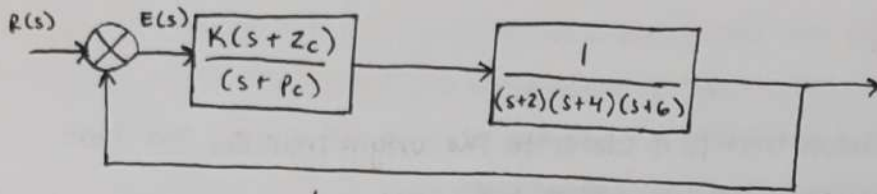


(Q.4) For the unity-feedback system of Figure P9.1, let $G(s) = \frac{K}{(s+2)(s+4)(s+6)}$

(a) Design a compensator that will not significantly change the position of the uncompensated dominant poles that will result in 10% overshoot but yields $K_p = 20$.

model of lag compensator:



$$\text{thus, } G(s) = \frac{1}{(s+2)(s+4)(s+6)} = \frac{1}{(s^2+6s+8)(s+6)}$$

$$G(s) = \frac{1}{s^3 + 6s^2 + 6s^2 + 36s + 8s + 48}$$

$$G(s) = \frac{1}{s^3 + 12s^2 + 44s + 48}$$

solving for 10% overshoot:

$$\xi = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = \frac{-\ln(0.1)}{\sqrt{\pi^2 + \ln^2(0.1)}}$$

$$\xi = .5911 \leftarrow \text{damping ratio}$$

angle of damping ratio: (from real positive axis)

$$\theta = 180^\circ - \cos^{-1}(\xi) = 180^\circ - \cos^{-1}(.5911)$$

$$\theta = 180^\circ - 53.76^\circ = 126.235^\circ$$

Find the value of K using the magnitude criterion:

$$K = \frac{1}{|G(s)|} = \frac{1}{|s^3 + 12s^2 + 44s + 48|}$$

we need to find the dominant poles, which can be done accurately with the use of MATLAB. with our MATLAB calculation, we see that the location of the dominant poles are:

$$s = -2.028 \pm 2.768j$$

$$\text{consequently, } K = |s^3 + 12s^2 + 44s + 48|_{s=-2.028 \pm 2.768j}$$

$$K = 45.546$$

(9.4a)

Edreese Basharyar

Solving for $K_{po} = \lim_{s \rightarrow 0} G(s)$:

$$\lim_{s \rightarrow 0} G(s) = \lim_{s \rightarrow 0} \frac{K}{(s+2)(s+4)(s+6)} = \frac{45.546}{48}$$

$$K_{po} = .948875$$

consequently, $\frac{z_c}{p_c} = \frac{K_{pn}}{K_{po}}$, where $K_{pn} = 20$

Since we are creating a lag compensator, we know that p_c is closer to the origin than z_c , and that $z_c \gg p_c$. So, if we create an arbitrary pole value where $p_c = 0.01$:

$$\frac{z_c}{0.01} = \frac{20}{.948875} \rightarrow z_c = \frac{20(0.01)}{.948875} = .21077$$

$$\begin{aligned} \bullet p_c &= 0.01 \\ \bullet z_c &= .21077 \end{aligned} \rightarrow G_c(s) = \frac{(s + .21077)}{(s + 0.01)}$$

therefore,

$$G_{LC}(s) = G(s) \cdot G_c(s) = \frac{K(s + .21077)}{(s + 0.01)(s + 2)(s + 4)(s + 6)}$$

$$G_{LC}(s) = \frac{K(s + .21077)}{(s + 0.01)(s^3 + 12s^2 + 44s + 48)} = \frac{K(s + .21077)}{s^4 + 12.01s^3 + 44.12s^2 + 48.44s + .48}$$

we can find the location of the dominant poles by using MATLAB software, as it is the most accurate way. With our MATLAB calculation, we see that the dominant poles are

$$s = -1.986 \pm 2.71j$$

now, to solve for the value of K :

$$K = \frac{1}{|G_{LC}(s)|} = \left| \frac{s^4 + 12.01s^3 + 44.12s^2 + 48.44s + .48}{s + .21077} \right|_{s = -1.986 \pm 2.71j}$$

$$K = 49.3$$

solving for $K_{po} = \lim_{s \rightarrow 0} G_{LC}(s)$

$$\lim_{s \rightarrow 0} G_{LC}(s) = \lim_{s \rightarrow 0} \frac{K(s + .21077)}{(s + 0.01)(s + 2)(s + 4)(s + 6)} = \frac{10.3906}{.48} = 21.647$$

if we review the characteristics between our uncompensated and lag-compensated systems

uncompensated

$$K = 45.546$$

$$K_p = .948875$$

$$e(\infty) = 1/.948875$$

$$e(\infty) = .513116$$

second-order poles

$$@ s = -2.028 \pm 2.765j$$

lag-compensated

$$K = 49.3$$

$$K_p = 21.647$$

$$e(\infty) = 1/21.647$$

$$e(\infty) = .044156$$

second order poles

$$@ s = -1.986 \pm 2.71j$$

★ based on these observations,

we can see that our lag compensator design has satisfied the conditions that were needed, such as yielding a $K_p = 20$ while not significantly changing the location of the dominant second order poles. Our steady state error has also improved by a lot and is almost zero.

EE 370 FINAL PROJECT

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(4.12) A unity feedback control system has the following forward transfer function: $G(s) = \frac{K}{s^2(s+4)(s+12)}$

(a) Design a lead compensator to yield a closed-loop step response with 20.5% overshoot and a settling time of 3 seconds. Be sure to specify the value of K .

damping ratio:

$$\xi = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = \frac{-\ln(.205)}{\sqrt{\pi^2 + \ln^2(.205)}} = .45038 = \xi$$

real part of lead compensated dominant second-order poles:

$$\sigma = 4/T_s = 4/3 = 1.333$$

imaginary part of lead compensated dominant second-order poles:

$$\omega_d = \sigma \tan(180^\circ - \cos^{-1}(\xi)) = 1.333 \tan(180^\circ - \cos^{-1}(.45038))$$

$$\omega_d = 2.6426$$

location of lead compensated dominant second-order poles: $(-\sigma \pm \omega_d j)$

$$s = -1.333 \pm 2.6426j$$

definition of lead compensator: $\frac{s+z_c}{s+p_c}$, $p_c \gg z_c$. assume $z_c = 0.01$

using angle criterion to find lead compensator pole:

$$-\angle s + p_c \big|_{s=-1.333+2.6426j} = -180^\circ + \angle \frac{s^2(s+4)(s+12)}{(s+0.01)} \bigg|_{s=-1.333+2.6426j}$$

$$-\tan^{-1}\left(\frac{2.6426}{p_c - 1.333}\right) = -184.44^\circ$$

$$\frac{2.6426}{p_c - 1.333} = 0.0776$$

$$p_c - 1.333$$

$$p_c - 1.333 = 34.054$$

$$p_c = 35.388$$

lead compensator with compensator pole and zero:

$$\frac{(s+0.01)}{(s+35.388)}, p_c \gg z_c, \text{ and } z_c \text{ is very close to origin.}$$

EE370 FINAL PROJECT

Edreese Basharyar

(9.12) transfer function with lead compensator:

$$G_{LC}(s) = \frac{K(s+0.01)}{s^2(s+4)(s+12)(s+35.388)}$$

magnitude criterion to solve for value of gain, K :

$$K = \frac{1}{|G_{LC}(s)|} = \left| \frac{s^2(s+4)(s+12)(s+35.388)}{(s+0.01)} \right|_{s=-1.333+2.6426j}$$

$$K = 4173.66$$

transfer function with lead compensator and $K = 4173.66$

$$G_{LC}(s) = \frac{4173.66(s+0.01)}{s^2(s+4)(s+12)(s+35.388)}$$

(b) Is your second-order approximation valid?

We can determine the validity by finding all poles of the closed-loop transfer function. using the characteristic equation, $1 + KG(s) = 0$:

$$1 + \frac{4173.66(s+0.01)}{s^2(s+4)(s+12)(s+35.388)} = 0 \rightarrow s^2(s+4)(s+12)(s+35.388) + 4173.66(s+0.01) = 0$$

$$s^2(s+4)(s+12)(s+35.388)$$

using MATLAB, the roots of the closed-loop transfer function are:

$$\begin{aligned} s_1 &= -0.01 & \cdot \text{since the value of the higher order poles is more than } 5 \times \\ s_2 &= -1.33 \pm 2.65j & \text{the value of the real part of the dominant poles, our second} \\ s_3 &= -13.488 & \text{order approximation is valid.} \\ s_4 &= -35.2247 \end{aligned}$$

(c) Use MATLAB or any other computer program to simulate and compare the transient response of the compensated system to the predicted transient response.

We need the closed-loop transfer function of the uncompensated and compensated system, and the values of K at their dominant poles.

$$K = \frac{1}{|G(s)|} = \left| \frac{s^2(s+4)(s+12)}{s} \right|_{s=-1.333 \pm 2.6426j}$$

$$K = 361.445$$

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Edreese Bashanyar

(9.15) Given the unity feedback system of Figure P9.1, with $G(s) = \frac{K}{(s+2)(s+4)(s+6)(s+8)}$

Find the transfer function of a lag-lead compensator that will yield a settling time 0.5 second shorter than that of the uncompensated system. The compensated system will also have a damping ratio of 0.5, and improve the steady-state error by a factor of 30. The compensator zero is -5 ; also, find the compensated system's gain. Justify any 2nd order approximations or verify the design through simulation.

settling time of uncompensated system: $T_s = 4/\zeta\omega_n$

dominant poles of uncompensated system (MATLAB):

$$s = -1.531 \pm 2.652j$$

$$T_s = 4/1.531 = 2.613s$$

settling time of compensated system = $2.613 - .50 = 2.113s$

real part of compensated second-order dominant poles:

$$\sigma = 4/T_s = 4/2.113 = 1.893$$

imaginary part of compensated second-order dominant poles:

$$\omega_d = 10 \tan(180^\circ - \cos^{-1}(\zeta)) = 1.893 \cdot \tan(180^\circ - \cos^{-1}(0.5))$$

$$\omega_d = -3.278j$$

location of second-order dominant poles (compensated):

$$s = -1.893 \pm 3.278j$$

magnitude criterion to solve for value of gain, K (uncompensated):

$$K = \frac{1}{|G(s)|} = \frac{1}{|(s+2)(s+4)(s+6)(s+8)|} \Big|_{s=-1.531 \pm 2.652j}$$

$$K = 354.14$$

angle criterion of compensated dominant poles to solve for lead-compensator pole (assume $z_c = -5$)

$$180^\circ - (\angle(s+2)(s+4)(s+6)(s+8) \Big|_{s=-1.893+3.278j} + \angle(s+p) \Big|_{s=-1.893+3.278j}) + \angle(s+5) \Big|_{s=-1.893+3.278j}$$

$$\tan^{-1}\left(\frac{3.278}{p_c - 1.893}\right) = 14.42^\circ$$

now, we have defined our lead compensator as:

$$\frac{3.278}{p_c - 1.893} = 0.257$$

$$\frac{(s+5)}{(s+14.64)}, \quad p_c \gg z_c, \quad z_c \text{ is closer to origin.}$$

$$12.75 = p_c - 1.893$$

$$p_c = 14.64$$

(9.15) transfer function with lead compensator:

$$G_{lc}(s) = \frac{K(s+5)}{(s+2)(s+4)(s+6)(s+8)(s+14.64)}$$

magnitude criterion to solve for value of gain, K (lead-compensated):

$$K = \frac{1}{|G_{lc}(s)|} = \left| \frac{(s+2)(s+4)(s+6)(s+8)(s+14.64)}{(s+5)} \right|_{s=-1.893+3.278j}$$

$$K = 1359.3$$

solving for lag-compensator by finding K_{pu} :

$$K_{pu} = \lim_{s \rightarrow 0} G(s) = \lim_{s \rightarrow 0} \frac{354.44}{(s+2)(s+4)(s+6)(s+8)} = \frac{354.44}{(2)(4)(6)(8)} = .9238$$

$$e(\infty) = \frac{1}{1+K_{pu}} = \frac{1}{1.9238} = .5186$$

solving for lag compensator by finding K_{pn} :

$$K_{pn} = \lim_{s \rightarrow 0} G_{lc}(s) = \lim_{s \rightarrow 0} \frac{1359.3(s+5)}{(s+2)(s+4)(s+6)(s+8)(s+14.64)} = \frac{(1359.3)(5)}{(2)(4)(6)(8)(14.64)}$$

$$K_{pn} = 1.209 \quad e(\infty) = \frac{1}{1+K_{pn}} = \frac{1}{2.209} = .4527$$

improvement of steady-state error with lead compensation: $\frac{K_{pn}}{K_{pu}} = \frac{.4527}{.5186} = .8728$ improvement of steady-state error by a factor of 30: $\frac{30}{.8728} = 34.3719$ if the lag compensator pole has an arbitrary value of $p_c = 0.01$, $z_c = 34.3719 \cdot 0.01 = .3437$ thus, the lag compensator is fully defined as: $(s+.3437)/(s+0.01)$

transfer function with lag and lead compensator:

$$G_{llc}(s) = \frac{K(s+5)(s+.3437)}{(s+0.01)(s+2)(s+4)(s+6)(s+8)(s+14.64)}$$

dominant poles of lag-lead transfer function (MATLAB):

$$s = -1.82 \pm 3.153j$$

magnitude criterion to solve for value of gain, K (lag-lead compensated):

$$K = \frac{1}{|G_{llc}(s)|} = \left| \frac{(s+0.01)(s+2)(s+4)(s+6)(s+8)(s+14.64)}{(s+5)(s+.3437)} \right|_{s=-1.82+3.153j}$$

$$K = 1359.1$$

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(9.20) For the unity feedback system in Figure P9.1, with $G(s) = \frac{K}{(s+4)(s+6)(s+10)}$ do the following:

(a) Design a controller that will yield no more than 25% overshoot and no more than a 2-second settling time for a step input and zero steady state error for step and ramp inputs.

damping ratio at 25% overshoot:

$$\xi = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = \frac{-\ln(.25)}{\sqrt{\pi^2 + \ln^2(.25)}} = .4037$$

location of uncompensated dominant poles (MATLAB):

$$s = -2.708 \pm 6.137j$$

magnitude criterion to solve for value of gain, K (uncompensated):

$$K = \frac{1}{|G(s)|} = \frac{1}{|(s+4)(s+6)(s+10)|} \Big|_{s=-2.708 \pm 6.137j}$$

$$K = 416.26$$

real part of PD compensated second order dominant poles:

$$T_s = 4/\sigma \rightarrow \sigma = 4/T_s \rightarrow \sigma = 4/2 = 2$$

imaginary part of PD compensated second-order dominant poles:

$$\omega_d = 10 \tan(180^\circ - \cos^{-1}(\xi))$$

$$\omega_d = 2 \tan(180^\circ - \cos^{-1}(.4037)) = -4.532j$$

location of PD compensated dominant poles ($-\sigma \pm \omega_d j$):

$$s = -2 \pm 4.532j$$

angle criterion to solve for PD compensator zero:

$$-180^\circ = \angle s + z_c \Big|_{s=-2 \pm 4.532j} - \angle (s)(s+4)(s+6)(s+10) \Big|_{s=-2 \pm 4.532j}$$

$$-180^\circ = \angle s + z_c \Big|_{s=-2 \pm 4.532j} - 258.07^\circ$$

$$78.07^\circ = \angle s + z_c \Big|_{s=-2 \pm 4.532j}$$

$$78.07^\circ = \tan^{-1} \left(\frac{4.532}{z_c - 2} \right)$$

$$4.733 = \frac{4.532}{z_c - 2}$$

$$z_c = 2.958$$

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(9.20) transfer function of PD-compensated :

$$G_{PD}(s) = \frac{K(s+2.957)}{s(s+4)(s+6)(s+10)}$$

magnitude criterion to solve for value of gain, K (PD compensated) :

$$K = \frac{1}{|G_{PD}(s)|} = \left| \frac{(s)(s+4)(s+6)(s+10)}{(s+2.957)} \right| \Big|_{s=-2+4.532j}$$

$$K = 294.58$$

solving for PI compensator : (assume $z_c = 0.01$) :

$$G_{PI}(s) = \frac{(s+0.01)}{s}$$

transfer function of PID controller :

$$G_{PID}(s) = \frac{K(s+0.01)(s+2.957)}{s^2(s+4)(s+6)(s+10)}$$

magnitude criterion to solve for value of gain, K (PID controller) :

$$K = \frac{1}{|G_{PID}(s)|} = \left| \frac{(s^2)(s+4)(s+6)(s+10)}{(s+0.01)(s+2.957)} \right| \Big|_{s=-2+4.532j}$$

$$K = 294.78$$

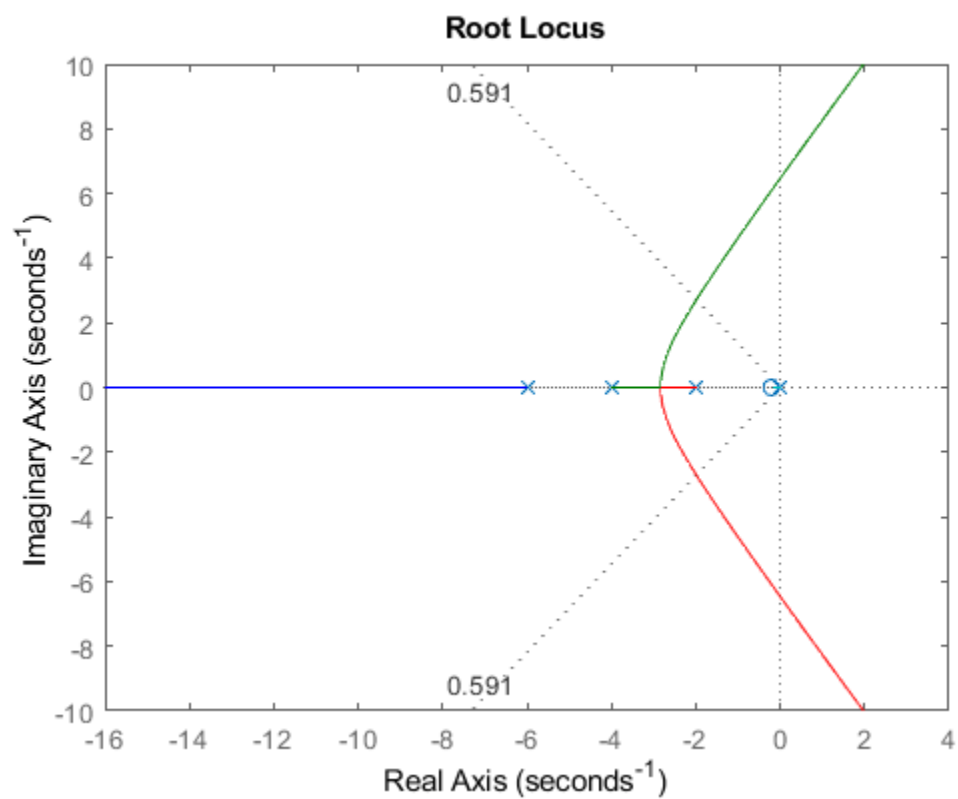
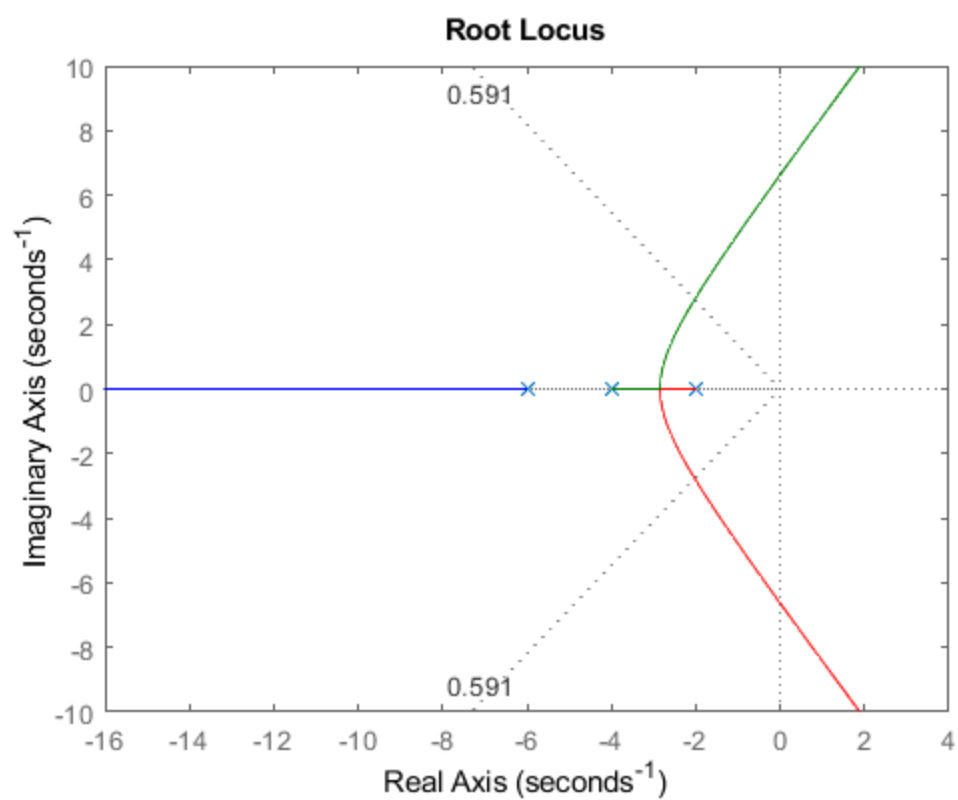
(b) Use MATLAB and verify your design.

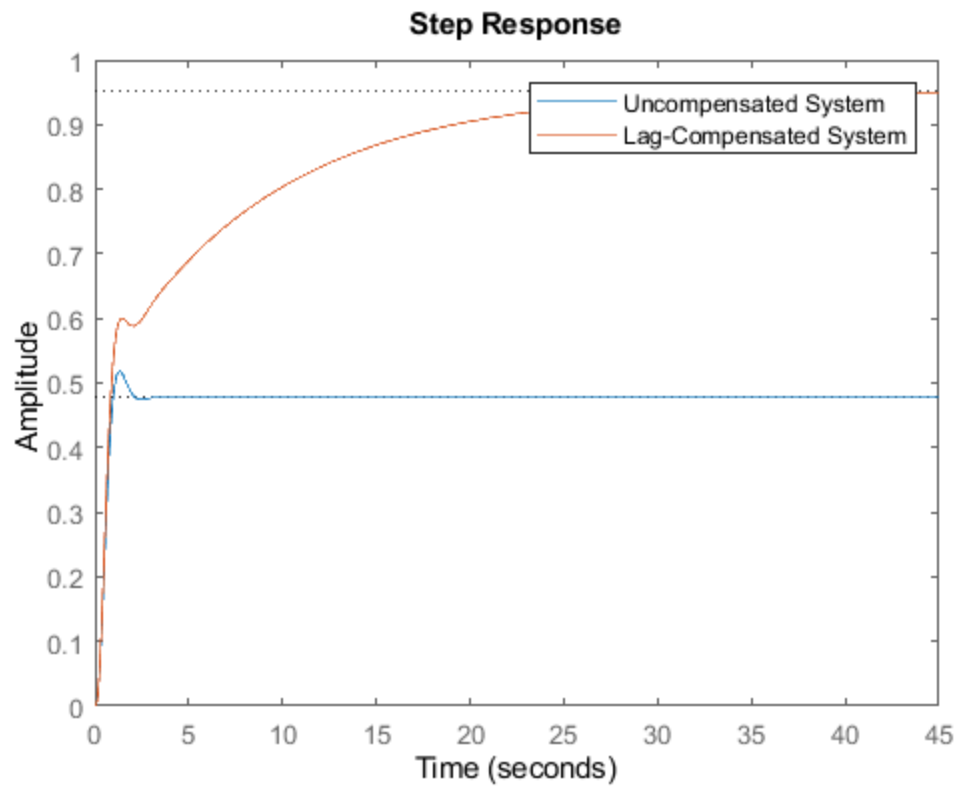
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Question 4 : Part B

```
Gs_uncompensated = tf(1, [1 12 44 48]);
figure(1)
rlocus(Gs_uncompensated)
damping = .591;
sgrid(damping, 0)
% Lag-Compensated Transfer Function
Gs_lagcompensated = tf([1 .21077], [1 12.01 44.12 48.44 .48]);
figure(2)
rlocus(Gs_lagcompensated)
sgrid(damping, 0)
% Gain of Uncompensated and Lag-Compensated Transfer Functions
Ku = 43.977;
Klc = 45.92;
% Closed-Loop Transfer Functions
Tu = feedback(Gs_uncompensated*Ku,1);
Tlc = feedback(Gs_lagcompensated*Klc,1);
% Step Response Plot of Closed-Loop Transfer Functions
figure(3)
step(Tu,Tlc)
legend("Uncompensated System","Lag-Compensated System")
```





Question 4: Part C

```
stepinfo(Tlc)
```

```
ans =
```

```
struct with fields:
```

```
    RiseTime: 13.6403  
    SettlingTime: 28.1310  
    SettlingMin: 0.8576  
    SettlingMax: 0.9510  
    Overshoot: 0  
    Undershoot: 0  
    Peak: 0.9510  
    PeakTime: 49.0345
```

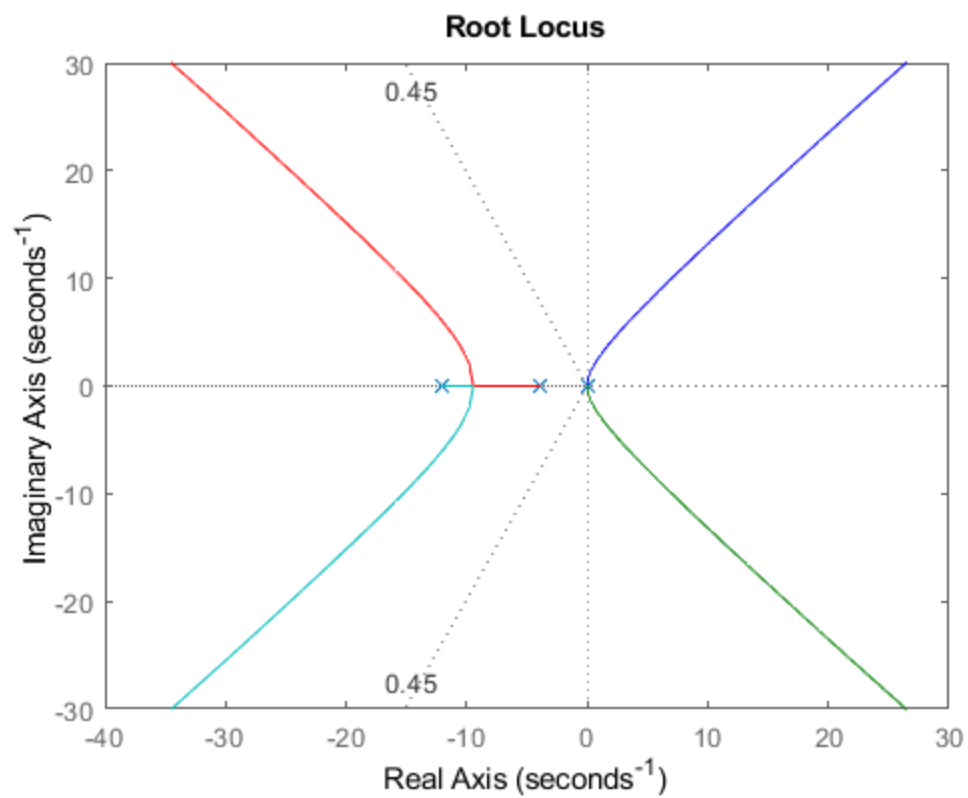
Question 12 : PART C

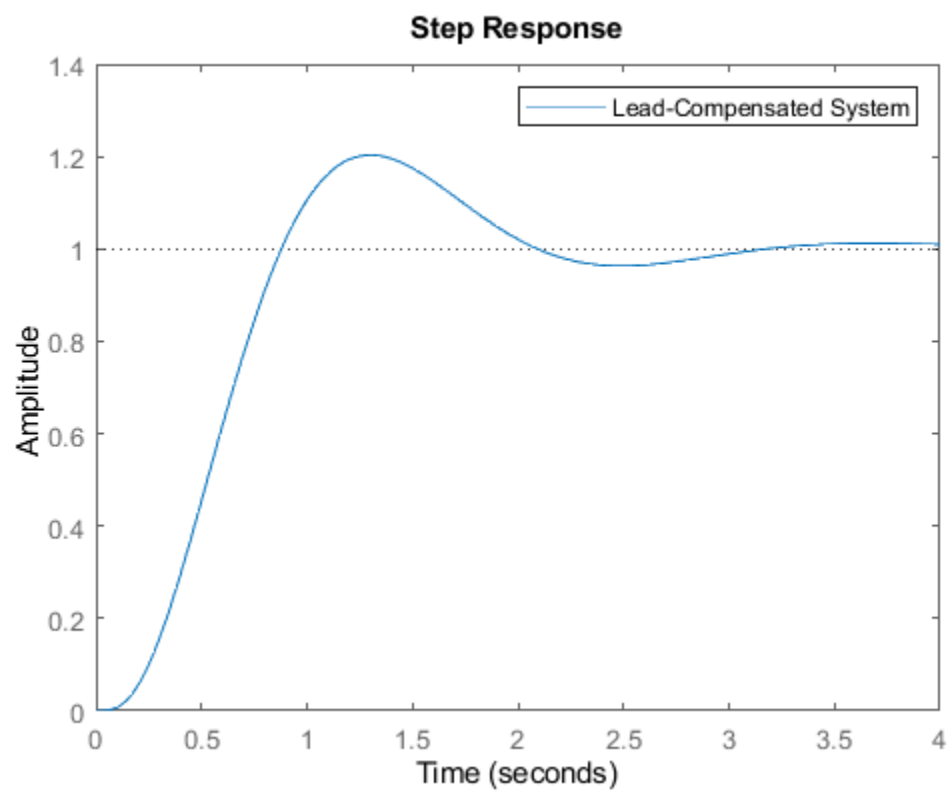
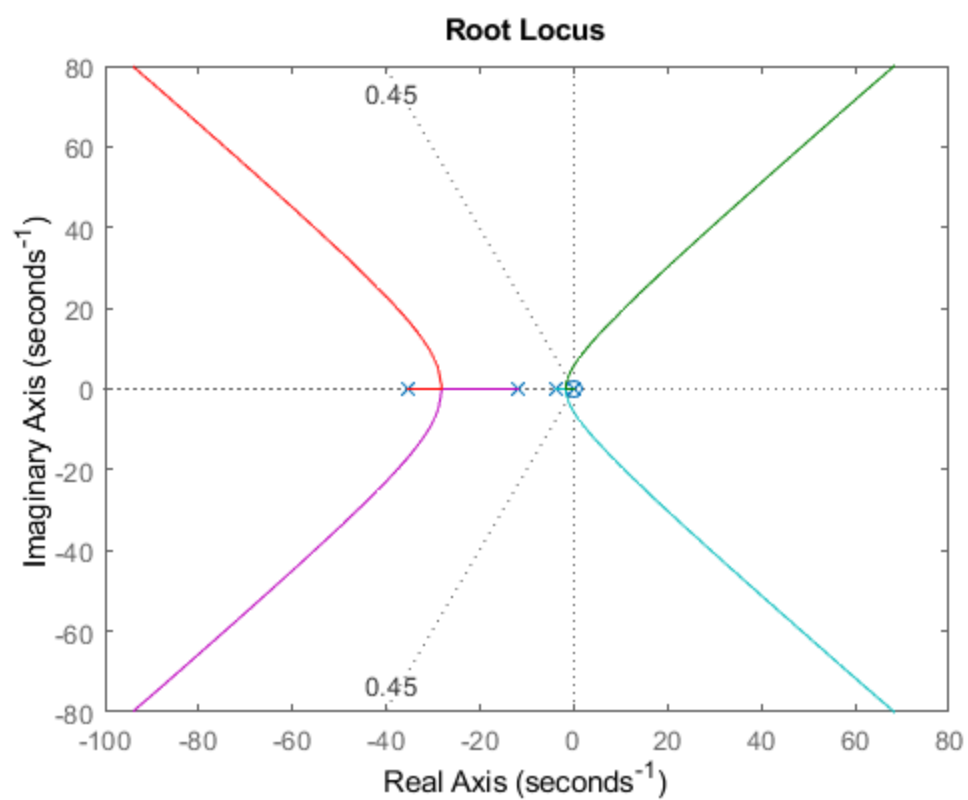
```
Gs_uncompensated = tf(1, [1 16 48 0 0]);  
figure(4)  
rlocus(Gs_uncompensated)  
damping = .45038;
```

```

sgrid(damping, 0)
Gs_leadcompensated = tf([1 0.01], [1 51.39 614.2 1699 0 0]);
figure(5)
rlocus(Gs_leadcompensated)
sgrid(damping, 0)
% Gain of Lead-Compensated Transfer Function
Kc = 4173.66;
% Closed-Loop Lead-Compensated Transfer Function
Tlc = feedback(Gs_leadcompensated*Kc,1);
% Step Response Plot of Closed-Loop Transfer Function
figure(6)
step(Tlc)
legend("Lead-Compensated System")

```





Question 15

Uncompensated Transfer Function

```
Gs_uncompensated = tf(1,[1 20 140 400 384]);
figure(7)
rlocus(Gs_uncompensated)
damping = .5;
sgrid(damping, 0)
% Lead Compensated Transfer Function
Gs_leadcompensated = tf([1,5], [1 34.64 432.8 2450 6240 5622]);
figure(8)
rlocus(Gs_leadcompensated)
sgrid(damping, 0)
% Lag-Lead Compensated Transfer Function
Gs_lagleadcompensated = tf([1 5.344 1.718], [1 34.65 433.1 2454 6264
5684 56.22]);
figure(9)
rlocus(Gs_lagleadcompensated)
sgrid(damping, 0)
% Gain of Uncompensated, Lead, and Lag-Lead Transfer Functions
Ku = 354.44;
Klc = 1359.3;
Kllc = 1359.1;
% Closed-Loop Transfer Functions
Tu = feedback(Gs_uncompensated*Ku,1);
Tlc = feedback(Gs_leadcompensated*Klc,1);
Tllc = feedback(Gs_lagleadcompensated*Kllc,1);
% Step Response Plots
figure(10)
step(Tu,Tlc,Tllc)
stepinfo(Tu)
stepinfo(Tlc)
stepinfo(Tllc)
legend("Uncompensated", "Lead-Compensated", "Lag-Lead Compensated");
```

ans =

struct with fields:

```
    RiseTime: 0.6054
  SettlingTime: 2.8199
  SettlingMin: 0.4346
  SettlingMax: 0.5497
    Overshoot: 14.5325
    Undershoot: 0
        Peak: 0.5497
    PeakTime: 1.4356
```

ans =

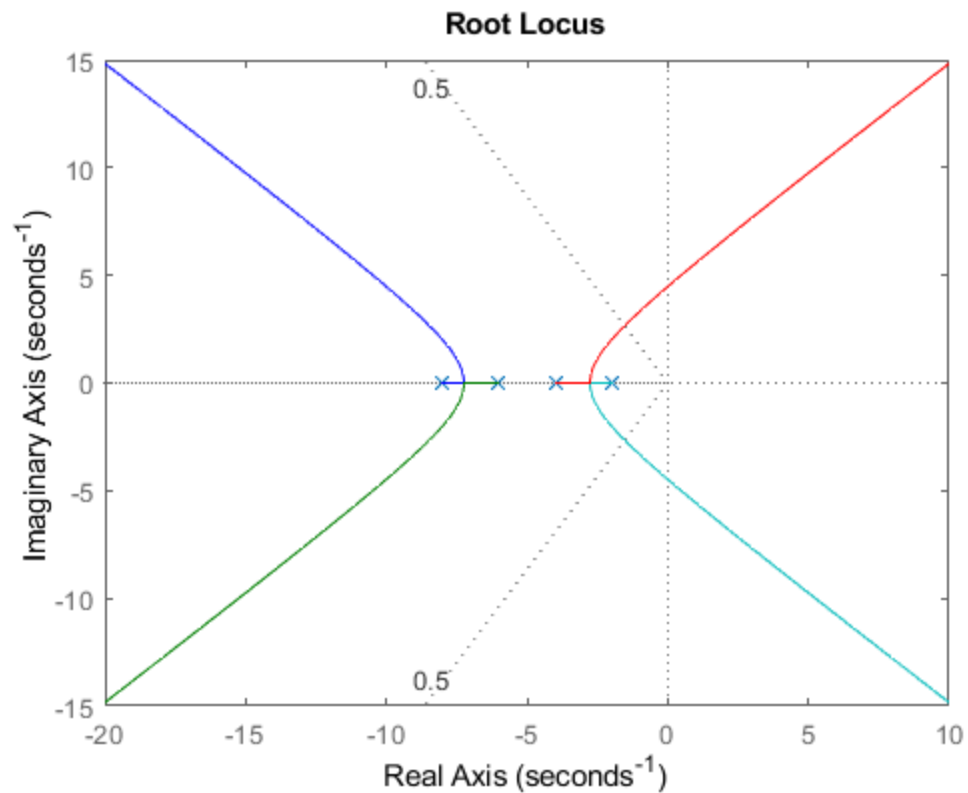
struct with fields:

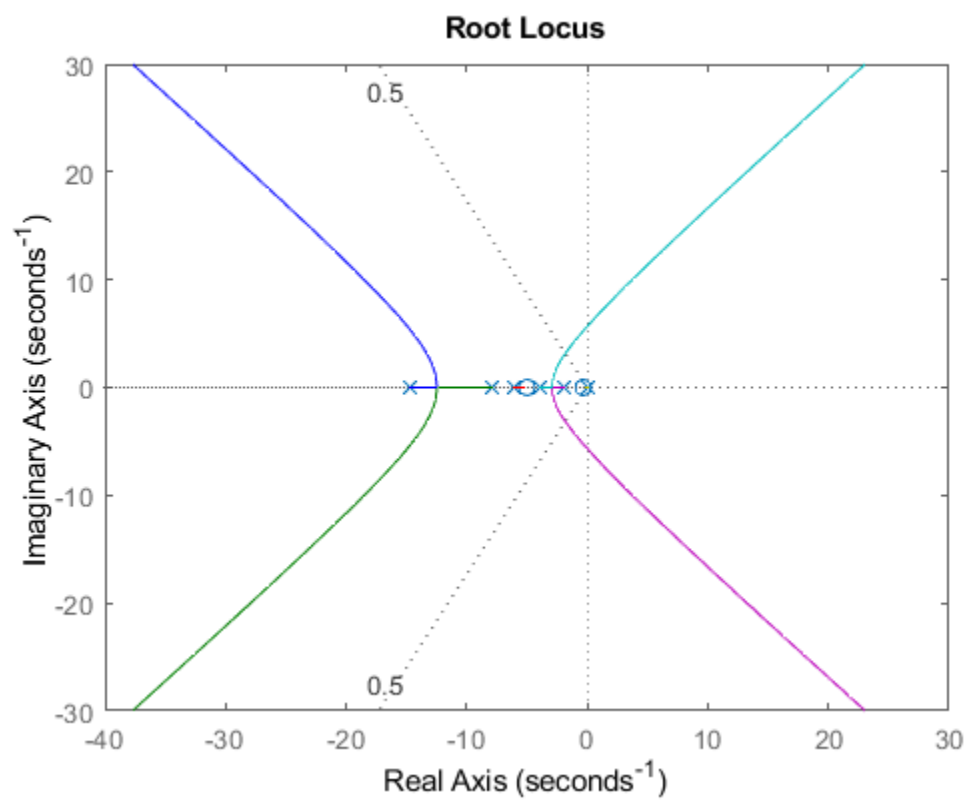
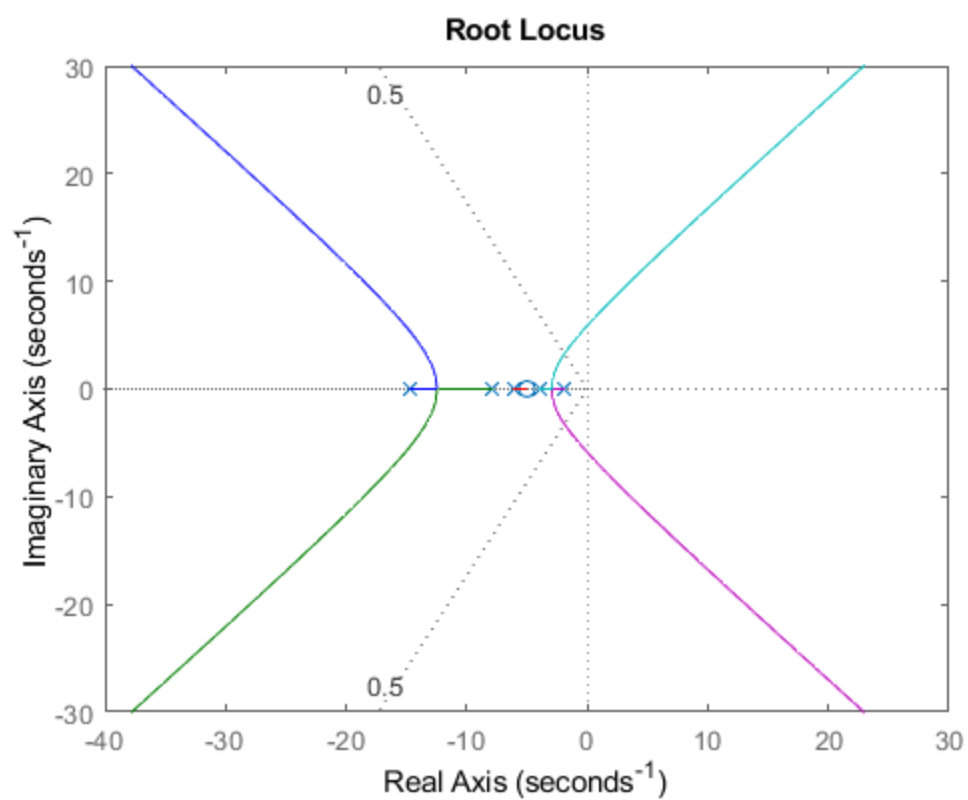
```
RiseTime: 0.4763
SettlingTime: 2.2683
SettlingMin: 0.4970
SettlingMax: 0.6297
Overshoot: 15.0622
Undershoot: 0
Peak: 0.6297
PeakTime: 1.1285
```

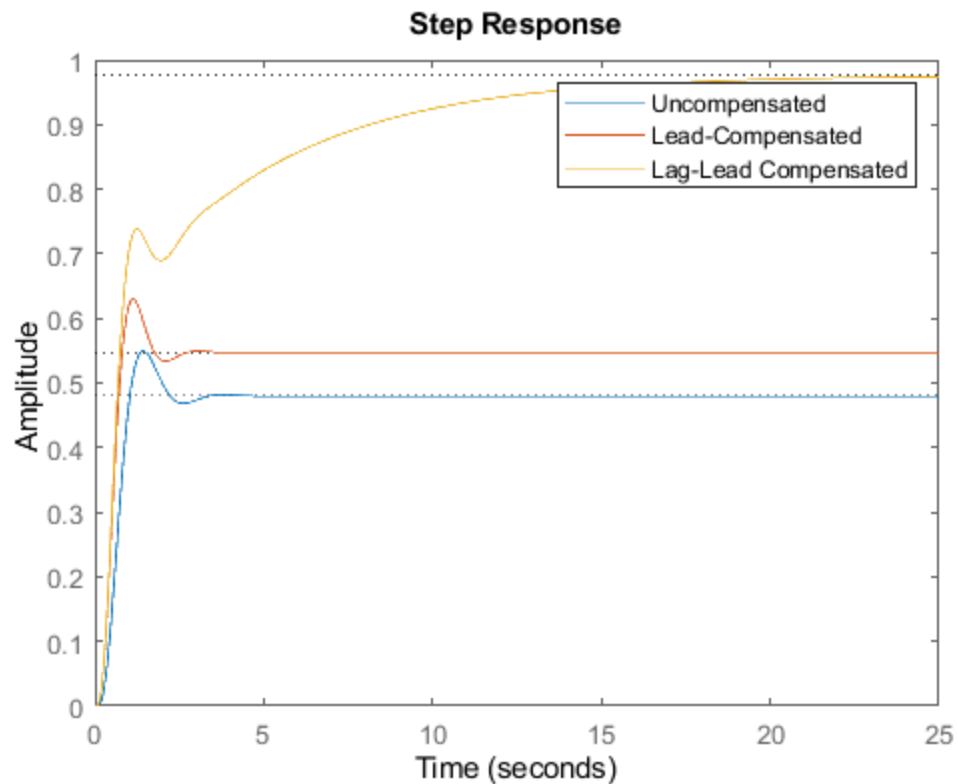
ans =

struct with fields:

```
RiseTime: 6.6664
SettlingTime: 14.6827
SettlingMin: 0.8789
SettlingMax: 0.9738
Overshoot: 0
Undershoot: 0
Peak: 0.9738
PeakTime: 24.1914
```







Question 20

```

Gs_uncompensated = tf(1, [1 20 124 240]);
figure(11)
rlocus(Gs_uncompensated)
damping = .4037;
sgrid(damping, 0)
% PD Compensated Transfer Function
Gs_PDcompensated = tf([1 2.957], [1 20 124 240 0]);
figure(12)
rlocus(Gs_PDcompensated)
sgrid(damping, 0)
% PID Compensated Transfer Function
Gs_PIDcompensated = tf([1 2.967 .02957],[1 20 124 240 0 0]);
figure(13)
rlocus(Gs_PIDcompensated)
sgrid(damping, 0)
% Gain of Uncompensated, PD, PID Transfer Functions
Ku    = 416.26;
Kpd    = 294.58;
Kpid   = 294.78;
% Closed-Loop Transfer Functions
Tu = feedback(Gs_uncompensated*Ku,1);
Tpd = feedback(Gs_PDcompensated*Kpd,1);
Tpid = feedback(Gs_PIDcompensated*Kpid,1);
  
```

```
% Step Response Plots
figure(14)
step(Tu,Tpd,Tpid)
stepinfo(Tu)
stepinfo(Tpd)
stepinfo(Tpid)
legend("Uncompensated", "PD Compensated", "PID Compensated");
```

```
ans =
```

```
struct with fields:
```

```

    RiseTime: 0.2473
    SettlingTime: 1.3204
    SettlingMin: 0.5736
    SettlingMax: 0.7744
    Overshoot: 22.0850
    Undershoot: 0
    Peak: 0.7744
    PeakTime: 0.5874
```

```
ans =
```

```
struct with fields:
```

```

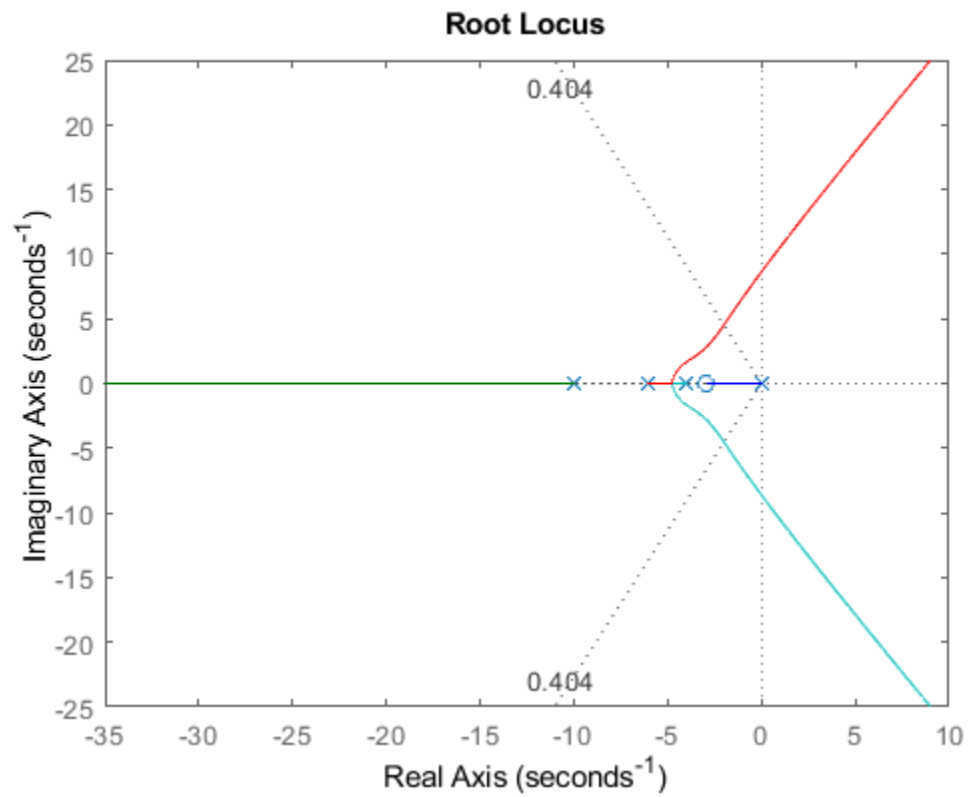
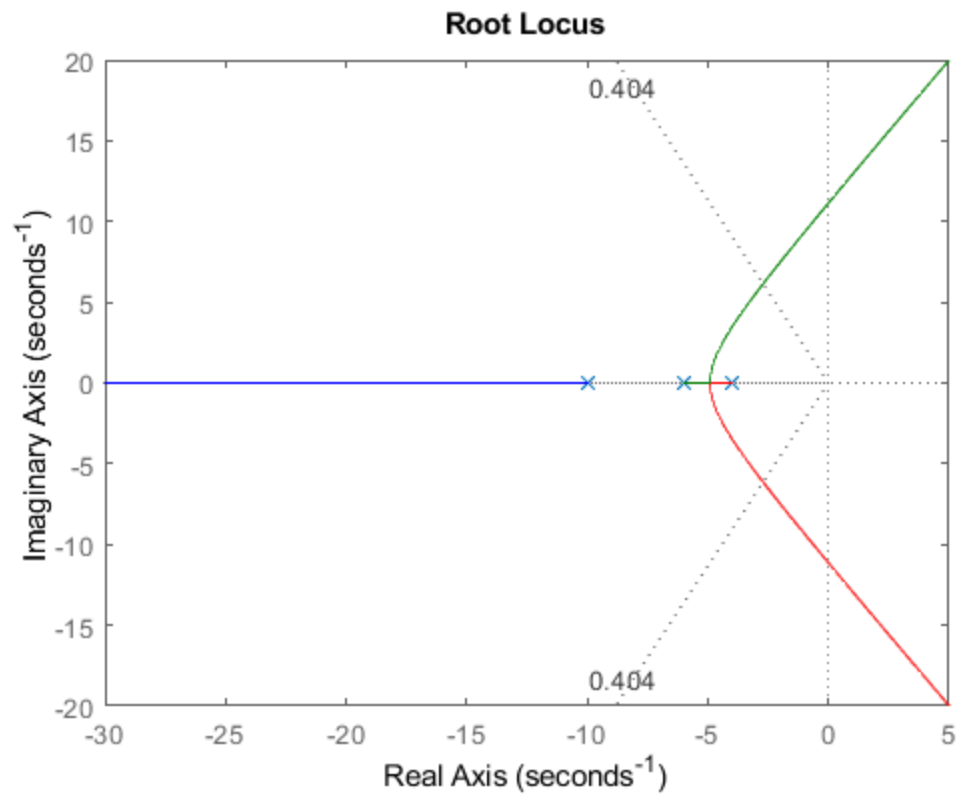
    RiseTime: 0.3455
    SettlingTime: 1.7793
    SettlingMin: 0.9040
    SettlingMax: 1.1869
    Overshoot: 18.6930
    Undershoot: 0
    Peak: 1.1869
    PeakTime: 0.8007
```

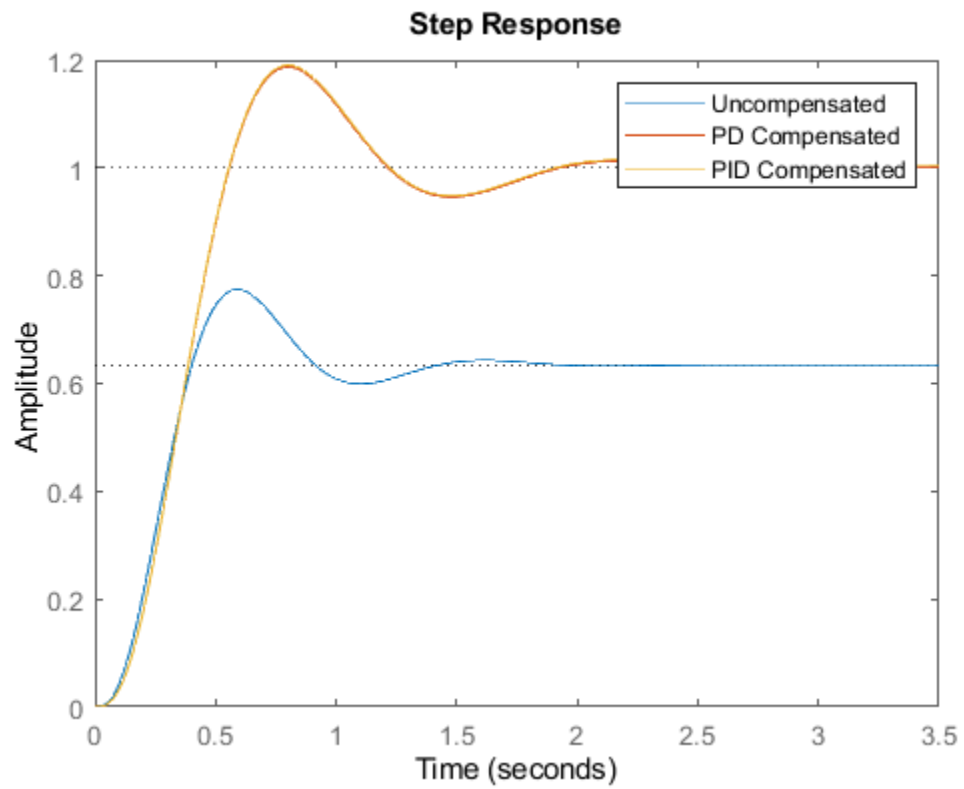
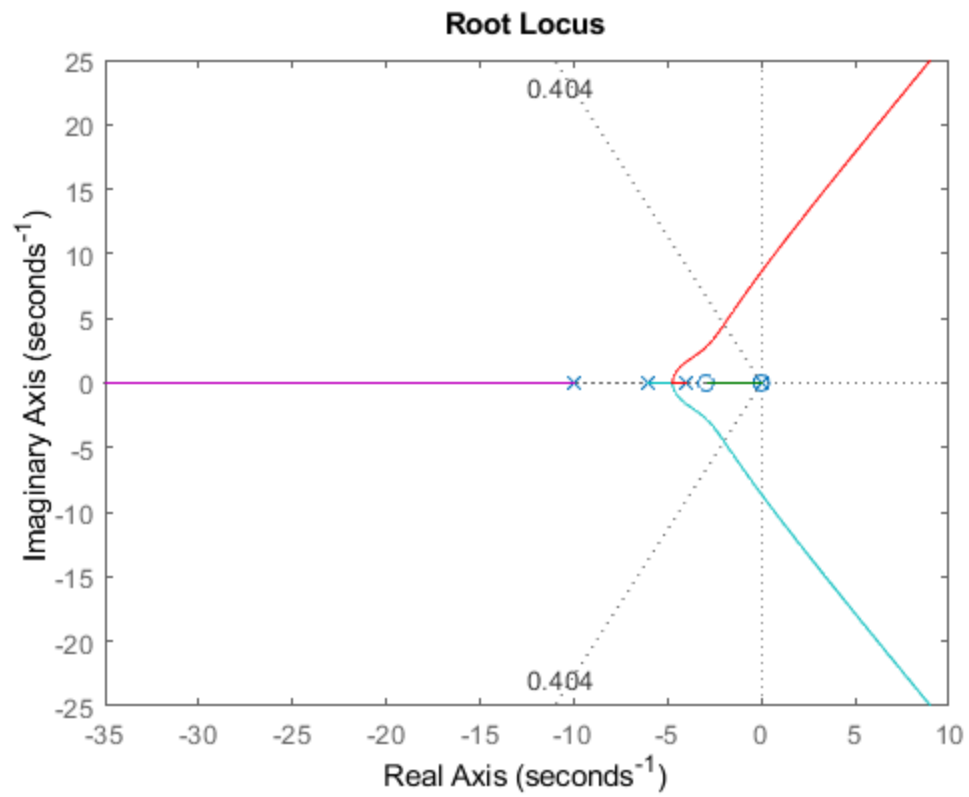
```
ans =
```

```
struct with fields:
```

```

    RiseTime: 0.3445
    SettlingTime: 1.7636
    SettlingMin: 0.9060
    SettlingMax: 1.1906
    Overshoot: 19.0553
    Undershoot: 0
    Peak: 1.1906
    PeakTime: 0.8008
```





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