

Non-Stationary Time Series

EC 421, Set 9

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Winter 2022

Prologue

Schedule

Last Time

Autocorrelation

Today

Brief introduction to nonstationarity

Upcoming

New assignment available.

Nonstationarity

Nonstationarity

Intro

Let's go back to our assumption of **weak dependence/persistence**

1. **Weakly persistent outcomes**—essentially, x_{t+k} in the distant period $t + k$ weakly correlates with x_t (when k is "big").

Nonstationarity

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We'll define this *good behavior* as **stationarity**.

Nonstationarity

Stationarity

Requirements for **stationarity** (a *stationary* time-series process):

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1. The **mean** of the distribution is independent of time, *i.e.*,

$$\mathbf{E}[x_t] = \mathbf{E}[x_{t-k}] \text{ for all } k$$

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$$\text{Var}(x_t) = \text{Var}(x_{t-k}) \text{ for all } k$$

3. The **covariance** between x_t and x_{t-k} depends only on k —**not on t** , *i.e.*,

$$\text{Cov}(x_t, x_{t-k}) = \text{Cov}(x_s, x_{s-k}) \text{ for all } t \text{ and } s$$

Nonstationarity

Random walks

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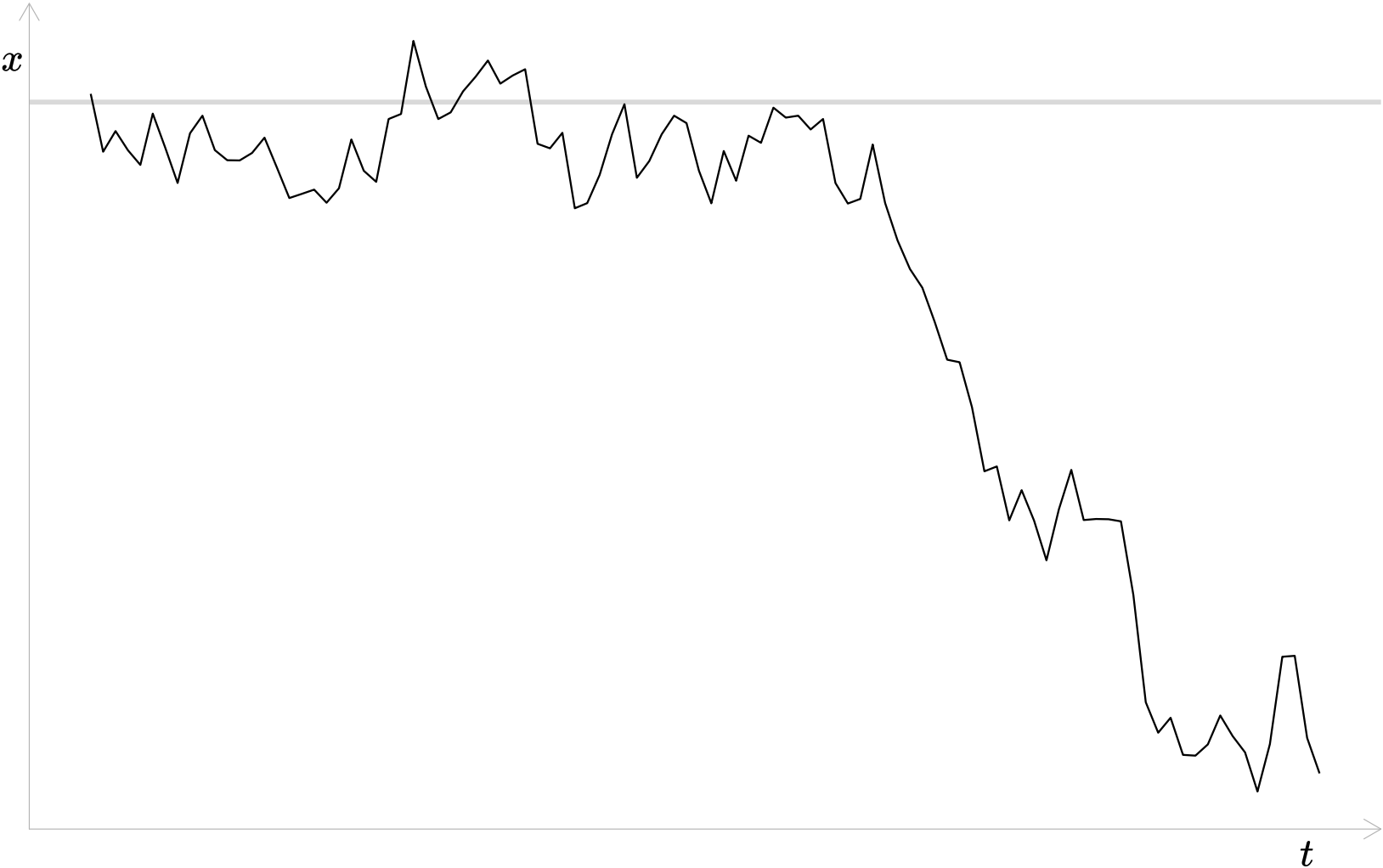
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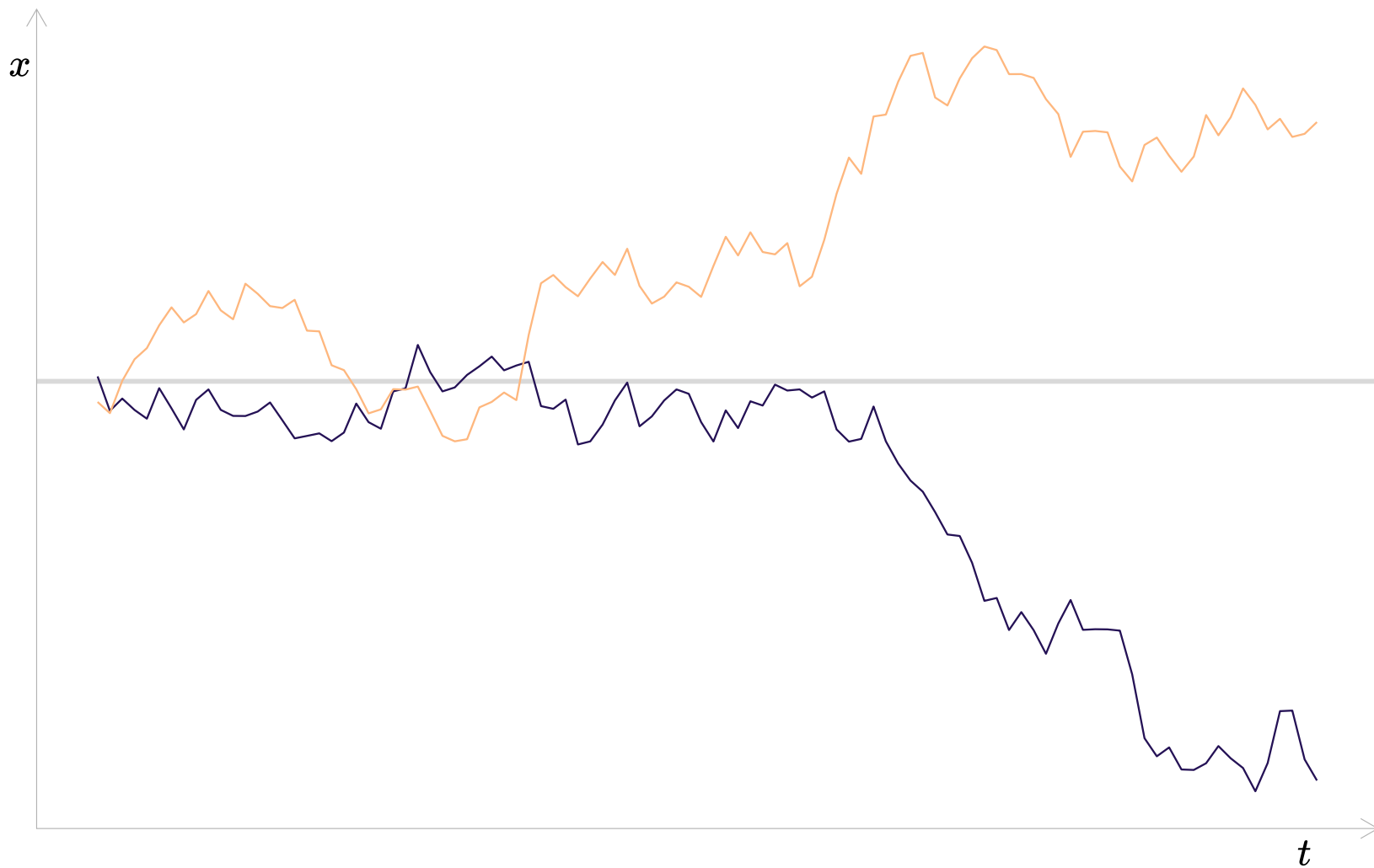
$$\begin{aligned}\text{Var}(x_t) &= \text{Var}(x_{t-1} + \varepsilon_t) \\ &= \text{Var}(x_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\ &= \text{Var}(x_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\ &\dots \\ &= \text{Var}(x_0 + \varepsilon_1 + \dots + \varepsilon_{t_2} + \varepsilon_{t-1} + \varepsilon_t) \\ &= \sigma_\varepsilon^2 + \dots + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \\ &= t\sigma_\varepsilon^2\end{aligned}$$

Q: What's the big deal with this violation?

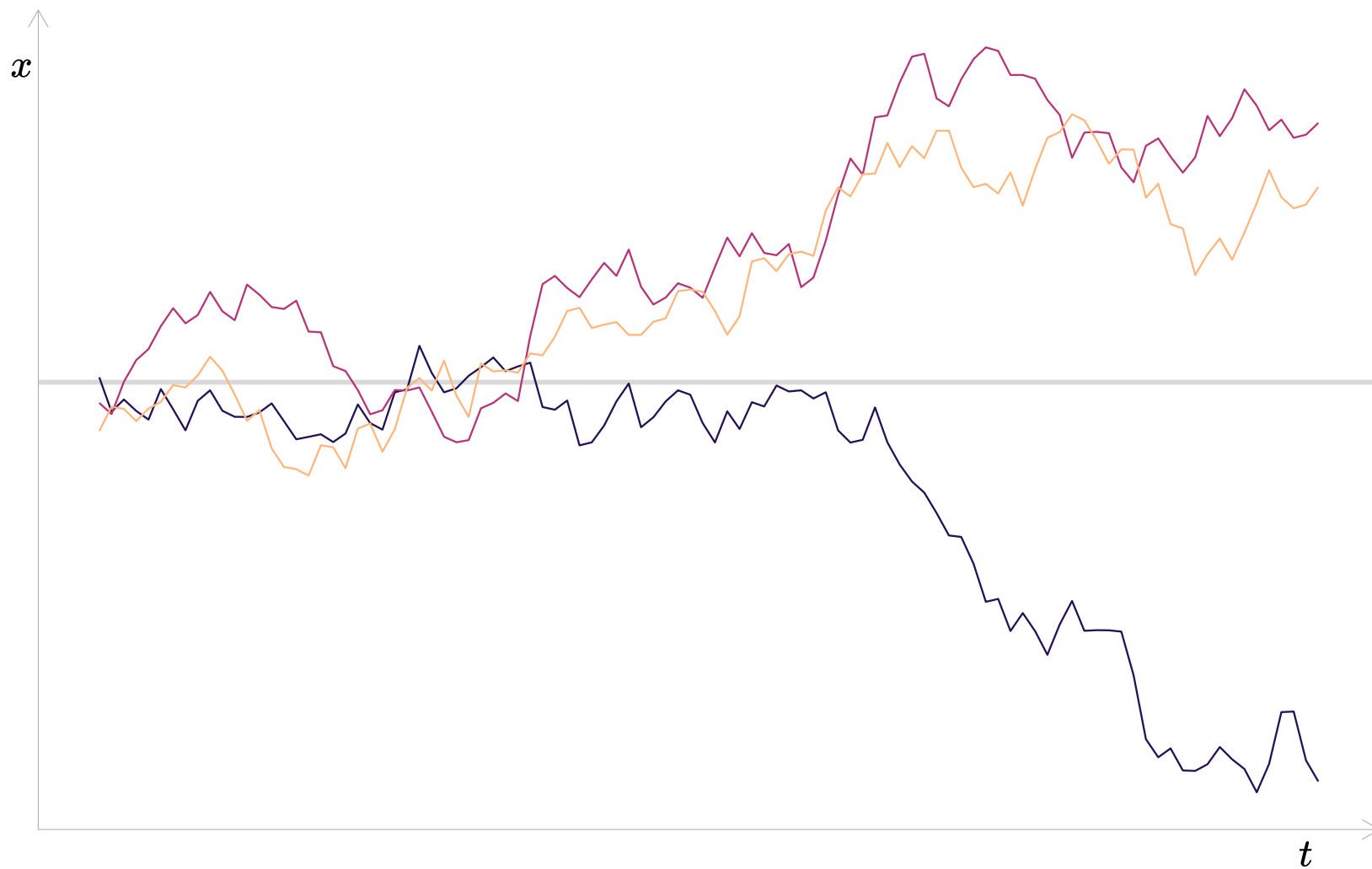
One 100-period random walk



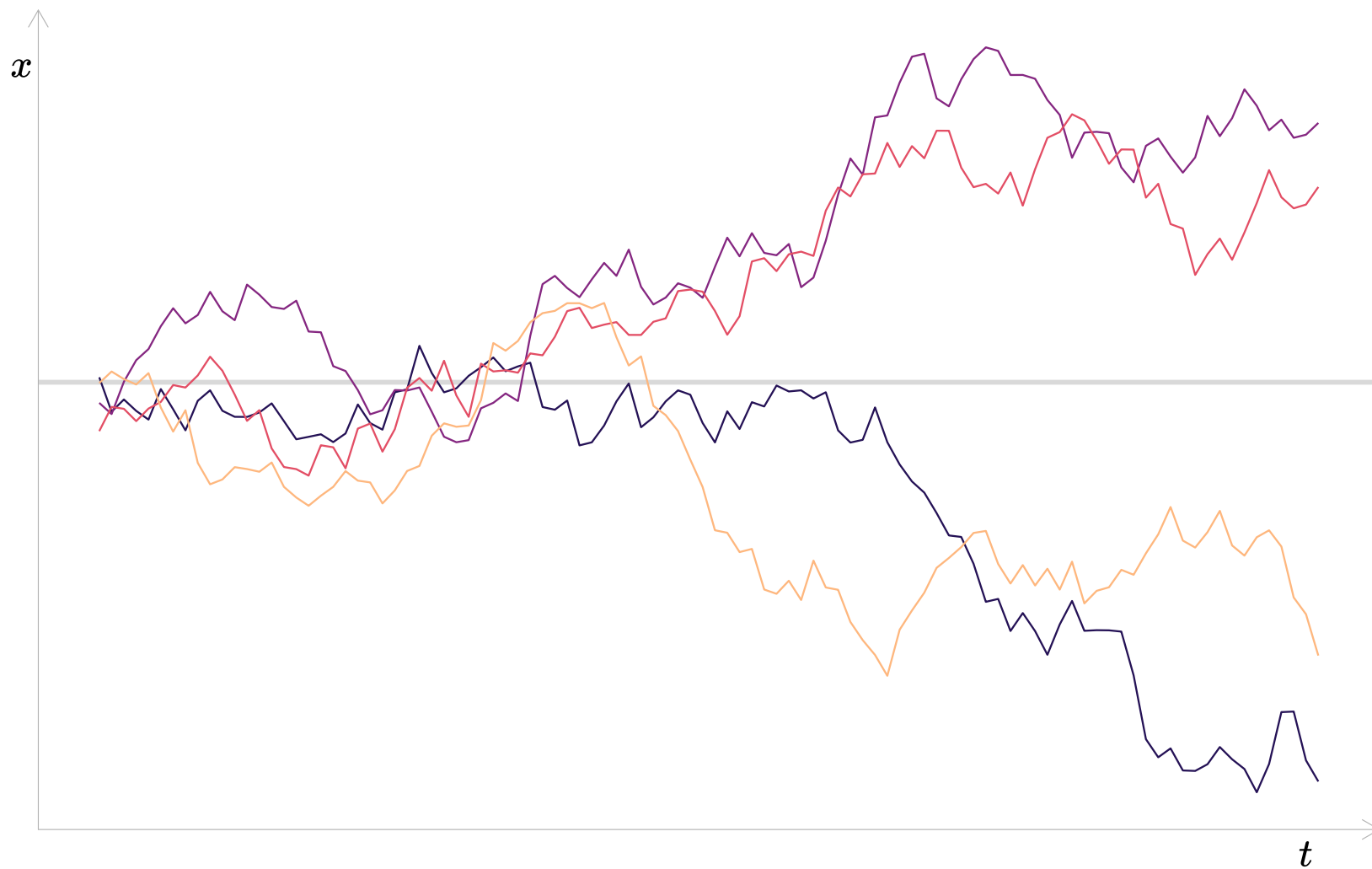
Two 100-period random walks



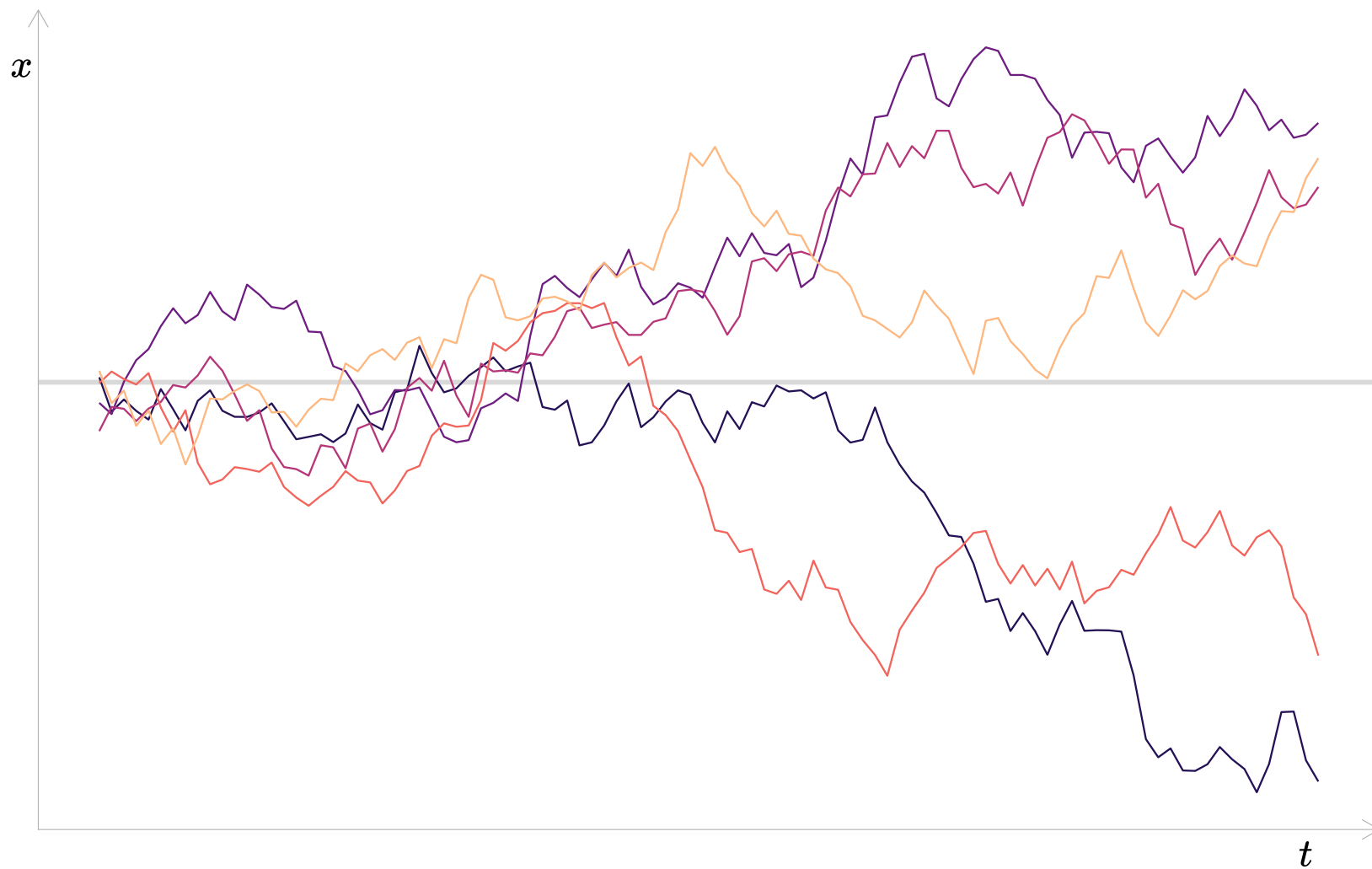
Three 100-period random walks



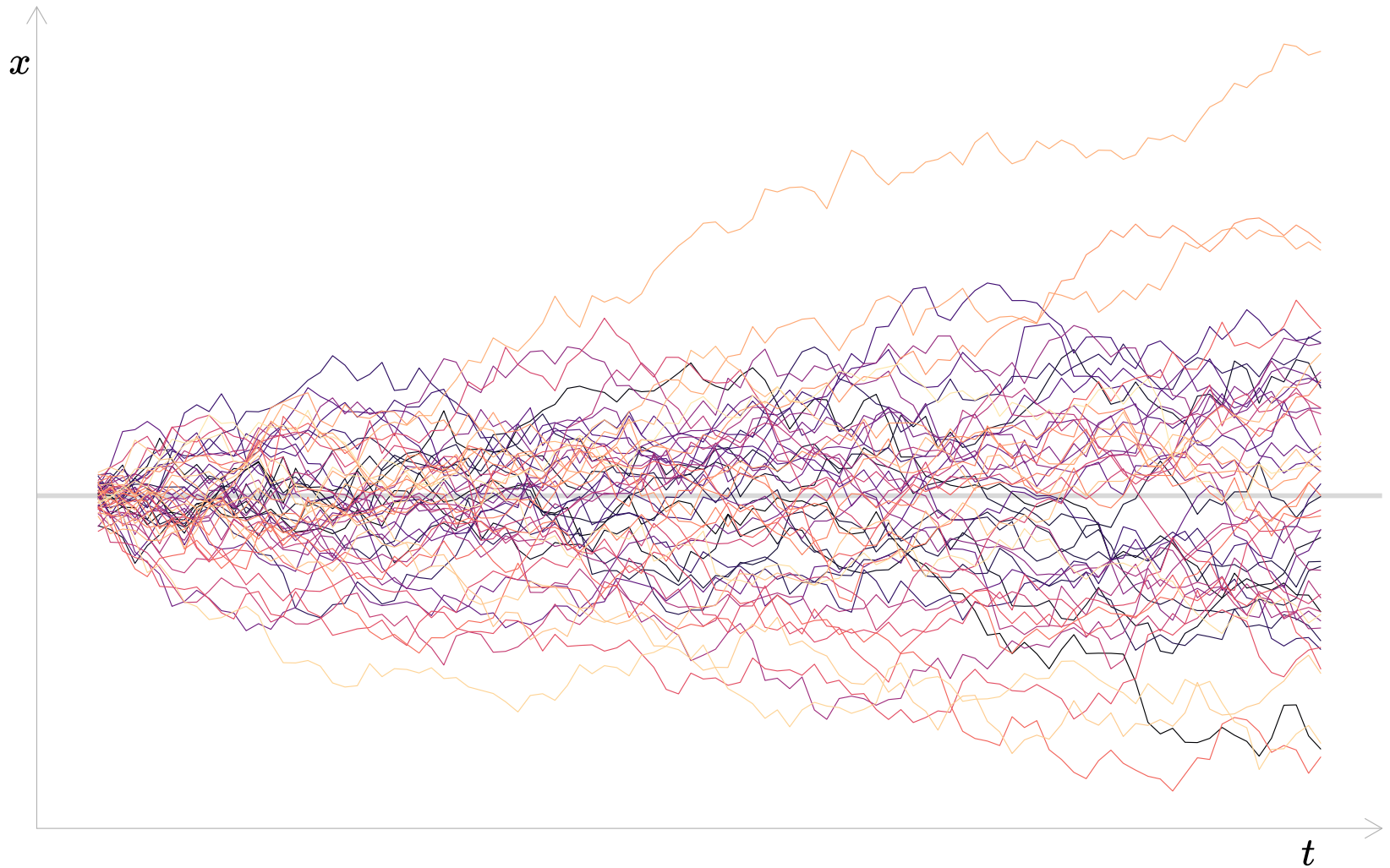
Four 100-period random walks



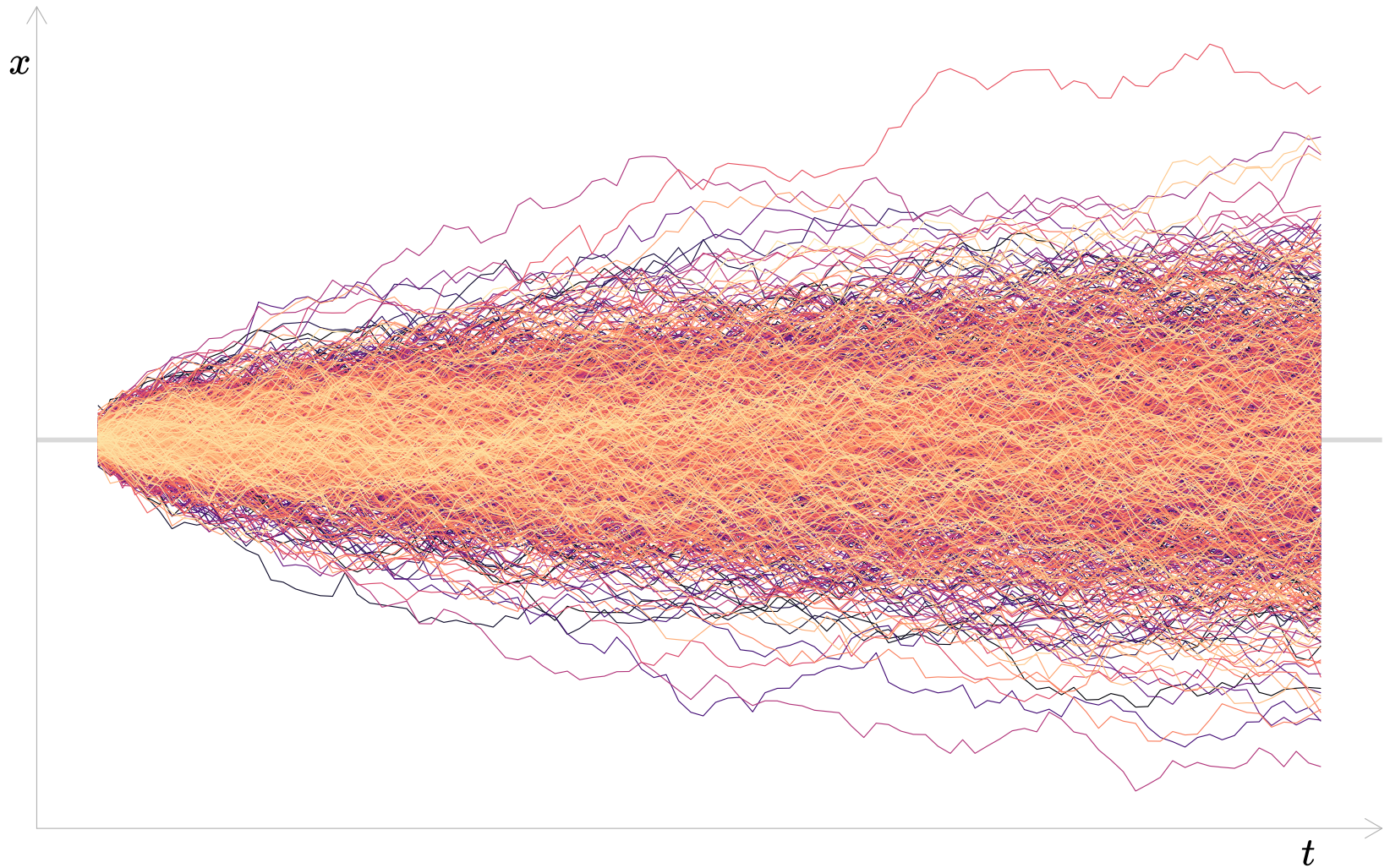
Five 100-period random walks



Fifty 100-period random walks



1,000 100-period random walks



Nonstationarity

Problem

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- apparently but not actually valid

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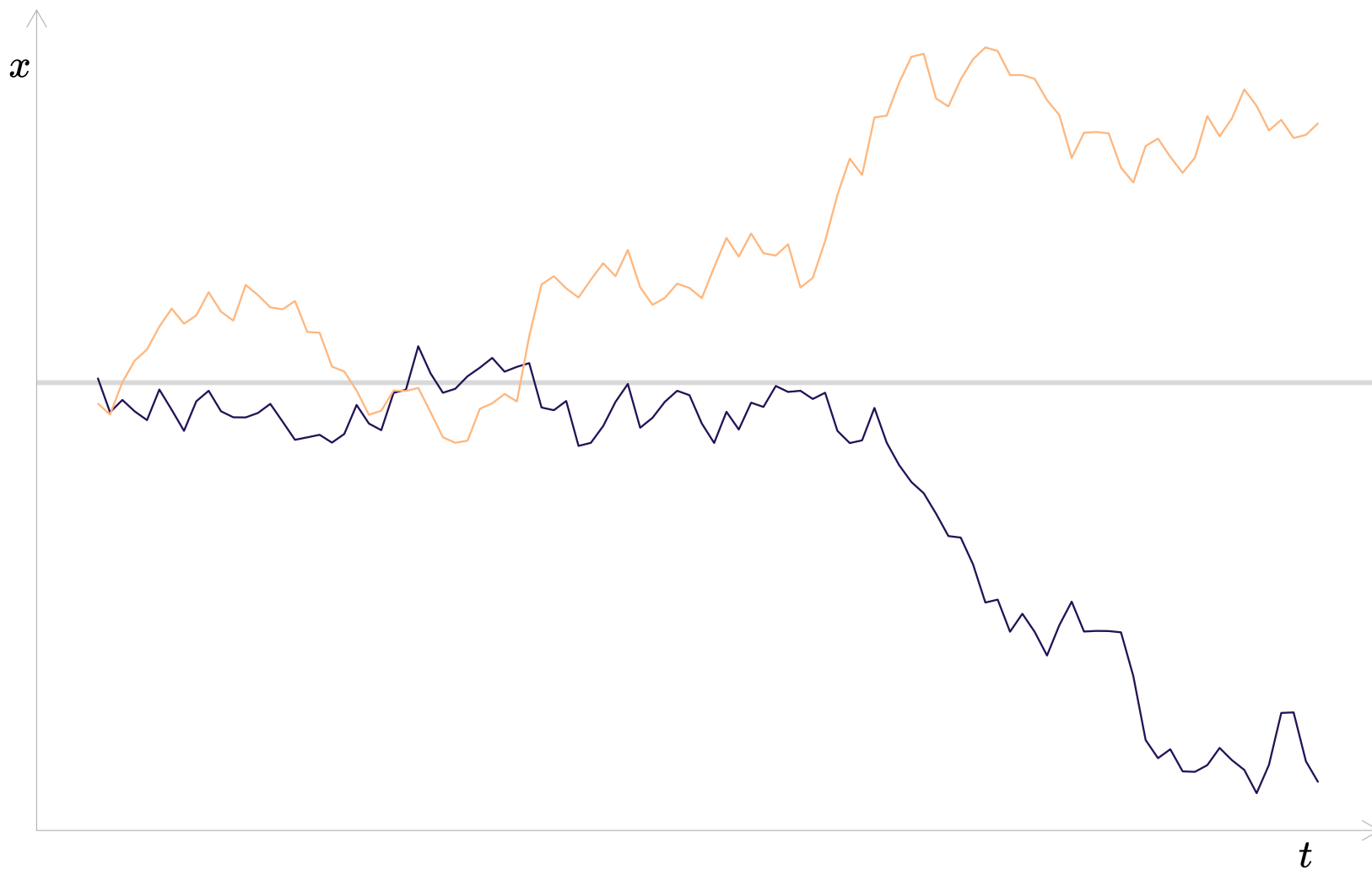
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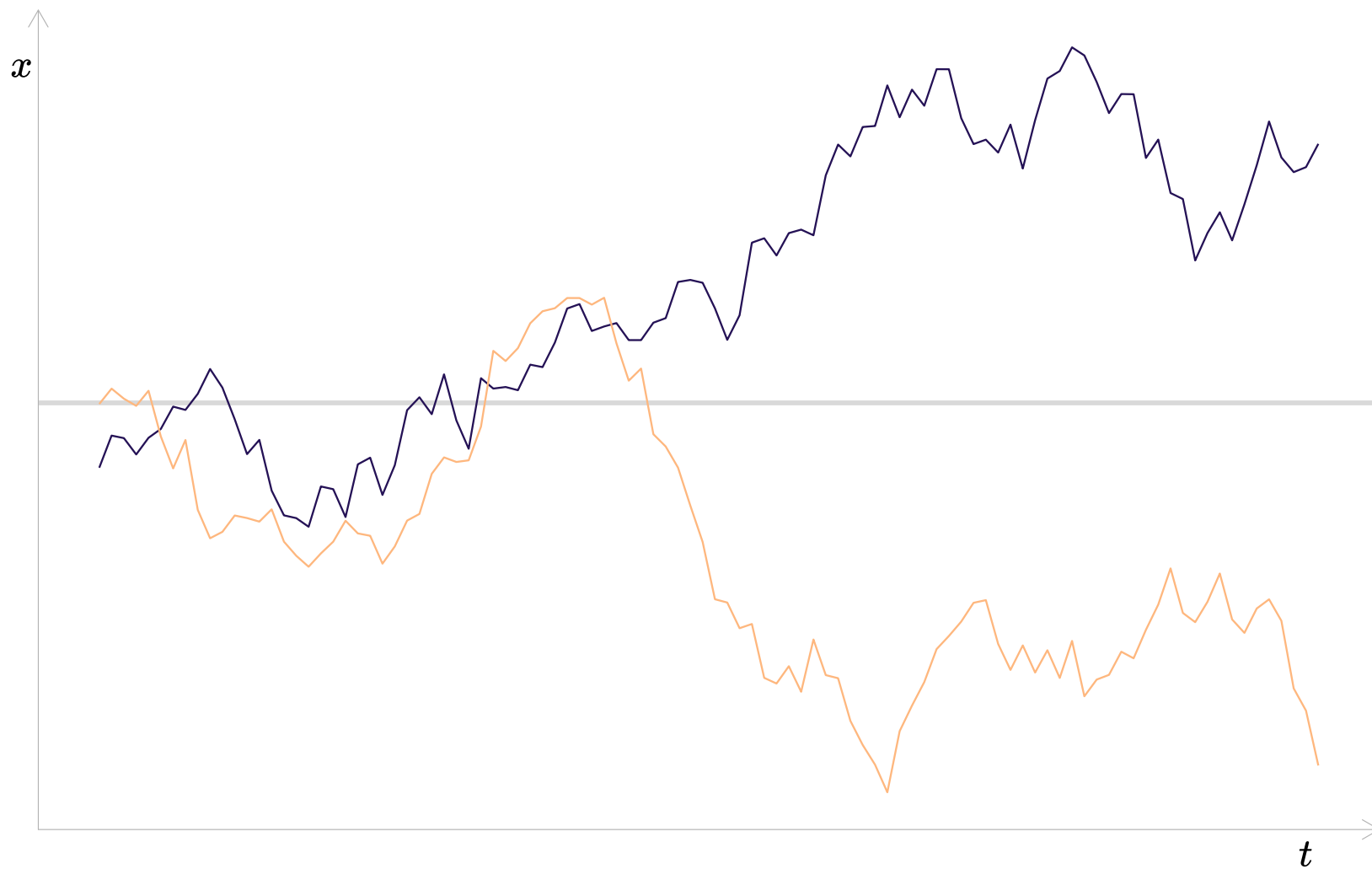
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Back in 1974, Granger and Newbold showed that when they **generated random walks** and **regressed the random walks on each other**, **77/100 regressions were statistically significant** at the 5% level (should have been approximately 5/100).

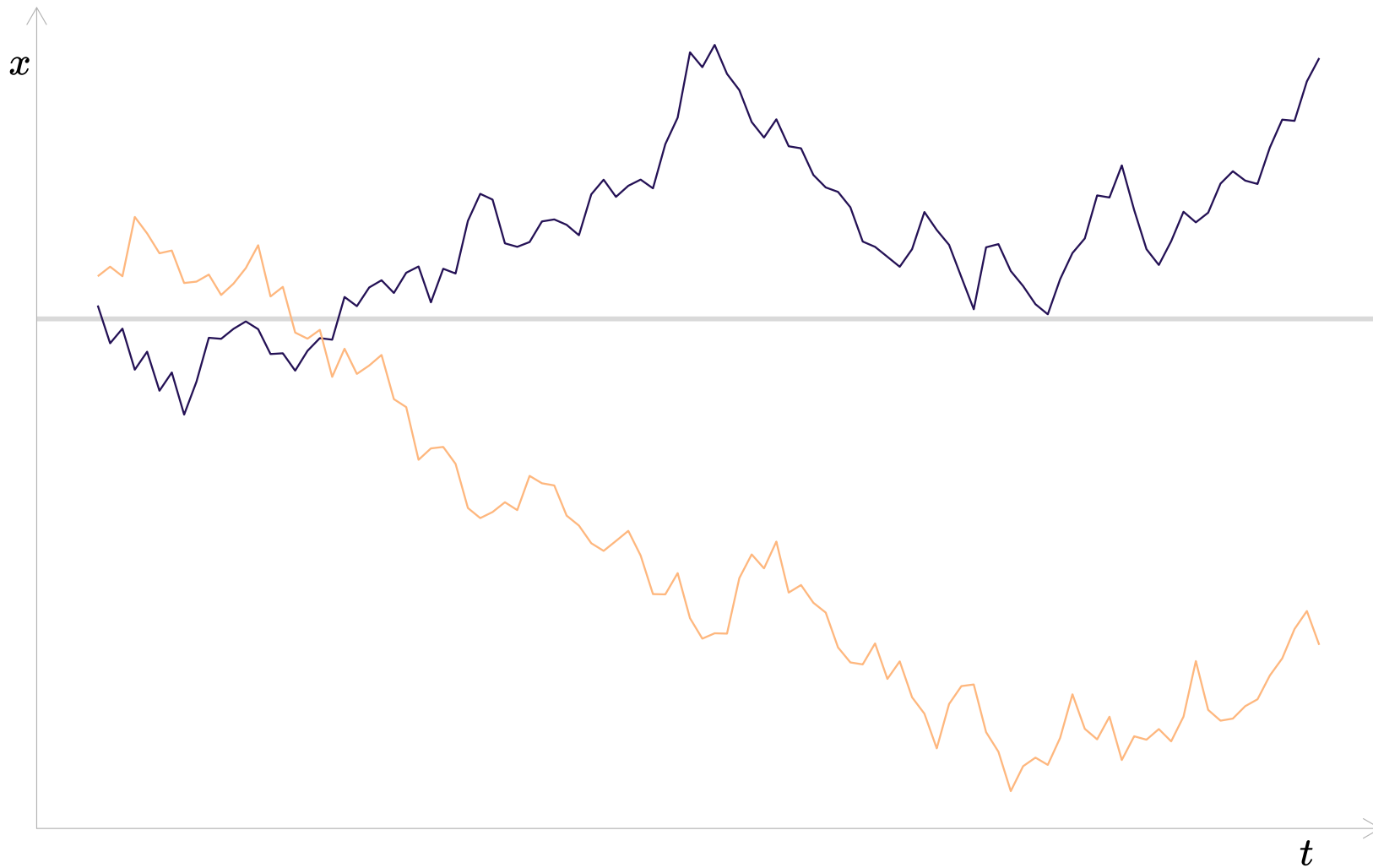
Granger and Newbold simulation example: t statistic ≈ -10.58



Granger and Newbold simulation example: t statistic ≈ -8.92



Granger and Newbold simulation example: t statistic ≈ -7.23



Nonstationarity

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Random walk with drift: $u_t = \alpha_0 + u_{t-1} + \varepsilon_t$

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Deterministic trend: $u_t = \alpha_0 + \beta_1 t + \varepsilon_t$

Nonstationarity

A potential solution

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$$\begin{aligned}y_t &= \beta_0 + \beta_1 x_t + u_t \\y_{t-1} &= \beta_0 + \beta_1 x_{t-1} + u_{t-1} \\y_t - y_{t-1} &= \beta_1 (x_t - x_{t-1}) + (u_t - u_{t-1}) \\\Delta y_t &= \beta_1 \Delta x_t + \Delta u_t\end{aligned}$$

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Dickey-Fuller tests compare

$H_0: y_t = \beta_0 + \beta_1 y_{t-1} + u_t$ with $|\beta_1| < 1$ (**stationarity**)

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using a t test that $|\beta_1| < 1$.[†]

[†] People often just test $\beta_1 < 1$.

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