Instrumental Variables

EC 421, Set 11

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07 March 2019

Prologue

Schedule

Last Time

Causality

Today

- Econ. Masters program
- Review: Causality
- New: Instrumental variables

Upcoming

Assignment soon.

Master's Program

Applied Economics

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- 1-year program including courses on applied econometrics, data science, and "big data".
- Awesome opportunity to focus on applying economic methods to real-world questions/scenarios.
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Causality

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but never both at the same time.

Causality

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Idea: Estimate the **average treatment effect** as the difference between the average outcomes in the treatment group and the control group, *i.e.*,

$$Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0)$$

where $D_i = 1$ if i received treatment, and $D_i = 0$ if i is in the control group.

Causality

Review

Result: We showed that even when the treatment effect is constant (meaning $\tau_i = \tau$ for all i),

$$\begin{aligned} & Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0) \\ & = \tau + \underbrace{Avg(y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0)}_{\text{Selection bias}} \end{aligned}$$

which says that the difference in the groups' means will give us a **biased estimate** for the causal effect of treatment **if we have selection bias.**

Causality

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A: (Formal) The average *untreated* outcome for a member of our **treatment group** (which we cannot observe) differs from the average *untreated* outcome for a member of our **control group**, *i.e.*,

$$Avg(y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0)$$

Causality

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Practical problem: Selection bias is also difficult to observe

$$\underbrace{Avg(y_{0,i} \mid D_i = 1)}_{\text{Unobservable}} - Avg(y_{0,i} \mid D_i = 0)$$

(back to the *fundamental problem of causal inference*)

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Bigger problem: If selection bias is present, our estimate for τ is biased, preventing us from understanding the causal effect of treatment.

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Bigger problem: If selection bias is present, our estimate for τ is biased, preventing us from understanding the causal effect of treatment.

Sounds a bit like omitted-variable bias, right? Our treatment variable is correlated with something that makese the two groups different.

Causality

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Example: Imagine we have two people—Al and Bri—and a single binary treatment, college. We interested in the effect of college on earnings.

$\text{Earn}_{1,\text{Al}} = \$60\text{K}$ $\qquad$ $\text{Earn}_{1,\text{Bri}} = \$140\text{K}$

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but any real-world estimate would have serious selection issues since $\text{Earn}_{0,\text{Al}} \neq \text{Earn}_{0,\text{Bri}}$.

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- **Option 1:** **Distribute treatment** in a way such that the treatment and control groups are essentially identical (experiments).

- **Option 2:** **Build a control** group that *matches* the treatment group (life with observational data).

Instrumental variables

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Intro

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- the **exogenous** part of x, which gives us unbiased estimates
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Instrumental variables attempts to separate out

- the **exogenous** part of x, which gives us unbiased estimates
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If we use only the exogenous (*good*) variation in x, then we can avoid selection bias/omitted-variable bias.

Introductory example

Example: If we want to estimate the effect of veteran status on earnings,

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And OLS will likely be biased for (1) due to selection/omitted-variable bias.

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&= \beta_0 + \beta_1 \textbf{Veteran}_i^{\text{Exog.}} + \underbrace{\beta_1 \textbf{Veteran}_i^{\text{Endog.}} + u_i}_{w_i} \\
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Q: What would exogenous variation in veteran status mean?

A$_1$: Choices to enlist in the military that are essentially random—or at least uncorrelated with omitted variables and the disturbance.

A$_2$: **No selection bias:**

$$Avg(\text{Earnings}_{0i} \mid \text{Veteran}_i = 1) - Avg(\text{Earnings}_{0i} \mid \text{Veteran}_i = 0) = 0$$

Instruments

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A: An **instrument** is a variable that is

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So if we want an instrument z_i for endogenous veteran status in

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$

1. **Relevant:** $\text{Cov}(\text{Veteran}_i,\, z_i) \neq 0$
2. **Exogenous:** $\text{Cov}(z_i,\, u_i) = 0$

Instruments: Relevance

Relevance: We need the instrument to cause a change in (correlate with) our endogenous explanatory variable.

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3. Vietnam War draft — **Relevant**
   being draw led to service

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2. Physical fitness — **Not exogenous**
   fitness correlates with many things
3. Vietnam War draft — **Exogenous**
   the lottery was random

Instrumental review

Let's recap...

- Our instrument must be **correlated with our endogenous variable**.

- Our instrument must be **uncorrelated with any other variable that affects the outcome**.

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- Our instrument must be **correlated with our endogenous variable**.
- Our instrument must be **uncorrelated with any other variable that affects the outcome**.

In other words:

The instrument only affects our outcome through the endogenous variable.

Instrumental variables

Back to our example

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3. Vietnam War draft	**Relevant** **Exogenous**

Back to our example

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1. Social security number — **Not relevant** **Exogenous**

2. Physical fitness — **Probably relevant** **Not exogenous**

3. Vietnam War draft — **Relevant** **Exogenous**

Thus, only the Vietnam War's draft lottery appears to be a ***valid* instrument**.

If we have a *valid* instrument (*e.g.*, the draft lottery), how do we use it?

Estimation

Recall: We want to estimate the effect of veteran status on earnings.

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$

Estimation

Recall: We want to estimate the effect of veteran status on earnings.

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Let's consider two related effects:

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1. The effect of the **instrument** on the **endogenous variable**, *e.g.*,

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Draft $\longrightarrow$ **Veteran status** $\longrightarrow$ **Earnings**

Using our assumptions on independence and exogeneity:

(Effect of **the draft** on **earnings**) =
(Effect of **the draft** on **veteran status**)×
(Effect of **veteran status** on **earnings**)

Instrumental variables

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We just wrote out an expression for the effect of **the draft** on **earnings**, *i.e.*,

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Instrumental variables

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$$\frac{\text{(Effect of \textbf{the draft} on \textbf{earnings})}}{\text{(Effect of \textbf{the draft} on \textbf{veteran status})}}$$

Our **instrument** consistently estimates both parts of this fraction!

Estimation: Bring it all together

By estimating two regressions involving our **instrument**,

1. The effect of the **instrument** on the **endogenous variable**, *e.g.*,

$$\text{Veteran}_i = \gamma_0 + \gamma_1 \text{Draft}_i + v_i$$

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we can estimate our desired effect:

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we can estimate our desired effect:

(Effect of **veteran status** on **earnings**) = $\dfrac{\pi_1}{\gamma_1}$

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So with instrumental variables, we estimate β_1 using

$$\hat{\beta}_1^{\mathrm{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1}$$

where $\hat{\pi}_1$ and $\hat{\gamma}_1$ come from the two equations we just discussed.

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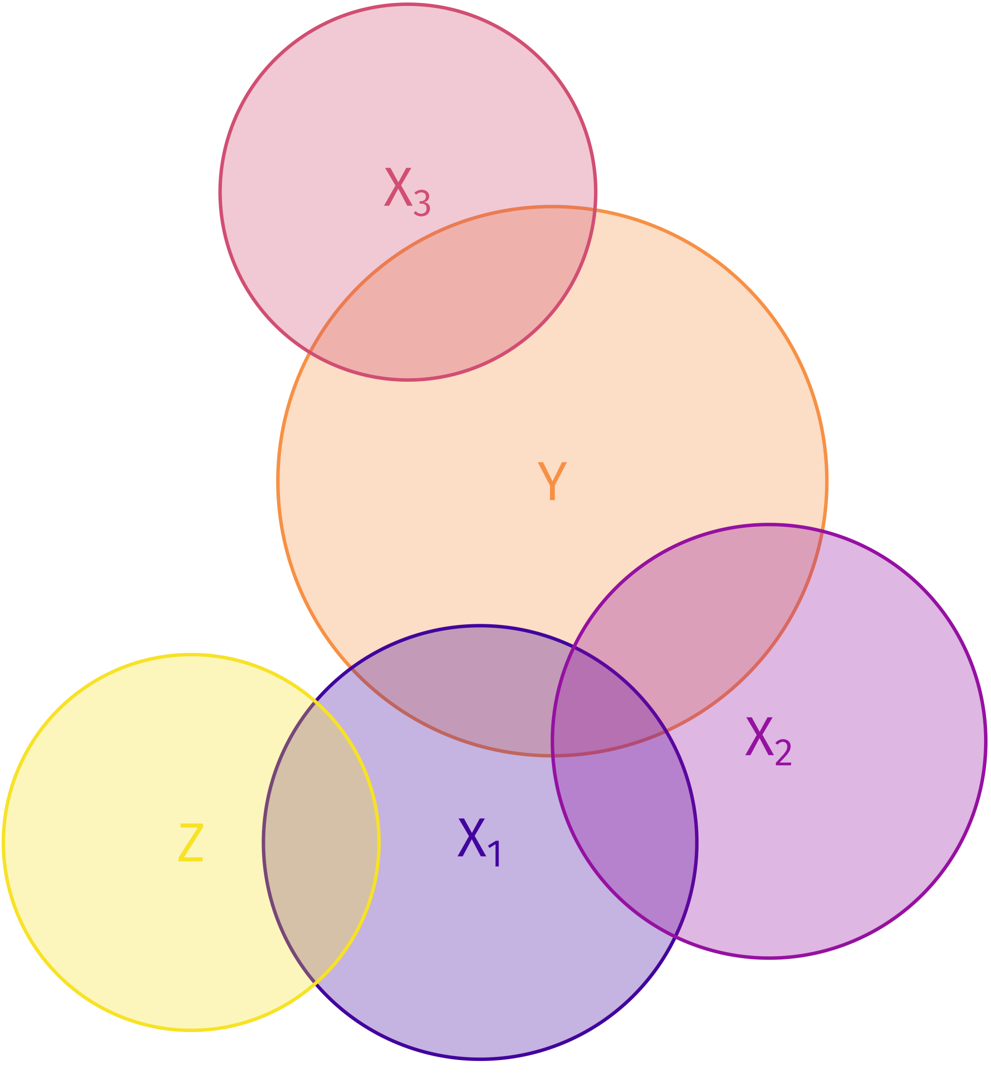
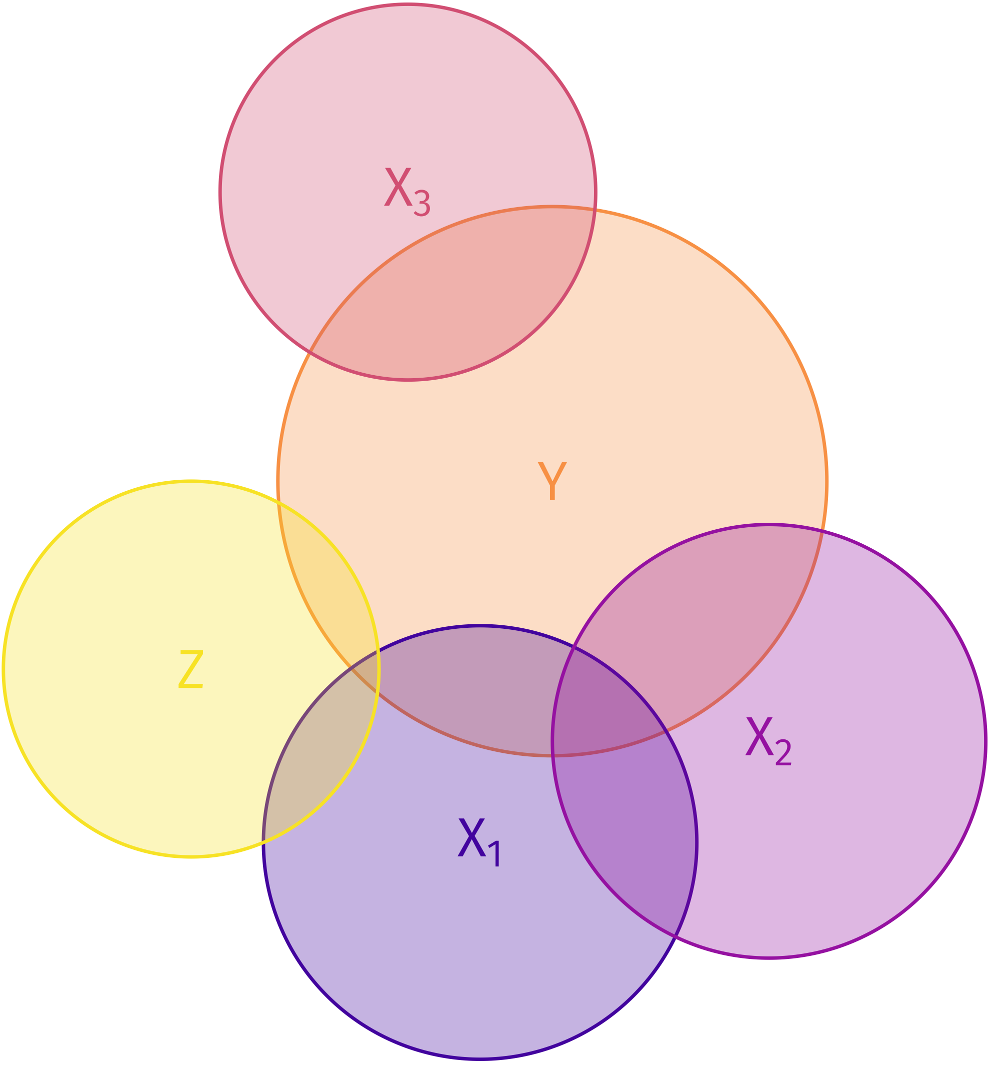
which equals β_1 as long as our instrument is **exogenous** (numerator) and **relevant** (denominator).



X_3
Y
X_2
Z
X_1



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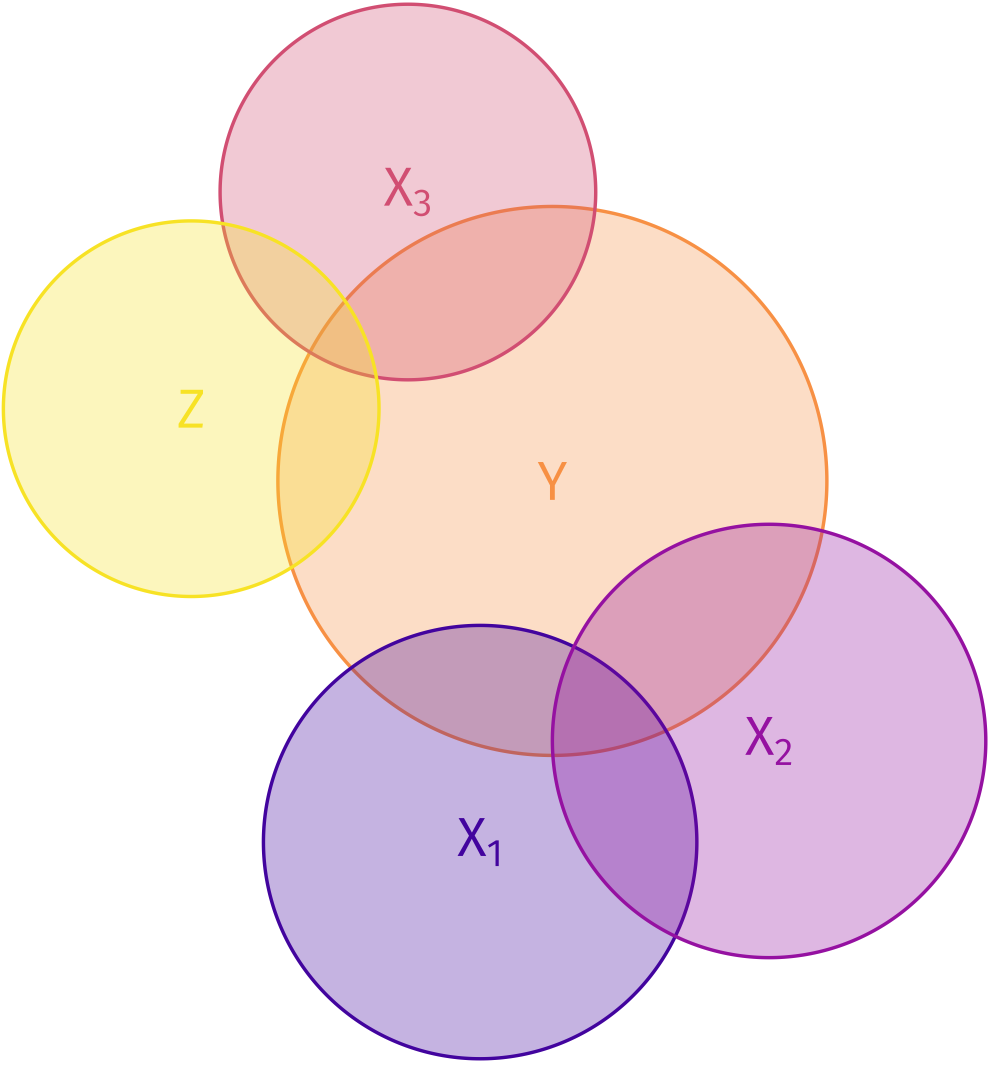
X_3
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Let's work an example in `R`.

Instrumental variables

Example in R

Back to our age-old battle to estimate the returns to education.

```
#> # A tibble: 722 x 4
#>     wage education education_dad education_mom
#>    <int>     <int>         <int>         <int>
#>  1   769        12             8             8
#>  2   808        18            14            14
#>  3   825        14            14            14
#>  4   650        12            12            12
#>  5   562        11            11             6
#>  6   600        10             8             8
#>  7  1154        15             5            14
#>  8  1000        12            11            12
#>  9   930        18            14            13
#> 10   900        15            12            12
#> # … with 712 more rows
```

Example in R

OLS for the returns to education with will likely (definitely) be biased...

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(Likely biased) OLS results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	176.504	89.152	1.98	0.0481
Education	**58.594**	**6.439**	**9.10**	**<0.0001**

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but what if mother's education provides a valid instrument?

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Term	**Est.**	**S.E.**	**t stat.**	**p-Value**
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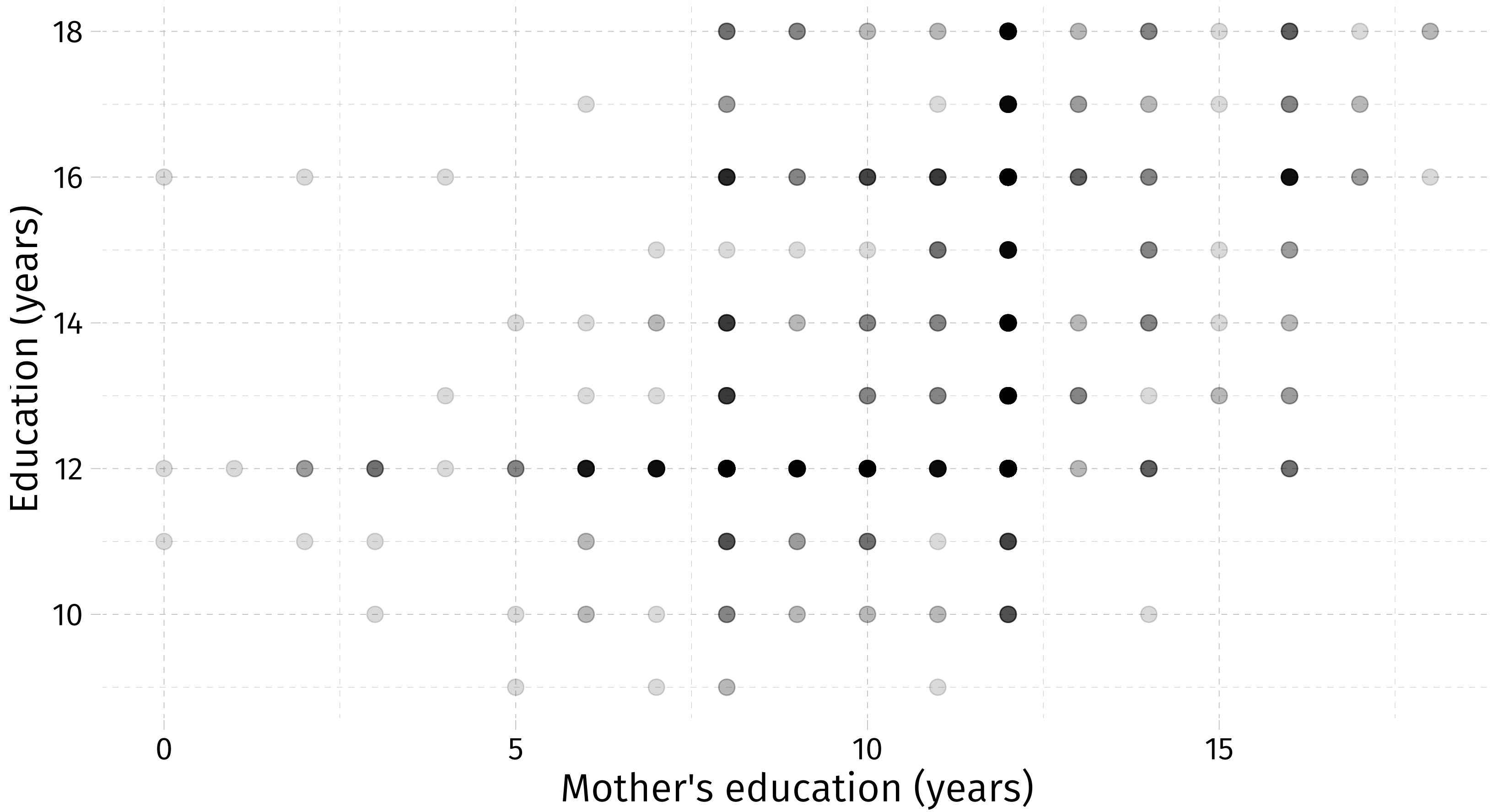
The *p*-value suggests a very strong relationship (very *relevant*).

Instrumental variables

Visualizing the first stage



Instrumental variables

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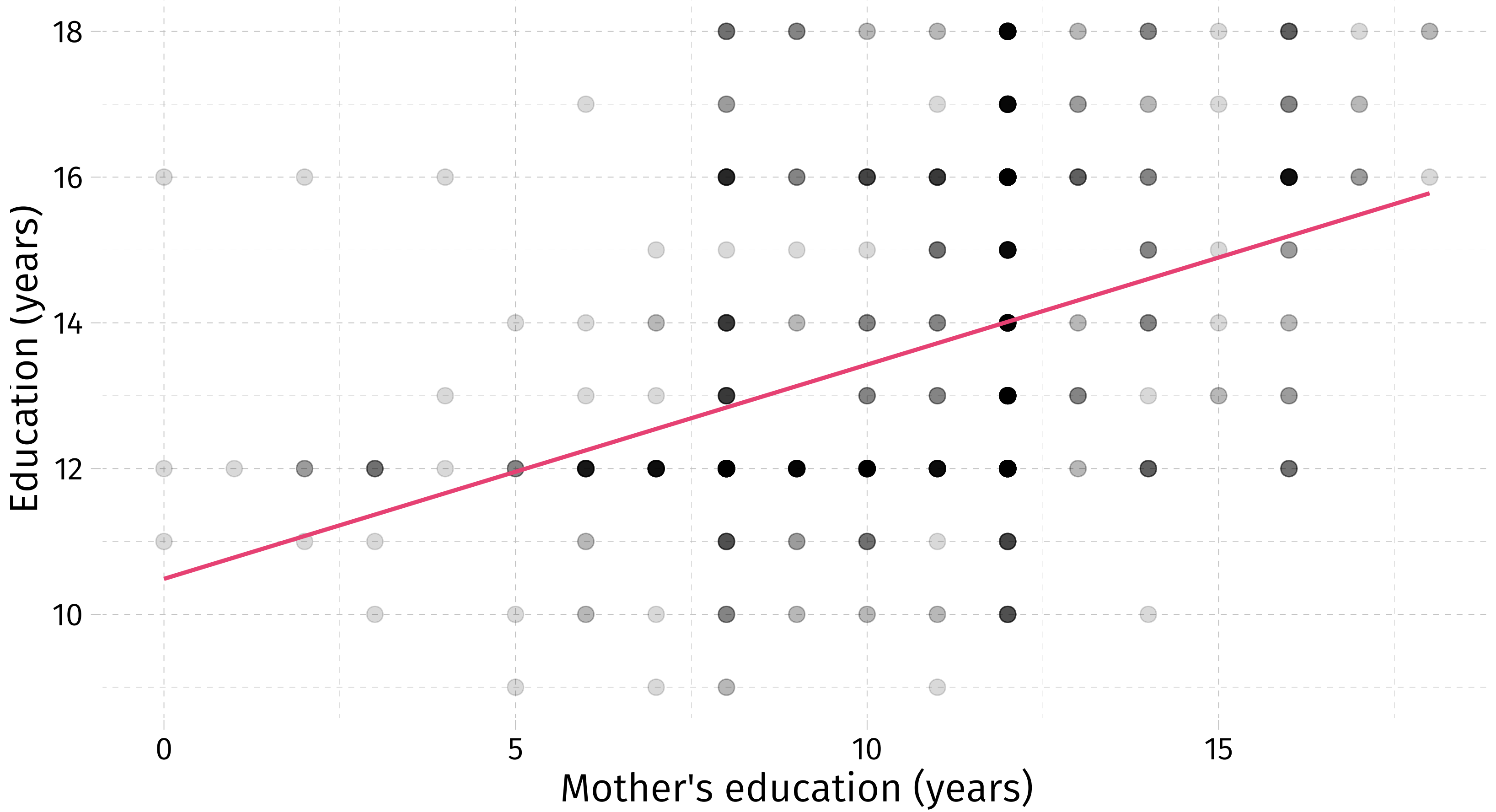

Instrumental variables

Exogeneity

Q: What does **exogeneity** mean in this case?

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Q: Does *mother's education* seem likely to satisfy exogeneity?

Example in R

Now, let's estimate the ***reduced form***:

The effect of the instrument on our outcome variable.

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Term	Est.	S.E.	t stat.	p-Value
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Q_1: How do we interpret this estimated coefficient ($\hat{\pi}_1$)?

Q_2: If our instrument is *valid*, can we interpret these estimates as **causal**?

Instrumental variables

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$$\implies \hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1} = \frac{31.81}{0.294} \approx \mathbf{108.2}$$

Example in R

Alternative: Use the function `iv_robust()` from the `estimatr` package.

This new function `iv_robust` works very similar to our good friend `lm`:

```
iv_robust(y ~ x | z, data = dataset)
```

- `formula` Specify the regression followed by `|` and your instrument (`z`).
- `data` You still need a dataset.

Note: As you might guess by its name, `iv_robust` calculates heteroskedasticity-robust standard errors by default.

Example in R

In practice...

```
# Estimate our IV regression
iv_est ← iv_robust(wage ~ education | education_mom, data = wage_df)
```

Term	Est.	S.E.	t stat.	p-Value
Intercept	-501.474	226.476	-2.21	0.0271
Education	**108.214**	**16.810**	**6.44**	**<0.0001**

Instrumental variables

More

So now we know how to "do" instrumental variables

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1. Estimate the reduced form (regress outcome var. on instrument).
2. Estimate the first stage (regress expl. var. on instrument).
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A: ~~Too bad.~~ Extend IV to **two-stage least squares (2SLS)**.

Two-stage least squares

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Intro

The intuition and insights of IV carry over into two-stage least squares.

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$$
\begin{aligned}
\text{Endogenous model} \qquad & \text{Outcome}_i = \beta_0 + \beta_1 \left(\text{Endog. var.}\right)_i + u_i \\
\text{First stage} \qquad & \left(\text{Endog. var.}\right)_i = \pi_0 + \pi_1 \text{Instrument}_i + v_i \\
\text{Second stage} \qquad & \text{Outcome}_i = \delta_0 + \delta_1 \widehat{\left(\text{Endog. var.}\right)}_i + \varepsilon_i \\
\text{Reduced form} \qquad & \text{Outcome}_i = \pi_0 + \pi_1 \text{Instrument}_i + w_i
\end{aligned}
$$

where $\widehat{\left(\text{Endog. var.}\right)}_i$ denotes the predicted values (*fitted values*) from the first-stage regression.

Two-stage least squares

Intro

Two-stage least squares is very flexible—we include other controls, additional endogenous variables, *and* have multiple instruments.

But don't get too distracted by this fancy flexiblity.

We still need **valid** instruments.

Two-stage least squares

In R

Back to our *returns to education* example.

$$\text{Wage}_i = \beta_0 + \beta_1 \text{Education}_i + u_i$$

Imagine that mother's *and* father's education are both valid instruments.

Then our **first-stage regression** is

$$\text{Education}_i = \gamma_0 + \gamma_1 (\text{Mother's education})_i + \gamma_2 (\text{Father's education})_i + v_i$$

which we can estimate via OLS.

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Q: Why?

Two-stage least squares

In R

$$\text{Education}_i = \gamma_0 + \gamma_1 (\text{Mother's education})_i + \gamma_2 (\text{Father's education})_i + v_i$$

```
stage1 <- lm(education ~ education_mom + education_dad, wage_df)
```

First-stage results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	9.845	0.305	32.31	<0.0001
Mother's Education	**0.149**	**0.032**	**4.62**	**<0.0001**
Father's Education	**0.216**	**0.028**	**7.84**	**<0.0001**

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Formally, we should jointly test them (*e.g.*, F test).

Two-stage least squares

In R

Using our estimated first stage, we grab the *fitted* endogenous variable

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```r
# Add fitted values from first stage
wage_df$education_hat ← stage1$fitted.values
```

Two-stage least squares

In R

Using our estimated first stage, we grab the *fitted* endogenous variable

$$\widehat{\text{Education}}_i = \widehat{\gamma}_0 + \widehat{\gamma}_1 (\text{Mother's education})_i + \widehat{\gamma}_2 (\text{Father's education})_i$$

```r
# Add fitted values from first stage
wage_df$education_hat <- stage1$fitted.values
```

Now we use OLS (again) to estimate the **second-stage regression**

$$\text{Wage}_i = \delta_0 + \delta_1 \widehat{\text{Education}}_i + \varepsilon_i$$

Two-stage least squares

In R

$$\text{Wage}_i = \delta_0 + \delta_1 \widehat{\text{Education}}_i + \varepsilon_i$$

```
stage2 <- lm(wage ~ education_hat, wage_df)
```

Second-stage results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	198.149	-2.29	0.022
Fitted Education	**104.789**	**14.462**	**7.25**	**<0.0001**

Ordinary least squares

Term	Est.	S.E.	t stat.	p-Value
Intercept	176.504	89.152	1.98	0.0481
Education	**58.594**	**6.439**	**9.10**	**<0.0001**

Instrumental variables

Term	Est.	S.E.	t stat.	p-Value
Intercept	-501.474	226.476	-2.21	0.0271
Education	**108.214**	**16.810**	**6.44**	**<0.0001**

Two-stage least squares w/ two instruments

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	198.149	-2.29	0.022
Education	**104.789**	**14.462**	**7.25**	**<0.0001**

Two-stage least squares

In `R`

As you probably guessed, `R` will do both of the stages for you.

Two-stage least squares

In R

As you probably guessed, R will do both of the stages for you.

```
iv_robust(y ~ x1 + x2 + ⋯ | z1 + z2 + ⋯, data)
```

Two-stage least squares

In R

As you probably guessed, R will do both of the stages for you.

```
iv_robust(y ~ x1 + x2 + ··· | z1 + z2 + ···, data)
```

In our case, we have

- one explanatory variable (`x`) (education)
- two instruments (`z`) (parents' educations)

```
iv_robust(wage ~ education | education_mom + education_dad, data = wage_df)
```

Term	**Est.**	**S.E.**	**t stat.**	**p-Value**
Intercept	-454.683	199.946	-2.27	0.0233
Education, fitted	**104.789**	**14.852**	**7.06**	**<0.0001**

Two-stage least squares

There's more!

Because 2SLS **isolates exogenous variation in an endogenous variable**, we apply it in other settings that are biased from *endogenous* relationships.

Two-stage least squares

There's more!

Because 2SLS **isolates exogenous variation in an endogenous variable**, we apply it in other settings that are biased from *endogenous* relationships.

Common applications

- **General causal inference** for observational data (as we've seen).
- **Experiments:** Randomize a treatment that affects an endog. variable.
- **Measurement error:** Regress noisy x_1 on noisy x_2 to capture signal.
- **Simultaneous relationships** (*e.g.*, p and q from supply and demand).

However, in any 2SLS/IV setting, you need to mind the requirements for **valid instruments**—exogeneity and relevance.

Table of contents