

Heterosk.

↳ violation of assump. homosk. ($\text{Var}(u_i) = \sigma^2$ for all i)

↳ Typically $\text{Var}(u_i) \neq \text{Var}(u_j)$ for some inds i and j

Problem: We don't observe u_i or $\text{Var}(u_i)$

Solution: Het.-robust SEs

Why care about heterosk.?

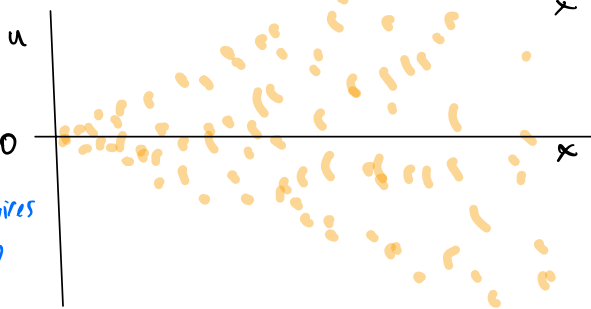
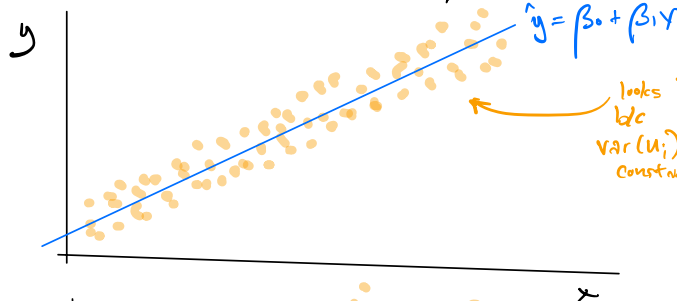
1. OLS is biased for SEs
↳ bad inferences (e.g. CIs, tests, ...)

2. OLS is inefficient for $\hat{\beta}$
WLS is more eff.

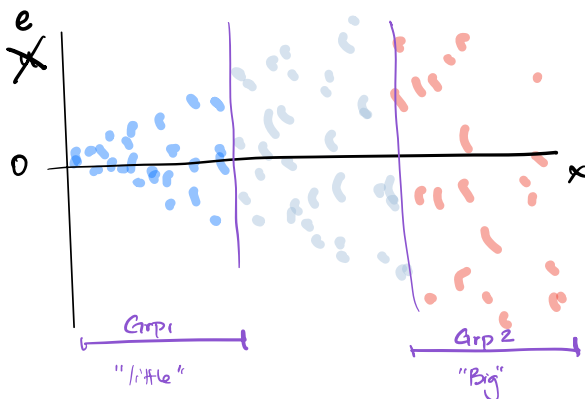
requires knowing $\sigma_i^2 = \text{Var}(u_i)$

Exog. requires $E[u|x] = 0$

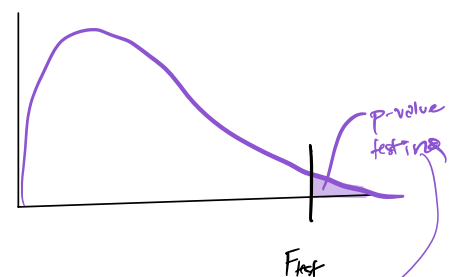
$$\text{disturbance} = y_i - \hat{y}_i = y_i - (\beta_0 + \beta_1 x)$$



Goldfeld - Quandt : use e to learn about u . Test SSE_{big} vs. $\text{SSE}_{\text{little}}$



$$F_{\text{test}} = \frac{\text{SSE}_{\text{big}}}{\text{SSE}_{\text{little}}} \sim F_{n-k, n-k}$$



$H_0: \text{Homosk.}$

White test for heterosk.

3²

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + u_i$$

get
resids...

Adequate test for heterosk.

$$e_i^2$$

estimate
of $\text{Var}(u_i)$

$$\alpha_0 + \alpha_1 x_{i1} + \alpha_2 x_{i2} + \alpha_3 x_{i3}$$

orig. regressors

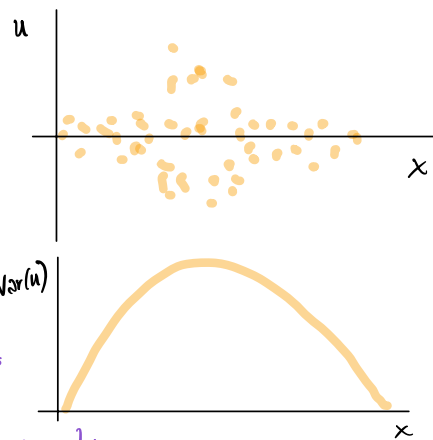
$$+ \alpha_4 x_{i1}^2 + \alpha_5 x_{i2}^2 + \alpha_6 x_{i3}^2$$

sq. regressors

$$+ \alpha_7 x_{i1} x_{i2} + \alpha_8 x_{i1} x_{i3} + \alpha_9 x_{i2} x_{i3}$$

interactions b/n
regressors

+ v



$\hookrightarrow n \times R^2 = \text{test stat against } \chi^2_9$

Asymptotics: $n \rightarrow \infty$

sample size $\rightarrow \infty$

probability limit $\hat{\beta}$: where does the estimator "end up" as $n \rightarrow \infty$

$$\text{plim } \hat{\beta}_{OLS} = \beta \Rightarrow \text{OLS is consistent}$$

$$\text{plim } \hat{\beta}_{OLS} = \beta + \frac{\text{Cov}(u, x)}{\text{Var}(x)}$$

exogeneity gives = 0

$\text{Var}(x) \neq 0$ is an assumption (but clear in data)

In the past:

$$E[\hat{\beta}] \leftarrow = \beta \text{ then unbiased}$$

$$\text{Var}(\hat{\beta}) \leftarrow \text{efficient or inefficient}$$

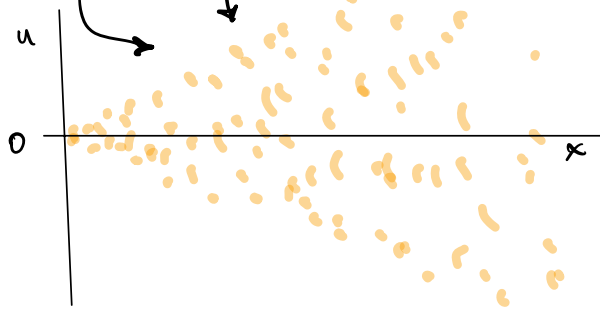
holding n (sample size) fixed,
repeating many samples (all size n)

$$\text{plim } f(x) = f(\text{plim } x)$$

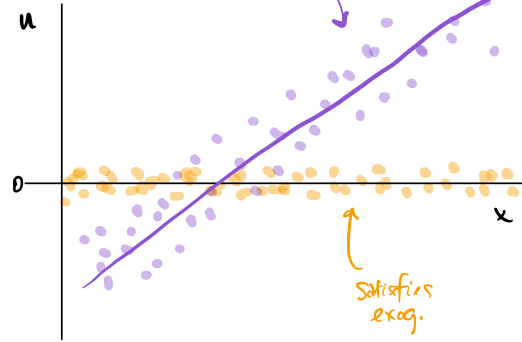
we need f to be continuous @ $\text{plim } x$

$$SSE = \sum e_i^2$$

Heteroskedasticity. Satisfies exogeneity.



violates exog.



Homosk.: $\text{Var}(u_i) = \sigma^2$ for every i

Exog.: $E[u_i | x_i] = 0$

violation: (omitted variable bias)

1. om. var. affect y

2. om. var. corr. w/ x

↳ biases OLS for β