - Lunar Free Return Trajectory
  - Impulsive Maneuvers
  - ΔV efficient trajectories

Background: Minimum Time Trajectories

- "Turn and Burn" Trajectory
  - Minimize the Time-of-Flight Maneuver
  - Satisfies Hamilton-Jacobi-Bellman Time optimality
  - Continuous-High-Thrust Maneuver
  - Not feasible outright with current technologies



[1]

[1] Clarke, Arthur C. *The Songs of Distant Earth.* New York: Del Rey Books, 1986.

Background: NTP Technology

- Nuclear Thermal Propulsion
  - Expand a working fluid (e.g. Hydrogen) using a small fission reactor
  - 800s – 1000s Isp
  - Two large pitfalls
    - Larger Engine Mass
    - Non-negligible ramp times

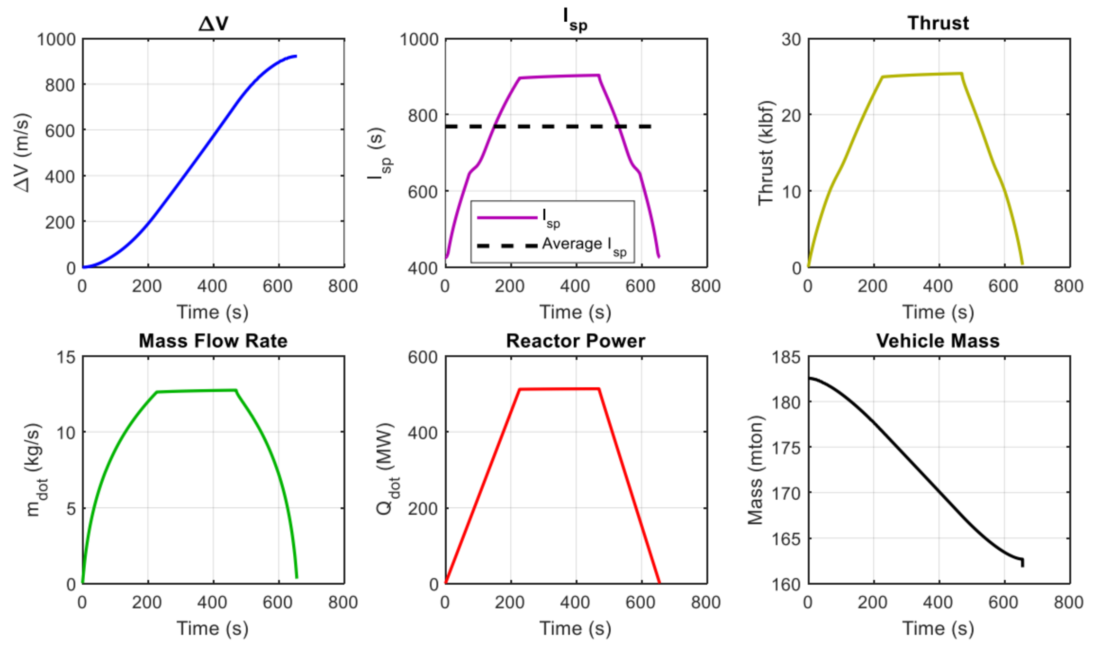
[2]

Specification	NTP	Chemical
Thrust Class	66,700 N	66,700 N
Isp	900 s	451 s
Total Engine Mass	4550 kg	510 kg
Total Engine Length	5.0 m	2.3 m
Total Engine Diameter	1.9 m	1.9 m

[2] Harnack, C., Grella, E., Greenhalge, S., and Duchek, M., "Capabilities of Sinlge Launch NTP and Chemical Spacecrafts for Cis-Lunar Tug Missions," Nuclear and Emerging Technologies for Space, 2023.

Motivation: NTP and Turn and Burn

- No ramp time hinderance on efficiencies
- Continuous Thrust would require less thrust magnitude so the engine would be smaller, reducing mass
  - NASA Artemis System [4]
    - 26.6 kN Thrust (Service Module)
    - ~15,000 kg Mass (Service Module)
  - Cont. Thrust
    - 300-400 N Thrust
    - 20,000 kg Spacecraft
    - 98% Decrease in thrust compared to Impulsive
    - 15-20% Increase in I_{sp} Compared to NTP impulsive



[3]

[3] Duchek, M. E., Nikitaeva, D., Harnack, C., Grella, E., and Greenhalge, S., "Parametric Modeling of NTP Engine Performance for a Crewed Mars Mission," ASCEND 2023, 2023.
[4] Bowman, A., Peters, E., "Orion Components", NASA 2024

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    - Assumptions
    - Dynamics
    - Numerical Methods
- Case Study Results
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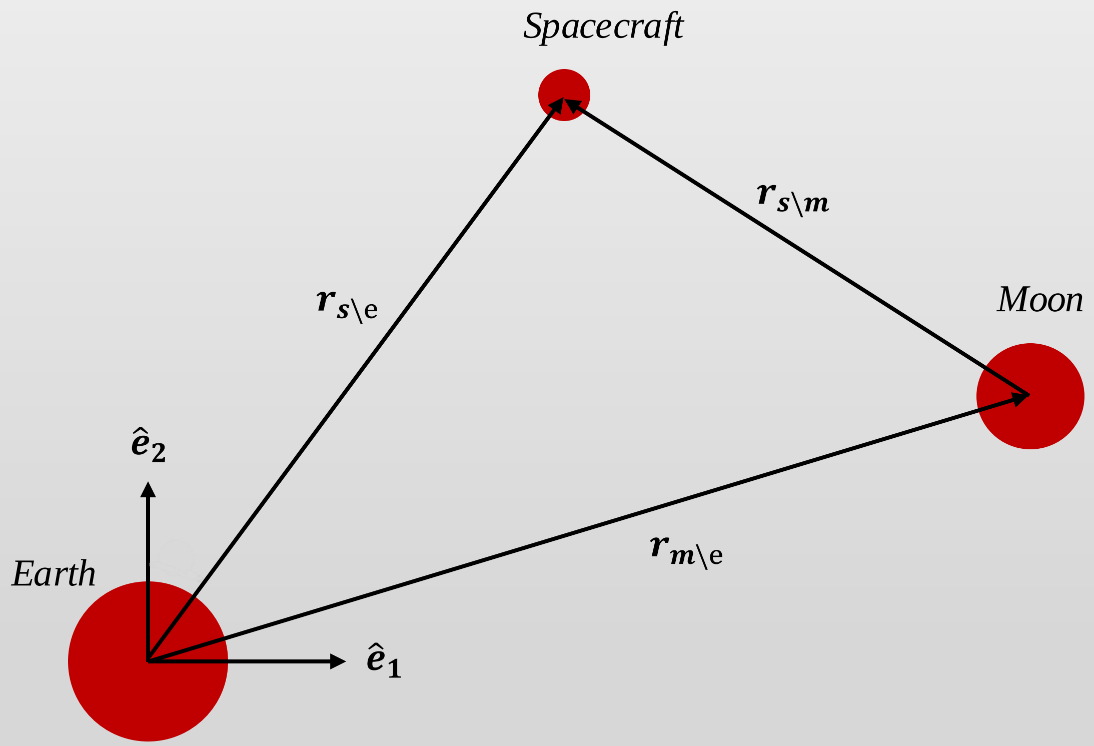
Assumptions

- Spacecraft considered in the Earth-Moon system
  - Spacecraft Parameters
    - Specific Impulse: 1000s
    - Wet Mass: 20,000 kg
    - Propellent Mass: 9,000 kg
  - Orbital Parameters
    - Starting Orbit: GEO
    - Target Lunar Orbit: LLO
    - 1 DU = 384,400 km (a_{moon})



Dynamics

- Gravitational interactions with Earth and Moon
- Ephemeris Model
- Force from propulsion system aligned with velocity vector

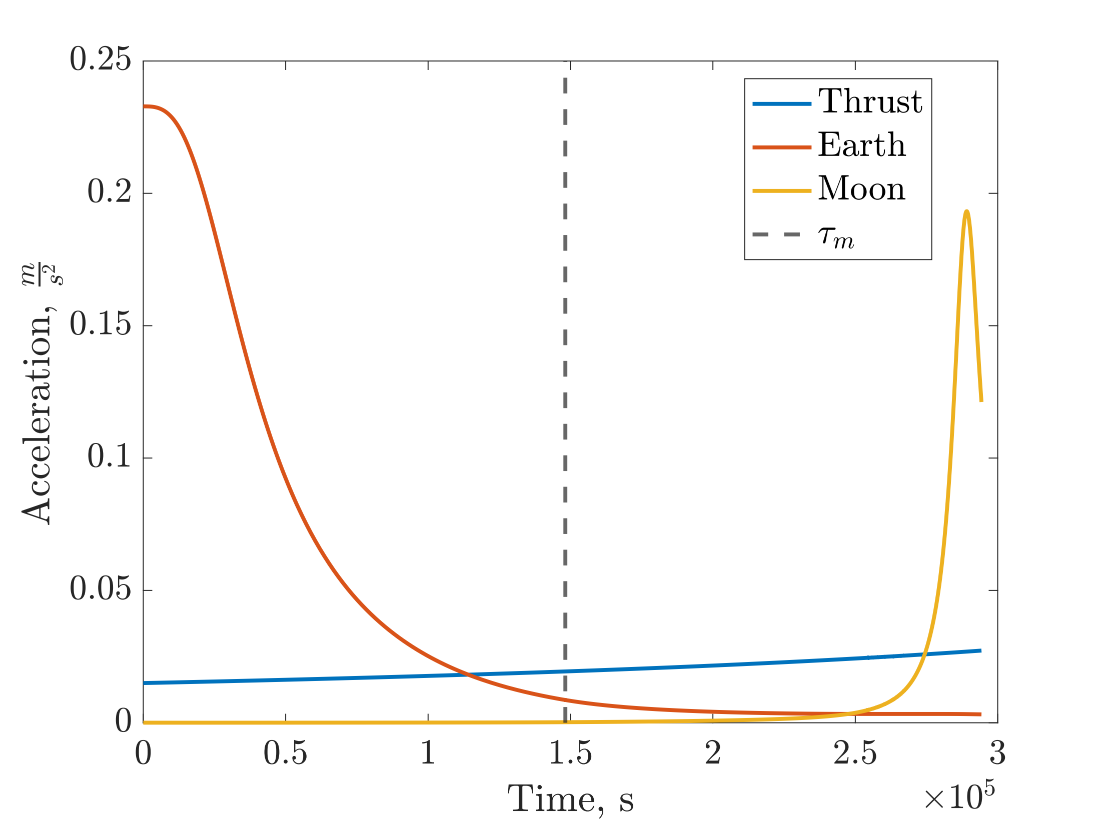
$$\frac{d^2}{dt^2}\boldsymbol{r_{s/e}} = \underbrace{-\frac{\mu_{earth}}{||\boldsymbol{r_{s/e}}||^3}\boldsymbol{r_{s/e}}}_{\text{Earth Dynamics}} \underbrace{-\frac{\mu_{moon}}{||\boldsymbol{r_{s/m}}||^3}\boldsymbol{r_{s/m}}}_{\text{Lunar Dynamics}} + \underbrace{\frac{\boldsymbol{T}}{m}}_{\text{Thrust}}$$

$$\boldsymbol{T} = \begin{cases} T\left(\frac{\boldsymbol{V_{s/e}}}{||\boldsymbol{V_{s/e}}||+0.2}\right) & \text{if } 0 \leq t \leq \tau_m \\ T\left(\frac{-\boldsymbol{V_{s/e}}}{||\boldsymbol{V_{s/e}}||\boxed{+0.2}}\right) & \text{if } \tau_m \leq t \leq \tau \end{cases}$$

For Numerical Stability at Zero Velocity

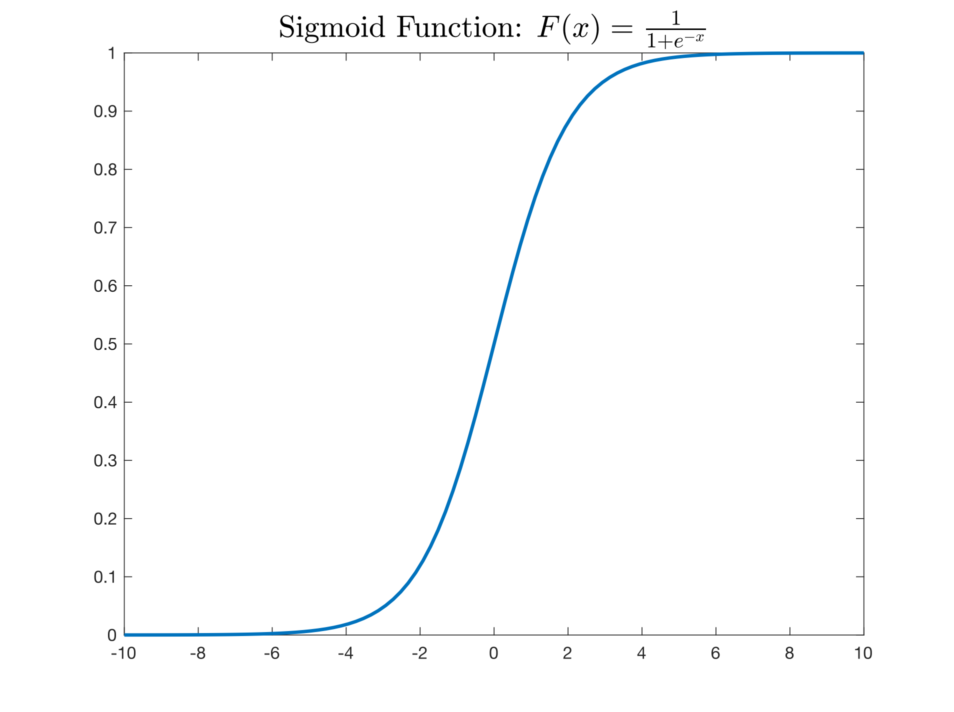
$$m = M_{tot} - \dot{m}t$$

Maneuver Flip time



Numerical Methods

- Adams-Bashforth-Moulton predictor-evaluator-corrector-evaluator solver
  - Max/Min orders of 13 and 1
  - Tolerances of 1 x 10^{-11}
- Nelder-Mead simplex algorithms described in [5] Lagarias et al, 1998
  - Optimality achieved by minimizing the cost function



$$J = \overbrace{c_1\tau^2}^{\text{Integrated Time}} + c_2 \overbrace{\frac{10}{1+\exp[-(19||\boldsymbol{r}_{s/m}|| - r_{LLO})]}}^{\text{Boundary Value Problem: Radial Position}} + c_3 \overbrace{\frac{10}{1+\exp[-(100e_{sc} - 75)]}}^{\text{Boundary Value Problem: Eccentricity}}$$

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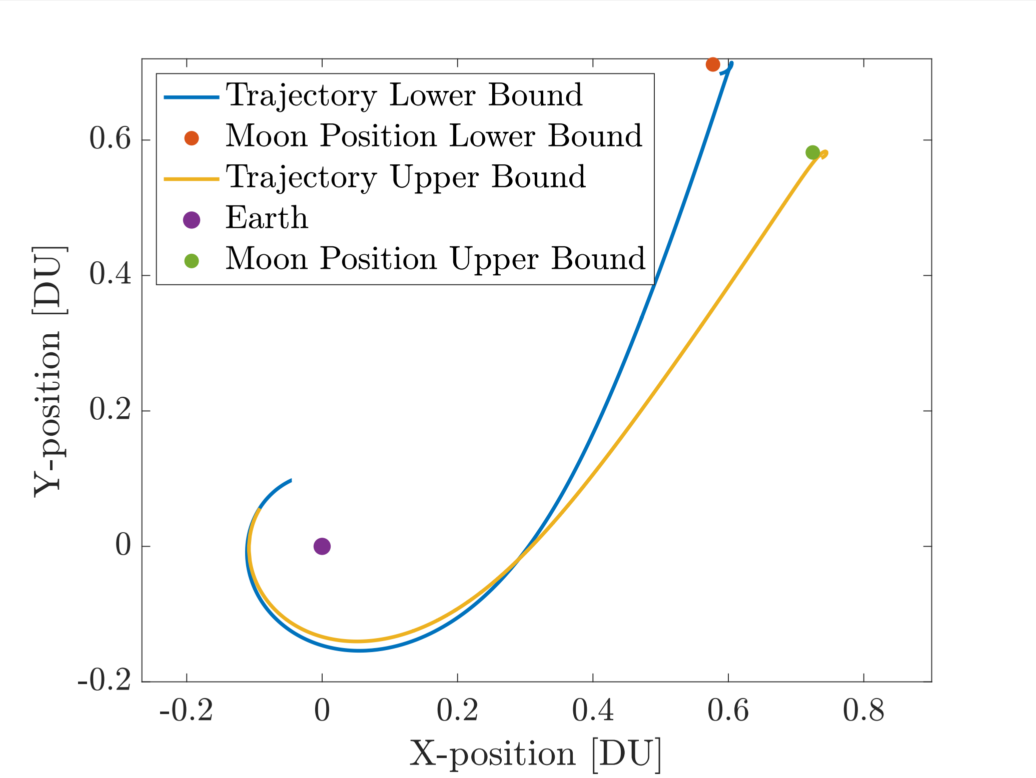
Results

- Case Study I
  - Exhaust Propellent
    - Utilize all the propellent mass
    - 300 N to 410 N test range
  - Parameters of Interest
    - Flight time τ
    - Normalized flip time τ_m/τ
    - Final eccentricity
  - Continuation Analysis
    - Use optimal results from last thrust level as an initial guess for the next

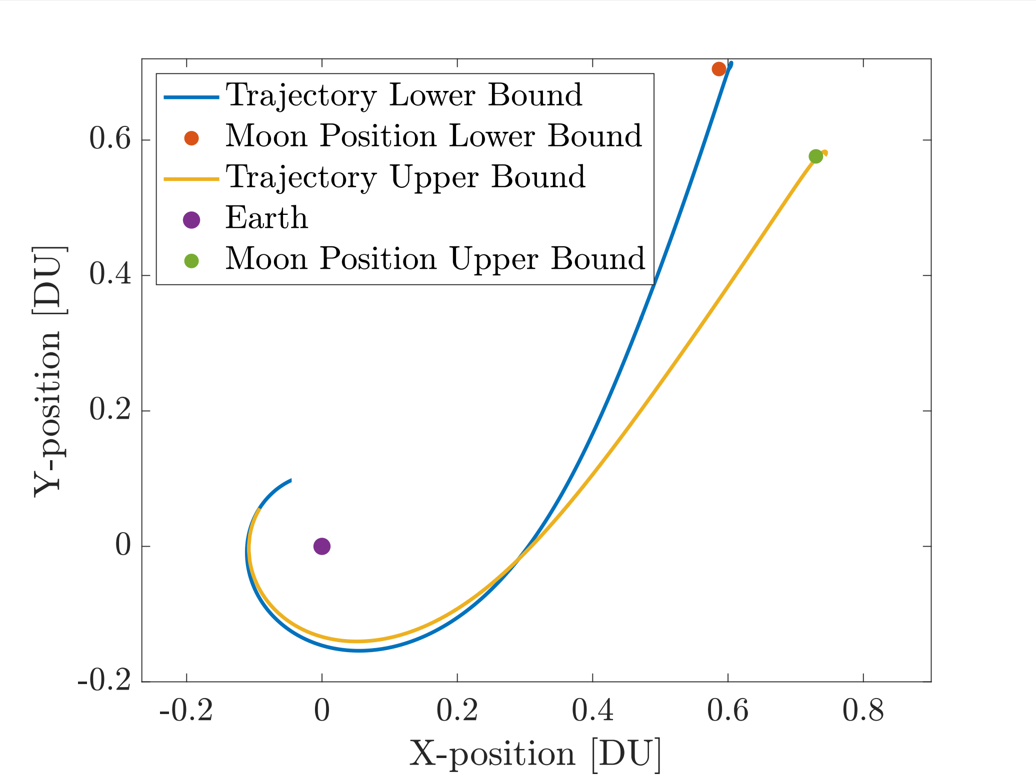
- Case Study II
  - Halt Integration at threshold
    - Halt integration when eccentricity = 0.6
    - 300 N to 410 N test range
    - Eccentricity should be constant
  - Parameters of Interest
    - Flight time τ
    - Normalized flip time τ_m/τ
    - ΔV and Propellent exhausted
  - "Continuation Analysis"
    - Use corresponding result from case study I to inform guess for case study II

Trajectory Bounds

Case Study 1

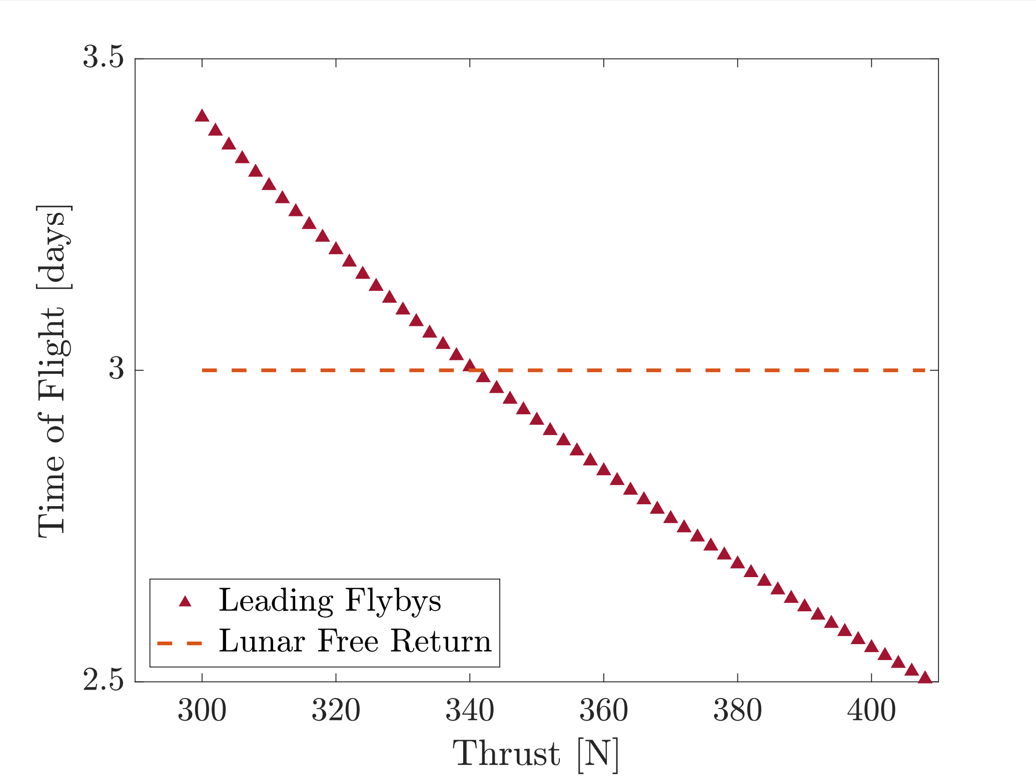


Case Study 2

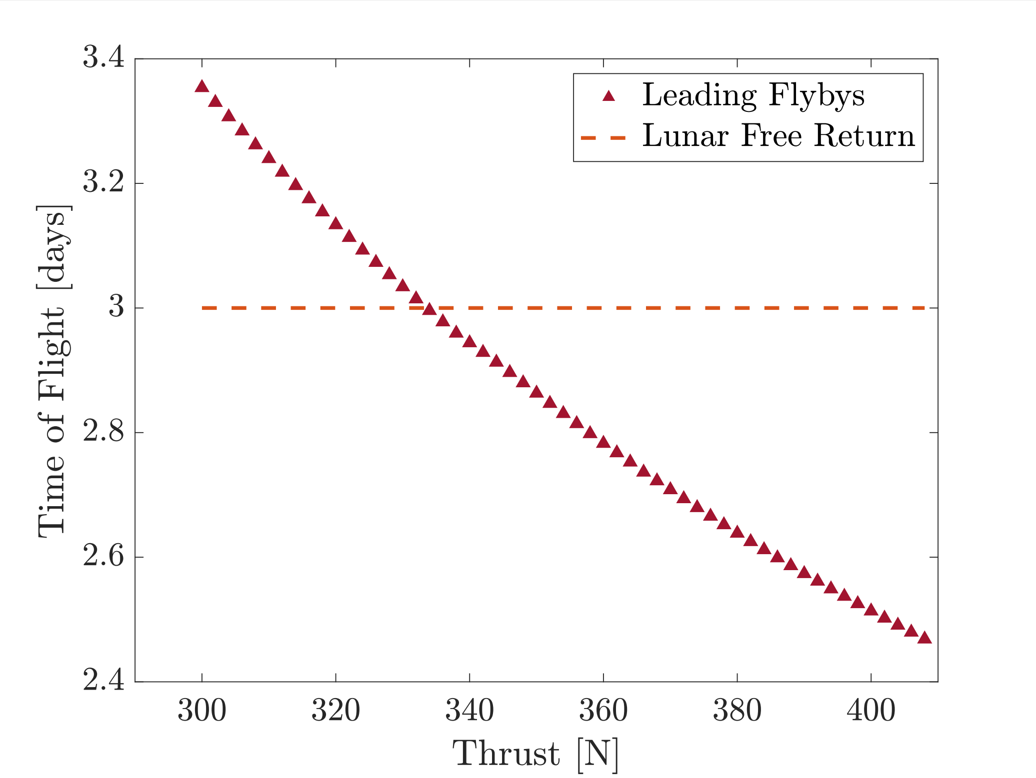


Time of Flight

Case Study 1

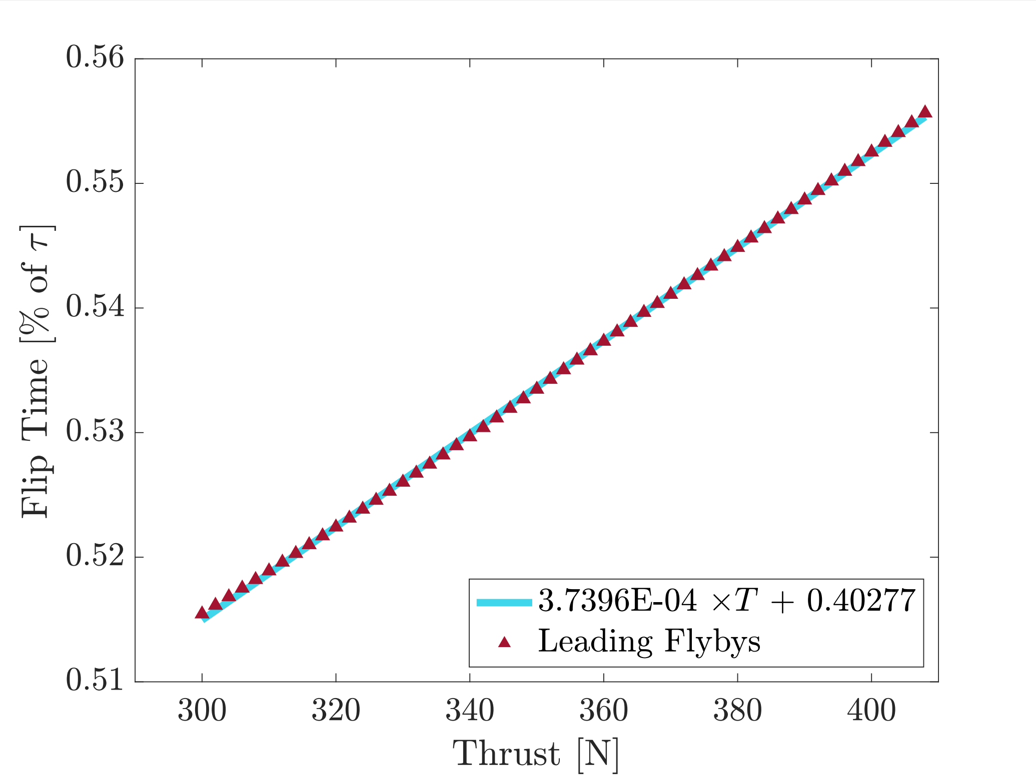


Case Study 2

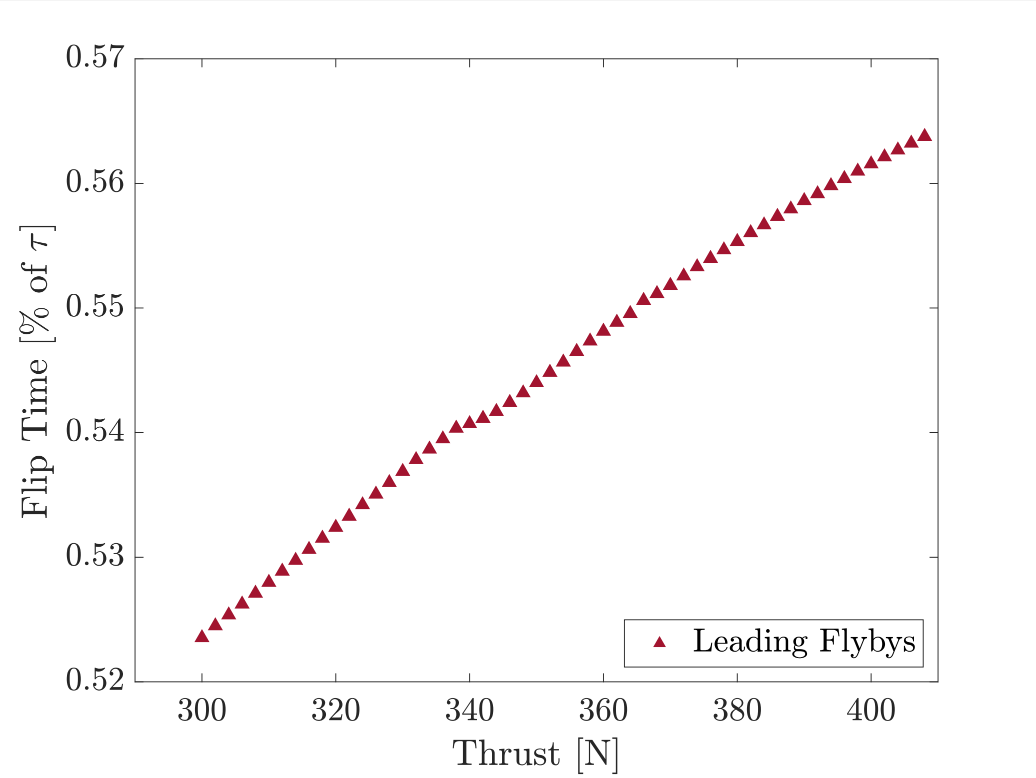


Flip Time Percentage

Case Study 1

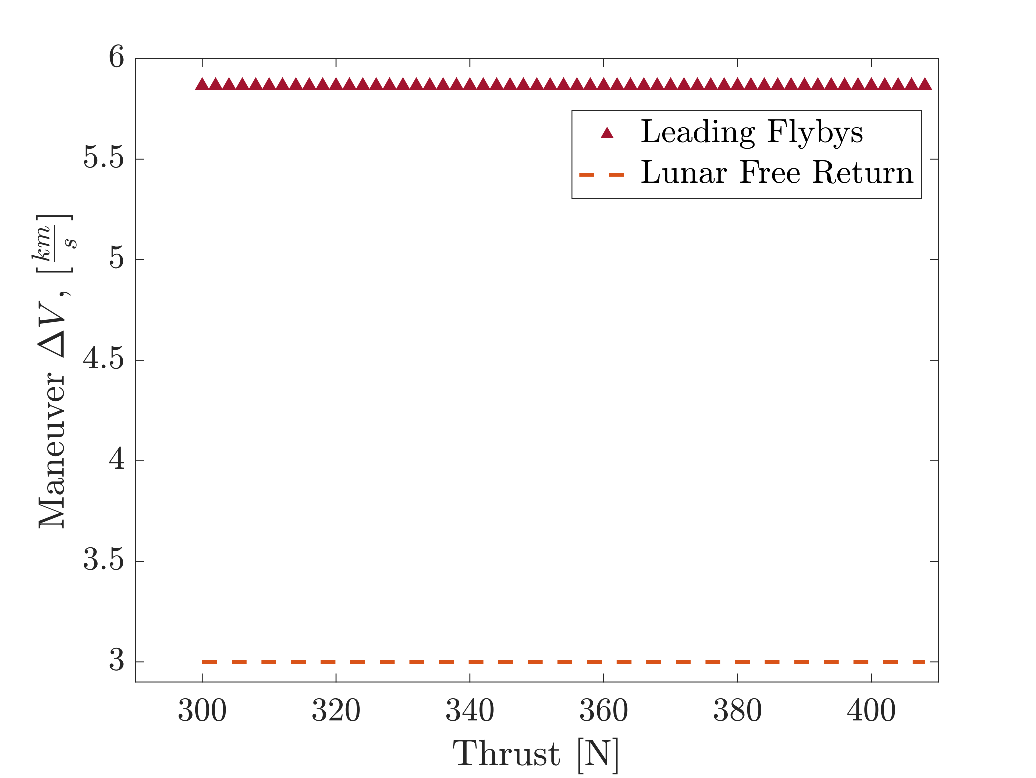


Case Study 2

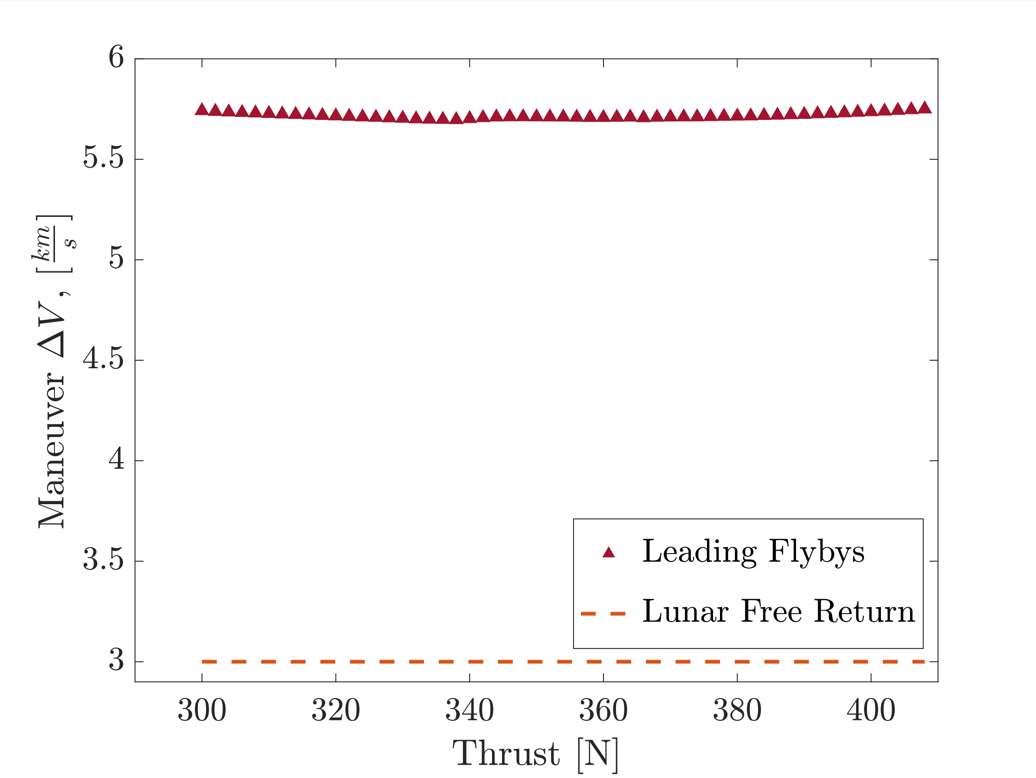


Maneuver ΔV

Case Study 1

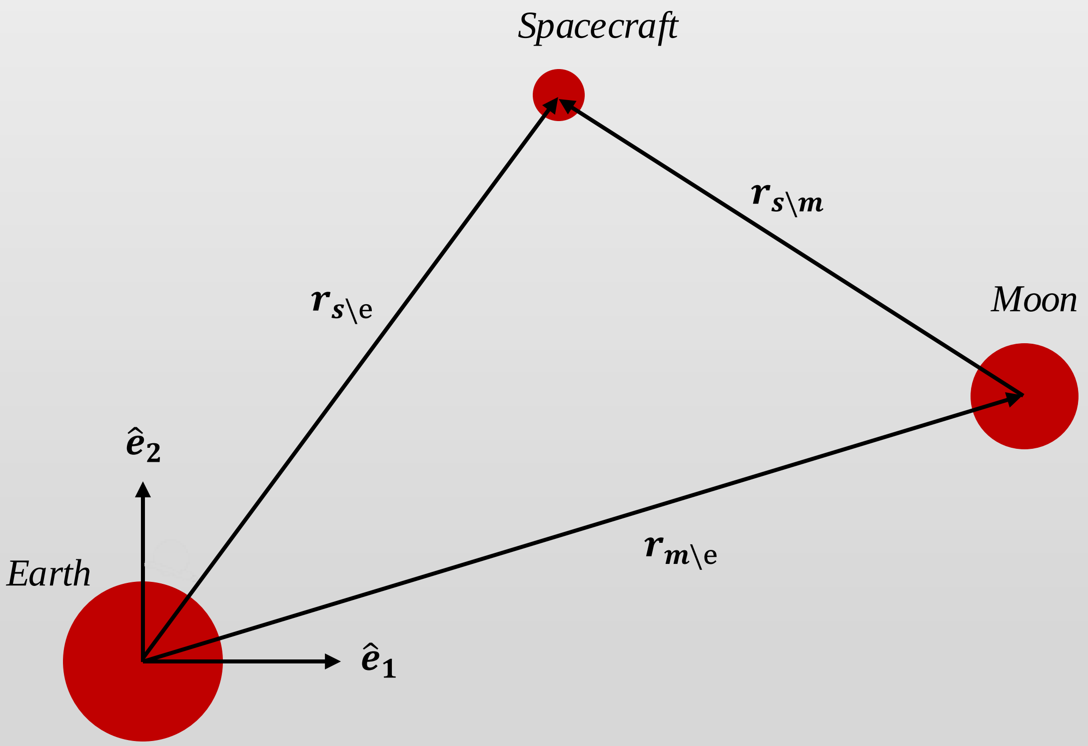


Case Study 2



Results

- Key Results
  - Trajectories
    - Max thrust from tested range achieves shorter TOF than Lunar free return
    - ΔV about twice lunar free return
    - Linear trends locally in $\frac{\tau_m}{\tau}$ for Case Study I
  - Engine Mass
    - Linear Scaling says order 20 kg engine mass
    - Would not really be linear → motivates engine *would* be much smaller



Conclusion

- Turn and Burn offers a new operational paradigm for NTP technology

- The inherent benefits of running the reactor continuously could also include providing more power to the other systems of the spacecraft.

- Order of magnitude smaller NTP engine

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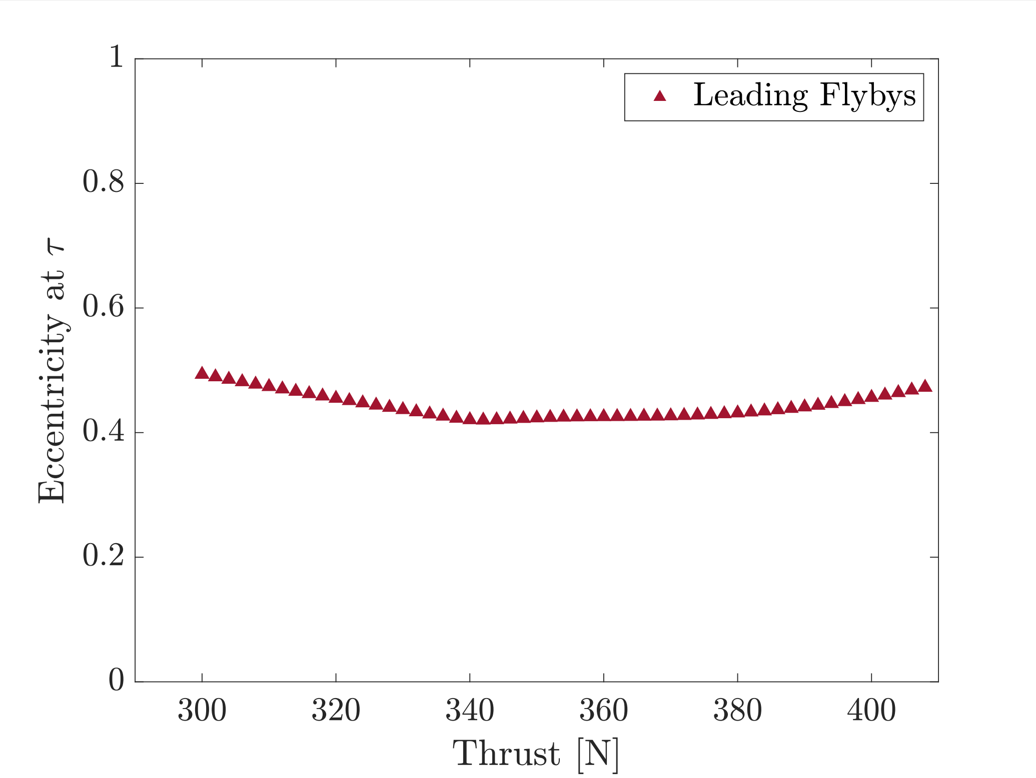
References

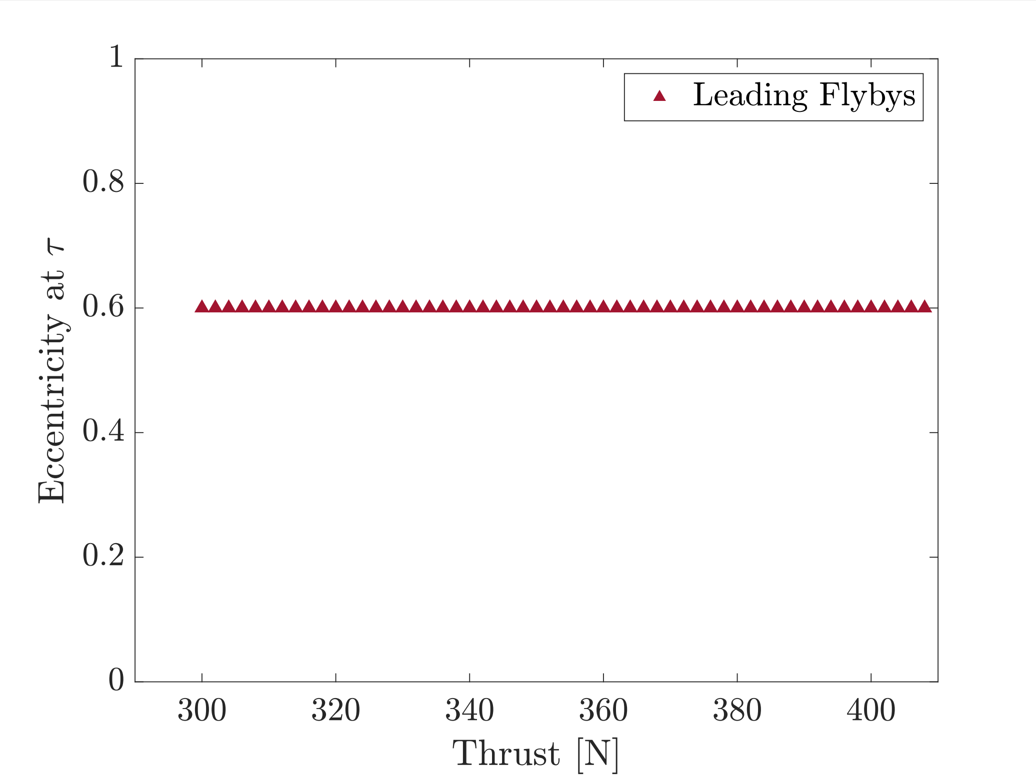
[1] Clark, A. C., 1986
[2] Harnack, C , 2023.
[3] Duchek, M. E., 2023.
[4] Bowman, A., 2024
[5]Lagarias et al, 1998

Final Eccentricity

Case Study 1

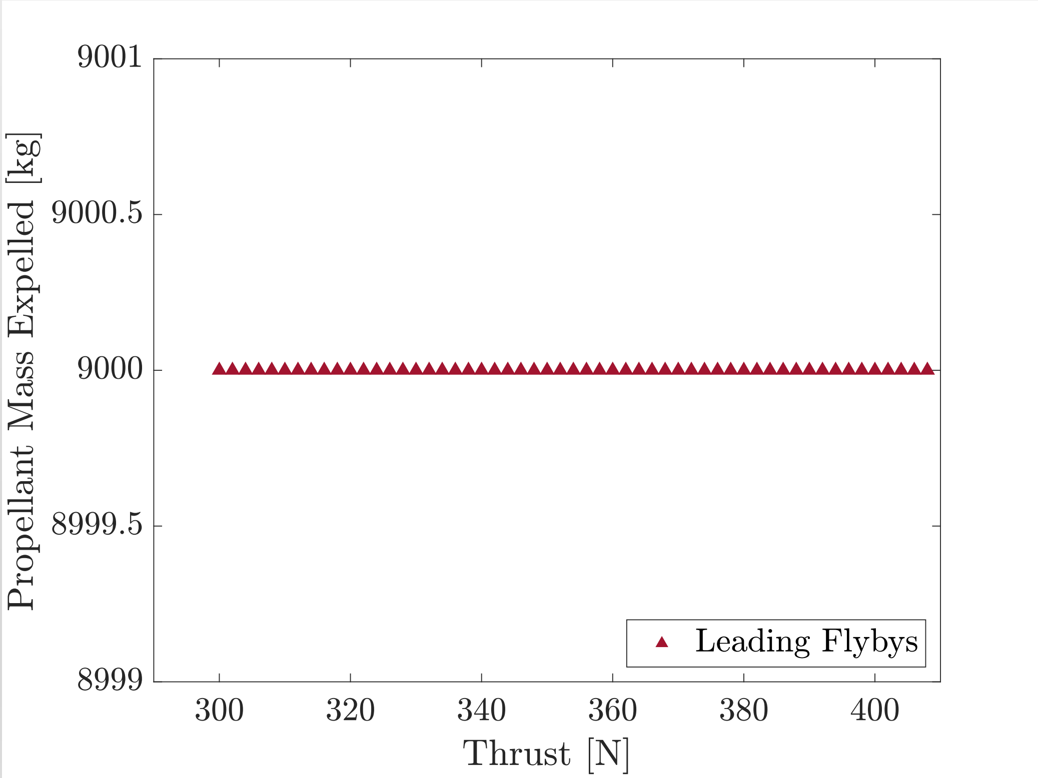


Case Study 2

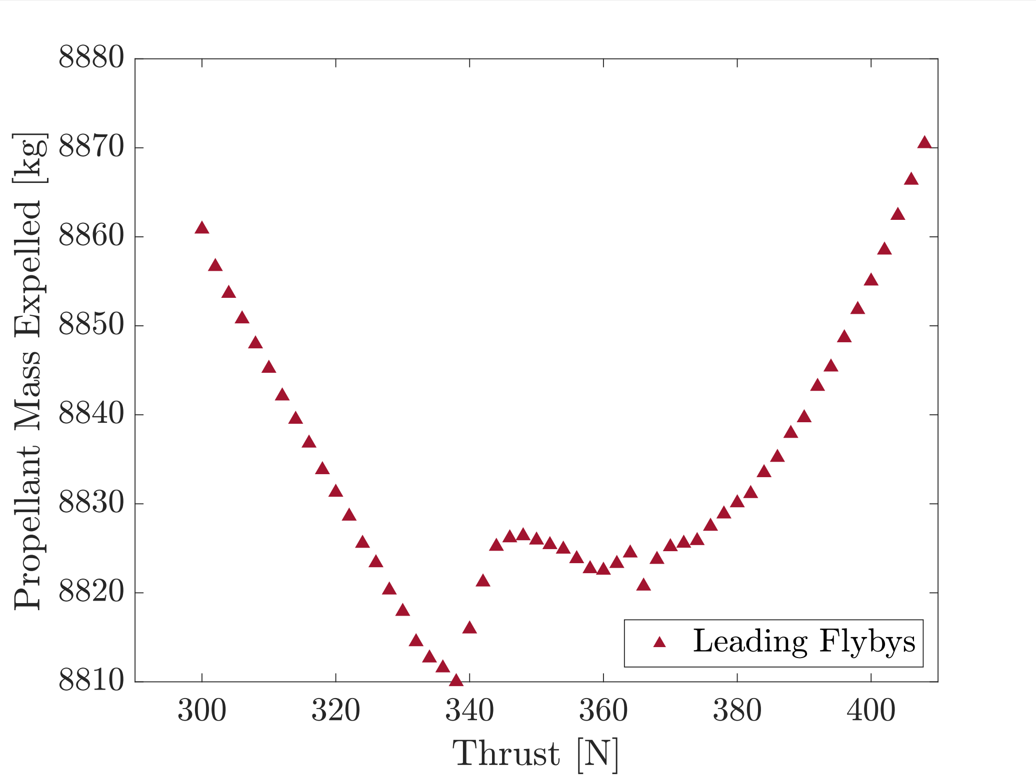


Propellent Mass Consumption

Case Study 1



Case Study 2



Backup

Analytical Expressions for Turn and Burn Maneuver

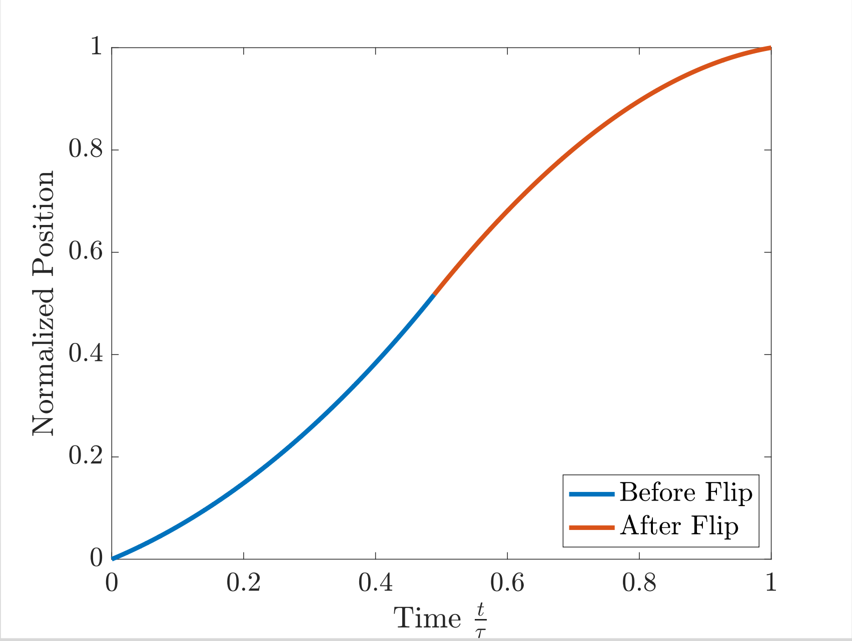
$$a(t) = \begin{cases} \dfrac{\dot{m} g I_{sp}}{M_{tot} - \dot{m} t} & \text{if } 0 \leq t \leq \tau_m \\ -\dfrac{\dot{m} g I_{sp}}{M_{tot} - \dot{m} t} & \text{if } \tau_m \leq t \leq \tau \end{cases}$$

$$\tau_m = \frac{1}{\dot{m}}\left(M_{tot} - \sqrt{({M_{tot}}^2 - M_{tot}\dot{m}\tau)e^{-\frac{V_f - V_0}{g I_{sp}}}}\right)$$

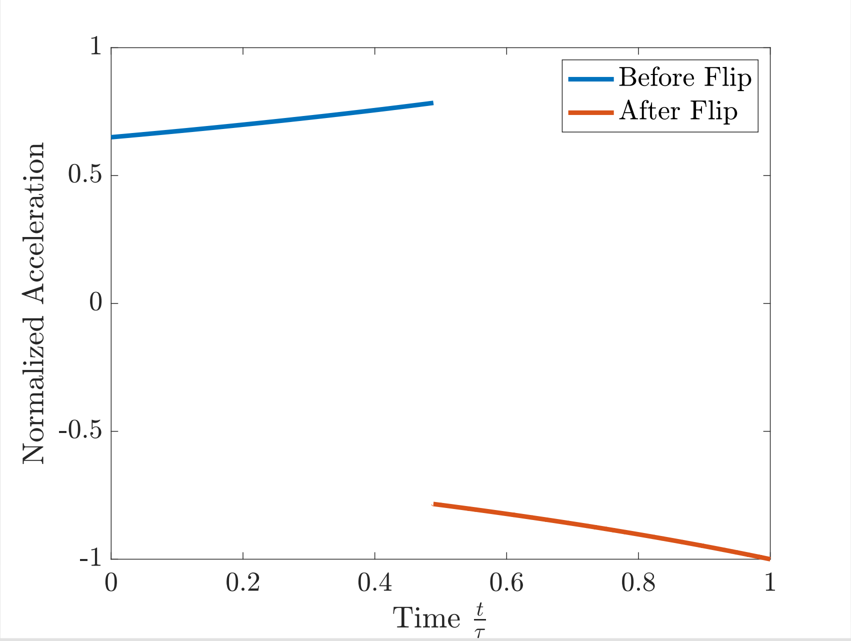
$$v(t) = \begin{cases} I_{sp} g \ln \dfrac{M_{tot}}{M_{tot} - \dot{m} t} + V_0 & \text{if } 0 \leq t \leq \tau_m \\ I_{sp} g \ln \dfrac{M_{tot}(M_{tot} - \dot{m} t)}{(M_{tot} - \dot{m}\tau_m)^2} + V_0 & \text{if } \tau_m \leq t \leq \tau \end{cases}$$

Hur, D., "Turn and Burn Closed Form Report (Cornell University internal document)," , 2023.

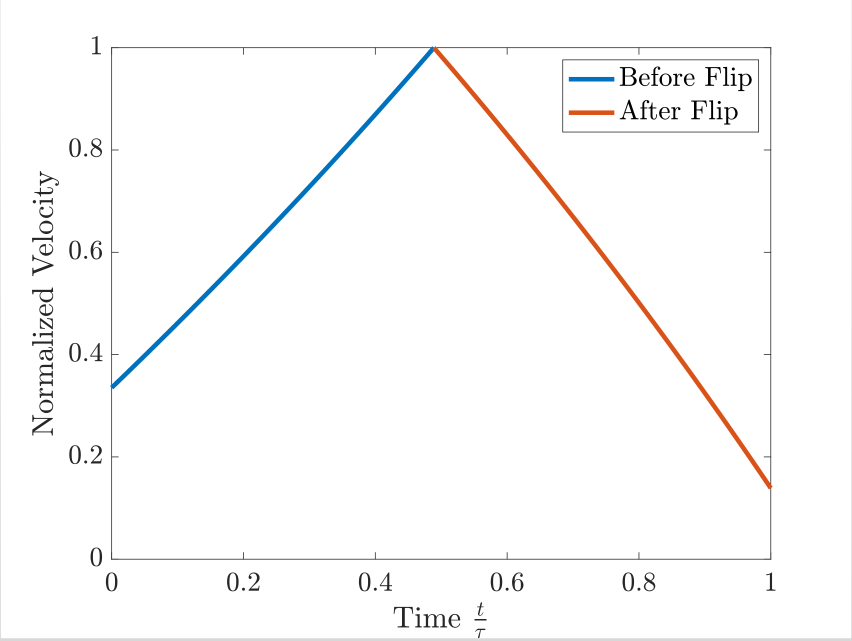
Backup



Normalized Position
Time $\frac{t}{\tau}$
Before Flip
After Flip



Normalized Acceleration
Time $\frac{t}{\tau}$
Before Flip
After Flip



Normalized Velocity
Time $\frac{t}{\tau}$
Before Flip
After Flip

Backup: Engine Mass Scaling

$$F = \dot{m} U_e$$

Constant of Specific Technology

Want half of this

Must also be halved

Constants of these Equations:

- All Stagnation Properties
- Mach Number
- γ

$$\frac{p_e}{p_0} = (1 + \frac{\gamma - 1}{2} M_e^2)^{\frac{-\gamma}{\gamma - 1}}$$

$$\frac{T_e}{T_0} = (1 + \frac{\gamma - 1}{2} M_e^2)^{-1}$$

$$\frac{A_e}{A^*} = \frac{1}{M_e}[\frac{2}{\gamma + 1}(1 + \frac{\gamma - 1}{2} M_e^2)]^{\frac{\gamma+1}{2(\gamma-1)}}$$

RHS of all Equations is constant, given constant stagnation properties, p_e and T_e will be constant as well

Constant Ratio

Backup: Engine Mass Scaling

$$\dot{m} = \sqrt{\gamma}\left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} A^* \frac{p_0}{\sqrt{RT_0}}$$

$$r^* \propto \sqrt{A^*}$$

$$\frac{A^*}{2} \Rightarrow \frac{r^*}{\sqrt{2}}$$

$$\dot{m} \propto A^*$$

Know this is halved

Must also be halved

Hoop Stress will be constant

$$\sigma_h = \frac{p_0 r}{t} \Rightarrow t \propto r^*$$

Backup: Engine Mass Scaling

Cylindrical Shell Volume formula

$$V = 2\pi r(r+t)l - 2\pi r^2 l = 2\pi t l r.$$

With both t and r^* scaling as $\frac{1}{\sqrt{2}}$ the whole volume and therefore mass is simply halved, thus implying a simple linear scaling

Backup

Final Conditions:

- a: 9.05e6 km
- e: 0.472
- Omega: 0 rad
- I: 0 rad
- omega: 0.5281 rad