

Biologically Inspired Computing

EECS 6180

Homework 5

Hopfield Networks

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Mar. 22, 2012

Use 1, -1 for black white

1. Design a Hopfield Neural Network to recognize the following two patterns.**a. Describe the network architecture. How many neurons will it require? 5pts**

This network will require as many neurons as there are pixels [16 neurons for 16 pixels]. The neurons will each have weight values representing the connections to all other neurons this will require each of the 16 neurons having 16 weights for a total number of 16^2 weights = 256 weights.

b. Calculate the weight matrix. 5pts

The weight matrix W is calculated using the following formula:

$$W = \sum_{m=1}^M Y_m Y_m^T - M * I$$

Where M is the total number of states (images) needed to be memorized and I is the identity matrix.

For calculating the weight matrix both patterns were made into a single row matrix so:

P1 = Patter 1 = [1 1 1 1 1 -1 -1 1 1 -1 -1 1 1 1 1 1]

P2 = Pattern 2 = [1 -1 -1 1 -1 1 1 -1 -1 1 1 -1 1 -1 -1 1]

%in matlab

```
P1 = [1 1 1 1 1 -1 -1 1 1 -1 -1 1 1 1 1 1];
P1t = P1';
```

```
P2 = [1 -1 -1 1 -1 1 1 -1 -1 1 1 -1 1 -1 -1 1];
P2t = P2';
```

```
Y1 = P1t*P1;
Y2 = P2t*P2;
```

```
W = Y1 + Y2 - 2*eye(16)=
```

| | | | | | | | | | | | | | | | | |
|---|----|----|---|----|----|----|----|----|----|----|----|---|----|----|---|---|
| 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 2 |
| 0 | 0 | 2 | 0 | 2 | -2 | -2 | 2 | 2 | -2 | -2 | 2 | 0 | 2 | 2 | 0 | 0 |
| 0 | 2 | 0 | 0 | 2 | -2 | -2 | 2 | 2 | -2 | -2 | 2 | 0 | 2 | 2 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 2 |
| 0 | 2 | 2 | 0 | 0 | -2 | -2 | 2 | 2 | -2 | -2 | 2 | 0 | 2 | 2 | 0 | 0 |
| 0 | -2 | -2 | 0 | -2 | 0 | 2 | -2 | -2 | 2 | 2 | -2 | 0 | -2 | -2 | 0 | 0 |
| 0 | -2 | -2 | 0 | -2 | 2 | 0 | -2 | -2 | 2 | 2 | -2 | 0 | -2 | -2 | 0 | 0 |
| 0 | 2 | 2 | 0 | 2 | -2 | -2 | 0 | 2 | -2 | -2 | 2 | 0 | 2 | 2 | 0 | 0 |
| 0 | 2 | 2 | 0 | 2 | -2 | -2 | 2 | 0 | -2 | -2 | 2 | 0 | 2 | 2 | 0 | 0 |
| 0 | -2 | -2 | 0 | -2 | 2 | 2 | -2 | -2 | 0 | 2 | -2 | 0 | -2 | -2 | 0 | 0 |
| 0 | -2 | -2 | 0 | -2 | 2 | 2 | -2 | -2 | 2 | 0 | -2 | 0 | -2 | -2 | 0 | 0 |
| 0 | 2 | 2 | 0 | 2 | -2 | -2 | 2 | 2 | -2 | -2 | 0 | 0 | 2 | 2 | 0 | 0 |
| 2 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| 0 | 2 | 2 | 0 | 2 | -2 | -2 | 2 | 2 | -2 | -2 | 2 | 0 | 0 | 2 | 0 | 0 |
| 0 | 2 | 2 | 0 | 2 | -2 | -2 | 2 | 2 | -2 | -2 | 2 | 0 | 2 | 0 | 0 | 0 |
| 2 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 |

c. Test if these two patterns are classified correctly with that weight matrix. 5pts

Testing Pattern1 with W:

```
%test P1
testP1 = W*P1t;

%sign function
for i = 1:length(testP1)
    if testP1(i) > 0
        testP1(i) = 1;
    elseif testP1(i) < 0
        testP1(i) = -1;
    else
        testP1(i) = 0;
    end
end
testP1' =
    1  1  1  1  1 -1 -1  1  1 -1 -1  1  1  1  1  1
P1 =
    1  1  1  1  1 -1 -1  1  1 -1 -1  1  1  1  1  1
```

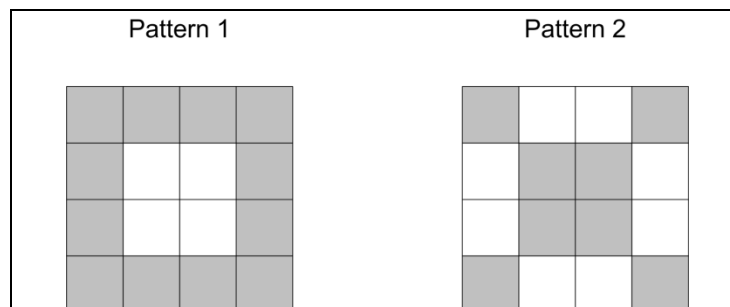
W works for Pattern 1 recognition

Testing Pattern2 with W:

```
%test second original image
testP2 = W*P2t;

%sign function
for i = 1:length(testP2)
    if testP2(i) > 0
        testP2(i) = 1;
    elseif testP2(i) < 0
        testP2(i) = -1;
    else
        testP2(i) = 0;
    end
end
testP2' =
    1 -1 -1  1 -1  1  1 -1 -1  1  1 -1  1 -1 -1  1
P2 =
    1 -1 -1  1 -1  1  1 -1 -1  1  1 -1  1 -1 -1  1
```

W works for Pattern 2 recognition



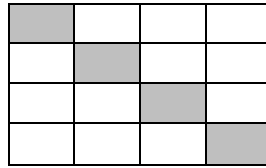


Fig. 2

d. If a new pattern is given as shown in Fig. 2, to which category it will be classified? 5pts

Testing figure 2 for recognition to Pattern 2 or Patter 1

For this test figure 2 was made into a row matrix in MATLAB:

```
fig2 = [1 -1 -1 -1 -1 1 -1 -1 -1 1 -1 -1 -1 -1 1];
fig2mat = vec2mat(fig2,4)
fig2 = fig2';
```

```
testfig2 = W*fig2;
test2mat = vec2mat(testfig2, 4)
```

fig2mat =

```

1      -1      -1      -1
-1       1      -1      -1
-1      -1       1      -1
-1      -1      -1       1
```

test2mat =

```

-2     -14     -14       2
-14      14      18     -14
-14      18      14     -14
 2      -14     -14     -2
```

```
% for iterations until converging to a pattern
%check sign of pixels
```

```
for i = 1:length(testfig2)
    if testfig2(i) > 0
        testfig2(i) = 1;
    elseif testfig2(i) < 0
        testfig2(i) = -1;
    else
        testfig2(i) = 0;
    end
end
```

```
resultmat = vec2mat(testfig2,4)
testfig2 = W*testfig2;
```

Iteration 1: resultmat =

| | | | |
|----|----|----|----|
| -1 | -1 | -1 | 1 |
| -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | -1 |
| 1 | -1 | -1 | -1 |

Iteration 2: resultmat =

| | | | |
|----|----|----|----|
| 1 | -1 | -1 | -1 |
| -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | -1 |
| -1 | -1 | -1 | 1 |

Iteration 3: resultmat =

| | | | |
|----|----|----|----|
| -1 | -1 | -1 | 1 |
| -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | -1 |
| 1 | -1 | -1 | -1 |

Iteration 4: resultmat =

| | | | |
|----|----|----|----|
| 1 | -1 | -1 | -1 |
| -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | -1 |
| -1 | -1 | -1 | 1 |

Iteration 5: resultmat =

| | | | |
|----|----|----|----|
| -1 | -1 | -1 | 1 |
| -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | -1 |
| 1 | -1 | -1 | -1 |

Iteration 6: resultmat =

| | | | |
|----|----|----|----|
| 1 | -1 | -1 | -1 |
| -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | -1 |
| -1 | -1 | -1 | 1 |

Iteration 7: resultmat =

| | | | |
|----|----|----|----|
| -1 | -1 | -1 | 1 |
| -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | -1 |
| 1 | -1 | -1 | -1 |

Iteration 8: resultmat =

| | | | |
|----|----|----|----|
| 1 | -1 | -1 | -1 |
| -1 | 1 | 1 | -1 |
| -1 | 1 | 1 | -1 |
| -1 | -1 | -1 | 1 |

After at 8 iterations of the sign check it seems that the network would converge to choosing Patter2 for figure 2. This will NOT be an exact match.