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Problem

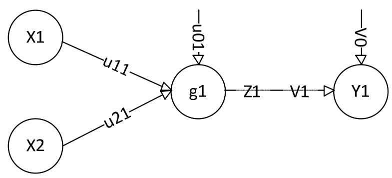
Take an MLP Neural Network with 3 layers with:

2 Inputs, 1 output, and any number of hidden neurons

Implement sample-by-sample training and show that it works

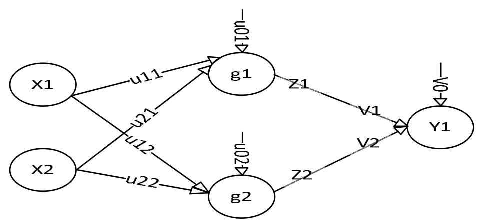
Design

1 hidden neuron

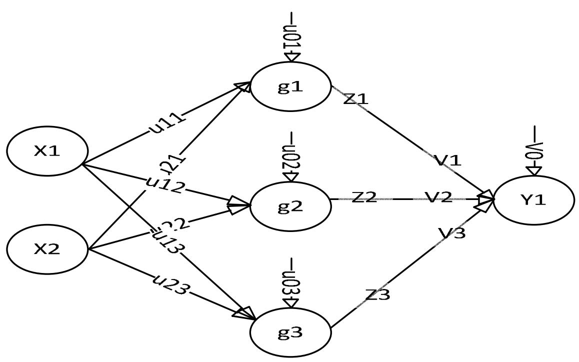


Y1= V0 + V1*Z1

2 hidden neurons



Y1= V0 + V1*Z1 + V2*Z2



3 hidden neurons; Y1= V0 + V1*Z1 + V2*Z2 + V3*Z3

g1= u01 + x1*u11+ x2*u21 g2= u02 + x1*u12+ x2*u22 g3= u03 + x1*u13+ x2*u23

Z1= 1/(1+exp(-g1)) Z2= 1/(1+exp(-g2)) Z3= 1/(1+exp(-g3))

User input data or randomized at runtime:

$$U = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & \dots & h \end{array} \\ \begin{array}{c} 1 \\ 2 \\ \dots \\ n+1 \end{array} & \begin{bmatrix} u11 & u12 & \dots & u1h \\ u21 & u22 & \cdots & u2h \\ \dots & \dots & \cdots & \dots \\ u01 & u02 & \cdots & u0h \end{bmatrix} \end{array} \qquad V = \begin{bmatrix} V1 \\ V2 \\ V3 \\ \dots \\ Vh \end{bmatrix} \qquad V0$$

Training Data output = x1 + x2

x1	x2	d
-3	-1	-4
-2	1.2	-0.8
-1	0	-1
0	-3	-3
1.2	2.2	3.4
2.2	-2	0.2
2.5	2.5	5

Error function and BP equations

Error = E = $\frac{1}{2}(y-d)^2$ $\frac{\mathrm{dE}}{\mathrm{dy}} = \mathrm{y} - \mathrm{d}$

Update function: $w_{new} = w_{old} - \eta \frac{dE}{dw}$; where w is any U, or V value

$\frac{dE}{dV0} = 1$ $\frac{dE}{dVh} = Z_h$ $\frac{dE}{du_{oh}} = V_h * Z_h(Z_h-1)$ $\frac{dE}{du_{nh}} = V_h * Z_h(Z_h-1) * x_n$

Results and Conclusion

I wrote my code as a function call with the following parameters:
(# hidden neurons, # epoch, hidden bias multiplier, eta)

The results reported below are for constant values:
#epoch = 1000
Hidden bias multiplier = 1
eta = 0.1

Errors for an NN with 2 and 3 hidden neurons were compared with 10 iterations each.
The reported error is the absolute error E from the equation given above.

2 Hidden	3 Hidden
0.4439	0.1181
0.4363	0.0076
1.1487	0.0666
0.2671	0.1915
0.8074	0.1225
0.1822	0.2432
0.0491	0.0991
1.1177	0.3026
0.3703	0.0313
0.4103	0.0953

The trials I ran with my code show that my addition function works better with 3 hidden neurons.

Code

```
function HW1(nhid, nepoch, ux, eta)
% clc;
% clear all;
%define number of input, hidden, and output neurons
nin=2; %2 input N
%nhid=2; %3 hidden H
nout=1; %1 output M

%define matrix for x input data:
%        1    2        3    ... number of tests
% x1     x11, x12, x13,   ... x1t
% x2     x21, x22, x23,   ... x2t
%
% number tests data = t= 5
t = 7;
x=zeros(2, t);

%matrix of weights between input -- hidden u
%      u1      u2      u3
%x1    u11     u12     u13
%x2    u21     u22     u23
%N+1   u01     u02     u03
%
u = zeros(nin+1, nhid);
%BP vector for du = zeros(nhid, nin+1)
du = zeros(nin+1, nhid);
%      u1      u2      u3
%x1    du11    du12    du13
%x2    du21    du22    du23
%N+1   du01    du02    du03
%


%derfine vector for gamma = g(size H)
g=zeros(nhid,1);

%define vector for Z = Z(size H)
Z=zeros(nhid, 1);

%define vector for V = V(size H)
V=zeros(nhid, 1);
%V0(nout, 1) = output bias
V0 = zeros(nout, 1);

%BP vector for dV = zeros(nhid,1)
dV = zeros(nhid,nout);
%BP vector for output bias dV0 = zeros(nout,1)
dV0 = zeros(nout,1);

%define output vector y = zeros(M,t)
y=zeros(nout,t);

%desired output vector d=(M,1)
d = zeros(nout, t);

%%____Define input matrix___ (test data input)
%define matrix for x input data:
%        1    2        3    ... number of tests
% x1     x11, x12, x13,   ... x1t
% x2     x21, x22, x23,   ... x2t
%
x= [-3 -2 -1 0 1.2 2.2 2.5; -1 1.2 0 -3 2.2 -2 2.5];

%define test data desired output
d= [-4 -0.8 -1 -3 3.4 0.2 5];

%randomize weights of links input -- hidden
%and randomize hidden layer bias (nin+1)
%matrix of weights between input -- hidden u
%      u1      u2      u3
```

```
%x1   u11     u12     u13
%x2   u21     u22     u23
%...    ...     ...
%N+1  u01     u02     u03
%
for i = 1:(nhid);
    for j = 1:1:(nin+1);
        u(j, i)= rand*ux;
    end
end
%u = [0.1985 0.1985 0.1985; 0.1979 0.1979 0.1979; 0.199 0.199 0.199];
%u = [0.1985; 0.1985; 0.1985];
u1 = u;

%Randomize V = zeros(H, M)
for j= 1:1:nhid;
    for i=1:nout;
        V(j,i) = rand;
    end
end
% V = [0.997; 0.996; 0.995];
V1 = V;
%randomize output bias V0 = zeros(nout,1)
for j = 1:1:nout
    V0(j, 1) = rand;
end
% V0=0.99;
V01 = V0;

%create Error vector E = zeros(nout, #tests
E=zeros(nout,t);
%dE = dE/dY = zeros(#out, training size)
dE = zeros(nout, t);

%********Epochs
%nepoch = 10000;
for epoch = 1:nepoch

    for test = 1:t;
        %+++++++++++ Feed-forward
        %calculate gamma vector
        %per testNUMBER test
        %per x(i, test)
        %for test = 1:1:t --> run through all test cases
        for j = 1:nhid
            g(j,1) = 0;
            for i = 1:nin+1;
                if i >nin;
                    g(j,1) = g(j,1) + u(i,j);
                else
                    g(j,1) = g(j,1) + u(i,j)*x(i,test);
                end
            end
        end

        %calculate Z vector Z(H,1)
        for j = 1:1:nhid;
            Z(j,1)=1/(1+exp(-g(j,1)));
        end

        %calculate output vector y(j,test) = V0(j,1)+Z(j)*V(j)
        y(1,test) = V0;
        for j = 1:1:nhid
            y(1,test) = y(1,test)+Z(j)*V(j);
        end
        %+++++++++++END feed-forward
        %Error Calculations 'E' = zeros(1, test)
        %columns = number of training data
        %e1=0.5*((y-d(1,test))^2);
        %E(1, test)=0.5*((y(1,test)-d(1,test))^2);
        %dE/dY = (y-d);
```

```
        for i = 1:nout
            dE(nout, test) = (y(1,test)-d(1,test));
        end

        %====== Back Propagation
        %find dV and dV0
        for j=1:nhid
            dV(j) = Z(j);
        end
        for j = 1:nout
            dV0(j) = 1;
        end

        %find du = zeros(nin+1, nhid);
        for j=1:(nin+1)
            for i=nhid
                if j >nin %hidden bias
                    du(j, i) = V(i)*Z(i)*(1-Z(i));
                else %hidden link (non-bias)
                    du(j,i) = V(i)*Z(i)*(1-Z(i))*x(j,t);
                end
            end
        end

        %update weights
        %w=w(-eta*dE/dW); eta = 0.1
        %eta = 0.1;
        u=u-eta*(dE(nout,test))*du;
        V=V-eta*(dE(nout,test))*dV;
        V0=V0-eta*(dE(nout,test))*dV0;
    end
end
u1
u
V1
V
V01
V0
x
d
y
E = abs(0.5*(y-d).^2)
Eavg= sum(E)/numel(E)
end
```