Anharmonic Group Elements as Generated by Machine

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\begin{aligned}
[-X, H_0] \quad = \quad & \lambda^2 \cdot (6 \cdot \beta_{21}) \cdot (B^6 + A^6) + \lambda^2 \cdot (6 \cdot \beta_{15}) \cdot (B^6 - A^6) \\
& + \lambda^2 \cdot (4 \cdot \beta_{22}) \cdot (B^5 A + B A^5) + \lambda^2 \cdot (4 \cdot \beta_{16}) \cdot (B^5 A - B A^5) \\
& + \lambda^2 \cdot (2 \cdot \beta_{23}) \cdot (B^4 A^2 + B^2 A^4) + \lambda^2 \cdot (2 \cdot \beta_{17}) \cdot (B^4 A^2 - B^2 A^4) \\
& + \lambda \cdot (4 \cdot \beta_8) \cdot (B^4 + A^4) + \lambda \cdot (4 \cdot \beta_5) \cdot (B^4 - A^4) \\
& + \lambda^2 \cdot (4 \cdot \beta_{24}) \cdot (B^4 + A^4) + \lambda^2 \cdot (4 \cdot \beta_{18}) \cdot (B^4 - A^4) \\
& + \lambda \cdot (2 \cdot \beta_9) \cdot (B^3 A + B A^3) + \lambda \cdot (2 \cdot \beta_6) \cdot (B^3 A - B A^3) \\
& + \lambda^2 \cdot (2 \cdot \beta_{25}) \cdot (B^3 A + B A^3) + \lambda^2 \cdot (2 \cdot \beta_{19}) \cdot (B^3 A - B A^3) \\
& + \lambda \cdot (2 \cdot \beta_{10}) \cdot (B^2 + A^2) + \lambda \cdot (2 \cdot \beta_7) \cdot (B^2 - A^2) \\
& + \lambda^2 \cdot (2 \cdot \beta_{26}) \cdot (B^2 + A^2) + \lambda^2 \cdot (2 \cdot \beta_{20}) \cdot (B^2 - A^2)
\end{aligned}
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\begin{aligned}
\tfrac{1}{2!}[-X, [-X, H_0]] \quad = \quad & \lambda^2 \cdot (-4 \cdot \beta_5 \cdot \beta_6 - 4 \cdot \beta_8 \cdot \beta_9 - 18 \cdot \beta_1 \cdot \beta_{15}) \cdot (B^6 + A^6) + \lambda^2 \cdot (-4 \cdot \beta_5 \cdot \beta_9 - 4 \cdot \beta_6 \cdot \beta_8 - 18 \cdot \beta_1 \cdot \\
& + \lambda^2 \cdot (-8 \cdot \beta_1 \cdot \beta_{16} - 16 \cdot \beta_4 \cdot \beta_5) \cdot (B^5 A + B A^5) + \lambda^2 \cdot (-8 \cdot \beta_1 \cdot \beta_{22} - 16 \cdot \beta_4 \cdot \beta_8) \cdot (B^5 A - B \\
& + \lambda^2 \cdot (-2 \cdot \beta_1 \cdot \beta_{17} - 4 \cdot \beta_4 \cdot \beta_6 - 36 \cdot \beta_5 \cdot \beta_6 + 36 \cdot \beta_8 \cdot \beta_9) \cdot (B^4 A^2 + B^2 A^4) + \lambda^2 \cdot (-2 \cdot \beta_1 \cdot \beta_2 \\
& + \lambda \cdot (-8 \cdot \beta_1 \cdot \beta_5) \cdot (B^4 + A^4) + \lambda \cdot (-8 \cdot \beta_1 \cdot \beta_8) \cdot (B^4 - A^4) \\
& + \lambda^2 \cdot (-8 \cdot \beta_1 \cdot \beta_{18} - 8 \cdot \beta_3 \cdot \beta_5 - 24 \cdot \beta_4 \cdot \beta_5) \cdot (B^4 + A^4) + \lambda^2 \cdot (-8 \cdot \beta_1 \cdot \beta_{24} - 8 \cdot \beta_3 \cdot \beta_8 - 24 \cdot \\
& + \lambda \cdot (-2 \cdot \beta_1 \cdot \beta_6) \cdot (B^3 A + B A^3) + \lambda \cdot (-2 \cdot \beta_1 \cdot \beta_9) \cdot (B^3 A - B A^3) \\
& + \lambda^2 \cdot (-2 \cdot \beta_3 \cdot \beta_6 - 108 \cdot \beta_5 \cdot \beta_6 + 108 \cdot \beta_8 \cdot \beta_9 - 2 \cdot \beta_1 \cdot \beta_{19} - 24 \cdot \beta_5 \cdot \beta_7 + 24 \cdot \beta_8 \cdot \beta_{10} - 6 \cdot \beta_4 \cdot \\
& + \lambda^2 \cdot (-72 \cdot \beta_5 \cdot \beta_6 + 72 \cdot \beta_8 \cdot \beta_9 - 36 \cdot \beta_5 \cdot \beta_7 + 36 \cdot \beta_8 \cdot \beta_{10} - 2 \cdot \beta_1 \cdot \beta_{20} - 2 \cdot \beta_3 \cdot \beta_7 - 2 \cdot \beta_4 \cdot \beta_7 \\
& + \lambda \cdot (-2 \cdot \beta_1 \cdot \beta_7) \cdot (B^2 + A^2) + \lambda \cdot (-2 \cdot \beta_1 \cdot \beta_{10}) \cdot (B^2 - A^2) \\
& + \lambda^2 \cdot (-64 \cdot \beta_5^2 + 64 \cdot \beta_8^2 - 16 \cdot \beta_6^2 + 16 \cdot \beta_9^2) \cdot B^3 A^3 \\
& + \lambda^2 \cdot (-288 \cdot \beta_5^2 + 288 \cdot \beta_8^2 - 36 \cdot \beta_6^2 - 24 \cdot \beta_6 \cdot \beta_7 + 36 \cdot \beta_9^2 + 24 \cdot \beta_9 \cdot \beta_{10}) \cdot B^2 A^2 \\
& + \lambda^2 \cdot (-384 \cdot \beta_5^2 + 384 \cdot \beta_8^2 - 12 \cdot \beta_6^2 - 24 \cdot \beta_6 \cdot \beta_7 + 12 \cdot \beta_9^2 + 24 \cdot \beta_9 \cdot \beta_{10} - 8 \cdot \beta_7^2 + 8 \cdot \beta_{10}^2) \cdot B \\
& + \lambda^2 \cdot (-96 \cdot \beta_5^2 + 96 \cdot \beta_8^2 - 4 \cdot \beta_7^2 + 4 \cdot \beta_{10}^2)
\end{aligned}
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\begin{aligned}
H_4 - U^\dagger H_0 U &= \Lambda_4 \\
&= \tfrac{\lambda}{4}(A+B)^4 - \big([-X,H_0] + \tfrac{1}{2!}[-X,[-X,H_0]]\big) \\
&= \lambda^2\cdot(-6\cdot\beta_{21} + 4\cdot\beta_5\cdot\beta_6 + 4\cdot\beta_8\cdot\beta_9 + 18\cdot\beta_1\cdot\beta_{15})\cdot(B^6 + A^6) \\
&\quad +\lambda^2\cdot(-6\cdot\beta_{15} + 4\cdot\beta_5\cdot\beta_9 + 4\cdot\beta_6\cdot\beta_8 + 18\cdot\beta_1\cdot\beta_{21})\cdot(B^6 - A^6) \\
&\quad +\lambda^2\cdot(-4\cdot\beta_{22} + 8\cdot\beta_1\cdot\beta_{16} + 16\cdot\beta_4\cdot\beta_5)\cdot(B^5A + BA^5) \\
&\quad +\lambda^2\cdot(-4\cdot\beta_{16} + 8\cdot\beta_1\cdot\beta_{22} + 16\cdot\beta_4\cdot\beta_8)\cdot(B^5A - BA^5) \\
&\quad +\lambda^2\cdot(-2\cdot\beta_{23} + 2\cdot\beta_1\cdot\beta_{17} + 4\cdot\beta_4\cdot\beta_6 + 36\cdot\beta_5\cdot\beta_6 - 36\cdot\beta_8\cdot\beta_9)\cdot(B^4A^2 + B^2A^4) \\
&\quad +\lambda^2\cdot(-2\cdot\beta_{17} + 2\cdot\beta_1\cdot\beta_{23} + 4\cdot\beta_4\cdot\beta_9 - 36\cdot\beta_5\cdot\beta_9 + 36\cdot\beta_6\cdot\beta_8)\cdot(B^4A^2 - B^2A^4) \\
&\quad +\lambda\cdot(0.25 - 4\cdot\beta_8 + 8\cdot\beta_1\cdot\beta_5)\cdot(B^4 + A^4) \\
&\quad +\lambda\cdot(-4\cdot\beta_5 + 8\cdot\beta_1\cdot\beta_8)\cdot(B^4 - A^4) \\
&\quad +\lambda^2\cdot(-4\cdot\beta_{24} + 8\cdot\beta_1\cdot\beta_{18} + 8\cdot\beta_3\cdot\beta_5 + 24\cdot\beta_4\cdot\beta_5)\cdot(B^4 + A^4) \\
&\quad +\lambda^2\cdot(-4\cdot\beta_{18} + 8\cdot\beta_1\cdot\beta_{24} + 8\cdot\beta_3\cdot\beta_8 + 24\cdot\beta_4\cdot\beta_8)\cdot(B^4 - A^4) \\
&\quad +\lambda\cdot(-2\cdot\beta_9 + 2\cdot\beta_1\cdot\beta_6)\cdot(B^3A + BA^3) \\
&\quad +\lambda\cdot(-2\cdot\beta_6 + 2\cdot\beta_1\cdot\beta_9)\cdot(B^3A - BA^3) \\
&\quad +\lambda^2\cdot(-2\cdot\beta_{25} + 2\cdot\beta_3\cdot\beta_6 + 108\cdot\beta_5\cdot\beta_6 - 108\cdot\beta_8\cdot\beta_9 + 2\cdot\beta_1\cdot\beta_{19} + 24\cdot\beta_5\cdot\beta_7 - 24\cdot\beta_8\cdot\beta_{10} + 6 \\
&\quad +\lambda^2\cdot(-2\cdot\beta_{19} + 2\cdot\beta_3\cdot\beta_9 - 108\cdot\beta_5\cdot\beta_9 + 108\cdot\beta_6\cdot\beta_8 + 2\cdot\beta_1\cdot\beta_{25} - 24\cdot\beta_5\cdot\beta_{10} + 24\cdot\beta_7\cdot\beta_8 + 6 \\
&\quad +\lambda\cdot(1.5 - 2\cdot\beta_{10} + 2\cdot\beta_1\cdot\beta_7)\cdot(B^2 + A^2) \\
&\quad +\lambda\cdot(-2\cdot\beta_7 + 2\cdot\beta_1\cdot\beta_{10})\cdot(B^2 - A^2) \\
&\quad +\lambda^2\cdot(-2\cdot\beta_{26} + 72\cdot\beta_5\cdot\beta_6 - 72\cdot\beta_8\cdot\beta_9 + 36\cdot\beta_5\cdot\beta_7 - 36\cdot\beta_8\cdot\beta_{10} + 2\cdot\beta_1\cdot\beta_{20} + 2\cdot\beta_3\cdot\beta_7 + 2\cdot\beta \\
&\quad +\lambda^2\cdot(-2\cdot\beta_{20} - 72\cdot\beta_5\cdot\beta_9 + 72\cdot\beta_6\cdot\beta_8 - 36\cdot\beta_5\cdot\beta_{10} + 36\cdot\beta_7\cdot\beta_8 + 2\cdot\beta_1\cdot\beta_{26} + 2\cdot\beta_3\cdot\beta_{10} + 2\cdot\beta \\
&\quad +\lambda^2\cdot(64\cdot\beta_5^2 - 64\cdot\beta_8^2 + 16\cdot\beta_6^2 - 16\cdot\beta_9^2)\cdot B^3A^3 \\
&\quad +\lambda\cdot(1.5)\cdot B^2A^2 \\
&\quad +\lambda^2\cdot(288\cdot\beta_5^2 - 288\cdot\beta_8^2 + 36\cdot\beta_6^2 + 24\cdot\beta_6\cdot\beta_7 - 36\cdot\beta_9^2 - 24\cdot\beta_9\cdot\beta_{10})\cdot B^2A^2 \\
&\quad +\lambda\cdot(3)\cdot BA \\
&\quad +\lambda^2\cdot(384\cdot\beta_5^2 - 384\cdot\beta_8^2 + 12\cdot\beta_6^2 + 24\cdot\beta_6\cdot\beta_7 - 12\cdot\beta_9^2 - 24\cdot\beta_9\cdot\beta_{10} + 8\cdot\beta_7^2 - 8\cdot\beta_{10}^2)\cdot BA \\
&\quad +\lambda\cdot(0.75) \\
&\quad +\lambda^2\cdot(96\cdot\beta_5^2 - 96\cdot\beta_8^2 + 4\cdot\beta_7^2 - 4\cdot\beta_{10}^2)
\end{aligned}
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