

# Lecture 006

## Classification

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Edward Rubin

Admin

# Admin

## Material

### Last time

- `tidymodels`
- Shrinkage methods
  - Ridge regression 
  - (The) lasso 
  - Elasticnet 

### Today Classification methods

- Introduction to classification
- Linear probability models
- Logistic regression

# Admin

## Upcoming

**Readings** *Today* ISL Ch. 4

### **Problem sets**

- *Cross validation* Due Wednesday? (2/15)
- *Shrinkage methods* Coming soon
- *Classification* After that

Classification

# Classification

## Intro

**Regression problems** seek to predict the number an outcome will take—integers (e.g., number of cats), reals (e.g., home/cat value), etc. <sup>†</sup>

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# Classification

## Intro

**Regression problems** seek to predict the number an outcome will take—integers (e.g., number of cats), reals (e.g., home/cat value), etc. <sup>†</sup>

**Classification problems** instead seek to predict the category of an outcome

- **Binary outcomes**

success/failure; true/false; A or B; cat or *not cat*; etc.

- **Multi-class outcomes**

yes, no, or *maybe*; colors; letters; type of cat;<sup>††</sup> etc.

This type of outcome is often called a *qualitative* or *categorical* response.

<sup>†</sup> Maybe: Binary indicators... <sup>††</sup> It turns out, all of machine learning is about cats.

# Classification

## Examples

For the past few weeks, we've been immersed in regression problems.

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Can we predict whether a defendant is **granted bail**?

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Can we predict whether a defendant is **granted bail**?
- Based upon a set of symptoms and observations:  
Can we predict a patient's **medical condition(s)**?

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For the past few weeks, we've been immersed in regression problems.

It's probably helpful to mention a few **examples of classification problems**.

- Using life/criminal history (and demographics?):  
Can we predict whether a defendant is **granted bail**?
- Based upon a set of symptoms and observations:  
Can we predict a patient's **medical condition(s)**?
- From the pixels in an image:  
Can we classify images as **bagel, puppy, or other**?

# Classification

## Approach

One can imagine two<sup>†</sup> related **approaches to classification**

1. Predict **which category** the outcome will take.
2. Estimate the **probability of each category** for the outcome.

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# Classification

## Approach

One can imagine two<sup>†</sup> related **approaches to classification**

1. Predict **which category** the outcome will take.
2. Estimate the **probability of each category** for the outcome.

That said, the general approach will

- Take a set of training observations  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$
- Build a classifier  $\hat{y}_o = f(x_o)$

all while balancing bias and variance.<sup>††</sup>

<sup>†</sup> At least. <sup>††</sup> Sound familiar?

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**Q** If everything is so similar, can't we use regression methods?

**A** *Sometimes.*

**Q** If everything is so similar, can't we use regression methods?

**A** *Sometimes. Other times:* No.



**Q** If everything is so similar, can't we use regression methods?

**A** *Sometimes. Other times:* No. Plus you still need new tools.

# Classification

## Why not regression?

Regression methods are not made to deal with **multiple categories**.

*Ex.* Consider three medical diagnoses: **stroke**, **overdose**, and **seizure**.

Regression needs a numeric outcome—how should we code our categories?

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$$Y = \begin{cases} 1 & \text{if } \text{stroke} \\ 2 & \text{if } \text{overdose} \\ 3 & \text{if } \text{seizure} \end{cases}$$

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Option 2

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Option 3

$$Y = \begin{cases} 1 & \text{if } \text{seizure} \\ 2 & \text{if } \text{stroke} \\ 3 & \text{if } \text{overdose} \end{cases}$$

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*Option 3*

$$Y = \begin{cases} 1 & \text{if } \text{seizure} \\ 2 & \text{if } \text{stroke} \\ 3 & \text{if } \text{overdose} \end{cases}$$

The categories' ordering is unclear—let alone the actual valuation.

The choice of ordering and valuation can affect predictions. 🐱

# Classification

## Why not regression?

As we've seen, **binary outcomes** are simpler.

# Classification

## Why not regression?

As we've seen, **binary outcomes** are simpler.

Ex If we are only choosing between **stroke** and **overdose**

*Option 1*

$$Y = \begin{cases} 0 & \text{if } \text{stroke} \\ 1 & \text{if } \text{overdose} \end{cases}$$

and

*Option 2*

$$Y = \begin{cases} 0 & \text{if } \text{overdose} \\ 1 & \text{if } \text{stroke} \end{cases}$$

will provide the same results.



# Classification

## Why not regression?

In these **binary outcome** cases, we *can* apply linear regression.

These models are called **linear probability models** (LPMs).

The **predictions** from an LPM

1. estimate the conditional probability  $y_i = 1$ , i.e.,  $\Pr(y_o = 1 \mid x_o)$
2. are not restricted to being between 0 and 1<sup>†</sup>
3. provide an ordering—and a reasonable estimate of probability

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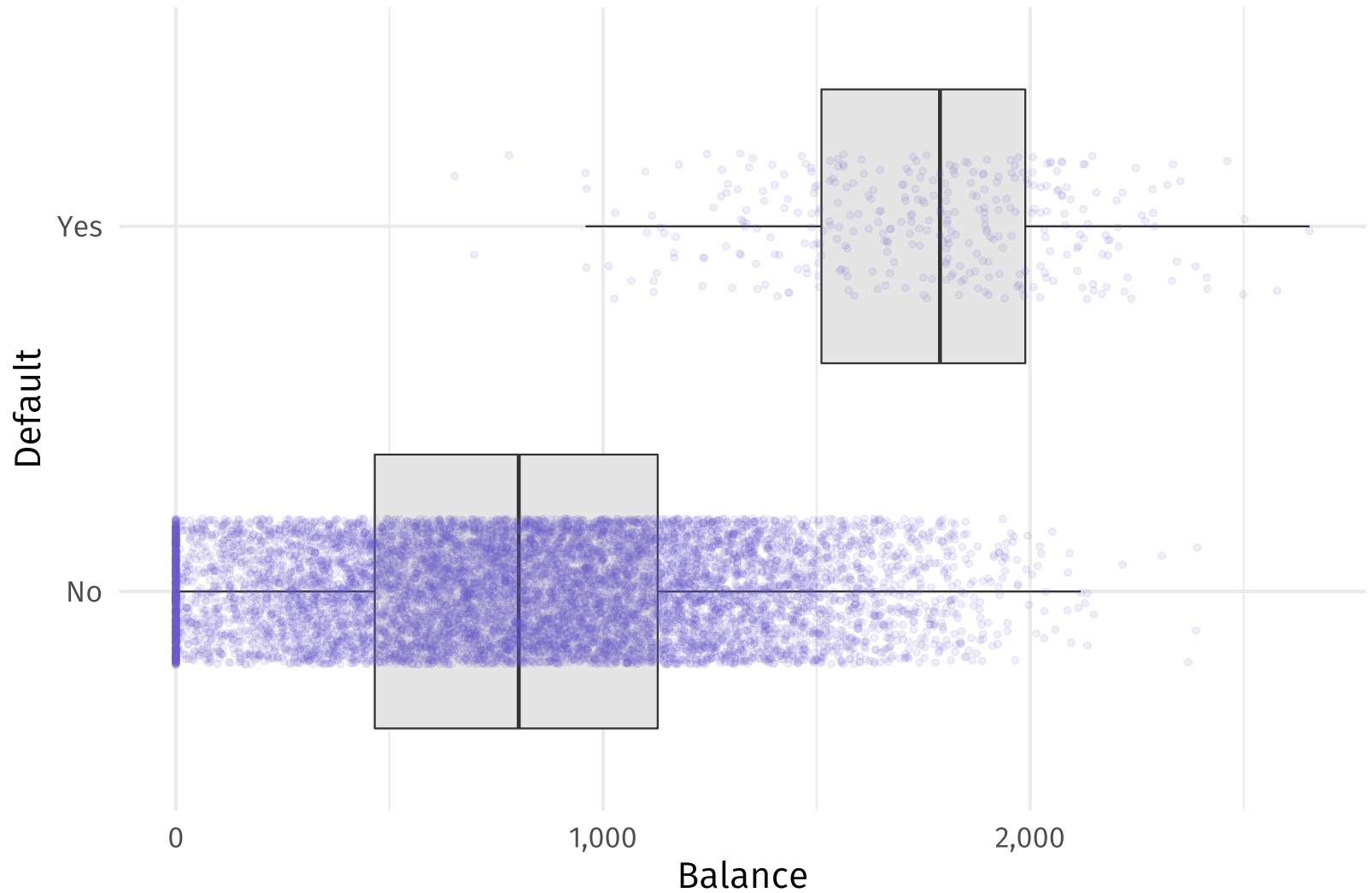
*Other benefits:* Coefficients are easily interpreted + we know how OLS works.

<sup>†</sup> Some people get very worked up about this point.

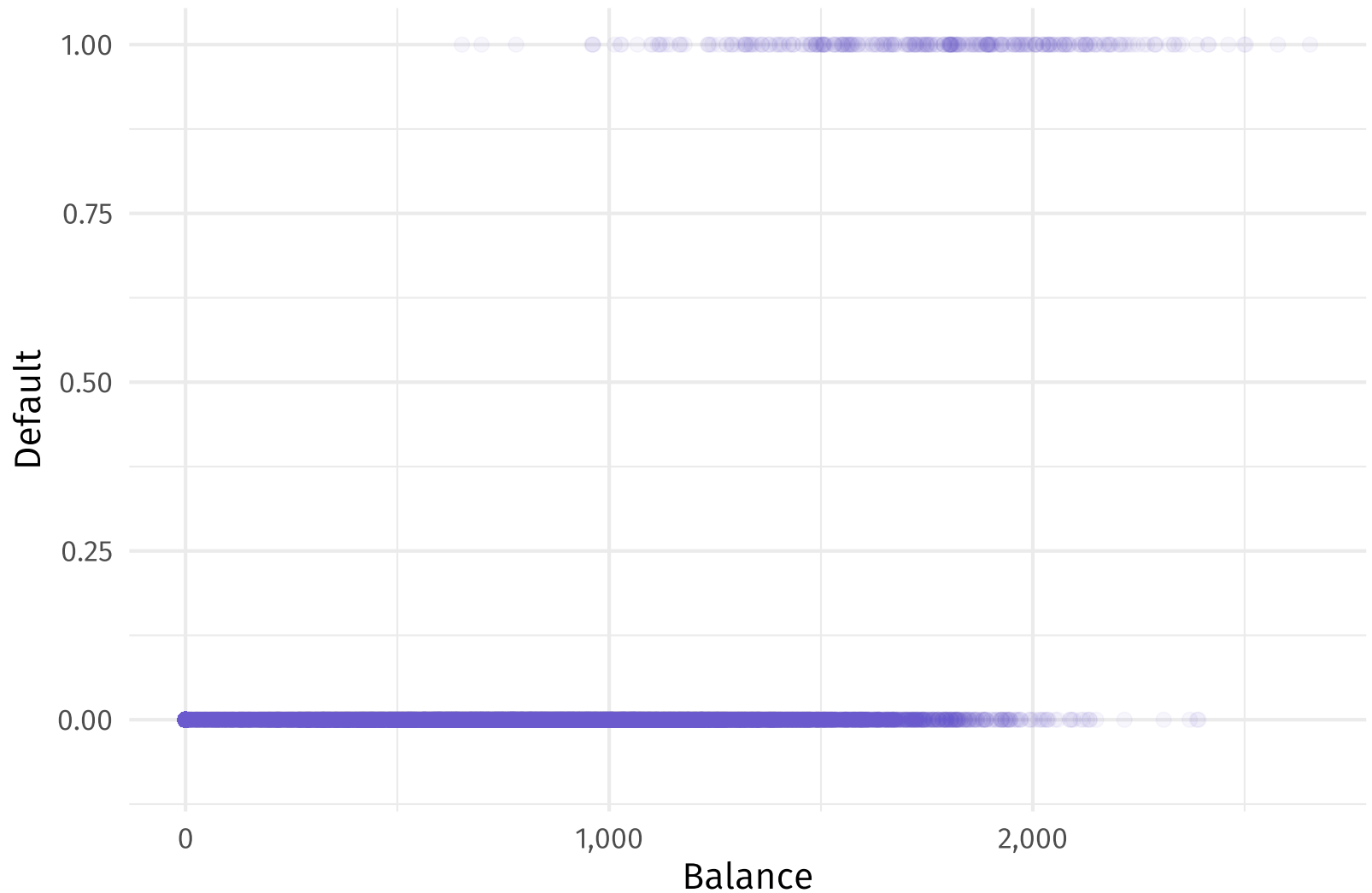
Let's consider an example: the `Default` dataset from `ISLR`

<b>default</b> ◆	<b>student</b> ◆	<b>balance</b> ◆	<b>income</b> ◆
No	No	939.10	45,519
No	Yes	397.54	22,711
Yes	No	1,511.61	53,507
No	No	301.32	51,540
No	No	878.45	29,562
Yes	No	1,673.49	49,310
No	No	310.13	37,697
No	No	1,272.05	44,896
No	No	887.20	41,641
No	No	230.87	32,799

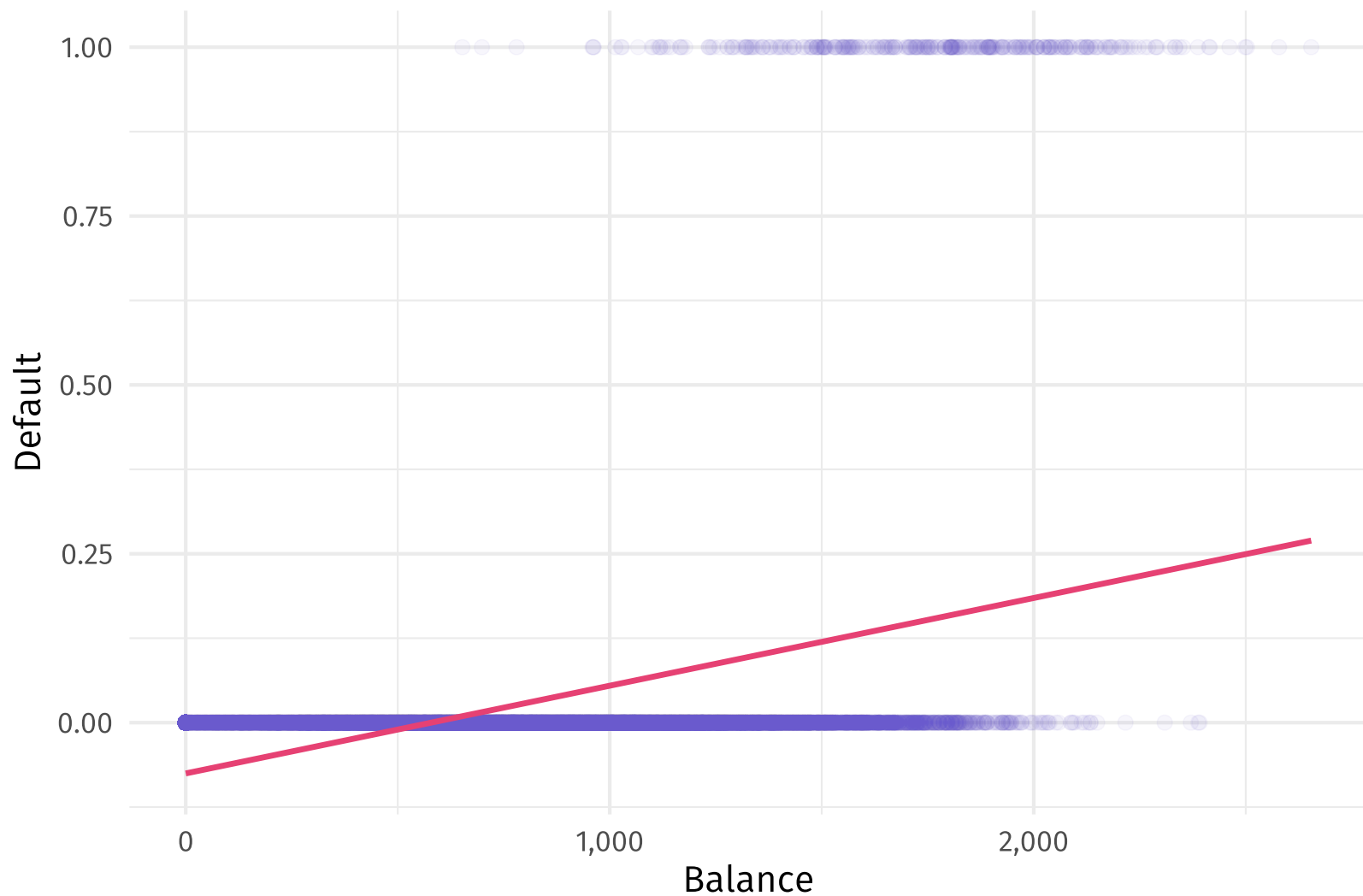
**The data:** The outcome, default, only takes two values (only 3.3% default).



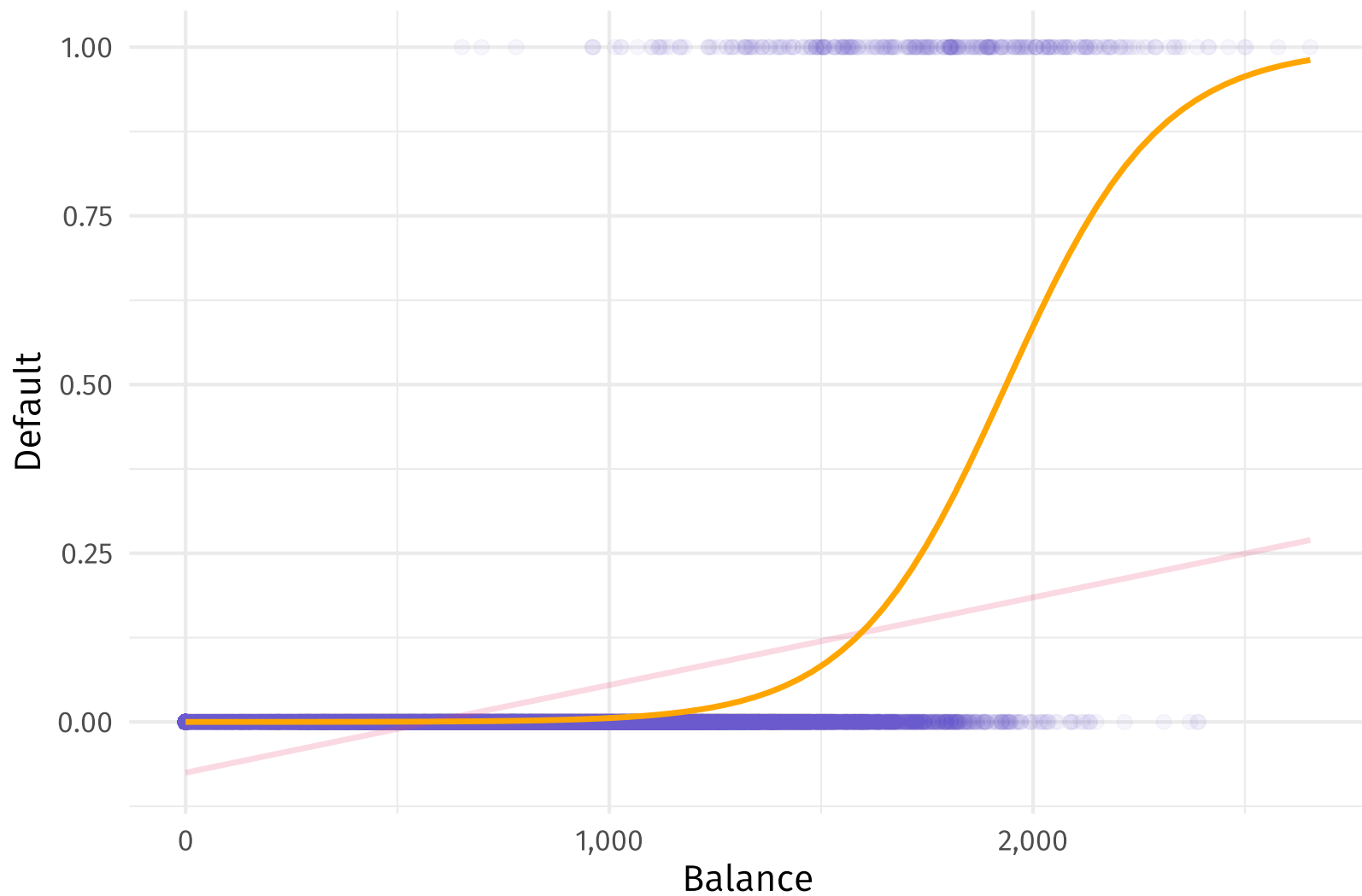
**The data:** The outcome, default, only takes two values (only 3.3% default).



**The linear probability model** struggles with prediction in this setting.



**Logistic regression** appears to offer an improvement.



So... what's logistic regression?



# Logistic regression

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## Intro

**Logistic regression** **models the probability** that our outcome  $Y$  belongs to a **specific category** (often whichever category we think of as `TRUE`).

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For example, we just saw a graph where

$$\Pr(\text{Default} = \text{Yes} | \text{Balance}) = p(\text{Balance})$$

we are modeling the probability of `default` as a function of `balance`.

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**Logistic regression models the probability** that our outcome  $Y$  belongs to a **specific category** (often whichever category we think of as `TRUE`).

For example, we just saw a graph where

$$\Pr(\text{Default} = \text{Yes} | \text{Balance}) = p(\text{Balance})$$

we are modeling the probability of `default` as a function of `balance`.

We use the **estimated probabilities** to **make predictions**, *e.g.*,

- if  $p(\text{Balance}) \geq 0.5$ , we could predict "Yes" for Default
- to be conservative, we could predict "Yes" if  $p(\text{Balance}) \geq 0.1$

# Logistic regression

## What's *logistic*?

We want to model probability as a function of the predictors ( $\beta_0 + \beta_1 X$ ).

### Linear probability model

linear transform. of predictors

$$p(X) = \beta_0 + \beta_1 X$$

### Logistic model

logistic transform. of predictors

$$p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}$$

What does this *logistic function* ( $\frac{e^x}{1+e^x}$ ) do?

1. ensures predictions are between 0 ( $x \rightarrow -\infty$ ) and 1 ( $x \rightarrow \infty$ )
2. forces an S-shaped curved through the data (not linear)

# Logistic regression

## What's *logistic*?

With a little math, you can show

$$p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}} \implies \log\left(\frac{p(X)}{1 - p(X)}\right) = \beta_0 + \beta_1 X$$

New definition: **log odds**<sup>†</sup> on the RHS and **linear predictors** on the LHS.

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New definition: **log odds**<sup>†</sup> on the RHS and **linear predictors** on the LHS.

1. **interpretation** of  $\beta_j$  is about **log odds**—not probability
2. **changes in probability** due to  $X$  depend on level of  $X$ <sup>††</sup>

<sup>†</sup> The "log odds" is sometimes called "logit". <sup>††</sup> It's nonlinear!



# Logistic regression

## Estimation

Before we can start predicting, we need to estimate the  $\beta_j$ s.

$$p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}} \implies \log\left(\frac{p(X)}{1 - p(X)}\right) = \beta_0 + \beta_1 X$$

We estimate logistic regression using **maximum likelihood estimation**.

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We estimate logistic regression using **maximum likelihood estimation**.

**Maximum likelihood estimation** (MLE) searches for the  $\beta_j$ s that make our data "most likely" given the model we've written.

# Logistic regression

## Maximum likelihood

**MLE** searches for the  $\beta_j$ s that make our data "most likely" using our model.

$$\log\left(\frac{p(X)}{1 - p(X)}\right) = \beta_0 + \beta_1 X$$

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2.  $\text{odds} = \frac{p(X)}{1 - p(X)}$ . If  $p(X) > 0.5$ , then  $\text{odds} > 1$  and **log odds**  $> 0$ .

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So we want choose  $\beta_j$  such that

- **log odds** are above zero for observations where  $y_i = 1$
- **log odds** even larger for areas of  $x_j$  where most  $i$ s have  $y_i = 1$

# Logistic regression

## Formally: The likelihood function

We estimate logistic regression by maximizing **the likelihood function**<sup>†</sup>

$$\ell(\beta_0, \beta_1) = \prod_{i:y_i=1} p(x_i) \prod_{i:y_i=0} (1 - p(x_i))$$

The likelihood function is maximized by

- making  $p(x_i)$  large for individuals with  $y_i = 1$
- making  $p(x_i)$  small for individuals with  $y_i = 0$

*Put simply:* Maximum likelihood maximizes a predictive performance, conditional on the model we have written down.

<sup>†</sup> Generally, we actually will maximize the *log* of the likelihood function.



# Logistic regression

## In R

In R, you can run logistic regression using the `glm()` function.

Also: `logistic_reg()` in the `tidymodels` galaxy (with the `"glm"` engine).

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*Aside:* Related to `lm`, `glm` stands for *generalized* (linear model).

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*Aside:* Related to `lm`, `glm` stands for *generalized* (linear model).

"Generalized" essentially means that we're applying some transformation to  $\beta_0 + \beta_1 X$  like logistic regression applies the logistic function.

More generally:

$$y = g^{-1}(X\beta) \iff g(y) = X\beta$$

# Logistic regression

## In R

In R, you can run logistic regression using the `glm()` function.

**Key arguments** (very similar to `lm()`)

- specify a formula,<sup>†</sup> e.g., `y ~ .` or `y ~ x + I(x^2)`
- define `family = "binomial"` (so R knows to run logistic regression)
- give the function some `data`

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```
est_logistic = glm(  
  i_default ~ balance,  
  family = "binomial",  
  data = default_df  
)
```

<sup>†</sup> Notice that we're back in the world of needing to select a model...

```
est_logistic %>% summary()
```

```
#>
#> Call:
#> glm(formula = i_default ~ balance, family = "binomial", data = default_df)
#>
#> Deviance Residuals:
#>      Min       1Q   Median       3Q      Max
#> -2.2697  -0.1465  -0.0589  -0.0221   3.7589
#>
#> Coefficients:
#>              Estimate Std. Error z value Pr(>|z|)
#> (Intercept) -1.065e+01  3.612e-01  -29.49  <2e-16 ***
#> balance      5.499e-03  2.204e-04   24.95  <2e-16 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
#> (Dispersion parameter for binomial family taken to be 1)
#>
#>      Null deviance: 2920.6  on 9999  degrees of freedom
#> Residual deviance: 1596.5  on 9998  degrees of freedom
#> AIC: 1600.5
#>
#> Number of Fisher Scoring iterations: 8
```

# Logistic regression

## Estimates and predictions

Thus, our estimates are  $\hat{\beta}_0 \approx -10.65$  and  $\hat{\beta}_1 \approx 0.0055$ .

*Remember:* These coefficients are for the **log odds**.

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If we want **to make predictions** for  $y_i$  (whether or not  $i$  defaults), then we first must **estimate the probability**  $p(\text{Balance})$

$$\hat{p}(\text{Balance}) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 \text{Balance}}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 \text{Balance}}} \approx \frac{e^{-10.65 + 0.0055 \cdot \text{Balance}}}{1 + e^{-10.65 + 0.0055 \cdot \text{Balance}}}$$



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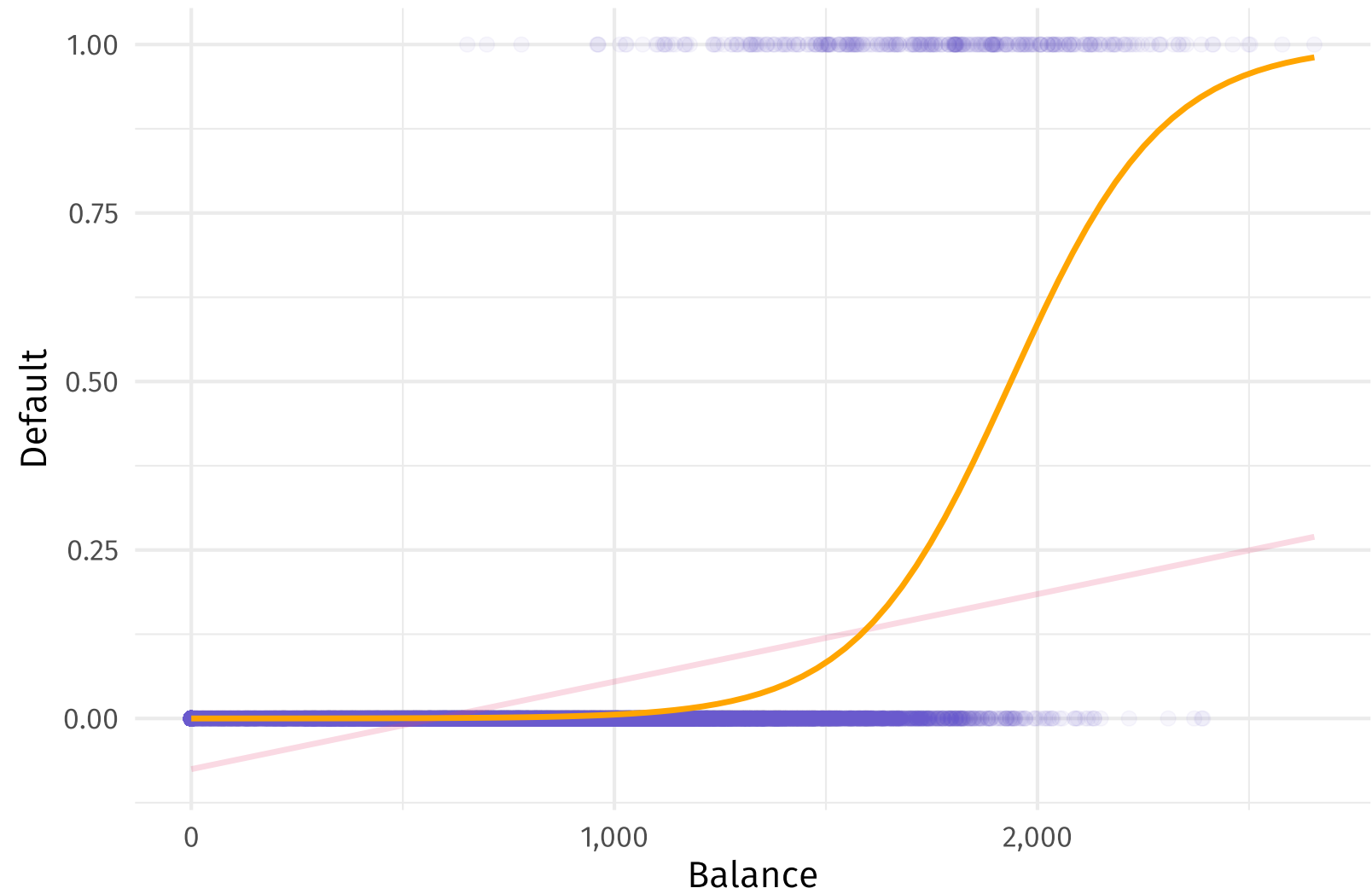
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- If  $\text{Balance} = 0$ , we then estimate  $\hat{p} \approx 0.000024$
- If  $\text{Balance} = 2,000$ , we then estimate  $\hat{p} \approx 0.586$
- If  $\text{Balance} = 3,000$ , we then estimate  $\hat{p} \approx 0.997$  <sup>†</sup>

<sup>†</sup> You get a sense of the nonlinearity of the predictors' effects.

Logistic regression's predictions of  $p(\text{Balance})$



*Note:* Everything we've done so far extends to models with many predictors.

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*Old news:* You can use `predict()` to get predictions out of `glm` objects.

**New and important:** `predict()` produces multiple `types` of predictions

1. `type = "response"` predicts *on the scale of the response variable* for logistic regression, this means **predicted probabilities** (0 to 1)
2. `type = "link"` predicts *on the scale of the linear predictors* for logistic regression, this means **predicted log odds** ( $-\infty$  to  $\infty$ )

**Beware:** The default is `type = "link"`, which you may not want.

# Logistic regression

## Prediction

Putting it all together, we can get (estimated) probabilities  $\hat{p}(X)$

```
# Predictions on scale of response (outcome) variable  
p_hat = predict(est_logistic, type = "response")
```

which we can use to make predictions on  $y$

```
# Predict '1' if p_hat is greater or equal to 0.5  
y_hat = as.numeric(p_hat ≥ 0.5)
```

So how did we do?

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<sup>†</sup> This idea is called the *null classifier*.

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Q How can we more formally assess our model's performance?

A All roads lead to the **confusion matrix**.

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# Assessment

## The confusion matrix

The **confusion matrix** is us a convenient way to display **correct** and **incorrect** predictions for each class of our outcome.

		Truth	
		No	Yes
Prediction	No	True Negative (TN)	False Negative (FN)
	Yes	False Positive (FP)	True Positive (TP)

# Assessment

## The confusion matrix

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$$\text{Accuracy} = (\text{TN} + \text{TP}) / (\text{TN} + \text{TP} + \text{FN} + \text{FP})$$

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This matrix also helps display many other measures of assessment.

# Assessment

## The confusion matrix

**Sensitivity:** the share of positive outcomes  $Y = 1$  that we correctly predict.

$$\text{Sensitivity} = \text{TP} / (\text{TP} + \text{FN})$$

		Truth	
		No	Yes
Prediction	No	True Negative (TN)	False Negative (FN)
	Yes	False Positive (FP)	True Positive (TP)

Sensitivity is also called **recall** and the **true-positive rate**.

One minus sensitivity is the **type-II error rate**.

# Assessment

## The confusion matrix

**Specificity:** the share of neg. outcomes ( $Y = 0$ ) that we correctly predict.

$$\text{Specificity} = \text{TN} / (\text{TN} + \text{FP})$$

		Truth	
		No	Yes
Prediction	No	True Negative (TN)	False Negative (FN)
	Yes	False Positive (FP)	True Positive (TP)

One minus specificity is the **false-positive rate** or **type-I error rate**.

# Assessment

## The confusion matrix

**Precision:** the share of predicted positives ( $\hat{Y} = 1$ ) that are correct.

$$\text{Precision} = \text{TP} / (\text{TP} + \text{FP})$$

		Truth	
		No	Yes
Prediction	No	True Negative (TN)	False Negative (FN)
	Yes	False Positive (FP)	True Positive (TP)

# Assessment

## Which assessment?

Q So *which* criterion should we use?

# Assessment

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**A** You should use the *right* criterion for your context.

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*Sensitivity* will be key.

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*Sensitivity* will be key.
- Do you want to have high confidence in predicted positives?

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## Which assessment?

**Q** So *which* criterion should we use?

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*Precision* is your friend

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*Precision* is your friend
- Are all errors equal?

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- Do you want to have high confidence in predicted positives?  
*Precision* is your friend
- Are all errors equal?  
*Accuracy* is perfect.

# Assessment

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**Q** So *which* criterion should we use?

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*Sensitivity* will be key.
- Do you want to have high confidence in predicted positives?  
*Precision* is your friend
- Are all errors equal?  
*Accuracy* is perfect.

There's a lot more, e.g., the **F<sub>1</sub> score** combines precision and sensitivity.

# Assessment

## Confusion in R

`conf_mat()` from `yardstick` (`tidymodels`) calculates the confusion matrix.

- `data`: a dataset ( `factor` variables) of true values and predictions
- `truth`: the name of the column (in `data`) of the truth values
- `estimate`: the name of the column (in `data`) of our predictions

# Assessment

## Confusion in R

`conf_mat()` from `yardstick` (`tidymodels`) calculates the confusion matrix.

- `data`: a dataset ( `factor` variables) of true values and predictions
- `truth`: the name of the column (in `data`) of the truth values
- `estimate`: the name of the column (in `data`) of our predictions

```
cm_logistic = conf_mat(  
  # Create a dataset of truth and predictions  
  data = tibble(  
    y_hat = y_hat %>% as.factor(),  
    y = default_df$i_default %>% as.factor()  
  ),  
  truth = y, estimate = y_hat  
)
```



# Assessment

## Confusion in R

`conf_mat()` from `yardstick` (`tidymodels`) calculates the confusion matrix.

- `data`: a dataset ( `factor` variables) of true values and predictions
- `truth`: the name of the column (in `data`) of the truth values
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```
#>           Truth
#> Prediction    0    1
#>           0 9625 233
#>           1   42 100
```

# Assessment

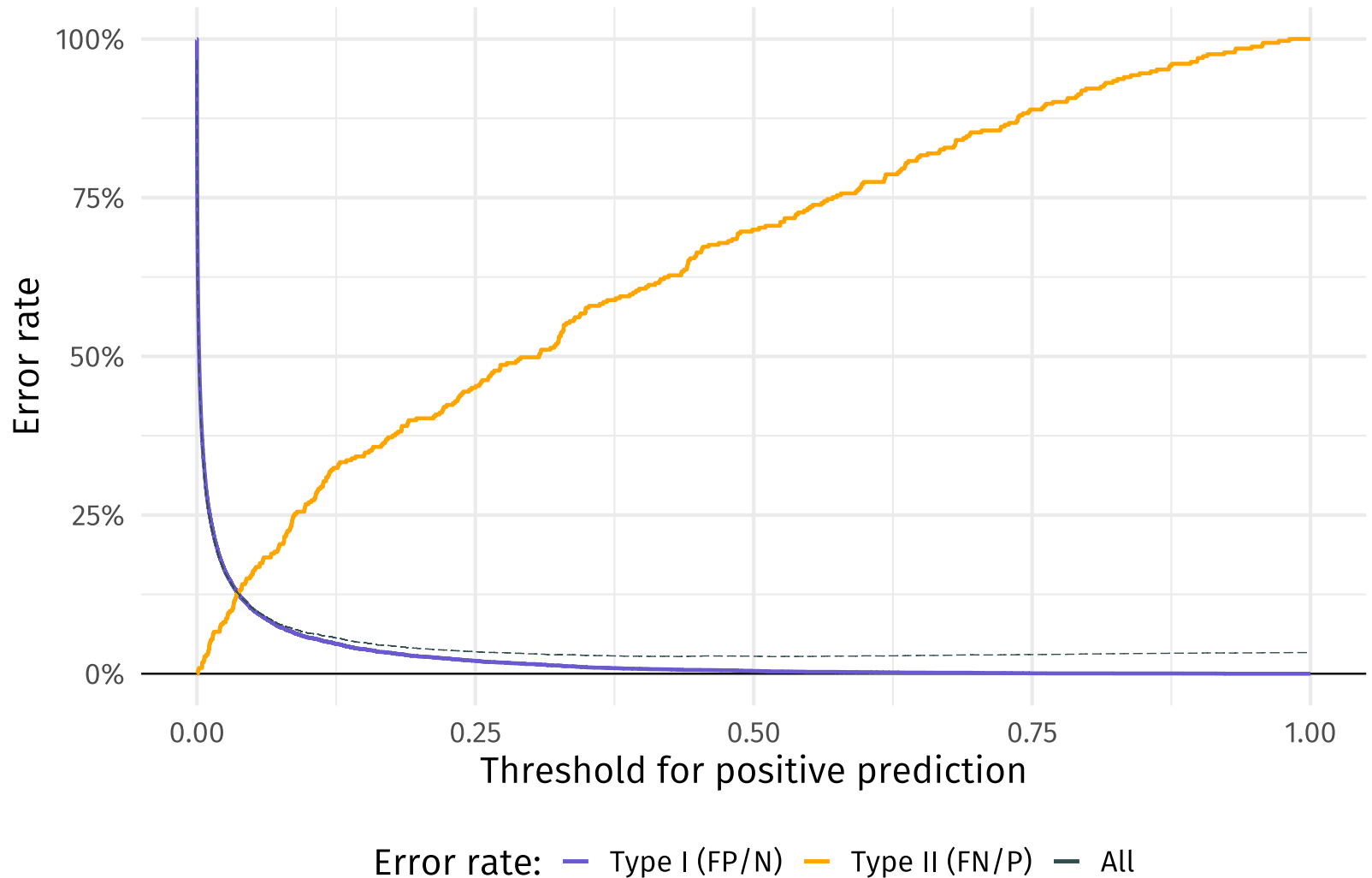
## Thresholds

Your setting also dictates the "optimal" threshold that moves a prediction from one class (e.g., Default = No) to another class (Default = Yes).

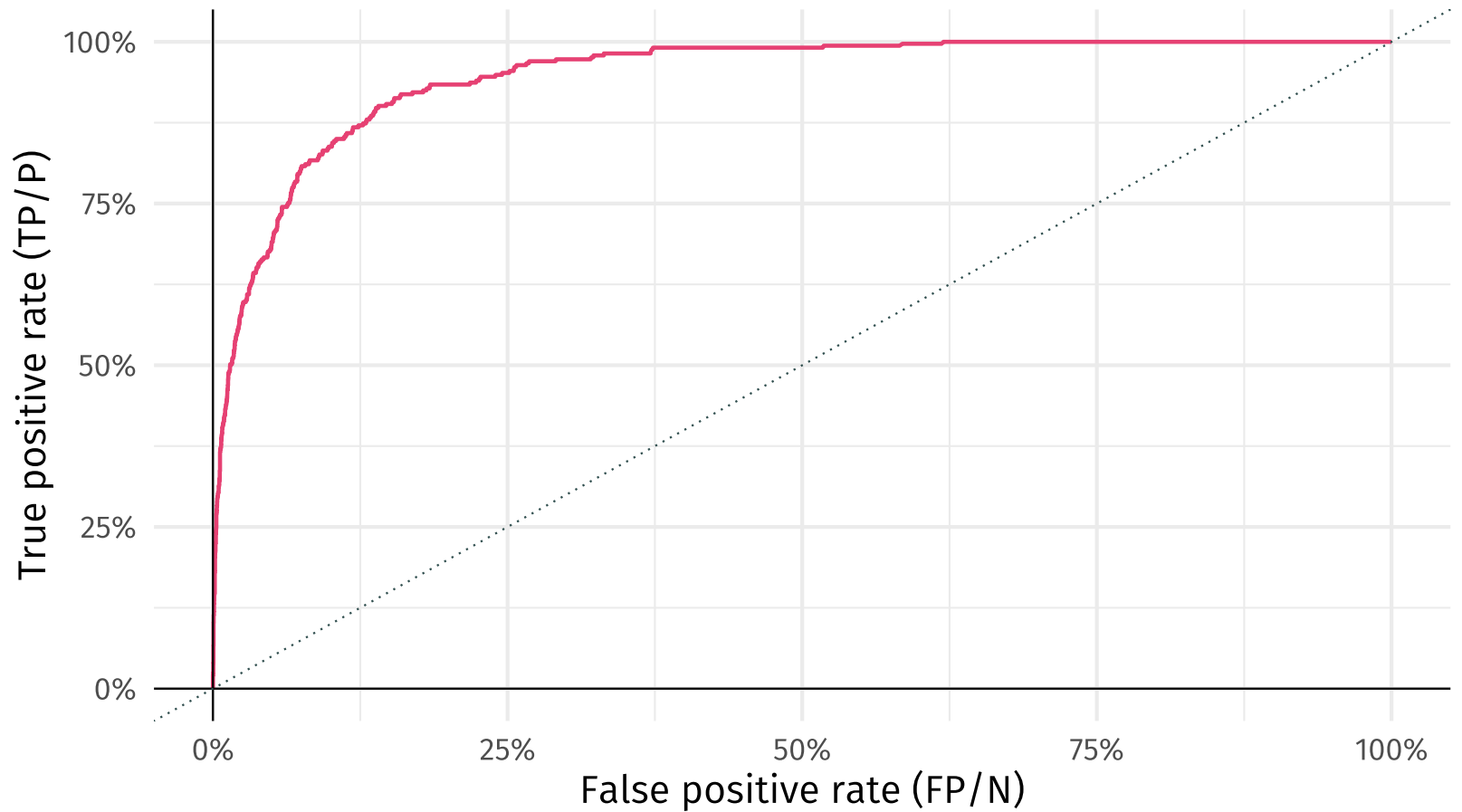
The Bayes classifier suggests a probability threshold of 0.5.

The Bayes classifier can't be beat in terms of *accuracy*, but if you have goals other than accuracy, you should consider other thresholds.

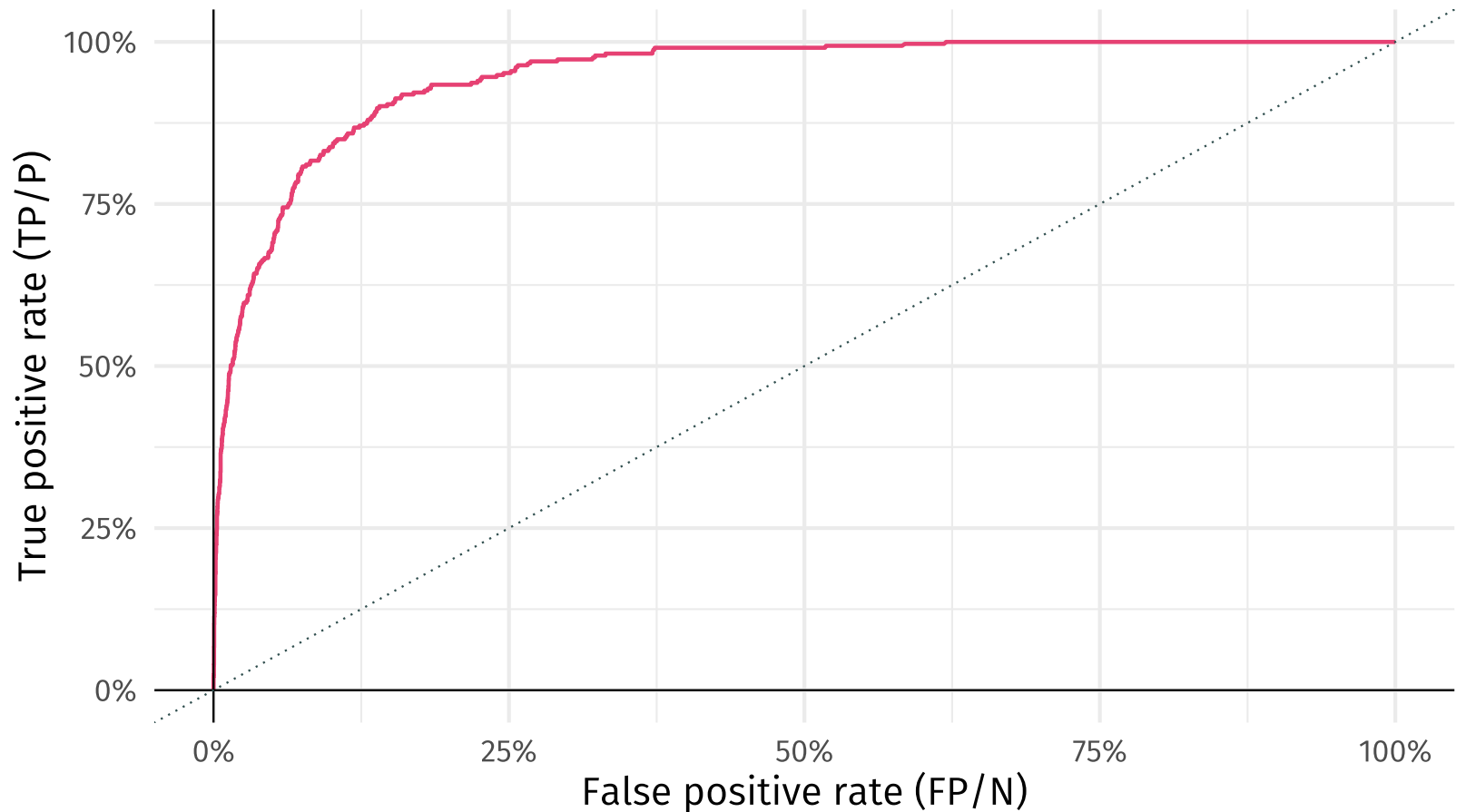
As we vary the threshold, our error rates (types I, II, and **overall**) change.



The **ROC curve** plots the true- (TP/P) and the false-positive rates (FP/N).

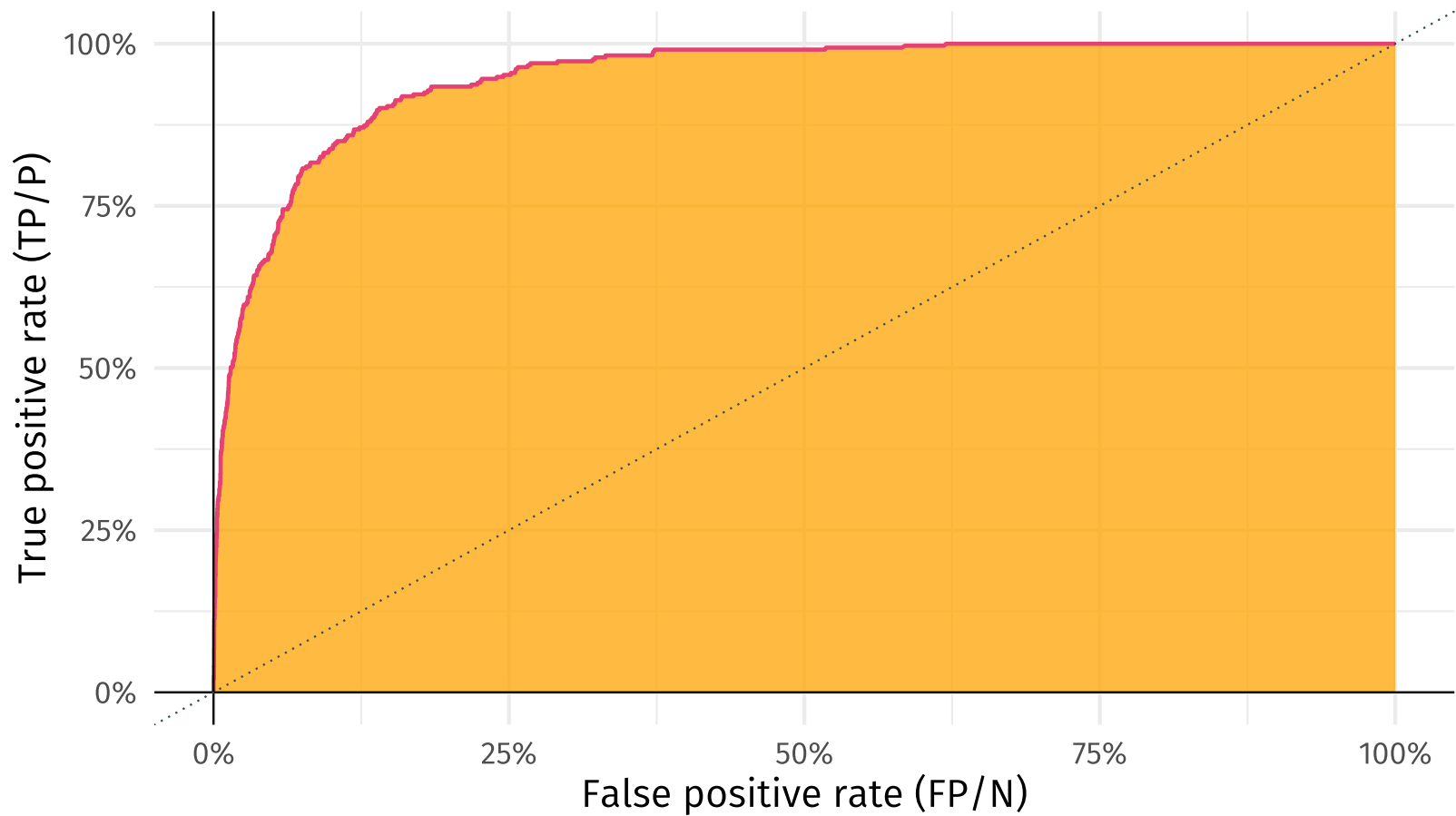


The **ROC curve** plots the true- (TP/P) and the false-positive rates (FP/N).

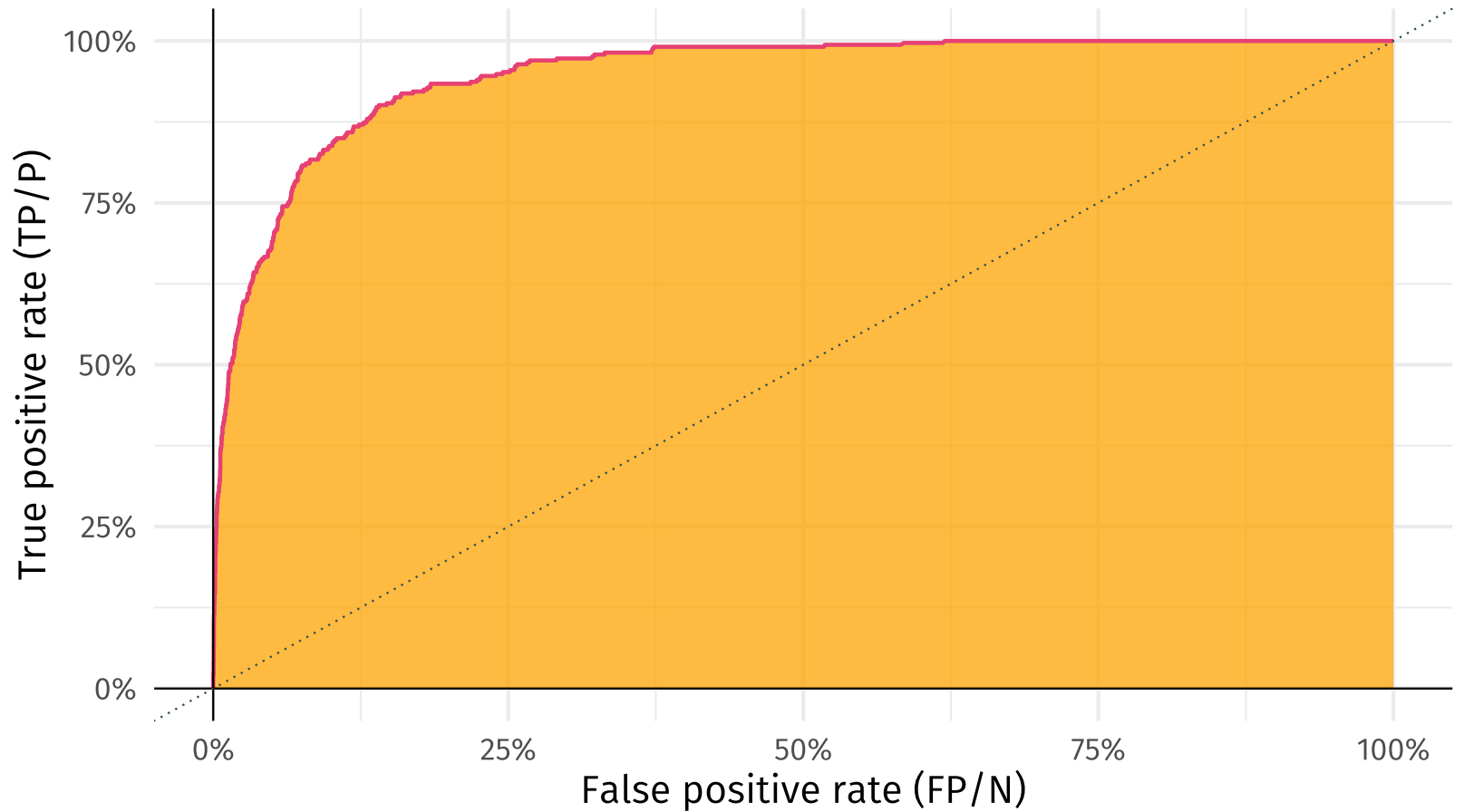


"Best performance" means the **ROC curve** hugs the top-left corner.

The **AUC** gives the area under the (ROC) curve.



The **AUC** gives the area under the (ROC) curve.



"Best performance" means the **AUC** is near 1. Random chance: 0.5

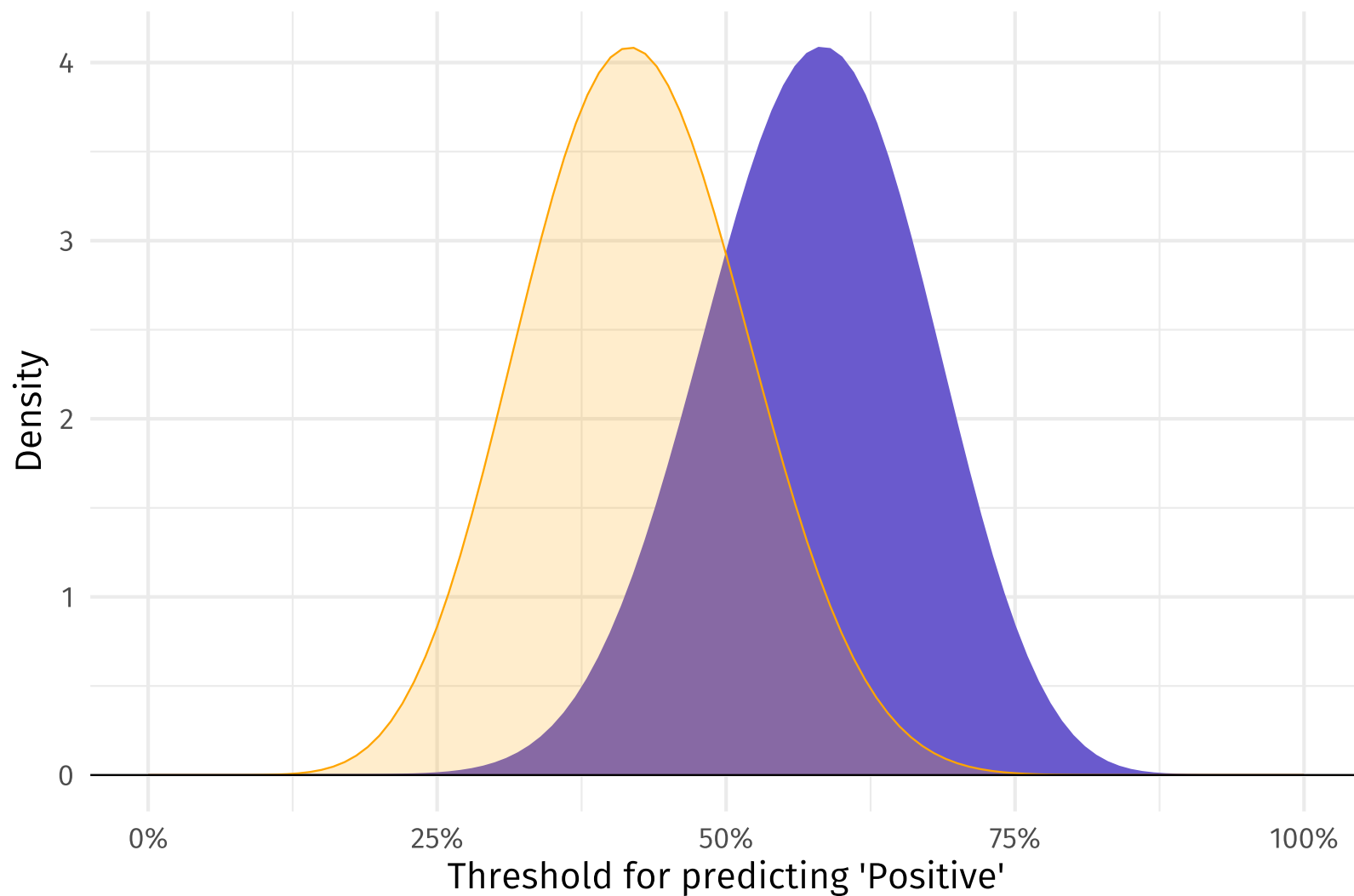
Q So what information is AUC telling us?



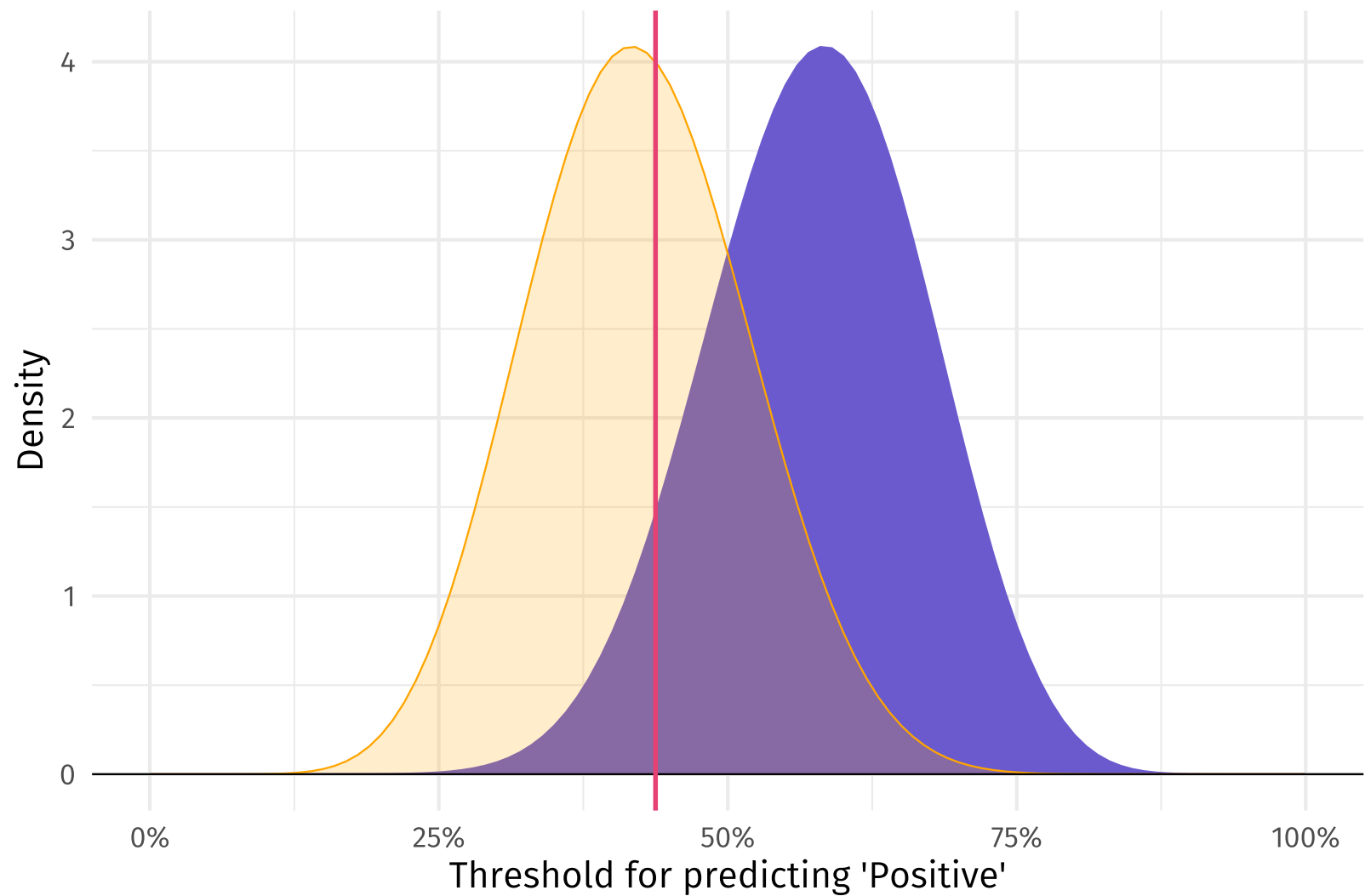
Q So what information is AUC telling us?

A AUC tells us how much we've **separated** the *positive* and *negative* labels.

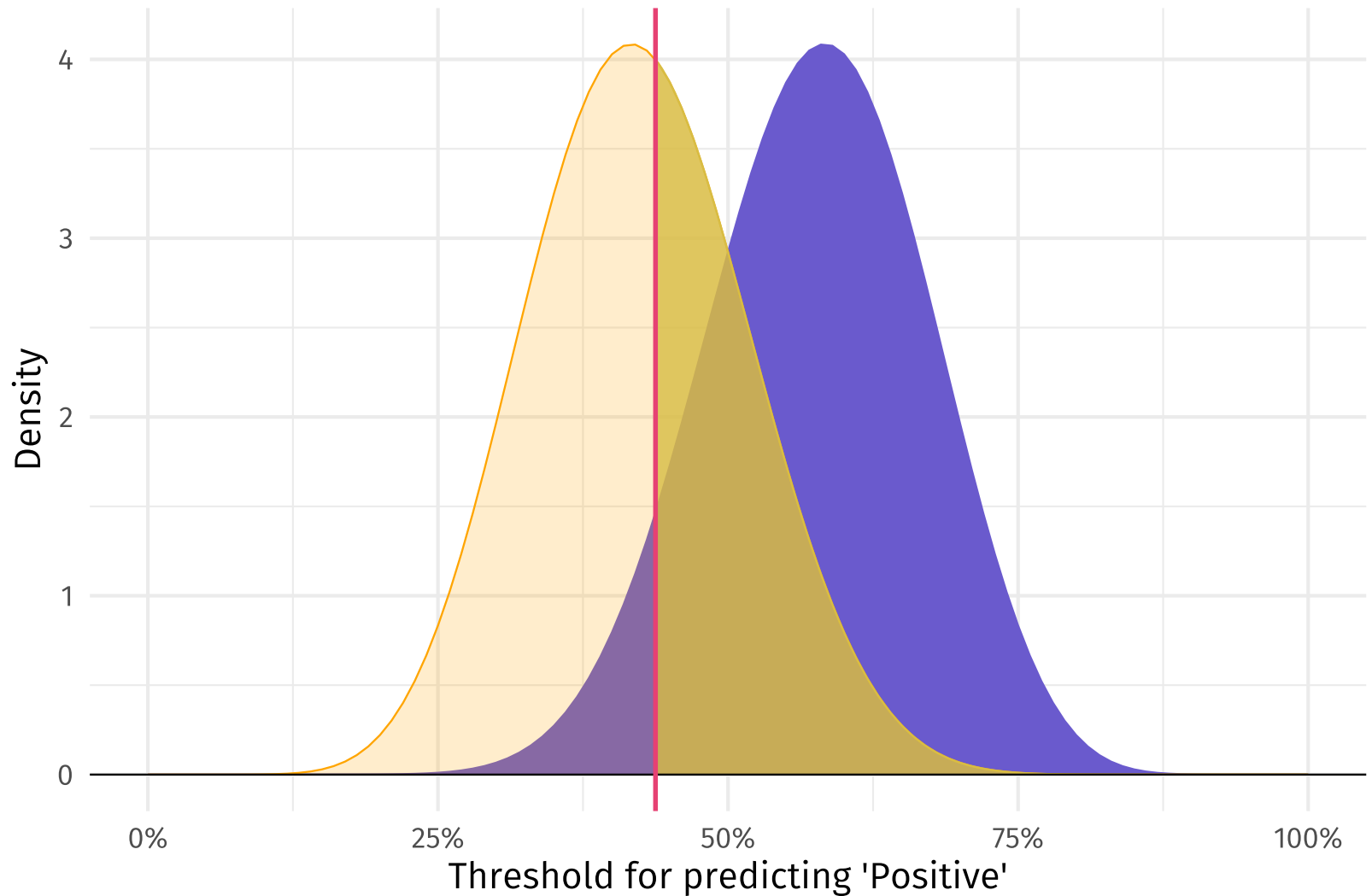
*Example:* Distributions of probabilities for **negative** and **positive** outcomes.



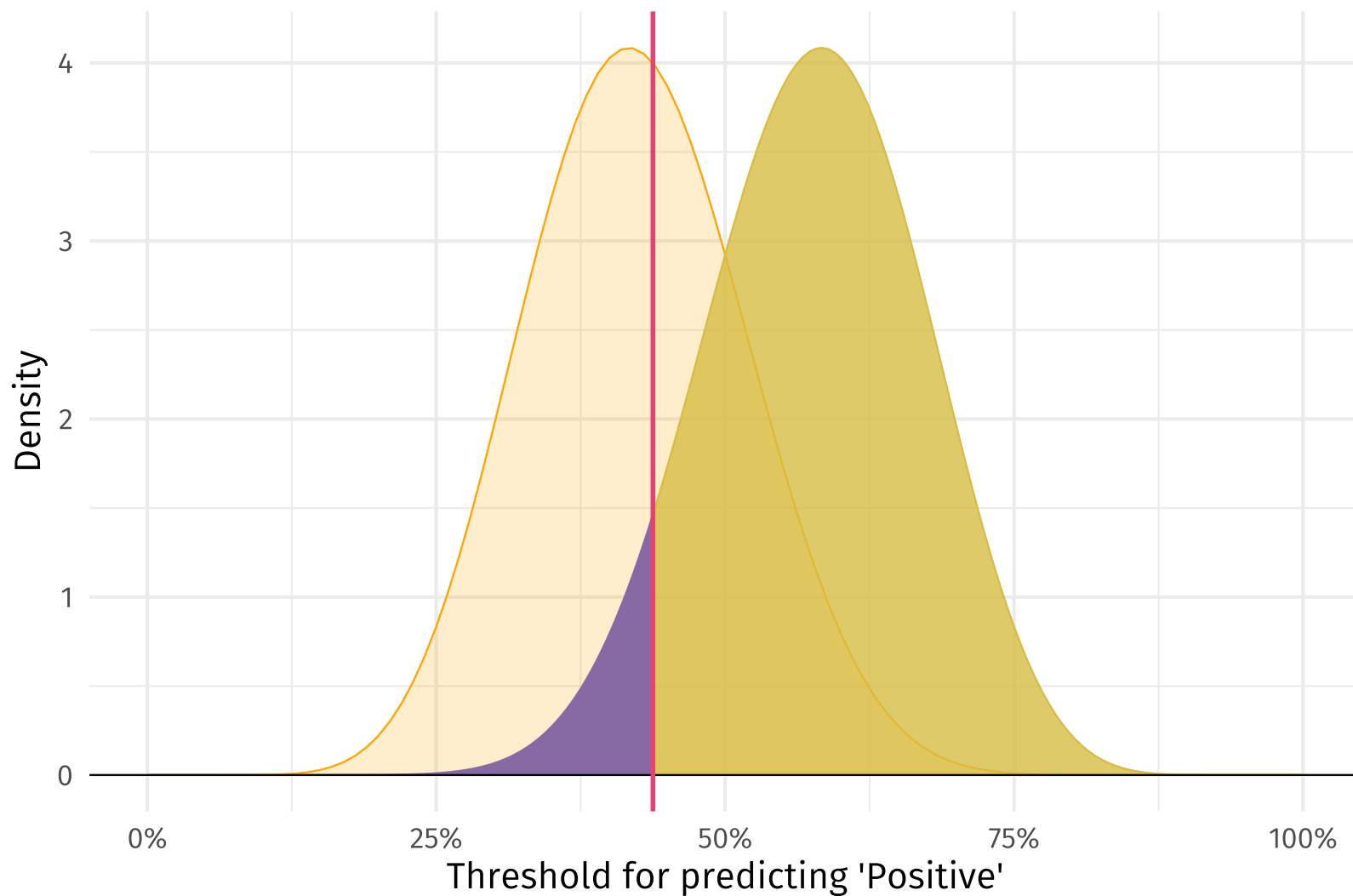
For any given **threshold**



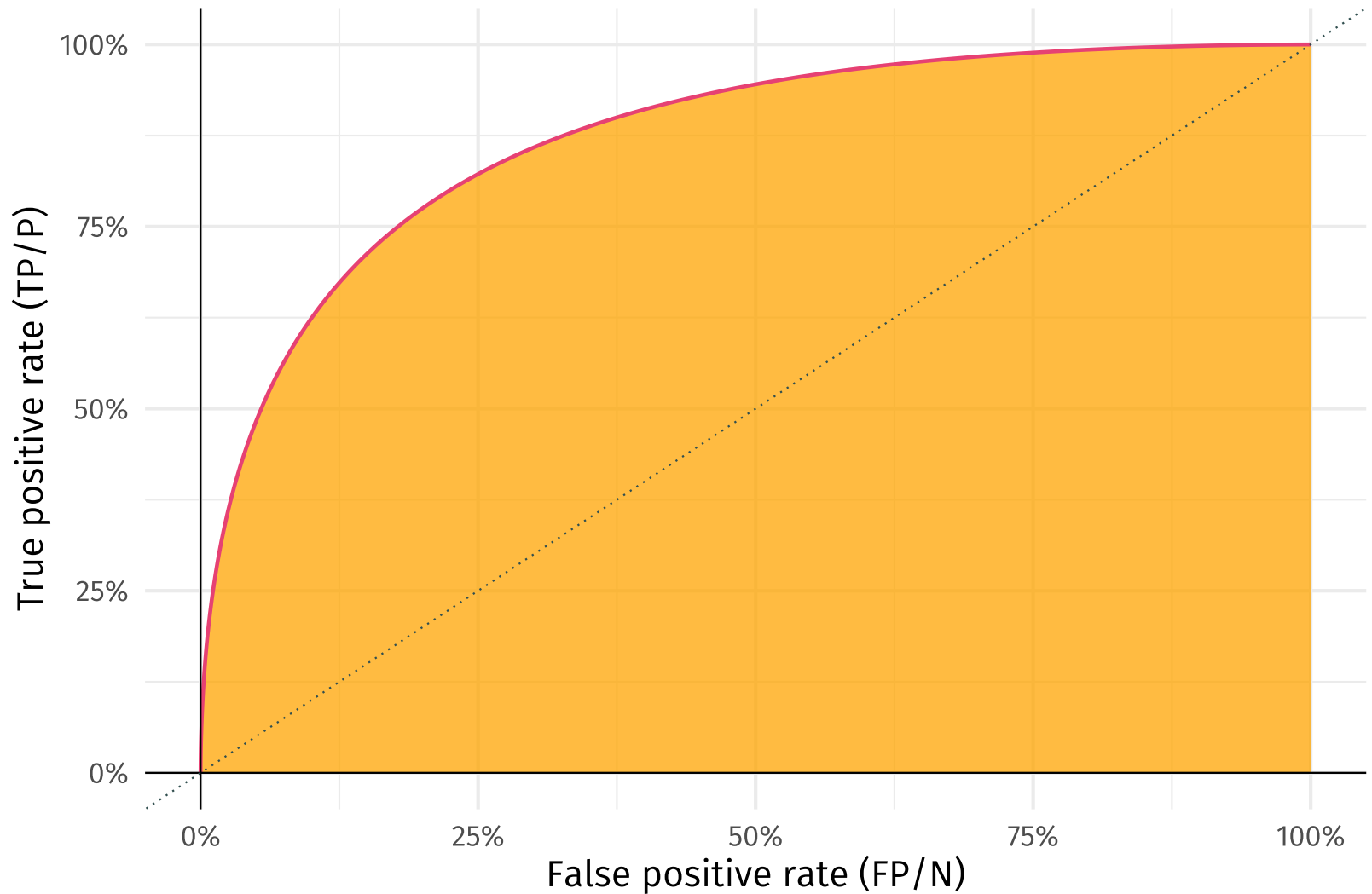
For any given **threshold**, we get **false positives**



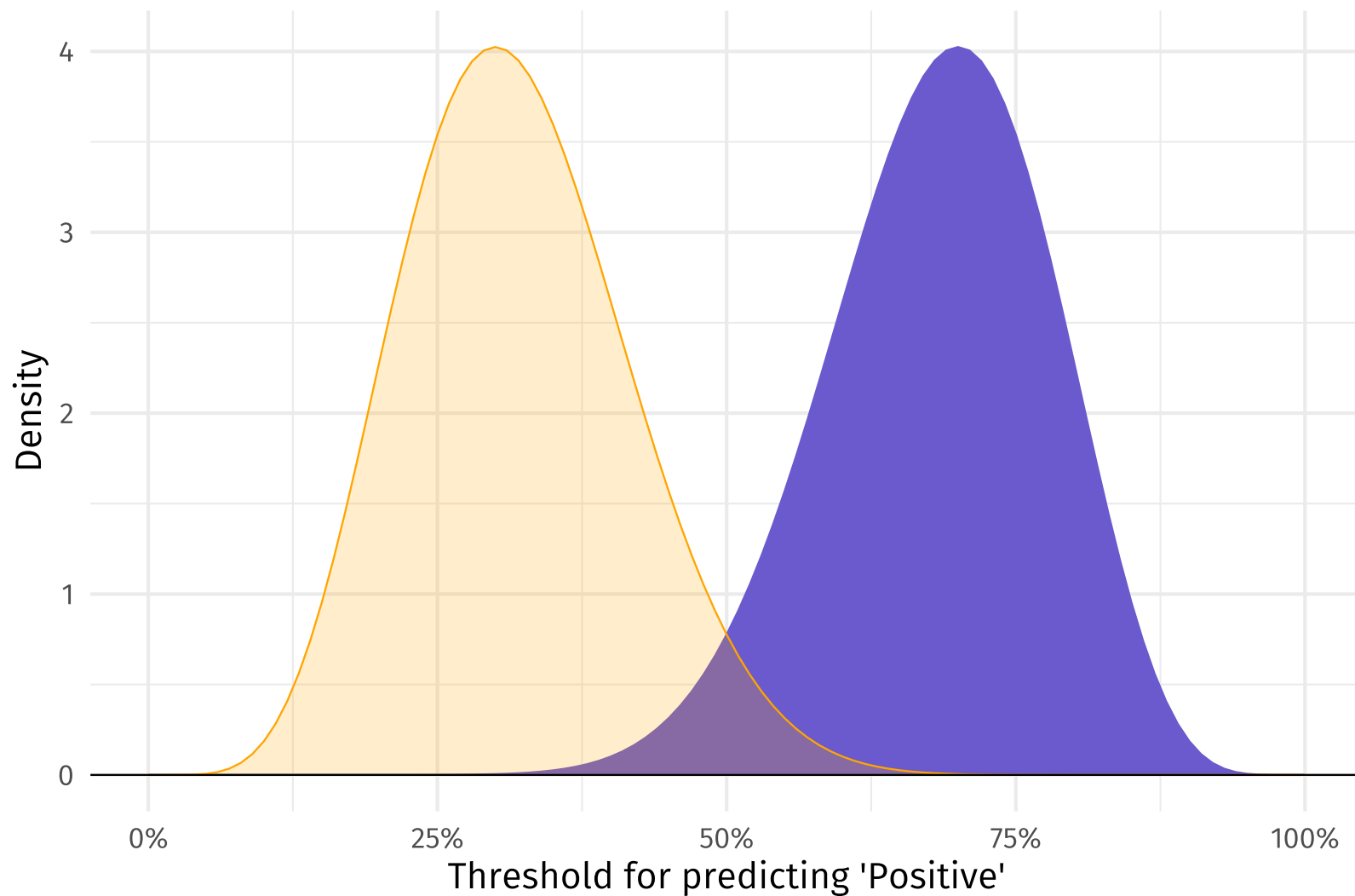
For any given **threshold**, we get false positives and **true positives**.



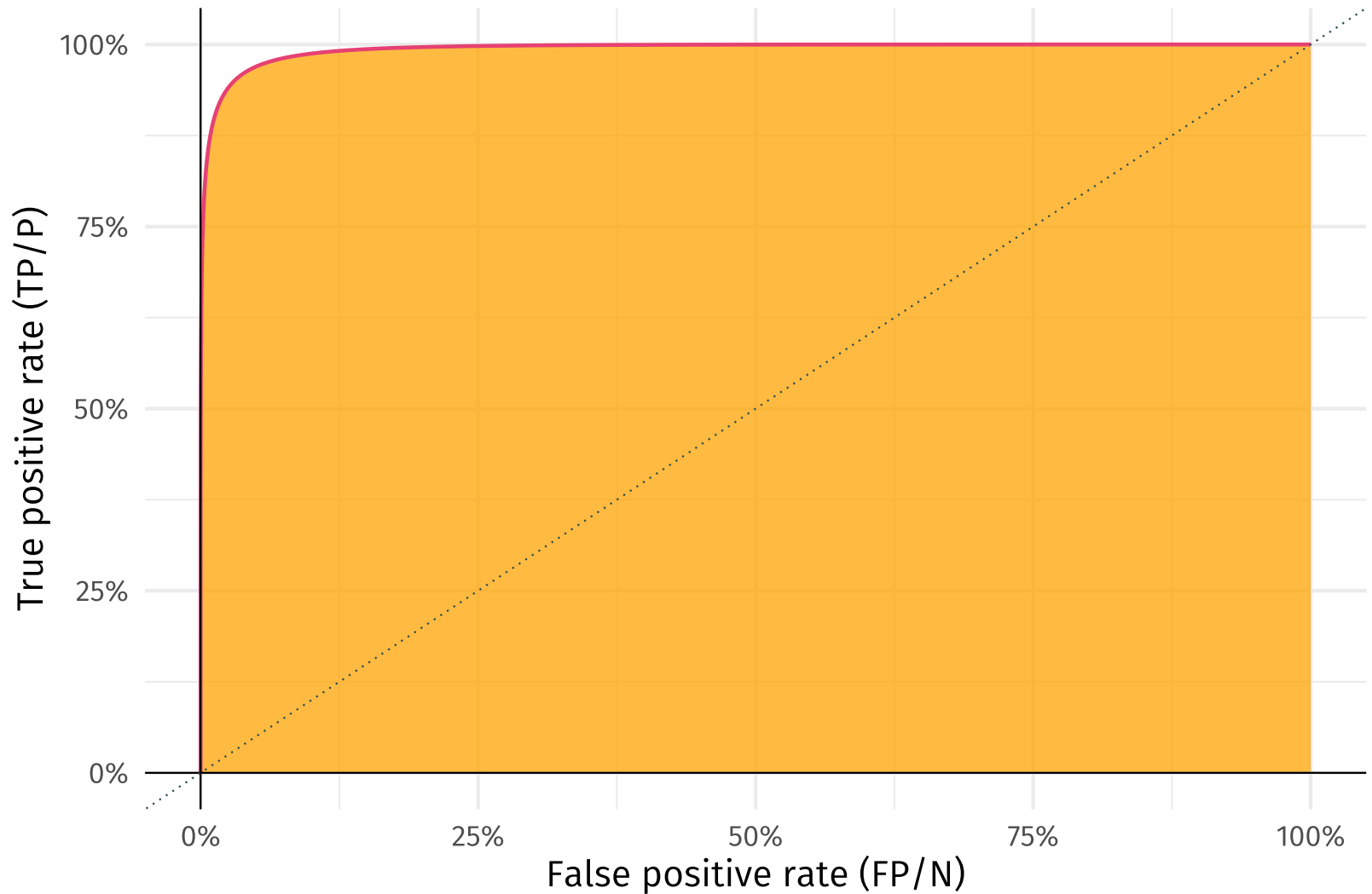
... moving through all possible thresholds generates the **ROC** (**AUC**  $\approx 0.872$ ).



Increasing separation between **negative** and **positive** outcomes...

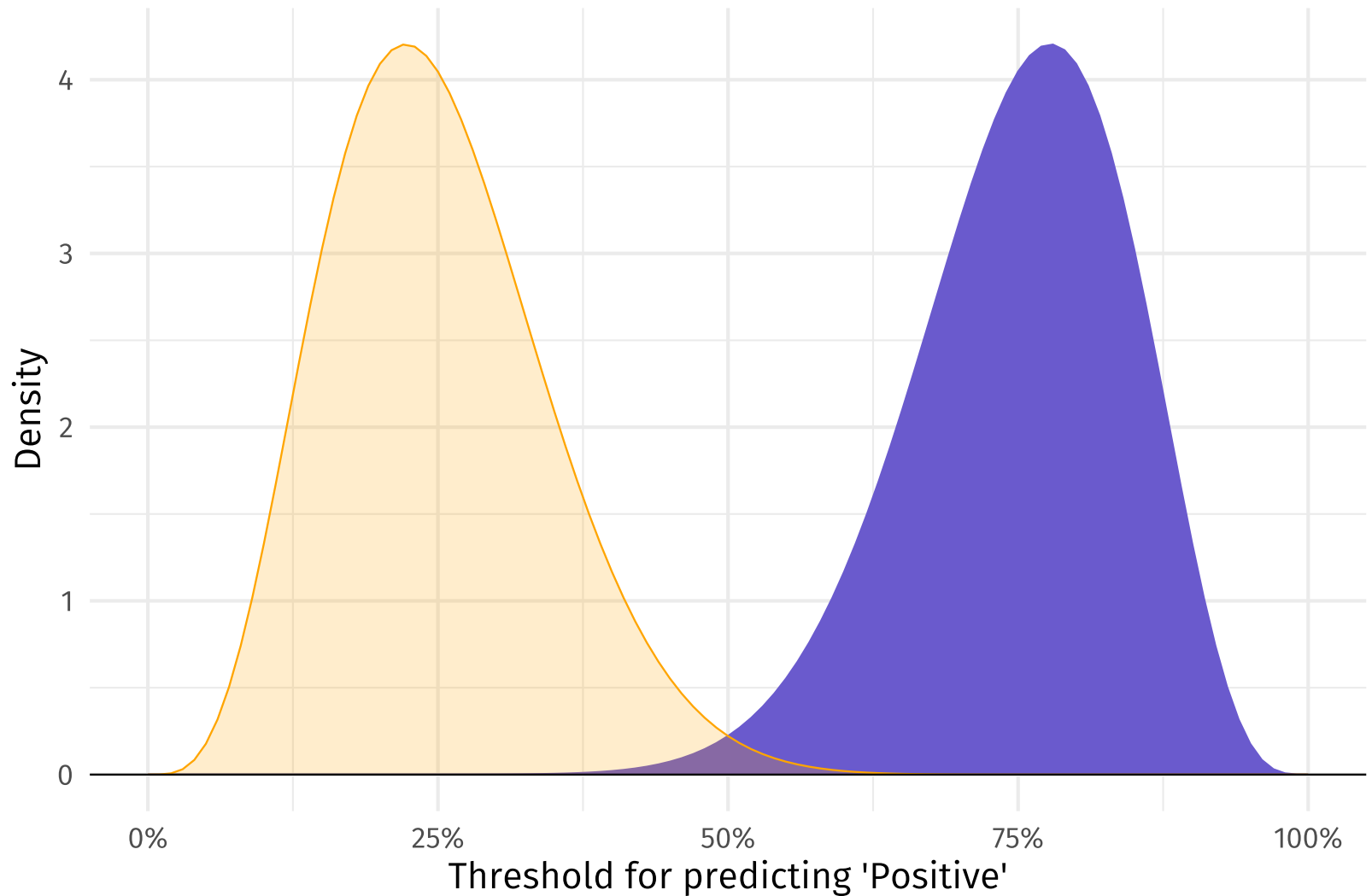


... reduces error (shifts **ROC**) and increases **AUC** ( $\approx 0.994$ ).

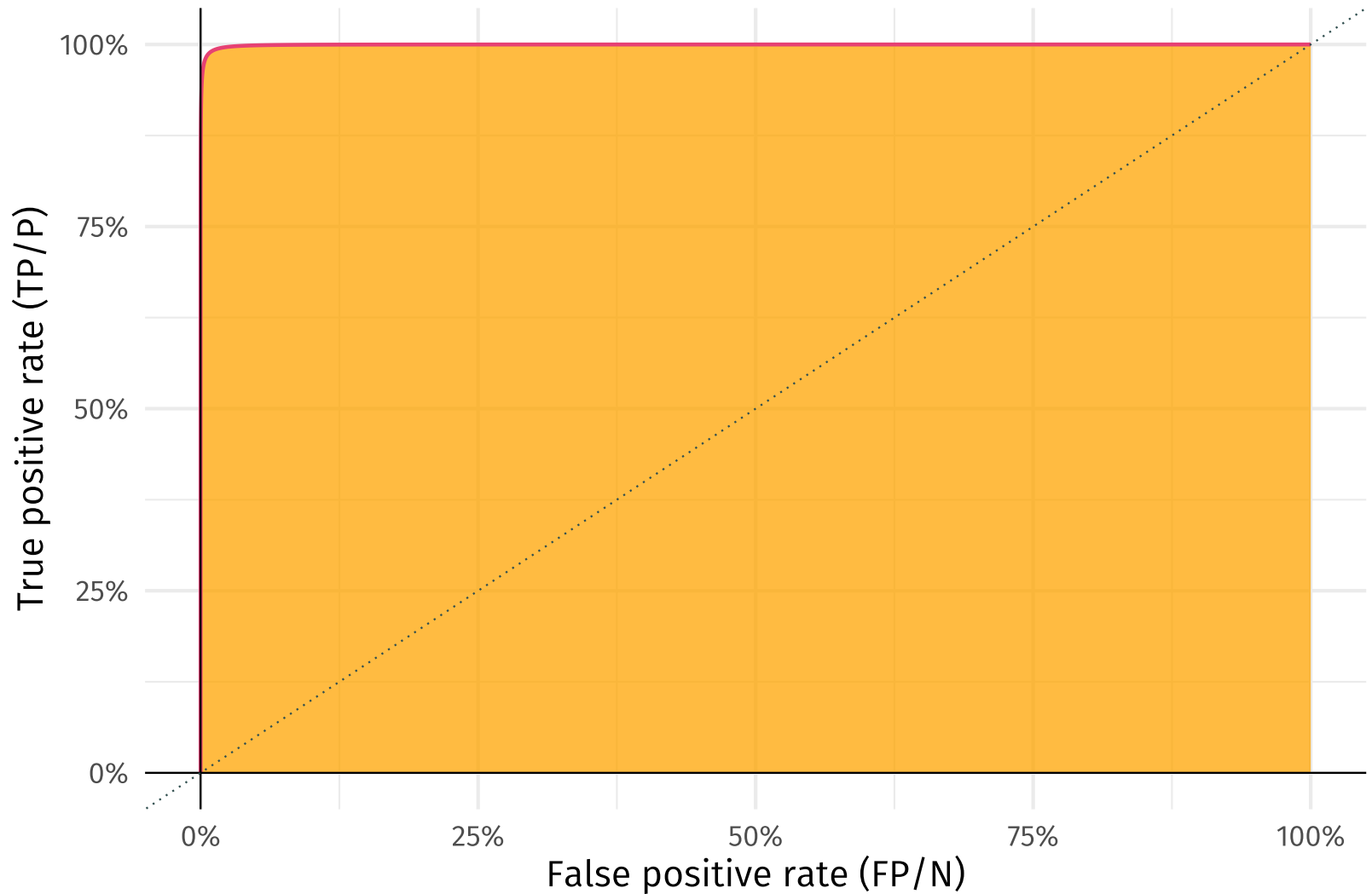




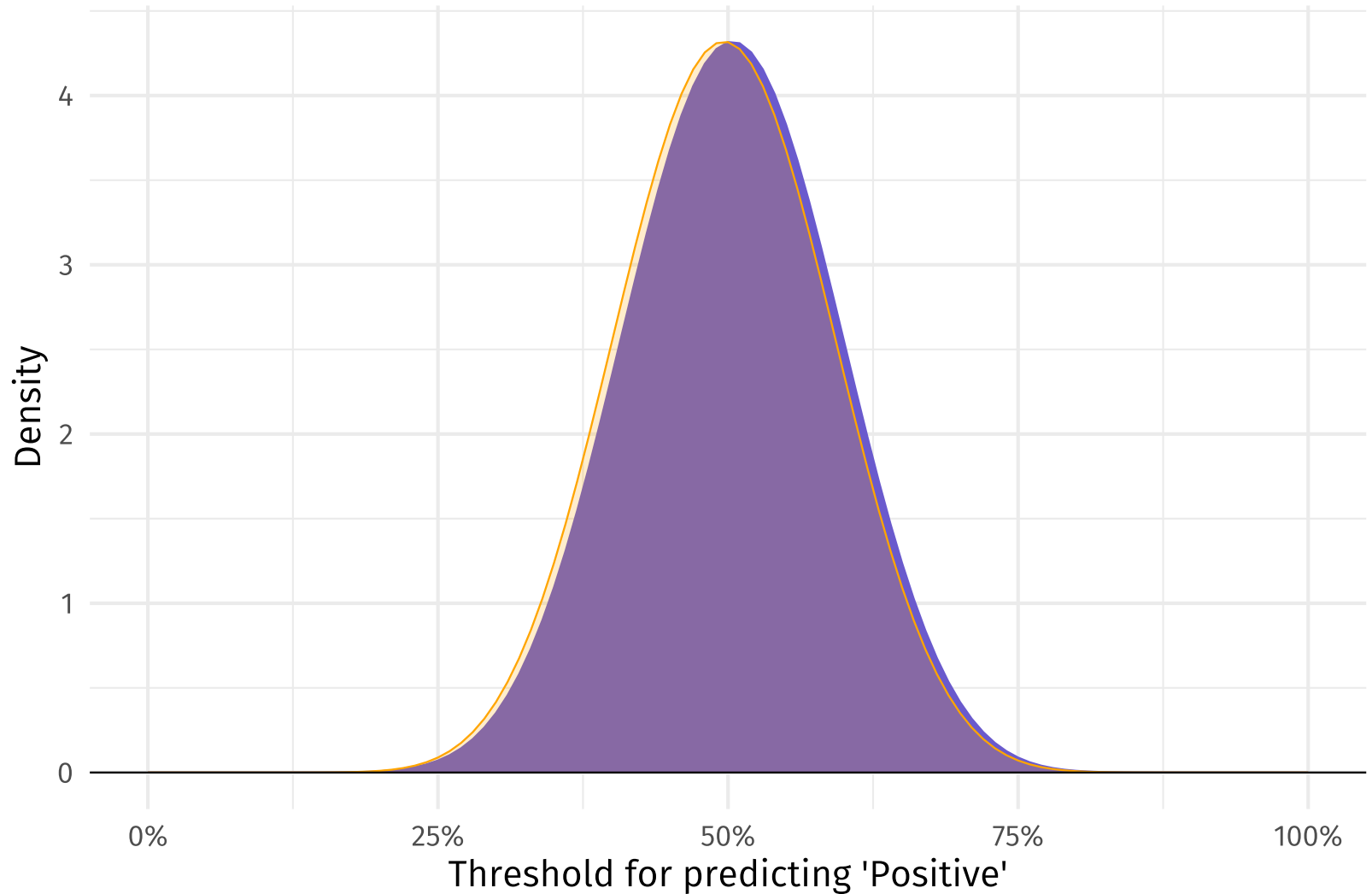
Further increasing separation between **negative** and **positive** outcomes...



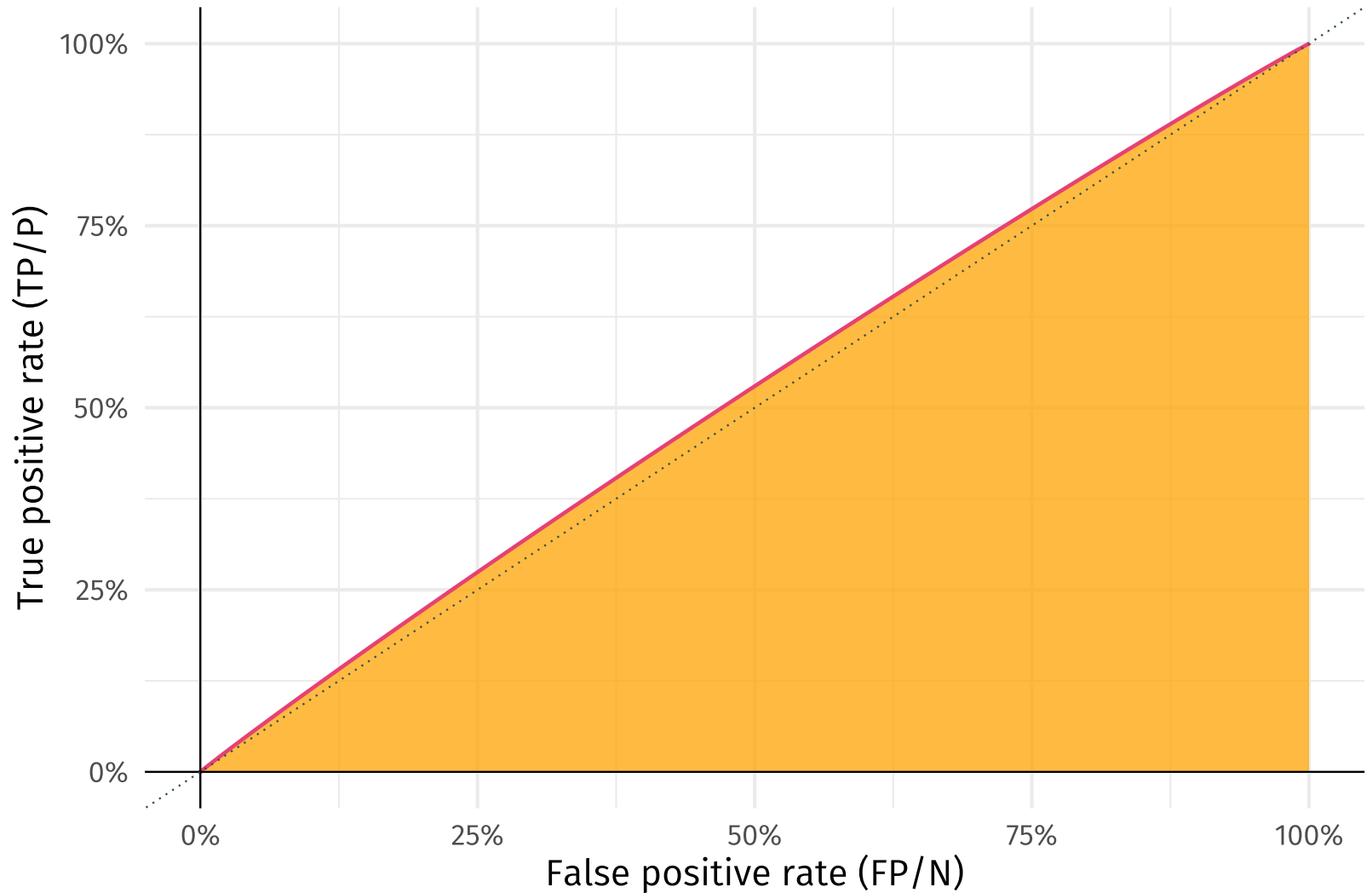
... reduces error (shifts **ROC**) and increases **AUC** ( $\approx 1$ ).



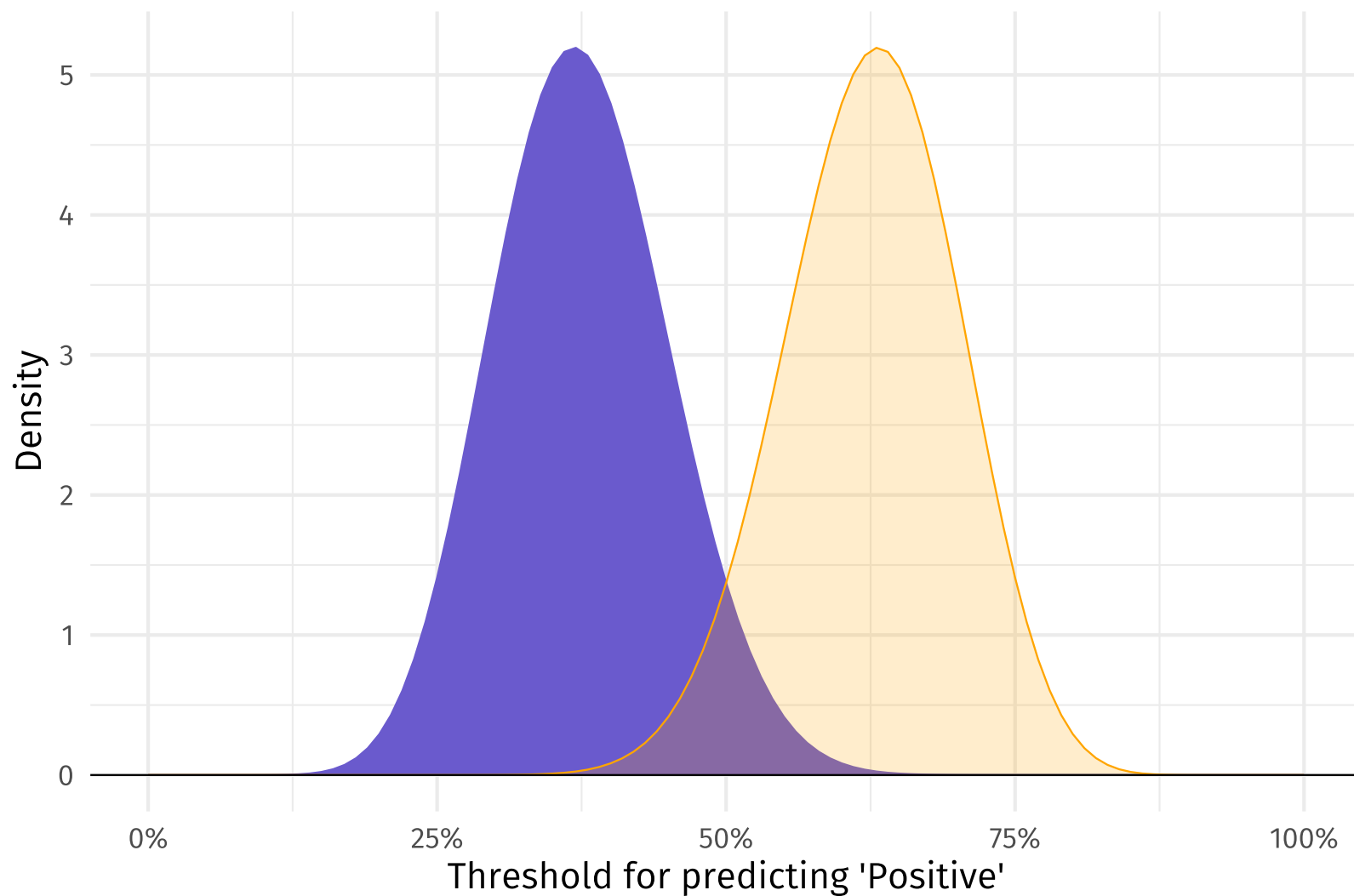
Tiny separation ("guessing") between **negative** and **positive** outcomes...



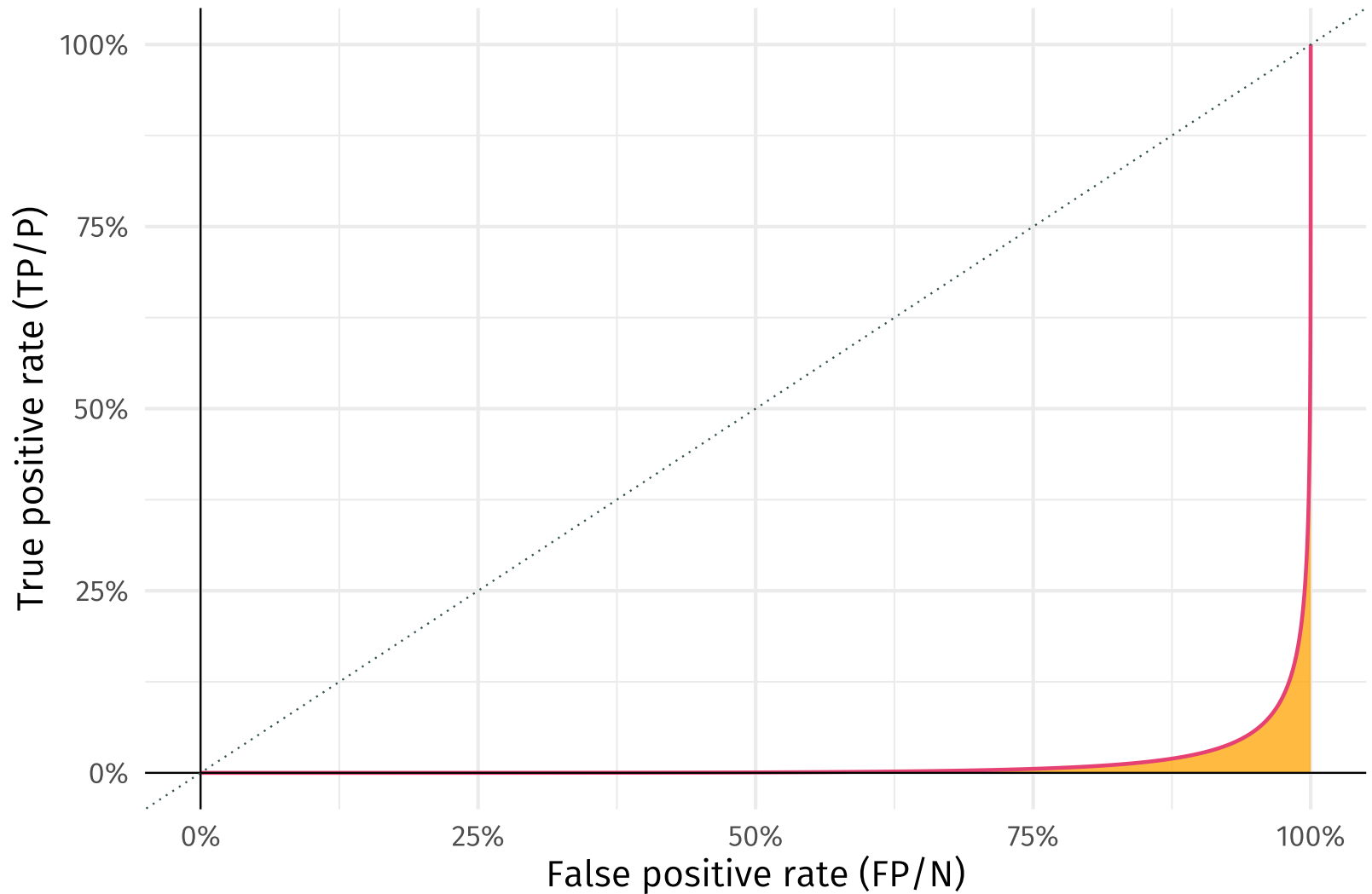
... increases error (shifts **ROC**) and pushes **AUC** toward 0.5 (here  $\approx 0.523$ ).



Getting **negative** and **positive** outcomes backwards...



... increases error (shifts **ROC**) and pushes **AUC** toward 0 (here  $\approx 0.012$ ).



# R extras

**AUC** You can calculate AUC in R using the `roc_auc()` function from `yardstick`. See the documentation for examples.

**Logistic elasticnet** `glmnet()` (for ridge, lasso, and elasticnet) extends to logistic regression<sup>†</sup> by specifying the `family` argument of `glmnet`, e.g.,

```
# Example of logistic regression with lasso
logistic_lasso = glmnet(
  y = y,
  x = x,
  family = "binomial",
  alpha = 1,
  lambda = best_lambda
)
```

You can also use the `"glmnet"` engine for `logistic_reg()` in `parsnip`.

<sup>†</sup> Or many other generalized linear models.

# Sources

These notes draw upon

- *An Introduction to Statistical Learning (ISL)*  
James, Witten, Hastie, and Tibshirani
- *Receiver Operating Characteristic Curves Demystified (in Python)*



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