Anharmonic Group Elements as Generated by Machine

Ed Rogers

March 2011

$$\begin{aligned}
\tfrac{\lambda}{4}(A+B)^4 \quad = \quad & \lambda\cdot(0.25)\cdot(B^4+A^4)\\
& +\lambda\cdot(B^3A+BA^3)\\
& +\lambda\cdot(1.5)\cdot(B^2+A^2)\\
& +\lambda\cdot(1.5)\cdot B^2A^2\\
& +\lambda\cdot(3)\cdot BA\\
& +\lambda\cdot(0.75)
\end{aligned}$$

$$
\begin{aligned}
[-X, H_0] \quad = \quad & \lambda^2 \cdot (6 \cdot \beta_{21}) \cdot (B^6 + A^6) + \lambda^2 \cdot (6 \cdot \beta_{15}) \cdot (B^6 - A^6) \\
& + \lambda^2 \cdot (4 \cdot \beta_{22}) \cdot (B^5 A + B A^5) + \lambda^2 \cdot (4 \cdot \beta_{16}) \cdot (B^5 A - B A^5) \\
& + \lambda^2 \cdot (2 \cdot \beta_{23}) \cdot (B^4 A^2 + B^2 A^4) + \lambda^2 \cdot (2 \cdot \beta_{17}) \cdot (B^4 A^2 - B^2 A^4) \\
& + \lambda \cdot (4 \cdot \beta_8) \cdot (B^4 + A^4) \\
& + \lambda^2 \cdot (4 \cdot \beta_{24}) \cdot (B^4 + A^4) + \lambda^2 \cdot (4 \cdot \beta_{18}) \cdot (B^4 - A^4) \\
& + \lambda \cdot (2 \cdot \beta_9) \cdot (B^3 A + B A^3) \\
& + \lambda^2 \cdot (2 \cdot \beta_{25}) \cdot (B^3 A + B A^3) + \lambda^2 \cdot (2 \cdot \beta_{19}) \cdot (B^3 A - B A^3) \\
& + \lambda \cdot (2 \cdot \beta_{10}) \cdot (B^2 + A^2) \\
& + \lambda^2 \cdot (2 \cdot \beta_{26}) \cdot (B^2 + A^2) + \lambda^2 \cdot (2 \cdot \beta_{20}) \cdot (B^2 - A^2)
\end{aligned}
$$

$$
\begin{aligned}
\tfrac{1}{2!}[-X,[-X,H_0]] \quad = \quad & \lambda^2\cdot(-4\cdot\beta_8\cdot\beta_9)\cdot(B^6+A^6) \\
& +\lambda^2\cdot(36\cdot\beta_8\cdot\beta_9)\cdot(B^4A^2+B^2A^4) \\
& +\lambda^2\cdot(108\cdot\beta_8\cdot\beta_9+24\cdot\beta_8\cdot\beta_{10})\cdot(B^3A+BA^3) \\
& +\lambda^2\cdot(72\cdot\beta_8\cdot\beta_9+36\cdot\beta_8\cdot\beta_{10})\cdot(B^2+A^2) \\
& +\lambda^2\cdot(64\cdot\beta_8^2+16\cdot\beta_9^2)\cdot B^3A^3 \\
& +\lambda^2\cdot(288\cdot\beta_8^2+36\cdot\beta_9^2+24\cdot\beta_9\cdot\beta_{10})\cdot B^2A^2 \\
& +\lambda^2\cdot(384\cdot\beta_8^2+12\cdot\beta_9^2+24\cdot\beta_9\cdot\beta_{10}+8\cdot\beta_{10}^2)\cdot BA \\
& +\lambda^2\cdot(96\cdot\beta_8^2+4\cdot\beta_{10}^2)
\end{aligned}
$$

$$
\begin{aligned}
&[-X, H_0] + \tfrac{1}{2!}[-X,[-X,H_0]] \\
&= \lambda^2\cdot(6\cdot\beta_{21} - 4\cdot\beta_8\cdot\beta_9)\cdot(B^6 + A^6) + \lambda^2\cdot(6\cdot\beta_{15})\cdot(B^6 - A^6) \\
&\quad +\lambda^2\cdot(4\cdot\beta_{22})\cdot(B^5A + BA^5) + \lambda^2\cdot(4\cdot\beta_{16})\cdot(B^5A - BA^5) \\
&\quad +\lambda^2\cdot(2\cdot\beta_{23} + 36\cdot\beta_8\cdot\beta_9)\cdot(B^4A^2 + B^2A^4) + \lambda^2\cdot(2\cdot\beta_{17})\cdot(B^4A^2 - B^2A^4) \\
&\quad +\lambda\cdot(4\cdot\beta_8)\cdot(B^4 + A^4) \\
&\quad +\lambda^2\cdot(4\cdot\beta_{24})\cdot(B^4 + A^4) + \lambda^2\cdot(4\cdot\beta_{18})\cdot(B^4 - A^4) \\
&\quad +\lambda\cdot(2\cdot\beta_9)\cdot(B^3A + BA^3) \\
&\quad +\lambda^2\cdot(2\cdot\beta_{25} + 108\cdot\beta_8\cdot\beta_9 + 24\cdot\beta_8\cdot\beta_{10})\cdot(B^3A + BA^3) + \lambda^2\cdot(2\cdot\beta_{19})\cdot(B^3A - BA^3) \\
&\quad +\lambda\cdot(2\cdot\beta_{10})\cdot(B^2 + A^2) \\
&\quad +\lambda^2\cdot(2\cdot\beta_{26} + 72\cdot\beta_8\cdot\beta_9 + 36\cdot\beta_8\cdot\beta_{10})\cdot(B^2 + A^2) + \lambda^2\cdot(2\cdot\beta_{20})\cdot(B^2 - A^2) \\
&\quad +\lambda^2\cdot(64\cdot\beta_8^2 + 16\cdot\beta_9^2)\cdot B^3A^3 \\
&\quad +\lambda^2\cdot(288\cdot\beta_8^2 + 36\cdot\beta_9^2 + 24\cdot\beta_9\cdot\beta_{10})\cdot B^2A^2 \\
&\quad +\lambda^2\cdot(384\cdot\beta_8^2 + 12\cdot\beta_9^2 + 24\cdot\beta_9\cdot\beta_{10} + 8\cdot\beta_{10}^2)\cdot BA \\
&\quad +\lambda^2\cdot(96\cdot\beta_8^2 + 4\cdot\beta_{10}^2)
\end{aligned}
$$

$$
\begin{aligned}
H_4 - U^\dagger H_0 U &= \Lambda_4 \\
&= \tfrac{\lambda}{4}(A+B)^4 - \big([-X,H_0] + \tfrac{1}{2!}[-X,[-X,H_0]]\big) \\
&= \lambda^2\cdot(-6\cdot\beta_{21} + 4\cdot\beta_8\cdot\beta_9)\cdot(B^6 + A^6) + \lambda^2\cdot(-6\cdot\beta_{15})\cdot(B^6 - A^6) \\
&\quad +\lambda^2\cdot(-4\cdot\beta_{22})\cdot(B^5A + BA^5) + \lambda^2\cdot(-4\cdot\beta_{16})\cdot(B^5A - BA^5) \\
&\quad +\lambda^2\cdot(-2\cdot\beta_{23} - 36\cdot\beta_8\cdot\beta_9)\cdot(B^4A^2 + B^2A^4) + \lambda^2\cdot(-2\cdot\beta_{17})\cdot(B^4A^2 - B^2A^4) \\
&\quad +\lambda\cdot(0.25 - 4\cdot\beta_8)\cdot(B^4 + A^4) \\
&\quad +\lambda^2\cdot(-4\cdot\beta_{24})\cdot(B^4 + A^4) + \lambda^2\cdot(-4\cdot\beta_{18})\cdot(B^4 - A^4) \\
&\quad +\lambda\cdot(1 - 2\cdot\beta_9)\cdot(B^3A + BA^3) \\
&\quad +\lambda^2\cdot(-2\cdot\beta_{25} - 108\cdot\beta_8\cdot\beta_9 - 24\cdot\beta_8\cdot\beta_{10})\cdot(B^3A + BA^3) + \lambda^2\cdot(-2\cdot\beta_{19})\cdot(B^3A - BA^3) \\
&\quad +\lambda\cdot(1.5 - 2\cdot\beta_{10})\cdot(B^2 + A^2) \\
&\quad +\lambda^2\cdot(-2\cdot\beta_{26} - 72\cdot\beta_8\cdot\beta_9 - 36\cdot\beta_8\cdot\beta_{10})\cdot(B^2 + A^2) + \lambda^2\cdot(-2\cdot\beta_{20})\cdot(B^2 - A^2) \\
&\quad +\lambda^2\cdot(-64\cdot\beta_8^2 - 16\cdot\beta_9^2)\cdot B^3A^3 \\
&\quad +\lambda\cdot(1.5)\cdot B^2A^2 \\
&\quad +\lambda^2\cdot(-288\cdot\beta_8^2 - 36\cdot\beta_9^2 - 24\cdot\beta_9\cdot\beta_{10})\cdot B^2A^2 \\
&\quad +\lambda\cdot(3)\cdot BA \\
&\quad +\lambda^2\cdot(-384\cdot\beta_8^2 - 12\cdot\beta_9^2 - 24\cdot\beta_9\cdot\beta_{10} - 8\cdot\beta_{10}^2)\cdot BA \\
&\quad +\lambda\cdot(0.75) \\
&\quad +\lambda^2\cdot(-96\cdot\beta_8^2 - 4\cdot\beta_{10}^2)
\end{aligned}
$$