word2vec (with a vengeance)

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Vector representation of words

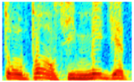
- Image and audio processing work with high-dimensional data

$$\text{encoded as vectors} \begin{cases} \textit{pixel intensities} \\ \textit{power spectra coefficients} \end{cases}$$

- For object and speech recognition, all info is in the raw data
  Traditionally, NLP systems treat words as atomic units

$$\text{arbitrary encodings} \begin{cases} cat \rightarrow \text{“}Id537\text{”} \\ dog \rightarrow \text{“}Id143\text{”} \end{cases}$$

  - No useful info on relationships that may exist between words
  - Representing words as unique, discrete IDs $\rightarrow$ sparsity



Audio Spectrogram

DENSE

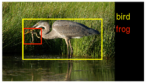
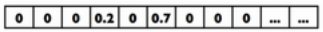


Image pixels

DENSE



Word, context, or document vectors

SPARSE

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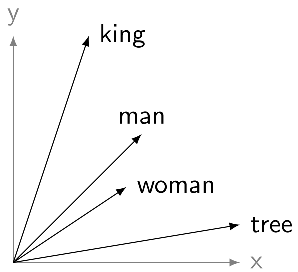
Vector representation of words (a.k.a. word embeddings)

Vector space models represent words in a continuous vector space, where semantically similar words are mapped to nearby points.

Distributional hypothesis:

- words with similar distributions have similar meanings
- words in similar contexts share semantic meaning

1. count-based methods (LSA)
   - compute statistics of neighbors co-occurrences from large text corpus
   - for each word, map statistics to a small and dense vector
2. predictive methods (NPL)
   - predict word from neighbors using learned small and dense vectors
   - embedding vectors are parameters of the model



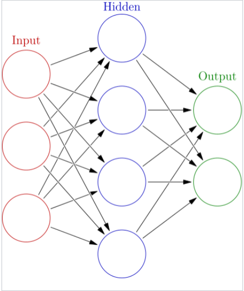
Learning vector representation of words

Language models:

- Feedforward neural network
- Recurrent neural network
- Continuous bag-of-words
- Continuous skip-gram

word2vec

Important architecture in neural network language models
Statistical approach coupled with machine learning



- Continuous bag-of-words (CBOW)
- Skip-gram

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CBOW based architecture

Log linear classifier with input averaged over past and future word vectors. Predicts a missing word from a given context of word sequence.

Example:

- latent Dirichlet **allocation**

Useful for these cases:

- missing word in sentence or long phrase
- meaningful bigrams $\rightarrow$ state – capital
- effective sentiment orientation

Skip-gram based architecture

Log linear classifier.
Predicts missing context of word sequence from a given word

Example:

- **latent** Dirichlet **allocation**

Vector representation of words (a.k.a. word embeddings)

Consider 3 sentences, vocabulary of 5 distinct words, and window size of 2 words.

S1: w1 w2 w3
S2: w2 w4
S3: w1 w5 w4

w1 w2	w2 w3
w2 w4	
w1 w5	w5 w4

	w1	**w2**	**w3**	**w4**	**w5**
w1	0	1	0	0	1
w2	0	0	1	1	0
w3	0	0	0	0	0
w4	0	0	0	0	0
w5	0	0	0	1	0

- window size = k-words
- number of rows = vocabulary size
- number of columns = vector dimensionality

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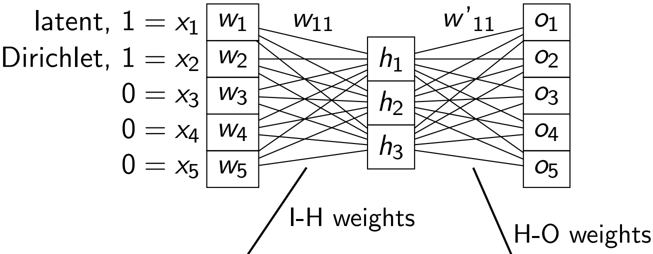
CBOW Example

- latent Dirichlet **allocation**



w_{11}	w_{12}	w_{13}
w_{21}	w_{22}	w_{23}
w_{31}	w_{32}	w_{33}
w_{41}	w_{42}	w_{43}
w_{51}	w_{52}	w_{53}

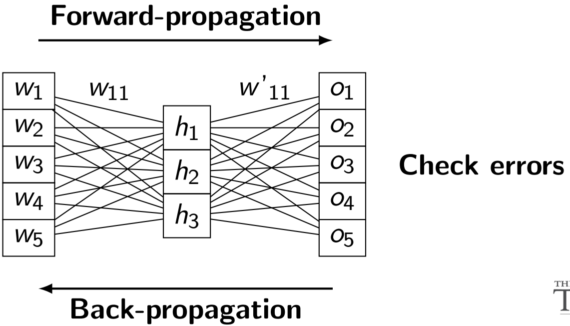
w'_{11}	w'_{12}	w'_{13}	w'_{14}	w'_{15}
w'_{21}	w'_{22}	w'_{23}	w'_{24}	w'_{25}
w'_{31}	w'_{32}	w'_{33}	w'_{34}	w'_{35}

Training CBOW model

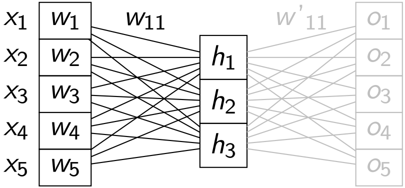
1. Forward-propagation (optimization objective function)
2. Check errors (stochastic gradient descent)
3. Back-propagation

Perform steps 1 – 3 until neuron weights are optimized.

- initially selected from uniform random distribution $[-1, 1]$.



Forward-propagation (input-to-hidden layer)

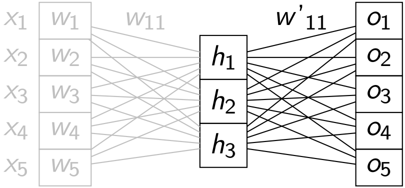


$$\mathbf{h} = \mathbf{W}^{\mathbf{T}}\mathbf{x}$$

$$h_1 = \left(w_{11}x_1 + w_{21}x_2 + \cdots + w_{51}x_5\right)$$
$$h_2 = \left(w_{12}x_1 + w_{22}x_2 + \cdots + w_{52}x_5\right)$$
$$h_3 = \left(w_{13}x_1 + w_{23}x_2 + \cdots + w_{53}x_5\right)$$

$$\begin{array}{|c|c|c|c|c|} \hline w_{11} & w_{21} & w_{31} & w_{41} & w_{51} \\ \hline w_{12} & w_{22} & w_{32} & w_{42} & w_{52} \\ \hline w_{13} & w_{23} & w_{33} & w_{43} & w_{53} \\ \hline \end{array} \quad \begin{array}{|c|} \hline x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline x_4 \\ \hline x_5 \\ \hline \end{array}$$

Forward-propagation (hidden-to-output layer)



$$\mathbf{Net(O)} = \mathbf{u} = \mathbf{W'^{T}h}$$

$$Net(o_1) = u_1 = (w'_{11}h_1 + w'_{21}h_2 + w'_{31}h_3)$$

$$Net(o_2) = u_2 = (w'_{12}h_1 + w'_{22}h_2 + w'_{32}h_3)$$

$$\vdots$$

$$Net(o_5) = u_5 = (w'_{15}h_1 + w'_{25}h_2 + w'_{35}h_3)$$

$$\begin{bmatrix} w'_{11} & w'_{21} & w'_{31} \\ w'_{12} & w'_{22} & w'_{32} \\ w'_{13} & w'_{23} & w'_{33} \\ w'_{14} & w'_{24} & w'_{34} \\ w'_{15} & w'_{25} & w'_{35} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

Forward-propagation (softmax classifier)

$$Out(o_1) = y_1 = \frac{e^{u_1}}{e^{u_1} + e^{u_2} + \cdots + e^{u_5}}$$

$$Out(o_2) = y_2 = \frac{e^{u_2}}{e^{u_1} + e^{u_2} + \cdots + e^{u_5}}$$

$$\vdots$$

$$Out(o_5) = y_5 = \frac{e^{u_5}}{e^{u_1} + e^{u_2} + \cdots + e^{u_5}}$$

Softmax output for word w_j w.r.t. context w_I $\rightarrow$ conditional probability

$$\mathbf{P(w_j|w_I) = y_j = \frac{e^{u_j}}{\sum_{j'=1}^{V} e^{u_{j'}}}}$$

Check errors

Assume:

- w_o = output word
- w_I = context words
- V = size of input context

$$max\ P(w_o|w_I) = max(y_{j^*}) = max(log(y_{j^*}))$$

where j^* = index of output word

Example:

$$E(o_4) = log(P(w_{o_4}|w_I)) = log_e\left(\frac{e^{u_j}}{\sum_{j'=1}^{V} e^{u_{j'}}}\right)$$

$$= u_4 - log_e\left(\sum_{j'=1}^{V} e^{u_{j'}}\right)$$

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Check errors

To minimize errors, $E = -log(P(w_o|w_I))$

$$E = log\left(\sum_{j'=1}^{V} e^{u_{j'}}\right) - u_{j^*}$$

Derivate of E w.r.t. u_4,

$$\frac{dE(o_4)}{d(u_4)} = Out(o_4) - \frac{du_4}{du_4} = y_4 - 1$$

$$\frac{dE}{dj} = y_j - t_j = e_j \begin{cases} t_j = 1, \ if \ t_j = t_{j^*} \\ t_j = 0, \ otherwise \end{cases}$$

Backward-propagation (output-to-hidden layer)

Take gradient of E, w.r.t. w'_{11}

$$\frac{dE(o_1)}{dw'_{11}} = \frac{dE(o_1)}{du_1} \times \frac{du_1}{dw'_{11}}$$

$$\frac{dE(o_1)}{du_1} = y_1 - 0 = e_1$$

$$\frac{du_1}{dw'_{11}} = \frac{d(w'_{11}h_1 + w'_{21}h_2 + w'_{31}h_3)}{dw'_{11}} = h_1$$

$$\mathbf{\frac{dE(o_1)}{dw'_{11}} = e_1 \times h_1}$$

Update weight of w'_{11}:

$$(\mathbf{w'_{11}})^{\mathbf{new}} = \mathbf{w'_{11}} - \eta(\mathbf{e_1} \times \mathbf{h_1})$$

$\eta = [0, 1] \rightarrow$ learning rate

Need to update all neurons w'_{ij}

Backward-propagation (hidden-to-input layer)

Take gradient of E, w.r.t. w_{11}

$$\frac{dE}{dw_{11}} = \frac{dE}{dh_1} \times \frac{dh_1}{dw_{11}}$$

$$\frac{dE}{dh_1} = \left(\frac{dE}{du_1} \times \frac{du_1}{dh_1}\right) + \cdots + \left(\frac{dE}{du_5} \times \frac{du_5}{dh_1}\right) = (e_1 w'_{11}) + \cdots + (e_5 w'_{15})$$

$$\frac{dh_1}{dw_{11}} = \frac{d(w_{11}x_1 + w_{21}x_2 + \cdots + w_{31}x_3)}{dw_{11}}$$

$$\mathbf{\frac{dE}{dw_{11}} = x_1 \frac{dE}{dh_1}}$$

Update weight of w_{11}:

$$(\mathbf{w_{11}})^{\mathbf{new}} = \mathbf{w_{11}} - \eta \mathbf{\frac{dE}{dw_{11}}}$$

$\eta = [0, 1] \rightarrow$ learning rate
Need to update all neurons w_{ij}

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Training optimizations

Problem: neural network is huge

- two weight matrices (hidden and output layers)
- $10K$ words $\times$ 300 size embedding vectors $= \frac{3\ G\ weights}{matrix}$
- need large amount of training data to tune weights
- training and gradient descent are slow on such NN

Training optimizations

Solutions:

- remove words that occur less than some minimum
- treat common pairs or phrases as single "words" - Boston Globe
- subsampling frequent words to decrease number of training examples

$$P(w_i) = \left(\sqrt{\frac{z(w_i)}{0.001}} + 1\right) \times \frac{0.001}{z(w_i)}$$

w_i is a word, $z(w_i)$ is w_i frequency in corpus, $P(w_i)$ is the probability of keeping w_i

Training optimizations

Solutions:

- modify the optimization objective function
  - negative sampling - Each training sample updates some weights. Negative samples are chosen from a "unigram distribution". The probability for selecting a word is related to its frequency.

$$P(w_i) = \frac{f(w_i)^{3/4}}{\sum_{j=0}^{n} f(w_j)^{3/4}}$$

  - hierarchical softmax layers - reduce output layer to $log_2|V|$

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THE END