Anharmonic Group Elements as Generated by Machine

Ed Rogers

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$$\begin{aligned}\tfrac{\lambda}{4}(A+B)^4 \quad = \quad & \lambda\cdot(0.25)\cdot(B^4+A^4)\\ & +\lambda\cdot(B^3A+BA^3)\\ & +\lambda\cdot(1.5)\cdot(B^2+A^2)\\ & +\lambda\cdot(1.5)\cdot B^2A^2\\ & +\lambda\cdot(3)\cdot BA\\ & +\lambda\cdot(0.75)\end{aligned}$$

$$
\begin{aligned}
[-X, H_0] \quad = \quad & \lambda^3 \cdot (8 \cdot \gamma_{42}) \cdot (B^8 + A^8) + \lambda^3 \cdot (8 \cdot \gamma_{32}) \cdot (B^8 - A^8) \\
& + \lambda^3 \cdot (6 \cdot \gamma_{43}) \cdot (B^7 A + B A^7) + \lambda^3 \cdot (6 \cdot \gamma_{33}) \cdot (B^7 A - B A^7) \\
& + \lambda^3 \cdot (4 \cdot \gamma_{44}) \cdot (B^6 A^2 + B^2 A^6) + \lambda^3 \cdot (4 \cdot \gamma_{34}) \cdot (B^6 A^2 - B^2 A^6) \\
& + \lambda^3 \cdot (2 \cdot \gamma_{45}) \cdot (B^5 A^3 + B^3 A^5) + \lambda^3 \cdot (2 \cdot \gamma_{35}) \cdot (B^5 A^3 - B^3 A^5) \\
& + \lambda^2 \cdot (6 \cdot \gamma_{21}) \cdot (B^6 + A^6) + \lambda^2 \cdot (6 \cdot \gamma_{15}) \cdot (B^6 - A^6) \\
& + \lambda^3 \cdot (6 \cdot \gamma_{46}) \cdot (B^6 + A^6) + \lambda^3 \cdot (6 \cdot \gamma_{36}) \cdot (B^6 - A^6) \\
& + \lambda^2 \cdot (4 \cdot \gamma_{22}) \cdot (B^5 A + B A^5) + \lambda^2 \cdot (4 \cdot \gamma_{16}) \cdot (B^5 A - B A^5) \\
& + \lambda^3 \cdot (4 \cdot \gamma_{47}) \cdot (B^5 A + B A^5) + \lambda^3 \cdot (4 \cdot \gamma_{37}) \cdot (B^5 A - B A^5) \\
& + \lambda^2 \cdot (2 \cdot \gamma_{23}) \cdot (B^4 A^2 + B^2 A^4) + \lambda^2 \cdot (2 \cdot \gamma_{17}) \cdot (B^4 A^2 - B^2 A^4) \\
& + \lambda^3 \cdot (2 \cdot \gamma_{48}) \cdot (B^4 A^2 + B^2 A^4) + \lambda^3 \cdot (2 \cdot \gamma_{38}) \cdot (B^4 A^2 - B^2 A^4) \\
& + \lambda \cdot (4 \cdot \gamma_8) \cdot (B^4 + A^4) + \lambda \cdot (4 \cdot \gamma_5) \cdot (B^4 - A^4) \\
& + \lambda^2 \cdot (4 \cdot \gamma_{24}) \cdot (B^4 + A^4) + \lambda^2 \cdot (4 \cdot \gamma_{18}) \cdot (B^4 - A^4) \\
& + \lambda^3 \cdot (4 \cdot \gamma_{49}) \cdot (B^4 + A^4) + \lambda^3 \cdot (4 \cdot \gamma_{39}) \cdot (B^4 - A^4) \\
& + \lambda \cdot (2 \cdot \gamma_9) \cdot (B^3 A + B A^3) + \lambda \cdot (2 \cdot \gamma_6) \cdot (B^3 A - B A^3) \\
& + \lambda^2 \cdot (2 \cdot \gamma_{25}) \cdot (B^3 A + B A^3) + \lambda^2 \cdot (2 \cdot \gamma_{19}) \cdot (B^3 A - B A^3) \\
& + \lambda^3 \cdot (2 \cdot \gamma_{50}) \cdot (B^3 A + B A^3) + \lambda^3 \cdot (2 \cdot \gamma_{40}) \cdot (B^3 A - B A^3) \\
& + \lambda \cdot (2 \cdot \gamma_{10}) \cdot (B^2 + A^2) + \lambda \cdot (2 \cdot \gamma_7) \cdot (B^2 - A^2) \\
& + \lambda^2 \cdot (2 \cdot \gamma_{26}) \cdot (B^2 + A^2) + \lambda^2 \cdot (2 \cdot \gamma_{20}) \cdot (B^2 - A^2) \\
& + \lambda^3 \cdot (2 \cdot \gamma_{51}) \cdot (B^2 + A^2) + \lambda^3 \cdot (2 \cdot \gamma_{41}) \cdot (B^2 - A^2)
\end{aligned}
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\begin{aligned}
\tfrac{1}{2!}[-X,[-X,H_0]] \quad = \quad & \lambda^3\cdot(-12\cdot\gamma_6\cdot\gamma_{15}-12\cdot\gamma_9\cdot\gamma_{21}-32\cdot\gamma_1\cdot\gamma_{32})\cdot(B^8+A^8)+\lambda^3\cdot(-12\cdot\gamma_9\cdot\gamma_{15}-12\cdot\gamma_6\cdot\gamma_{21}- \\
& +\lambda^3\cdot(-18\cdot\gamma_1\cdot\gamma_{33}-2\cdot\gamma_6\cdot\gamma_{16}-2\cdot\gamma_9\cdot\gamma_{22}-8\cdot\gamma_5\cdot\gamma_{17}-8\cdot\gamma_8\cdot\gamma_{23}-36\cdot\gamma_4\cdot\gamma_{15})\cdot(B^7A+ \\
& +\lambda^3\cdot(-8\cdot\gamma_1\cdot\gamma_{34}-16\cdot\gamma_4\cdot\gamma_{16}-72\cdot\gamma_6\cdot\gamma_{15}+72\cdot\gamma_9\cdot\gamma_{21}-24\cdot\gamma_5\cdot\gamma_{14})\cdot(B^6A^2+B^2A^6)- \\
& +\lambda^3\cdot(-2\cdot\gamma_1\cdot\gamma_{35}-42\cdot\gamma_6\cdot\gamma_{16}+42\cdot\gamma_9\cdot\gamma_{22}-4\cdot\gamma_4\cdot\gamma_{17}-48\cdot\gamma_5\cdot\gamma_{17}+48\cdot\gamma_8\cdot\gamma_{23}-120\cdot\gamma \\
& +\lambda^2\cdot(-4\cdot\gamma_5\cdot\gamma_6-4\cdot\gamma_8\cdot\gamma_9-18\cdot\gamma_1\cdot\gamma_{15})\cdot(B^6+A^6)+\lambda^2\cdot(-4\cdot\gamma_5\cdot\gamma_9-4\cdot\gamma_6\cdot\gamma_8-18\cdot\gamma_1 \\
& +\lambda^3\cdot(2\cdot\gamma_7\cdot\gamma_{16}+2\cdot\gamma_{10}\cdot\gamma_{22}-12\cdot\gamma_5\cdot\gamma_{17}-12\cdot\gamma_8\cdot\gamma_{23}-4\cdot\gamma_6\cdot\gamma_{18}-4\cdot\gamma_9\cdot\gamma_{24}-4\cdot\gamma_5\cdot\gamma_{19} \\
& +\lambda^2\cdot(-8\cdot\gamma_1\cdot\gamma_{16}-16\cdot\gamma_4\cdot\gamma_5)\cdot(B^5A+BA^5)+\lambda^2\cdot(-8\cdot\gamma_1\cdot\gamma_{22}-16\cdot\gamma_4\cdot\gamma_8)\cdot(B^5A-B \\
& +\lambda^3\cdot(-8\cdot\gamma_3\cdot\gamma_{16}-8\cdot\gamma_1\cdot\gamma_{37}-360\cdot\gamma_6\cdot\gamma_{15}+360\cdot\gamma_9\cdot\gamma_{21}-48\cdot\gamma_7\cdot\gamma_{15}+48\cdot\gamma_{10}\cdot\gamma_{21}-40 \\
& +\lambda^2\cdot(-2\cdot\gamma_1\cdot\gamma_{17}-4\cdot\gamma_4\cdot\gamma_6-36\cdot\gamma_5\cdot\gamma_6+36\cdot\gamma_8\cdot\gamma_9)\cdot(B^4A^2+B^2A^4)+\lambda^2\cdot(-2\cdot\gamma_1\cdot\gamma_{23} \\
& +\lambda^3\cdot(-2\cdot\gamma_3\cdot\gamma_{17}-10\cdot\gamma_4\cdot\gamma_{17}-216\cdot\gamma_5\cdot\gamma_{17}+216\cdot\gamma_8\cdot\gamma_{23}-2\cdot\gamma_1\cdot\gamma_{38}-900\cdot\gamma_5\cdot\gamma_{15}+90 \\
& +\lambda^3\cdot(-480\cdot\gamma_6\cdot\gamma_{15}+480\cdot\gamma_9\cdot\gamma_{21}-120\cdot\gamma_7\cdot\gamma_{15}+120\cdot\gamma_{10}\cdot\gamma_{21}-8\cdot\gamma_1\cdot\gamma_{39}-8\cdot\gamma_3\cdot\gamma_{18}- \\
& +\lambda\cdot(-8\cdot\gamma_1\cdot\gamma_5)\cdot(B^4+A^4)+\lambda\cdot(-8\cdot\gamma_1\cdot\gamma_8)\cdot(B^4-A^4) \\
& +\lambda^2\cdot(-8\cdot\gamma_1\cdot\gamma_{18}-8\cdot\gamma_3\cdot\gamma_5-24\cdot\gamma_4\cdot\gamma_5)\cdot(B^4+A^4)+\lambda^2\cdot(-8\cdot\gamma_1\cdot\gamma_{24}-8\cdot\gamma_3\cdot\gamma_8-24\cdot\gamma_ \\
& +\lambda^3\cdot(-288\cdot\gamma_5\cdot\gamma_{17}+288\cdot\gamma_8\cdot\gamma_{23}-2400\cdot\gamma_5\cdot\gamma_{15}+2400\cdot\gamma_8\cdot\gamma_{21}-2\cdot\gamma_6\cdot\gamma_{12}-180\cdot\gamma_6\cdot\gamma_1 \\
& +\lambda\cdot(-2\cdot\gamma_1\cdot\gamma_6)\cdot(B^3A+BA^3)+\lambda\cdot(-2\cdot\gamma_1\cdot\gamma_9)\cdot(B^3A-BA^3) \\
& +\lambda^2\cdot(-2\cdot\gamma_3\cdot\gamma_6-108\cdot\gamma_5\cdot\gamma_6+108\cdot\gamma_8\cdot\gamma_9-2\cdot\gamma_1\cdot\gamma_{19}-24\cdot\gamma_5\cdot\gamma_7+24\cdot\gamma_8\cdot\gamma_{10}-6\cdot\gamma_4\cdot\gamma_6 \\
& +\lambda^3\cdot(-72\cdot\gamma_5\cdot\gamma_{17}+72\cdot\gamma_8\cdot\gamma_{23}-1800\cdot\gamma_5\cdot\gamma_{15}+1800\cdot\gamma_8\cdot\gamma_{21}-72\cdot\gamma_6\cdot\gamma_{18}+72\cdot\gamma_9\cdot\gamma_{24}+ \\
& +\lambda^2\cdot(-72\cdot\gamma_5\cdot\gamma_6+72\cdot\gamma_8\cdot\gamma_9-36\cdot\gamma_5\cdot\gamma_7+36\cdot\gamma_8\cdot\gamma_{10}-2\cdot\gamma_1\cdot\gamma_{20}-2\cdot\gamma_3\cdot\gamma_7-2\cdot\gamma_4\cdot\gamma_7)\cdot \\
& +\lambda\cdot(-2\cdot\gamma_1\cdot\gamma_7)\cdot(B^2+A^2)+\lambda\cdot(-2\cdot\gamma_1\cdot\gamma_{10})\cdot(B^2-A^2) \\
& +\lambda^3\cdot(-160\cdot\gamma_5\cdot\gamma_{16}+160\cdot\gamma_8\cdot\gamma_{22}-40\cdot\gamma_6\cdot\gamma_{17}+40\cdot\gamma_9\cdot\gamma_{23})\cdot B^4A^4 \\
& +\lambda^3\cdot(-960\cdot\gamma_5\cdot\gamma_{16}+960\cdot\gamma_8\cdot\gamma_{22}-144\cdot\gamma_6\cdot\gamma_{17}-32\cdot\gamma_7\cdot\gamma_{17}+144\cdot\gamma_9\cdot\gamma_{23}+32\cdot\gamma_{10}\cdot\gamma_{23} \\
& +\lambda^2\cdot(-64\cdot\gamma_5^2+64\cdot\gamma_8^2-16\cdot\gamma_6^2+16\cdot\gamma_9^2)\cdot B^3A^3 \\
& +\lambda^3\cdot(-1920\cdot\gamma_5\cdot\gamma_{16}+1920\cdot\gamma_8\cdot\gamma_{22}-96\cdot\gamma_6\cdot\gamma_{17}-48\cdot\gamma_7\cdot\gamma_{17}+96\cdot\gamma_9\cdot\gamma_{23}+48\cdot\gamma_{10}\cdot\gamma_{23} \\
& +\lambda^2\cdot(-288\cdot\gamma_5^2+288\cdot\gamma_8^2-36\cdot\gamma_6^2-24\cdot\gamma_6\cdot\gamma_7+36\cdot\gamma_9^2+24\cdot\gamma_9\cdot\gamma_{10})\cdot B^2A^2 \\
& +\lambda^3\cdot(-960\cdot\gamma_5\cdot\gamma_{16}+960\cdot\gamma_8\cdot\gamma_{22}-768\cdot\gamma_5\cdot\gamma_{18}+768\cdot\gamma_8\cdot\gamma_{24}-24\cdot\gamma_6\cdot\gamma_{19}-24\cdot\gamma_6\cdot\gamma_{20}- \\
& +\lambda^2\cdot(-384\cdot\gamma_5^2+384\cdot\gamma_8^2-12\cdot\gamma_6^2-24\cdot\gamma_6\cdot\gamma_7+12\cdot\gamma_9^2+24\cdot\gamma_9\cdot\gamma_{10}-8\cdot\gamma_7^2+8\cdot\gamma_{10}^2)\cdot B \\
& +\lambda^2\cdot(-96\cdot\gamma_5^2+96\cdot\gamma_8^2-4\cdot\gamma_7^2+4\cdot\gamma_{10}^2) \\
& +\lambda^3\cdot(-192\cdot\gamma_5\cdot\gamma_{18}+192\cdot\gamma_8\cdot\gamma_{24}-8\cdot\gamma_7\cdot\gamma_{20}+8\cdot\gamma_{10}\cdot\gamma_{26})
\end{aligned}
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\begin{aligned}
[-X,[-X,[-X,H_0]]] \quad = \quad & \lambda^3\cdot(-256\cdot\gamma_4\cdot\gamma_5\cdot\gamma_8+384\cdot\gamma_1\cdot\gamma_9\cdot\gamma_{15}+384\cdot\gamma_1\cdot\gamma_6\cdot\gamma_{21}+512\cdot\gamma_1^2\cdot\gamma_{42}+96\cdot\gamma_5\cdot\gamma_6\cdot\gamma_9 \\
& +\lambda^3\cdot(216\cdot\gamma_1^2\cdot\gamma_{43}+48\cdot\gamma_1\cdot\gamma_6\cdot\gamma_{22}+48\cdot\gamma_1\cdot\gamma_9\cdot\gamma_{16}+192\cdot\gamma_1\cdot\gamma_5\cdot\gamma_{23}+192\cdot\gamma_1\cdot\gamma_8\cdot\gamma_{17} \\
& +\lambda^3\cdot(-1536\cdot\gamma_5^2\cdot\gamma_8+1536\cdot\gamma_8^3-384\cdot\gamma_6^2\cdot\gamma_8+384\cdot\gamma_8\cdot\gamma_9^2+64\cdot\gamma_1^2\cdot\gamma_{44}+256\cdot\gamma_1\cdot\gamma_4 \\
& +\lambda^3\cdot(-1728\cdot\gamma_5^2\cdot\gamma_9+1728\cdot\gamma_8^2\cdot\gamma_9-192\cdot\gamma_6^2\cdot\gamma_9+192\cdot\gamma_9^3+8\cdot\gamma_1^2\cdot\gamma_{45}+336\cdot\gamma_1\cdot\gamma_6\cdot\gamma \\
& +\lambda^2\cdot(96\cdot\gamma_1\cdot\gamma_5\cdot\gamma_9+96\cdot\gamma_1\cdot\gamma_6\cdot\gamma_8+216\cdot\gamma_1^2\cdot\gamma_{21})\cdot(B^6+A^6)+\lambda^2\cdot(96\cdot\gamma_1\cdot\gamma_5\cdot\gamma_6+9 \\
& +\lambda^3\cdot(-48\cdot\gamma_1\cdot\gamma_7\cdot\gamma_{22}-96\cdot\gamma_4\cdot\gamma_7\cdot\gamma_8-48\cdot\gamma_1\cdot\gamma_{10}\cdot\gamma_{16}-96\cdot\gamma_4\cdot\gamma_5\cdot\gamma_{10}+288\cdot\gamma_1\cdot\gamma_5\cdot\gamma \\
& +\lambda^3\cdot(-9216\cdot\gamma_5^2\cdot\gamma_8+9216\cdot\gamma_8^3-1440\cdot\gamma_6^2\cdot\gamma_8+1440\cdot\gamma_8\cdot\gamma_9^2-576\cdot\gamma_6\cdot\gamma_7\cdot\gamma_8+576\cdot \\
& +\lambda^2\cdot(64\cdot\gamma_1^2\cdot\gamma_{22}+256\cdot\gamma_1\cdot\gamma_4\cdot\gamma_8)\cdot(B^5A+BA^5)+\lambda^2\cdot(64\cdot\gamma_1^2\cdot\gamma_{16}+256\cdot\gamma_1\cdot\gamma_4\cdot\gamma_5) \\
& +\lambda^3\cdot(-10944\cdot\gamma_5^2\cdot\gamma_9+10944\cdot\gamma_8^2\cdot\gamma_9-864\cdot\gamma_6^2\cdot\gamma_9+864\cdot\gamma_9^3-1344\cdot\gamma_5^2\cdot\gamma_{10}+1344\cdot \\
& +\lambda^2\cdot(8\cdot\gamma_1^2\cdot\gamma_{23}+32\cdot\gamma_1\cdot\gamma_4\cdot\gamma_9-288\cdot\gamma_1\cdot\gamma_5\cdot\gamma_9+288\cdot\gamma_1\cdot\gamma_6\cdot\gamma_8)\cdot(B^4A^2+B^2A^4)+ \\
& +\lambda^3\cdot(-13056\cdot\gamma_5^2\cdot\gamma_8+13056\cdot\gamma_8^3-480\cdot\gamma_6^2\cdot\gamma_8+480\cdot\gamma_8\cdot\gamma_9^2-960\cdot\gamma_6\cdot\gamma_7\cdot\gamma_8+960\cdot \\
& +\lambda\cdot(64\cdot\gamma_1^2\cdot\gamma_8)\cdot(B^4+A^4)+\lambda\cdot(64\cdot\gamma_1^2\cdot\gamma_5)\cdot(B^4-A^4) \\
& +\lambda^2\cdot(64\cdot\gamma_1^2\cdot\gamma_{24}+128\cdot\gamma_1\cdot\gamma_3\cdot\gamma_8+384\cdot\gamma_1\cdot\gamma_4\cdot\gamma_8)\cdot(B^4+A^4)+\lambda^2\cdot(64\cdot\gamma_1^2\cdot\gamma_{18}+12 \\
& +\lambda^3\cdot(-17760\cdot\gamma_5^2\cdot\gamma_9+17760\cdot\gamma_8^2\cdot\gamma_9-672\cdot\gamma_6^2\cdot\gamma_9+672\cdot\gamma_9^3-5376\cdot\gamma_5^2\cdot\gamma_{10}+5376\cdot \\
& +\lambda\cdot(8\cdot\gamma_1^2\cdot\gamma_9)\cdot(B^3A+BA^3)+\lambda\cdot(8\cdot\gamma_1^2\cdot\gamma_6)\cdot(B^3A-BA^3) \\
& +\lambda^2\cdot(16\cdot\gamma_1\cdot\gamma_3\cdot\gamma_9-864\cdot\gamma_1\cdot\gamma_5\cdot\gamma_9+864\cdot\gamma_1\cdot\gamma_6\cdot\gamma_8+8\cdot\gamma_1^2\cdot\gamma_{25}-192\cdot\gamma_1\cdot\gamma_5\cdot\gamma_{10}+1 \\
& +\lambda^3\cdot(-4704\cdot\gamma_5^2\cdot\gamma_{10}+4704\cdot\gamma_8^2\cdot\gamma_{10}-192\cdot\gamma_6^2\cdot\gamma_{10}-192\cdot\gamma_6\cdot\gamma_7\cdot\gamma_{10}+192\cdot\gamma_9^2\cdot\gamma_{10}+ \\
& +\lambda^2\cdot(-576\cdot\gamma_1\cdot\gamma_5\cdot\gamma_9+576\cdot\gamma_1\cdot\gamma_6\cdot\gamma_8-288\cdot\gamma_1\cdot\gamma_5\cdot\gamma_{10}+288\cdot\gamma_1\cdot\gamma_7\cdot\gamma_8+8\cdot\gamma_1^2\cdot\gamma_{26}- \\
& +\lambda\cdot(8\cdot\gamma_1^2\cdot\gamma_{10})\cdot(B^2+A^2)+\lambda\cdot(8\cdot\gamma_1^2\cdot\gamma_7)\cdot(B^2-A^2) \\
& +\lambda^3\cdot(-2880\cdot\gamma_5\cdot\gamma_6\cdot\gamma_9+1440\cdot\gamma_6^2\cdot\gamma_8+1440\cdot\gamma_8\cdot\gamma_9^2)\cdot B^4A^4 \\
& +\lambda^3\cdot(8640\cdot\gamma_6^2\cdot\gamma_8-17280\cdot\gamma_5\cdot\gamma_6\cdot\gamma_9+1920\cdot\gamma_6\cdot\gamma_7\cdot\gamma_8-1920\cdot\gamma_5\cdot\gamma_7\cdot\gamma_9+8640\cdot\gamma_8\cdot \\
& +\lambda^3\cdot(12960\cdot\gamma_6^2\cdot\gamma_8-25920\cdot\gamma_5\cdot\gamma_6\cdot\gamma_9+6912\cdot\gamma_6\cdot\gamma_7\cdot\gamma_8-6912\cdot\gamma_5\cdot\gamma_7\cdot\gamma_9+12960\cdot\gamma \\
& +\lambda^3\cdot(4320\cdot\gamma_6^2\cdot\gamma_8+5184\cdot\gamma_6\cdot\gamma_7\cdot\gamma_8-8640\cdot\gamma_5\cdot\gamma_6\cdot\gamma_9-5184\cdot\gamma_5\cdot\gamma_6\cdot\gamma_{10}+1152\cdot\gamma_7^2\cdot \\
& +\lambda^3\cdot(576\cdot\gamma_6\cdot\gamma_7\cdot\gamma_8+288\cdot\gamma_7^2\cdot\gamma_8-576\cdot\gamma_5\cdot\gamma_7\cdot\gamma_9-576\cdot\gamma_5\cdot\gamma_7\cdot\gamma_{10}-576\cdot\gamma_5\cdot\gamma_6\cdot\gamma_{10}
\end{aligned}
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$$6 \cdot \left([-X,H_0] + \tfrac{1}{2!}[-X,[-X,H_0]] + \tfrac{1}{3!}[-X,[-X,[-X,H_0]]]\right)$$

$$\begin{aligned}
= \quad & \lambda^3\cdot(48\cdot\gamma_{42} - 72\cdot\gamma_6\cdot\gamma_{15} - 72\cdot\gamma_9\cdot\gamma_{21} - 192\cdot\gamma_1\cdot\gamma_{32} - 256\cdot\gamma_4\cdot\gamma_5\cdot\gamma_8 + 384\cdot\gamma_1\cdot\gamma_9\cdot\gamma_{15} + 384\cdot\gamma_1\cdot\gamma_6\cdot\gamma_{21} + 51 \\
& +\lambda^3\cdot(36\cdot\gamma_{43} - 108\cdot\gamma_1\cdot\gamma_{33} - 12\cdot\gamma_6\cdot\gamma_{16} - 12\cdot\gamma_9\cdot\gamma_{22} - 48\cdot\gamma_5\cdot\gamma_{17} - 48\cdot\gamma_8\cdot\gamma_{23} - 216\cdot\gamma_4\cdot\gamma_{15} + 216\cdot\gamma_1^2\cdot\gamma_{43} \\
& +\lambda^3\cdot(24\cdot\gamma_{44} - 48\cdot\gamma_1\cdot\gamma_{34} - 96\cdot\gamma_4\cdot\gamma_{16} - 432\cdot\gamma_6\cdot\gamma_{15} + 432\cdot\gamma_9\cdot\gamma_{21} - 144\cdot\gamma_5\cdot\gamma_{14} - 1536\cdot\gamma_5^2\cdot\gamma_8 + 1536\cdot\gamma_8^3 \\
& +\lambda^3\cdot(12\cdot\gamma_{45} - 12\cdot\gamma_1\cdot\gamma_{35} - 252\cdot\gamma_6\cdot\gamma_{16} + 252\cdot\gamma_9\cdot\gamma_{22} - 24\cdot\gamma_4\cdot\gamma_{17} - 288\cdot\gamma_5\cdot\gamma_{17} + 288\cdot\gamma_8\cdot\gamma_{23} - 720\cdot\gamma_5\cdot\gamma \\
& +\lambda^2\cdot(36\cdot\gamma_{21} - 24\cdot\gamma_5\cdot\gamma_6 - 24\cdot\gamma_8\cdot\gamma_9 - 108\cdot\gamma_1\cdot\gamma_{15} + 96\cdot\gamma_1\cdot\gamma_5\cdot\gamma_9 + 96\cdot\gamma_1\cdot\gamma_6\cdot\gamma_8 + 216\cdot\gamma_1^2\cdot\gamma_{21})\cdot(B^6 + A^6 \\
& +\lambda^3\cdot(36\cdot\gamma_{46} + 12\cdot\gamma_7\cdot\gamma_{16} + 12\cdot\gamma_{10}\cdot\gamma_{22} - 72\cdot\gamma_5\cdot\gamma_{17} - 72\cdot\gamma_8\cdot\gamma_{23} - 24\cdot\gamma_6\cdot\gamma_{18} - 24\cdot\gamma_9\cdot\gamma_{24} - 24\cdot\gamma_5\cdot\gamma_{19} - \\
& +\lambda^2\cdot(24\cdot\gamma_{22} - 48\cdot\gamma_1\cdot\gamma_{16} - 96\cdot\gamma_4\cdot\gamma_5 + 64\cdot\gamma_1^2\cdot\gamma_{22} + 256\cdot\gamma_1\cdot\gamma_4\cdot\gamma_8)\cdot(B^5A + BA^5) + \lambda^2\cdot(24\cdot\gamma_{16} - 48\cdot\gamma_1\cdot \\
& +\lambda^3\cdot(24\cdot\gamma_{47} - 48\cdot\gamma_3\cdot\gamma_{16} - 48\cdot\gamma_1\cdot\gamma_{37} - 2160\cdot\gamma_6\cdot\gamma_{15} + 2160\cdot\gamma_9\cdot\gamma_{21} - 288\cdot\gamma_7\cdot\gamma_{15} + 288\cdot\gamma_{10}\cdot\gamma_{21} - 240\cdot\gamma \\
& +\lambda^2\cdot(12\cdot\gamma_{23} - 12\cdot\gamma_1\cdot\gamma_{17} - 24\cdot\gamma_4\cdot\gamma_6 - 216\cdot\gamma_5\cdot\gamma_6 + 216\cdot\gamma_8\cdot\gamma_9 + 8\cdot\gamma_1^2\cdot\gamma_{23} + 32\cdot\gamma_1\cdot\gamma_4\cdot\gamma_9 - 288\cdot\gamma_1\cdot\gamma_5\cdot\gamma_9 \\
& +\lambda^3\cdot(12\cdot\gamma_{48} - 12\cdot\gamma_3\cdot\gamma_{17} - 60\cdot\gamma_4\cdot\gamma_{17} - 1296\cdot\gamma_5\cdot\gamma_{17} + 1296\cdot\gamma_8\cdot\gamma_{23} - 12\cdot\gamma_1\cdot\gamma_{38} - 5400\cdot\gamma_5\cdot\gamma_{15} + 5400\cdot \\
& +\lambda\cdot(24\cdot\gamma_8 - 48\cdot\gamma_1\cdot\gamma_5 + 64\cdot\gamma_1^2\cdot\gamma_8)\cdot(B^4 + A^4) + \lambda\cdot(24\cdot\gamma_5 - 48\cdot\gamma_1\cdot\gamma_8 + 64\cdot\gamma_1^2\cdot\gamma_5)\cdot(B^4 - A^4) \\
& +\lambda^2\cdot(24\cdot\gamma_{24} - 48\cdot\gamma_1\cdot\gamma_{18} - 48\cdot\gamma_3\cdot\gamma_5 - 144\cdot\gamma_4\cdot\gamma_5 + 64\cdot\gamma_1^2\cdot\gamma_{24} + 128\cdot\gamma_1\cdot\gamma_3\cdot\gamma_8 + 384\cdot\gamma_1\cdot\gamma_4\cdot\gamma_8)\cdot(B^4 + A \\
& +\lambda^3\cdot(24\cdot\gamma_{49} - 2880\cdot\gamma_6\cdot\gamma_{15} + 2880\cdot\gamma_9\cdot\gamma_{21} - 720\cdot\gamma_7\cdot\gamma_{15} + 720\cdot\gamma_{10}\cdot\gamma_{21} - 48\cdot\gamma_1\cdot\gamma_{39} - 48\cdot\gamma_3\cdot\gamma_{18} - 144\cdot\gamma \\
& +\lambda\cdot(12\cdot\gamma_9 - 12\cdot\gamma_1\cdot\gamma_6 + 8\cdot\gamma_1^2\cdot\gamma_9)\cdot(B^3A + BA^3) + \lambda\cdot(12\cdot\gamma_6 - 12\cdot\gamma_1\cdot\gamma_9 + 8\cdot\gamma_1^2\cdot\gamma_6)\cdot(B^3A - BA^3) \\
& +\lambda^2\cdot(12\cdot\gamma_{25} - 12\cdot\gamma_3\cdot\gamma_6 - 648\cdot\gamma_5\cdot\gamma_6 + 648\cdot\gamma_8\cdot\gamma_9 - 12\cdot\gamma_1\cdot\gamma_{19} - 144\cdot\gamma_5\cdot\gamma_7 + 144\cdot\gamma_8\cdot\gamma_{10} - 36\cdot\gamma_4\cdot\gamma_6 - 2 \\
& +\lambda^3\cdot(12\cdot\gamma_{50} - 1728\cdot\gamma_5\cdot\gamma_{17} + 1728\cdot\gamma_8\cdot\gamma_{23} - 14400\cdot\gamma_5\cdot\gamma_{15} + 14400\cdot\gamma_8\cdot\gamma_{21} - 12\cdot\gamma_6\cdot\gamma_{12} - 1080\cdot\gamma_6\cdot\gamma_{16} - \\
& +\lambda\cdot(12\cdot\gamma_{10} - 12\cdot\gamma_1\cdot\gamma_7 + 8\cdot\gamma_1^2\cdot\gamma_{10})\cdot(B^2 + A^2) + \lambda\cdot(12\cdot\gamma_7 - 12\cdot\gamma_1\cdot\gamma_{10} + 8\cdot\gamma_1^2\cdot\gamma_7)\cdot(B^2 - A^2) \\
& +\lambda^2\cdot(12\cdot\gamma_{26} - 432\cdot\gamma_5\cdot\gamma_6 + 432\cdot\gamma_8\cdot\gamma_9 - 216\cdot\gamma_5\cdot\gamma_7 + 216\cdot\gamma_8\cdot\gamma_{10} - 12\cdot\gamma_1\cdot\gamma_{20} - 12\cdot\gamma_3\cdot\gamma_7 - 12\cdot\gamma_4\cdot\gamma_7 - 5 \\
& +\lambda^3\cdot(12\cdot\gamma_{51} - 432\cdot\gamma_5\cdot\gamma_{17} + 432\cdot\gamma_8\cdot\gamma_{23} - 10800\cdot\gamma_5\cdot\gamma_{15} + 10800\cdot\gamma_8\cdot\gamma_{21} - 432\cdot\gamma_6\cdot\gamma_{18} + 432\cdot\gamma_9\cdot\gamma_{24} - 4 \\
& +\lambda^3\cdot(-960\cdot\gamma_5\cdot\gamma_{16} + 960\cdot\gamma_8\cdot\gamma_{22} - 240\cdot\gamma_6\cdot\gamma_{17} + 240\cdot\gamma_9\cdot\gamma_{23} - 2880\cdot\gamma_5\cdot\gamma_6\cdot\gamma_9 + 1440\cdot\gamma_6^2\cdot\gamma_8 + 1440\cdot\gamma_8\cdot\gamma \\
& +\lambda^3\cdot(-5760\cdot\gamma_5\cdot\gamma_{16} + 5760\cdot\gamma_8\cdot\gamma_{22} - 864\cdot\gamma_6\cdot\gamma_{17} - 192\cdot\gamma_7\cdot\gamma_{17} + 864\cdot\gamma_9\cdot\gamma_{23} + 192\cdot\gamma_{10}\cdot\gamma_{23} - 768\cdot\gamma_5\cdot\gamma_{18} \\
& +\lambda^2\cdot(-384\cdot\gamma_5^2 + 384\cdot\gamma_8^2 - 96\cdot\gamma_6^2 + 96\cdot\gamma_9^2)\cdot B^3A^3 \\
& +\lambda^3\cdot(-11520\cdot\gamma_5\cdot\gamma_{16} + 11520\cdot\gamma_8\cdot\gamma_{22} - 576\cdot\gamma_6\cdot\gamma_{17} - 288\cdot\gamma_7\cdot\gamma_{17} + 576\cdot\gamma_9\cdot\gamma_{23} + 288\cdot\gamma_{10}\cdot\gamma_{23} - 3456\cdot\gamma_5 \\
& +\lambda^2\cdot(-1728\cdot\gamma_5^2 + 1728\cdot\gamma_8^2 - 216\cdot\gamma_6^2 - 144\cdot\gamma_6\cdot\gamma_7 + 216\cdot\gamma_9^2 + 144\cdot\gamma_9\cdot\gamma_{10})\cdot B^2A^2 \\
& +\lambda^3\cdot(-5760\cdot\gamma_5\cdot\gamma_{16} + 5760\cdot\gamma_8\cdot\gamma_{22} - 4608\cdot\gamma_5\cdot\gamma_{18} + 4608\cdot\gamma_8\cdot\gamma_{24} - 144\cdot\gamma_6\cdot\gamma_{19} - 144\cdot\gamma_6\cdot\gamma_{20} + 144\cdot\gamma_9\cdot\gamma \\
& +\lambda^2\cdot(-2304\cdot\gamma_5^2 + 2304\cdot\gamma_8^2 - 72\cdot\gamma_6^2 - 144\cdot\gamma_6\cdot\gamma_7 + 72\cdot\gamma_9^2 + 144\cdot\gamma_9\cdot\gamma_{10} - 48\cdot\gamma_7^2 + 48\cdot\gamma_{10}^2)\cdot BA \\
& +\lambda^2\cdot(-576\cdot\gamma_5^2 + 576\cdot\gamma_8^2 - 24\cdot\gamma_7^2 + 24\cdot\gamma_{10}^2) \\
& +\lambda^3\cdot(-1152\cdot\gamma_5\cdot\gamma_{18} + 1152\cdot\gamma_8\cdot\gamma_{24} - 48\cdot\gamma_7\cdot\gamma_{20} + 48\cdot\gamma_{10}\cdot\gamma_{26} + 576\cdot\gamma_6\cdot\gamma_7\cdot\gamma_8 + 288\cdot\gamma_7^2\cdot\gamma_8 - 576\cdot\gamma_5\cdot\gamma_7\cdot\gamma
\end{aligned}$$

$$
\begin{aligned}
6 \cdot \left(H_4 - U^\dagger H_0 U\right) &= 6 * (\Lambda_4) \\
&= 6 \cdot \left(\tfrac{\lambda}{4}(A+B)^4 - \left([-X,H_0] + \tfrac{1}{2!}[-X,[-X,H_0]] + \tfrac{1}{3!}[-X,[-X,[-X,H_0]]]\right)\right) \\
&= \lambda^3 \cdot (-48 \cdot \gamma_{42} + 72 \cdot \gamma_6 \cdot \gamma_{15} + 72 \cdot \gamma_9 \cdot \gamma_{21} + 192 \cdot \gamma_1 \cdot \gamma_{32} + 256 \cdot \gamma_4 \cdot \gamma_5 \cdot \gamma_8 - 384 \cdot \gamma_1 \cdot \gamma_9 \cdot \gamma_{15} \\
&\quad + \lambda^3 \cdot (-36 \cdot \gamma_{43} + 108 \cdot \gamma_1 \cdot \gamma_{33} + 12 \cdot \gamma_6 \cdot \gamma_{16} + 12 \cdot \gamma_9 \cdot \gamma_{22} + 48 \cdot \gamma_5 \cdot \gamma_{17} + 48 \cdot \gamma_8 \cdot \gamma_{23} + 216 \\
&\quad + \lambda^3 \cdot (-24 \cdot \gamma_{44} + 48 \cdot \gamma_1 \cdot \gamma_{34} + 96 \cdot \gamma_4 \cdot \gamma_{16} + 432 \cdot \gamma_6 \cdot \gamma_{15} - 432 \cdot \gamma_9 \cdot \gamma_{21} + 144 \cdot \gamma_5 \cdot \gamma_{14} + 1 \\
&\quad + \lambda^3 \cdot (-12 \cdot \gamma_{45} + 12 \cdot \gamma_1 \cdot \gamma_{35} + 252 \cdot \gamma_6 \cdot \gamma_{16} - 252 \cdot \gamma_9 \cdot \gamma_{22} + 24 \cdot \gamma_4 \cdot \gamma_{17} + 288 \cdot \gamma_5 \cdot \gamma_{17} - 2 \\
&\quad + \lambda^2 \cdot (-36 \cdot \gamma_{21} + 24 \cdot \gamma_5 \cdot \gamma_6 + 24 \cdot \gamma_8 \cdot \gamma_9 + 108 \cdot \gamma_1 \cdot \gamma_{15} - 96 \cdot \gamma_1 \cdot \gamma_5 \cdot \gamma_9 - 96 \cdot \gamma_1 \cdot \gamma_6 \cdot \gamma_8 - 2 \\
&\quad + \lambda^3 \cdot (-36 \cdot \gamma_{46} - 12 \cdot \gamma_7 \cdot \gamma_{16} - 12 \cdot \gamma_{10} \cdot \gamma_{22} + 72 \cdot \gamma_5 \cdot \gamma_{17} + 72 \cdot \gamma_8 \cdot \gamma_{23} + 24 \cdot \gamma_6 \cdot \gamma_{18} + 24 \cdot \\
&\quad + \lambda^2 \cdot (-24 \cdot \gamma_{22} + 48 \cdot \gamma_1 \cdot \gamma_{16} + 96 \cdot \gamma_4 \cdot \gamma_5 - 64 \cdot \gamma_1^2 \cdot \gamma_{22} - 256 \cdot \gamma_1 \cdot \gamma_4 \cdot \gamma_8) \cdot (B^5 A + B A^5) + \\
&\quad + \lambda^3 \cdot (-24 \cdot \gamma_{47} + 48 \cdot \gamma_3 \cdot \gamma_{16} + 48 \cdot \gamma_1 \cdot \gamma_{37} + 2160 \cdot \gamma_6 \cdot \gamma_{15} - 2160 \cdot \gamma_9 \cdot \gamma_{21} + 288 \cdot \gamma_7 \cdot \gamma_{15} - \\
&\quad + \lambda^2 \cdot (-12 \cdot \gamma_{23} + 12 \cdot \gamma_1 \cdot \gamma_{17} + 24 \cdot \gamma_4 \cdot \gamma_6 + 216 \cdot \gamma_5 \cdot \gamma_6 - 216 \cdot \gamma_8 \cdot \gamma_9 - 8 \cdot \gamma_1^2 \cdot \gamma_{23} - 32 \cdot \gamma_1 \cdot \\
&\quad + \lambda^3 \cdot (-12 \cdot \gamma_{48} + 12 \cdot \gamma_3 \cdot \gamma_{17} + 60 \cdot \gamma_4 \cdot \gamma_{17} + 1296 \cdot \gamma_5 \cdot \gamma_{17} - 1296 \cdot \gamma_8 \cdot \gamma_{23} + 12 \cdot \gamma_1 \cdot \gamma_{38} + \\
&\quad + \lambda \cdot (1.5 - 24 \cdot \gamma_8 + 48 \cdot \gamma_1 \cdot \gamma_5 - 64 \cdot \gamma_1^2 \cdot \gamma_8) \cdot (B^4 + A^4) + \lambda \cdot (-24 \cdot \gamma_5 + 48 \cdot \gamma_1 \cdot \gamma_8 - 64 \cdot \gamma \\
&\quad + \lambda^2 \cdot (-24 \cdot \gamma_{24} + 48 \cdot \gamma_1 \cdot \gamma_{18} + 48 \cdot \gamma_3 \cdot \gamma_5 + 144 \cdot \gamma_4 \cdot \gamma_5 - 64 \cdot \gamma_1^2 \cdot \gamma_{24} - 128 \cdot \gamma_1 \cdot \gamma_3 \cdot \gamma_8 - 38 \\
&\quad + \lambda^3 \cdot (-24 \cdot \gamma_{49} + 2880 \cdot \gamma_6 \cdot \gamma_{15} - 2880 \cdot \gamma_9 \cdot \gamma_{21} + 720 \cdot \gamma_7 \cdot \gamma_{15} - 720 \cdot \gamma_{10} \cdot \gamma_{21} + 48 \cdot \gamma_1 \cdot \gamma_{39} \\
&\quad + \lambda \cdot (6 - 12 \cdot \gamma_9 + 12 \cdot \gamma_1 \cdot \gamma_6 - 8 \cdot \gamma_1^2 \cdot \gamma_9) \cdot (B^3 A + B A^3) + \lambda \cdot (-12 \cdot \gamma_6 + 12 \cdot \gamma_1 \cdot \gamma_9 - 8 \cdot \gamma_1^2 \\
&\quad + \lambda^2 \cdot (-12 \cdot \gamma_{25} + 12 \cdot \gamma_3 \cdot \gamma_6 + 648 \cdot \gamma_5 \cdot \gamma_6 - 648 \cdot \gamma_8 \cdot \gamma_9 + 12 \cdot \gamma_1 \cdot \gamma_{19} + 144 \cdot \gamma_5 \cdot \gamma_7 - 144 \cdot \\
&\quad + \lambda^3 \cdot (-12 \cdot \gamma_{50} + 1728 \cdot \gamma_5 \cdot \gamma_{17} - 1728 \cdot \gamma_8 \cdot \gamma_{23} + 14400 \cdot \gamma_5 \cdot \gamma_{15} - 14400 \cdot \gamma_8 \cdot \gamma_{21} + 12 \cdot \gamma_6 \\
&\quad + \lambda \cdot (9 - 12 \cdot \gamma_{10} + 12 \cdot \gamma_1 \cdot \gamma_7 - 8 \cdot \gamma_1^2 \cdot \gamma_{10}) \cdot (B^2 + A^2) + \lambda \cdot (-12 \cdot \gamma_7 + 12 \cdot \gamma_1 \cdot \gamma_{10} - 8 \cdot \gamma_1^2 \\
&\quad + \lambda^2 \cdot (-12 \cdot \gamma_{26} + 432 \cdot \gamma_5 \cdot \gamma_6 - 432 \cdot \gamma_8 \cdot \gamma_9 + 216 \cdot \gamma_5 \cdot \gamma_7 - 216 \cdot \gamma_8 \cdot \gamma_{10} + 12 \cdot \gamma_1 \cdot \gamma_{20} + 12 \cdot \\
&\quad + \lambda^3 \cdot (-12 \cdot \gamma_{51} + 432 \cdot \gamma_5 \cdot \gamma_{17} - 432 \cdot \gamma_8 \cdot \gamma_{23} + 10800 \cdot \gamma_5 \cdot \gamma_{15} - 10800 \cdot \gamma_8 \cdot \gamma_{21} + 432 \cdot \gamma_6 \cdot \\
&\quad + \lambda^3 \cdot (960 \cdot \gamma_5 \cdot \gamma_{16} - 960 \cdot \gamma_8 \cdot \gamma_{22} + 240 \cdot \gamma_6 \cdot \gamma_{17} - 240 \cdot \gamma_9 \cdot \gamma_{23} + 2880 \cdot \gamma_5 \cdot \gamma_6 \cdot \gamma_9 - 1440 \cdot \gamma \\
&\quad + \lambda^3 \cdot (5760 \cdot \gamma_5 \cdot \gamma_{16} - 5760 \cdot \gamma_8 \cdot \gamma_{22} + 864 \cdot \gamma_6 \cdot \gamma_{17} + 192 \cdot \gamma_7 \cdot \gamma_{17} - 864 \cdot \gamma_9 \cdot \gamma_{23} - 192 \cdot \gamma_{10} \\
&\quad + \lambda^2 \cdot (384 \cdot \gamma_5^2 - 384 \cdot \gamma_8^2 + 96 \cdot \gamma_6^2 - 96 \cdot \gamma_9^2) \cdot B^3 A^3 \\
&\quad + \lambda \cdot (9) \cdot B^2 A^2 \\
&\quad + \lambda^3 \cdot (11520 \cdot \gamma_5 \cdot \gamma_{16} - 11520 \cdot \gamma_8 \cdot \gamma_{22} + 576 \cdot \gamma_6 \cdot \gamma_{17} + 288 \cdot \gamma_7 \cdot \gamma_{17} - 576 \cdot \gamma_9 \cdot \gamma_{23} - 288 \cdot \gamma \\
&\quad + \lambda^2 \cdot (1728 \cdot \gamma_5^2 - 1728 \cdot \gamma_8^2 + 216 \cdot \gamma_6^2 + 144 \cdot \gamma_6 \cdot \gamma_7 - 216 \cdot \gamma_9^2 - 144 \cdot \gamma_9 \cdot \gamma_{10}) \cdot B^2 A^2 \\
&\quad + \lambda \cdot (18) \cdot BA \\
&\quad + \lambda^3 \cdot (5760 \cdot \gamma_5 \cdot \gamma_{16} - 5760 \cdot \gamma_8 \cdot \gamma_{22} + 4608 \cdot \gamma_5 \cdot \gamma_{18} - 4608 \cdot \gamma_8 \cdot \gamma_{24} + 144 \cdot \gamma_6 \cdot \gamma_{19} + 144 \cdot \gamma \\
&\quad + \lambda^2 \cdot (2304 \cdot \gamma_5^2 - 2304 \cdot \gamma_8^2 + 72 \cdot \gamma_6^2 + 144 \cdot \gamma_6 \cdot \gamma_7 - 72 \cdot \gamma_9^2 - 144 \cdot \gamma_9 \cdot \gamma_{10} + 48 \cdot \gamma_7^2 - 48 \cdot \gamma \\
&\quad + \lambda \cdot (4.5) \\
&\quad + \lambda^2 \cdot (576 \cdot \gamma_5^2 - 576 \cdot \gamma_8^2 + 24 \cdot \gamma_7^2 - 24 \cdot \gamma_{10}^2) \\
&\quad + \lambda^3 \cdot (1152 \cdot \gamma_5 \cdot \gamma_{18} - 1152 \cdot \gamma_8 \cdot \gamma_{24} + 48 \cdot \gamma_7 \cdot \gamma_{20} - 48 \cdot \gamma_{10} \cdot \gamma_{26} - 576 \cdot \gamma_6 \cdot \gamma_7 \cdot \gamma_8 - 288 \cdot \gamma_7^2
\end{aligned}
$$