

# Instrumental Variables

EC 607, Set 9

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Edward Rubin  
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# Prologue

# Schedule

## Last time

Matching and propensity-score methods

- Conditional independence
- Overlap

## Today

Instrumental variables (and two-stage least squares)

## Upcoming

Assignment 2

# Research designs

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## Selection on observables and/or unobservables

We've been focusing on **selection-on-observables designs**, i.e.,

$$(Y_{0i}, Y_{1i}) \perp\!\!\!\perp D_i | X_i$$

for **observable** variables  $X_i$ .

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for **observable** variables  $X_i$ .

**Selection-on-unobservable designs** replace this assumption with two new (but related) assumptions

1.  $(Y_{0i}, Y_{1i}) \perp Z_i$
2.  $\text{Cov}(Z_i, D_i) \neq 0$

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## Selection on observables and/or unobservables

Our main goal in causal-inference minded (applied) econometrics boils down to isolating **"good" variation** in  $D_i$  (exogenous/as-good-as-random) from **"bad" variation** (the part of  $D_i$  correlated with  $Y_{0i}$  and  $Y_{1i}$ ).

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Seems more plausible. Possible to validate. May be underpowered.

# Instrumental variables

## Introduction

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Consider some model (structural equation)

$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{D}_i + \varepsilon_i \quad (1)$$

To guarantee consistent OLS estimates for  $\beta_1$ , want  $\text{Cov}(\mathbf{D}_i, \varepsilon_i) = 0$ .  
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*Alternative:* Estimate  $\beta_1$  via instrumental variables.

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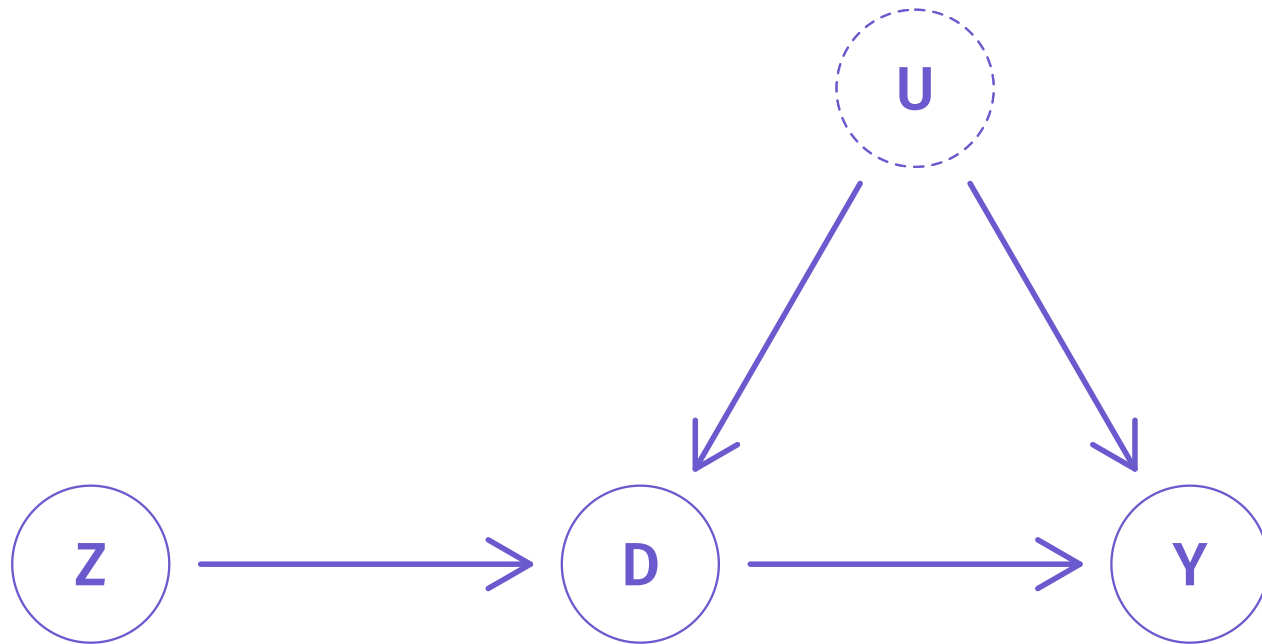
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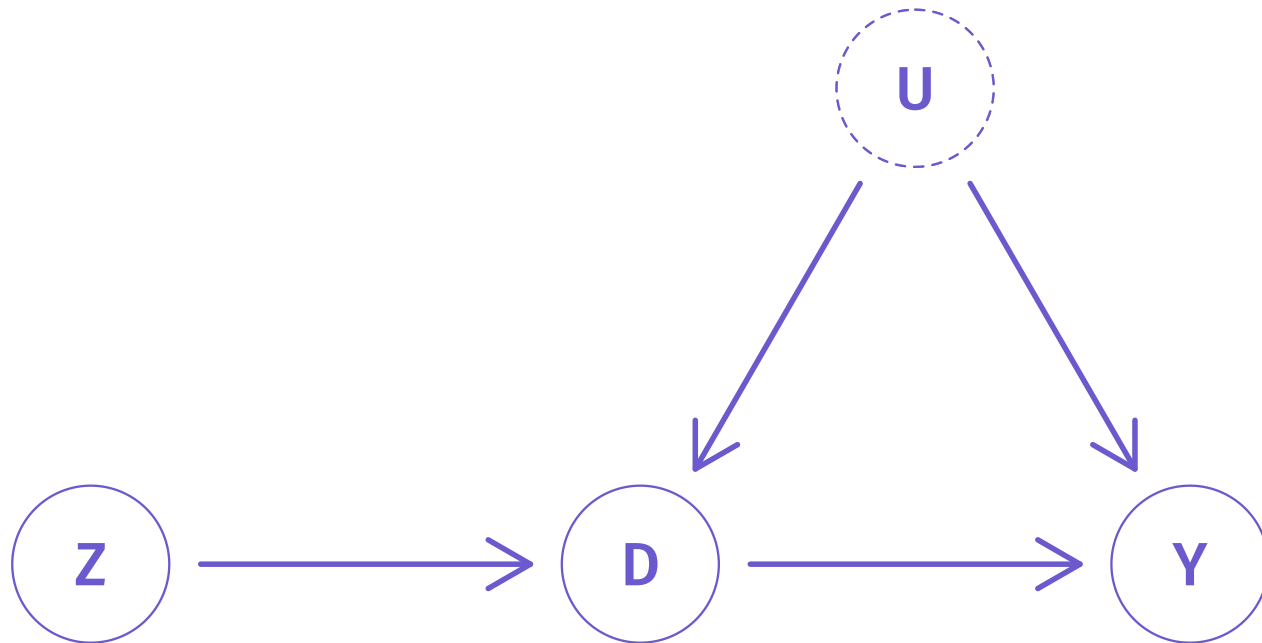
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## The DAG



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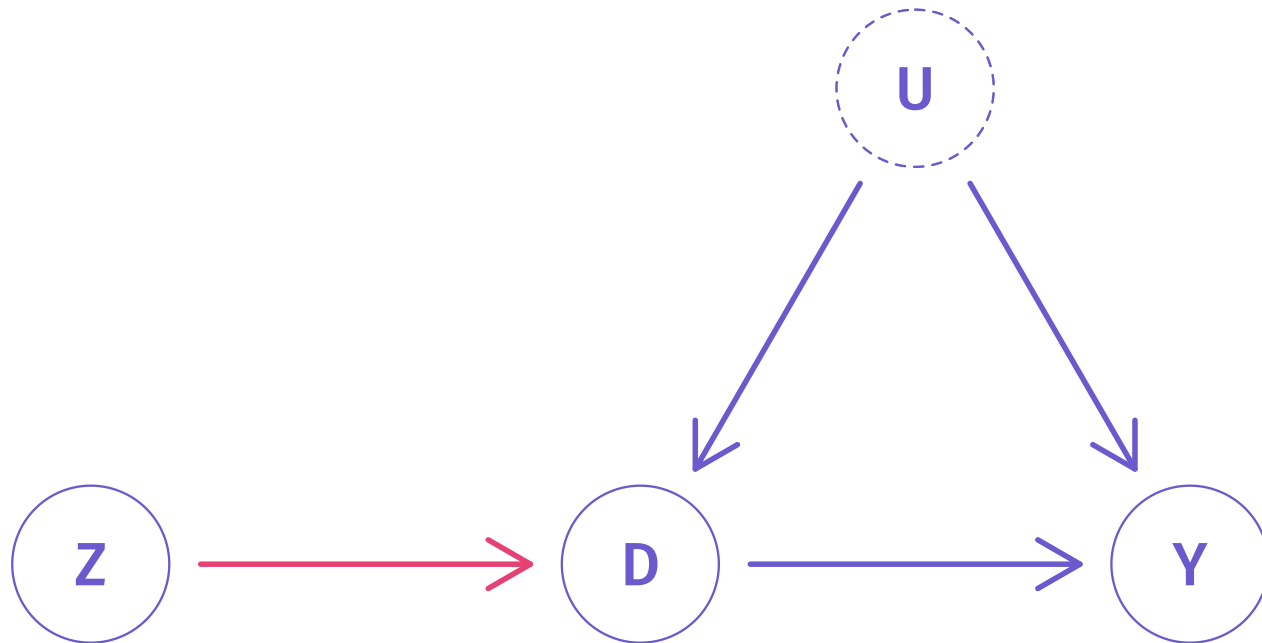
## The DAG



Q How does this DAG illustrate the requirements and identification of IV?

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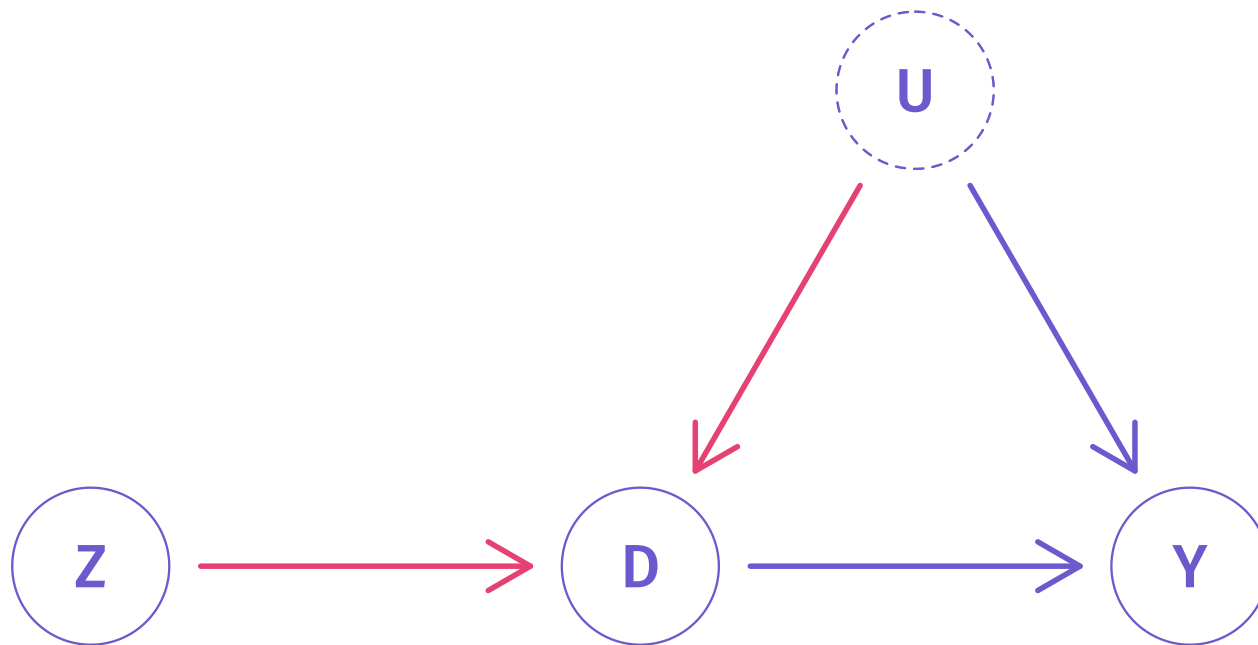
## The DAG



**Relevance:** **Z** causes an effect in **D**.

# Instrumental variables

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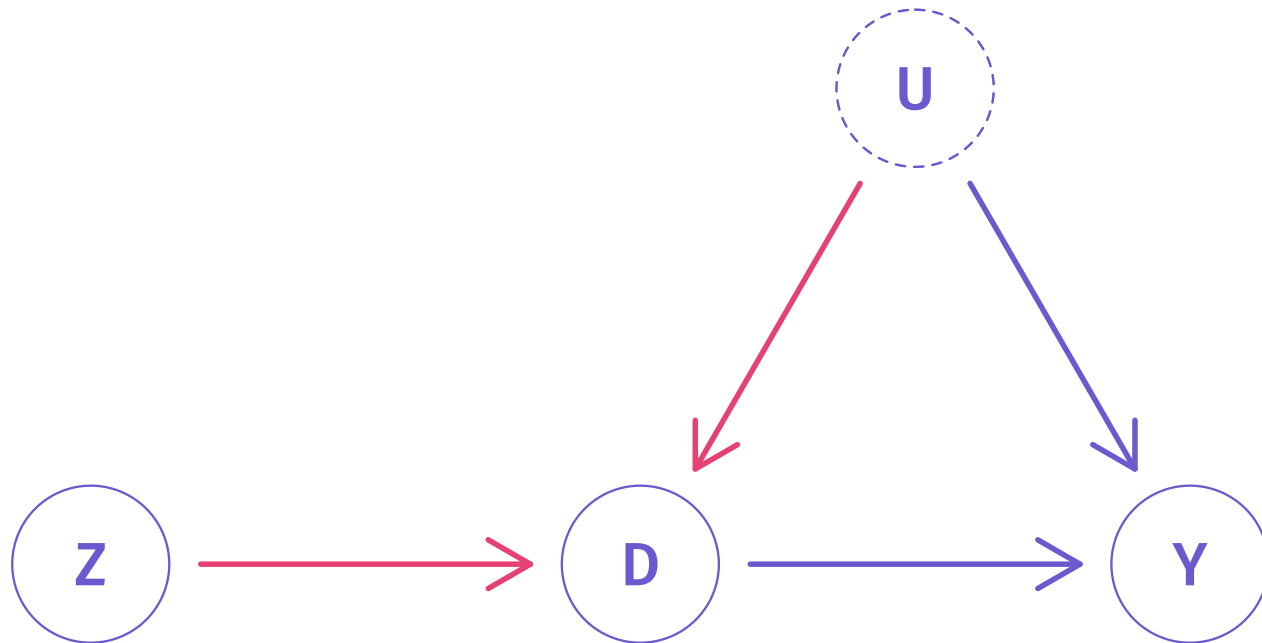


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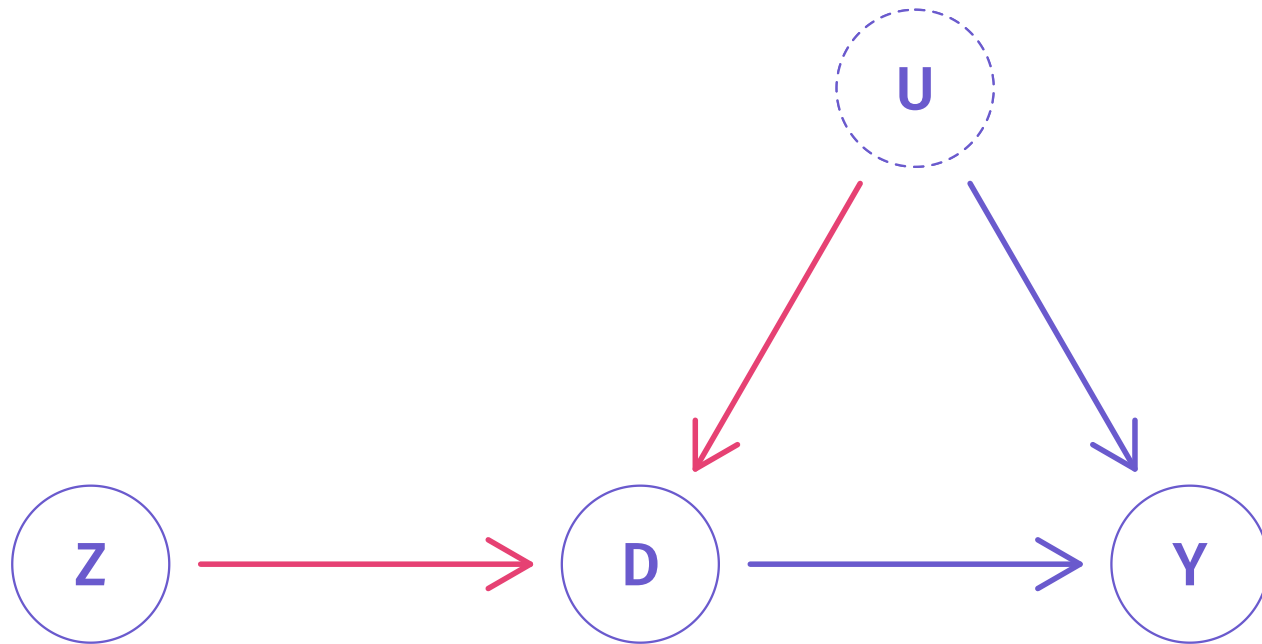
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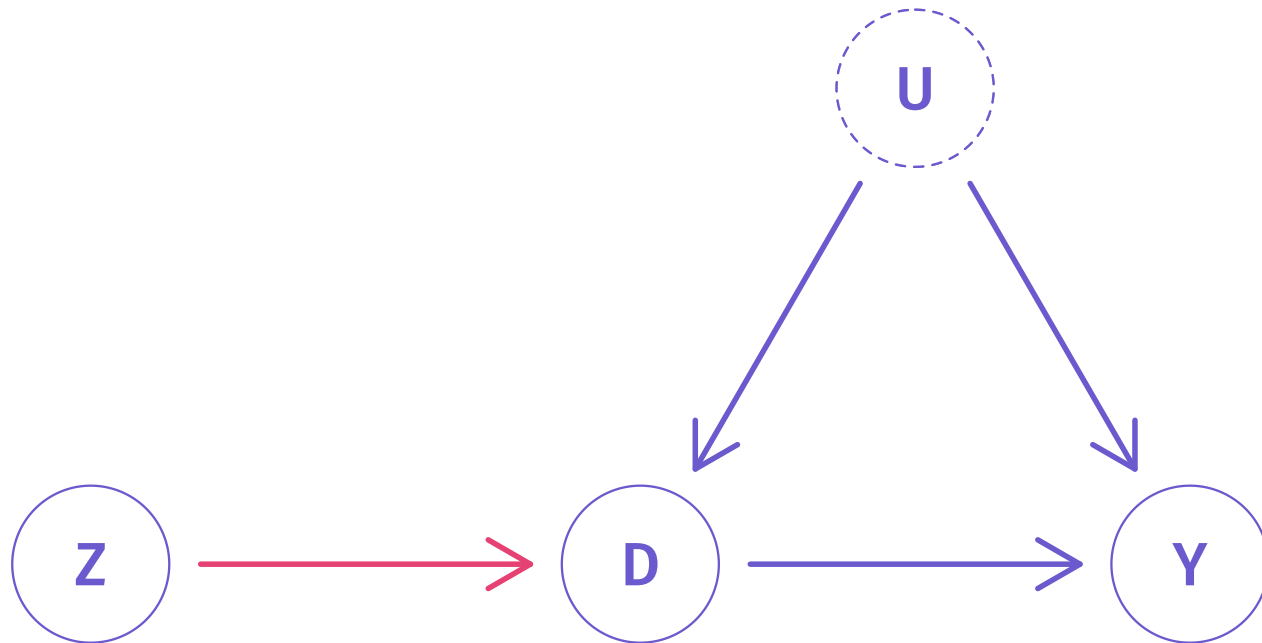


### Exclusion restriction:

1. **Z** is **exogenous** (not associated with) **U** because **D** is a collider.  
I.e.,  $Z \rightarrow D \leftarrow U$  is closed without conditioning.

# Instrumental variables

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### Exclusion restriction:

1. **Z** is **exogenous** (not associated with) **U** because **D** is a collider.
2. Also: **Z** does not directly cause **Y**.

# Instrumental variables

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Back to the returns to a college degree,

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Let **Lottery**<sub>*i*</sub> denote an indicator for whether *i* won a lottery scholarship.<sup>†</sup>

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2.  $\text{Cov}(\text{Lottery}_i, \varepsilon_i) = 0$  since the lottery is randomized.

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# Instrument variables

## The IV estimator

The IV estimator for our model

$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{D}_i + \varepsilon_i \quad (1)$$

with (valid) instrument  $\mathbf{Z}_i$  is

$$\hat{\beta}_{\text{IV}} = (\mathbf{Z}'\mathbf{D})^{-1} (\mathbf{Z}'\mathbf{Y})$$



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If you have no covariates, then

$$\hat{\beta}_{IV} = \frac{\text{Cov}(Z_i, Y_i)}{\text{Cov}(Z_i, D_i)}$$

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If you have additional (exogenous) covariates  $X_i$ , then

$$Z = [Z_i \quad X_i]$$

$$D = [D_i \quad X_i]$$

# Instrumental variables

## Proof: Consistency

With a valid instrument  $\mathbf{Z}_i$ ,  $\hat{\beta}_{IV}$  is a consistent estimator for  $\beta_1$  in

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$$= \beta \quad \checkmark$$

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$$\mathbf{D}_i = \gamma_1 \mathbf{Z}_i + \gamma_2 \mathbf{X}_i + u_i$$

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**Second stage** Estimate the model we wanted—but only using the variation in  $\mathbf{D}_i$  that correlates with  $\mathbf{Z}_i$ , i.e.,  $\hat{\mathbf{D}}_i$ .

$$\mathbf{Y}_i = \beta_1 \hat{\mathbf{D}}_i + \beta_2 \mathbf{X}_i + \varepsilon_i$$

*Note* The controls  $\mathbf{X}_i$  must match in the first and second stages.

# Two-stage least squares

## IV estimation

This two-step procedure, with a valid instrument, produces an estimator  $\hat{\beta}_1$  that is consistent for  $\beta_1$ .

$$\hat{\beta}_{2SLS} = (D'P_Z D)^{-1} (D'P_Z Y)$$

$$P_Z = Z(Z'Z)^{-1}Z'$$

where  $D$  is a matrix of our treatment and predetermined covariates ( $X_i$ ) and  $Z$  is a matrix of our instrument and our predetermined covariates.

# Two-stage least squares

## IV estimation

### Important notes

- The controls ( $\mathbf{X}_i$ ) must match in the first and second stages.
- *Related:* Nonlinear first stages can mess things up.
- If you have exactly **one instrument** and exactly **one endogenous variable**, then 2SLS and IV are identical.
- Your second-stage standard errors are not correct.

# Two-stage least squares

## The reduced form

In addition to the regressions within the two stages of 2SLS

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The **reduced form** regresses the outcome  $Y_i$  (LHS of the second stage) on our instrument  $Z_i$  and covariates  $X_i$  (RHS of the first stage).

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Thus, the reduced form provides a consistent estimate of the causal effect of our instrument on the outcome.



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## The reduced form, continued

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$$\hat{\beta}_1^{2SLS} = \frac{\hat{\pi}_1}{\hat{\gamma}_1}$$

when you have exactly one instrument.

# Two-stage least squares

## The reduced form, intuition

This expression for the 2SLS (and IV) estimator can be very helpful.

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$\hat{\gamma}_1$  estimates the effect of winning the scholarship lottery on graduation—the share of winners who graduated due to winning. We can scale with  $\hat{\gamma}_1$ !

# Two-stage least squares

## The reduced form, example

To see why this scaling makes sense, imagine that 50% of lottery winners graduate from college due to the lottery, *i.e.*,  $\hat{\gamma}_1 = 0.50$ .<sup>†</sup>

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Our reduced-form estimate of  $\hat{\pi}_1 = \$5,000$  says that lottery winners make \$5,000 more than the control group, on average.

However, half of the winners did not graduate, so  $\hat{\pi}_1$  "underestimates" the effect of college graduation by combining graduates by nongraduates.

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However, half of the winners did not graduate, so  $\hat{\pi}_1$  "underestimates" the effect of college graduation by combining graduates by nongraduates.

Thus, we want to double  $\hat{\pi}_1$ , *i.e.*, divide by  $\hat{\gamma}_1$ :  $\hat{\pi}_1 / \hat{\gamma}_1 = \$5,000 / 0.5 = \$10,000$ .

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Let's push a bit deeper into IV's mechanics and intuition.

# IV: Mechanics and intuition

## Setup

In this section, we'll use medical trials as a working example.<sup>†</sup>

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$D_i$  indicates whether  $i$  takes the treatment (medication).  $\varepsilon_i$  captures all other factors that affect  $Y_i$ .

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$$\begin{aligned} Y_i &= Y_{1i} D_i + Y_{0i} (1 - D_i) \\ Y_{0i} &= \beta_0 + \varepsilon_i \\ Y_{1i} &= Y_{0i} + \beta_1 \end{aligned}$$

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# IV: Mechanics and intuition

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*Analysis 2* **IV!** Instrument medication  $D_i$  with intention to treat  $Z_i$ .

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$\therefore \mathbf{Z}_i$  is a valid instrument for  $\mathbf{D}_i$  and IV consistently estimates  $\beta_1$ .

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Let's see how IV "solves" this problems.

First, assume noncompliance only affects treated individuals—*i.e.*, treated folks sometimes don't take their pills; control folks never take pills.

# IV: Mechanics and intuition

## Noncompliance, continued

The **first stage** recovers the share of treatment individuals who take the pill

$$D_i = \gamma_1 Z_i + u_i$$

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which we know IV rescales using the first stage

$$\hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1} = \frac{\hat{\pi}_1}{0.50} = 2 \times \hat{\pi}_1$$



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## Noncompliance, continued

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If everyone perfectly complies, then  $\hat{\gamma}_1 = 1$  and  $\hat{\beta}_1^{\text{IV}} = \hat{\pi}_1/1 = \hat{\beta}_1^{\text{ITT}}$ .

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Further example  $N_{\text{Trt}} = 10$ ; trt. compliance = 50%; ctrl. compliance = 100%.

$$\bar{Y}_{\text{Trt}} = \frac{5(\beta_0 + \beta_1) + 5(\beta_0)}{10} = \beta_0 + \frac{\beta_1}{2}$$

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So our reduced-form estimate (the ITT) is  $\hat{\gamma}_1 = \frac{\beta_1}{2}$  (half the true effect).

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So our reduced-form estimate (the ITT) is  $\hat{\gamma}_1 = \frac{\beta_1}{2}$  (half the true effect).

IV consistently estimates  $\beta_1$  via rescaling the ITT by the rate of compliance

$$\hat{\beta}_1^{\text{IV}} = \frac{\pi}{\gamma} = \frac{\beta_1/2}{1/2} = \beta_1$$

# IV: Mechanics and intuition

## Takeaways

### Main points

1. IV **rescales** the causal effect of  $Z_i$  on  $Y_i$  by the causal effect of  $Z_i$  on  $D_i$ .

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1. IV **rescales** the causal effect of  $Z_i$  on  $Y_i$  by the causal effect of  $Z_i$  on  $D_i$ .
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### Main points

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2. IV **does not** compare treated compliers to untreated compliers.  
Such a comparison/estimator would re-introduce selection bias.

Thus far, we assumed homogeneous treatment effects.

Q What happens **when treatment effects are heterogeneous**?

**A** Let's recall what our instruments are doing (with Venn diagrams!).

*Credit* Glen Waddell introduced me to IV via Venn.

Figure 1

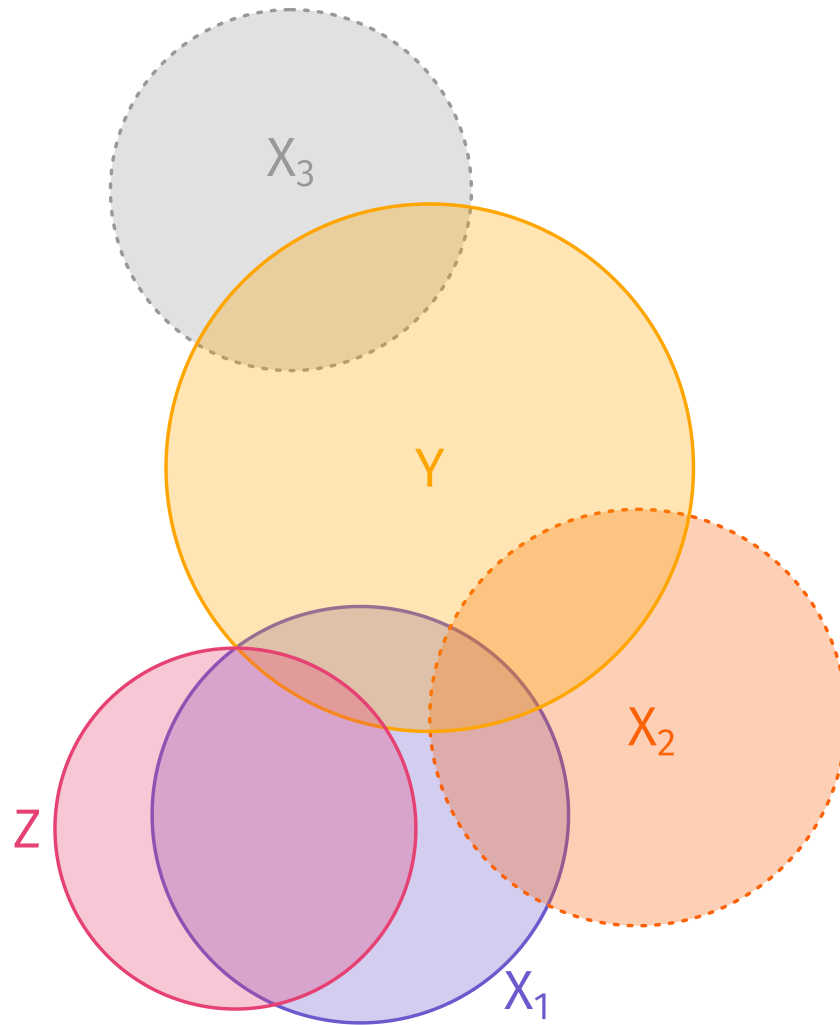


Figure 2

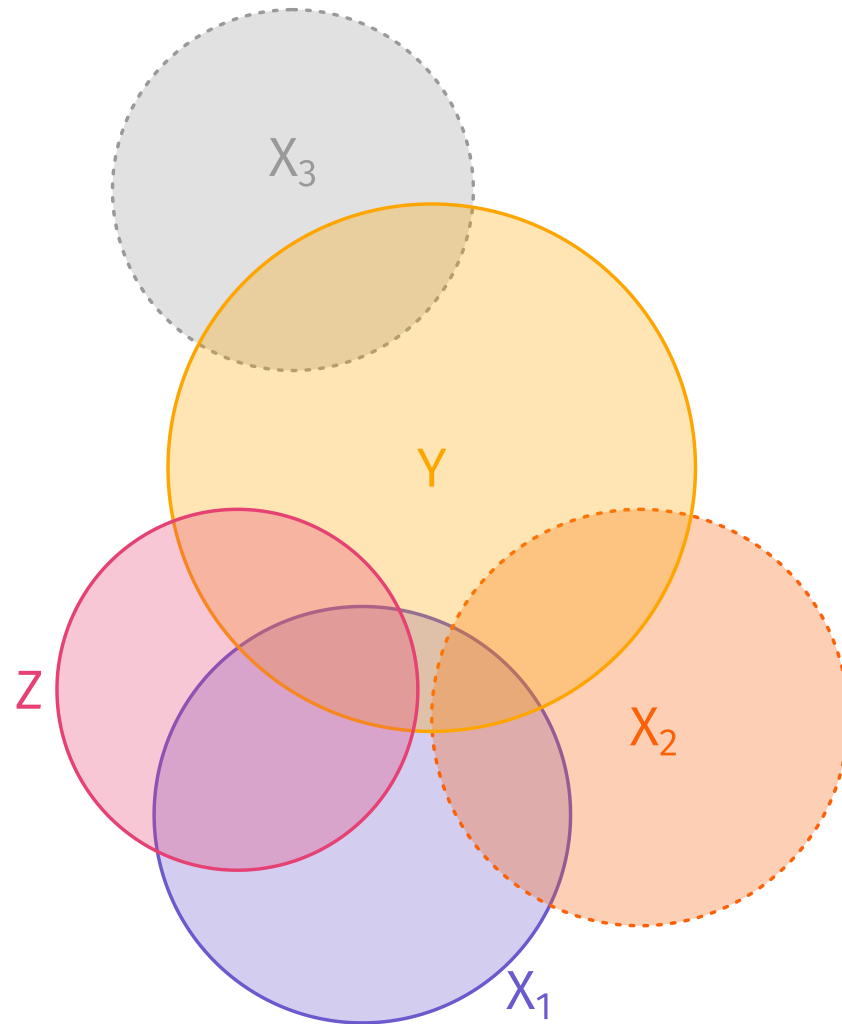


Figure 3

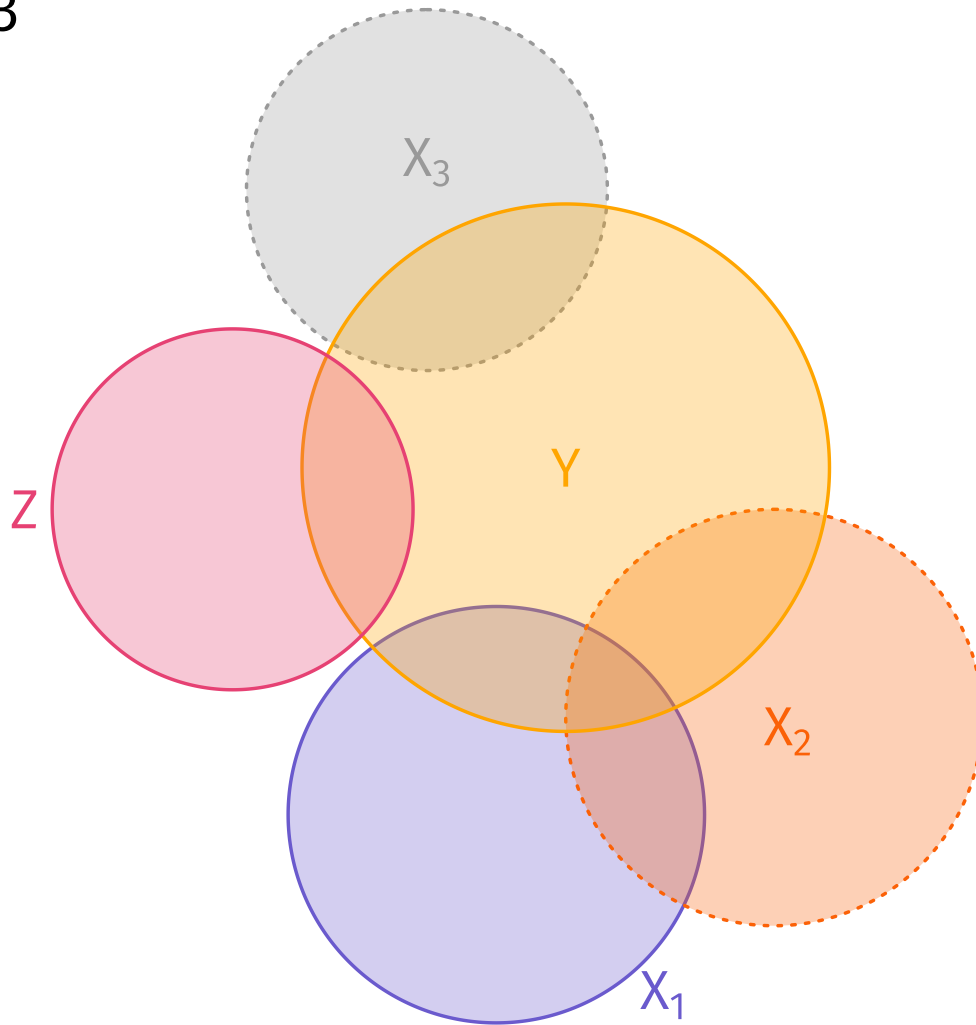


Figure 4

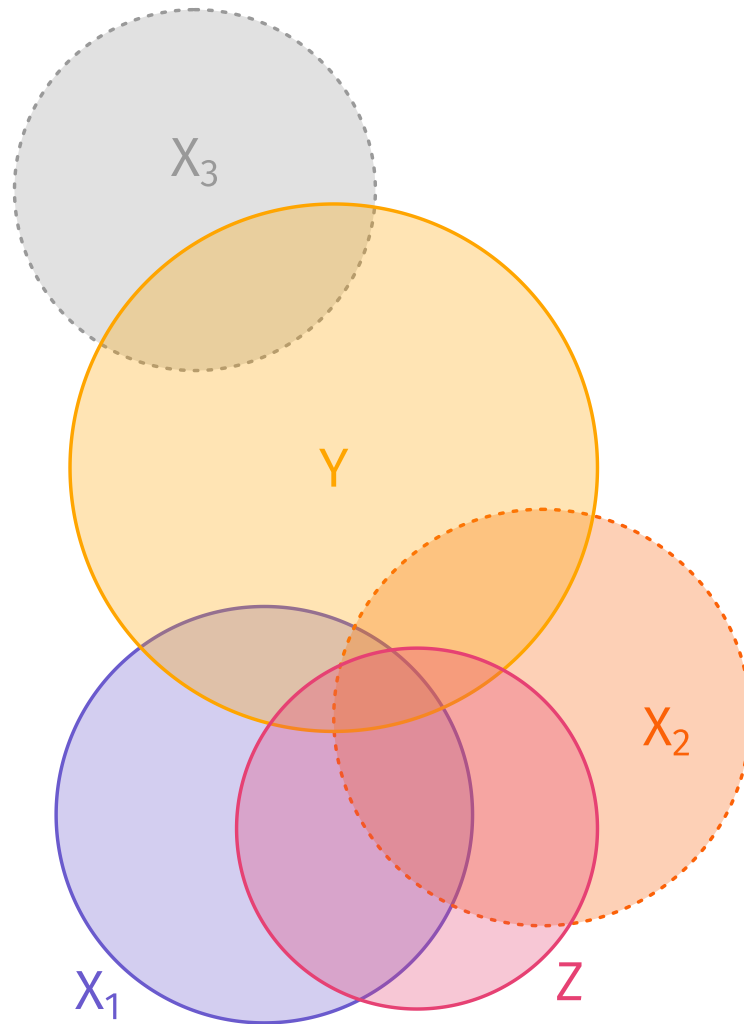
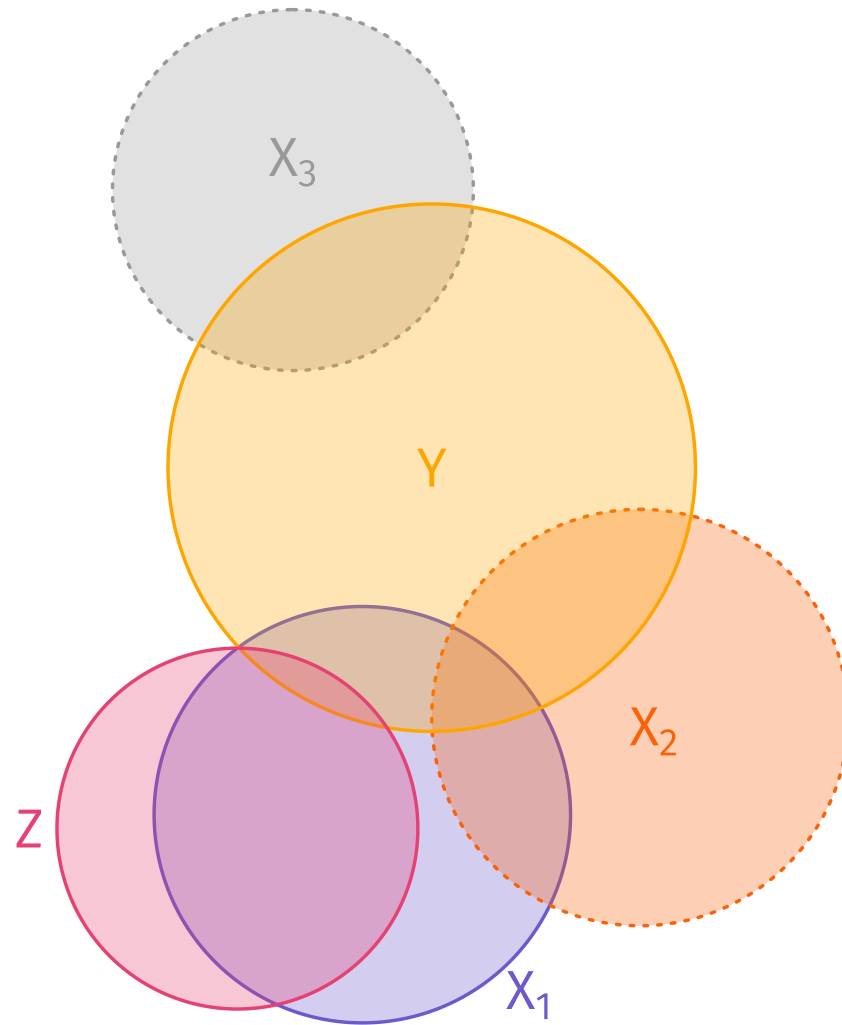


Figure 1





Can you draw the DAGs?

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**A** Not ATE. And not TOT. They estimate the LATE.<sup>†</sup>

<sup>†</sup> See Angrist, Imbens, and Rubin (1996).

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In other words: IV focuses on the individuals whose  $\mathbf{D}_i$  changes due to  $\mathbf{Z}_i$ .

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However, *compliers* are only one of four possible groups.

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2. **Always-takers**  $D_i = 1 \forall Z_i$ .
3. **Never-takers**  $D_i = 0 \forall Z_i$ .
4. **Defiers**  $D_i = 1$  iff  $Z_i = 0$ .

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Hence the "local" in *local average treatment effect*.

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Imagine treatment works for some ( $\beta_{1,i} < 0$ ) and not for others ( $\beta_{1,j} = 0$ ).

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- $\beta_{1,j} = 0$  individuals only take the pill when treated.

Then our compliers will be individuals for whom  $\beta_{1,j} = 0$ .

Thus, IV's LATE will indicate no treatment effect ( $\hat{\beta}_1^{\text{IV}} = 0$ ).

# IV + heterogeneity

## The LATE

Q So is IV actually inconsistent?



# IV + heterogeneity

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**Q** So is IV actually inconsistent?

**A** It depends what you are trying to estimate (and how you interpret it).

IV doesn't estimate the ATE or TOT, so it would be inconsistent for them.<sup>†</sup>

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*Takeaway* Because IV identifies off of compliers, it estimates an average treatment effect for these individuals (who *comply* with the instrument).

*Takeaway*<sub>2</sub> Different instruments have different LATEs.

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# IV + heterogeneity

## Monotonicity

We've already written down the two classical IV/2SLS assumptions

- *First stage*:  $\text{Cov}(Z_i, D_i) > 0$
- *Exclusion restriction*:  $\text{Cov}(Z_i, \varepsilon_i) = 0$

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but we need a third assumption to get ensure IV's complier-based LATE interpretation.

- **Monotonicity (Uniformity)**:  $D_i(z) \geq D_i(z')$  or  $D_i(z) \leq D_i(z') \quad \forall i$   
**Heckman**: *Uniformity of responses across persons.*  
**Imbens and Angrist (1994)**: Instrument has monotone effect on  $D_i$ .

# IV + heterogeneity

## Monotonicity

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In this case, the IV estimand is

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Example  $\tau_c = 1$  and  $\tau_d = 2$ .  $\Pr(\text{complier}) = 2/3$  and  $\Pr(\text{defier}) = 1/3$ .

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Example  $\tau_c = 1$  and  $\tau_d = 2$ .  $\Pr(\text{complier}) = 2/3$  and  $\Pr(\text{defier}) = 1/3$ .

Then the "LATE" is 0.<sup>†</sup>

<sup>†</sup> Some people would instead say that there is no LATE when you violate monotonicity.

Until now, we've focused on using a single instrument.

The 2SLS estimator accomodates multiple instruments.<sup>†</sup>

<sup>†</sup> Whether you can find multiple valid instruments is another question.

Multiple instruments

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Using terminology from the *system-of-equations* literature,

- one instrument for one endogenous variable: **just identified**
- multiple instruments for one endogenous variable: **over identified**

# Multiple instruments

## In practice

With (valid) instruments  $\mathbf{Z}_{1i}$  and  $\mathbf{Z}_{2i}$ , or first stage becomes

$$\mathbf{D}_i = \gamma_0 + \gamma_1 \mathbf{Z}_{1i} + \gamma_2 \mathbf{Z}_{2i} + \gamma_3 \mathbf{X}_i + u_i$$



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With (valid) instruments  $\mathbf{Z}_{1i}$  and  $\mathbf{Z}_{2i}$ , or first stage becomes

$$\mathbf{D}_i = \gamma_0 + \gamma_1 \mathbf{Z}_{1i} + \gamma_2 \mathbf{Z}_{2i} + \gamma_3 \mathbf{X}_i + u_i$$

while our second stage is still

$$\mathbf{Y}_i = \beta_0 + \beta_1 \hat{\mathbf{D}}_i + \beta_2 \mathbf{X}_i + v_i$$

# Multiple instruments

## Example: Quarter of birth

Back to our quest to estimate the returns to education.

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Angrist and Krueger (1991) proposed *quarter of birth* as a set of instruments for years of schooling.

Accordingly, their first stage looks something like<sup>†</sup>

$$\begin{aligned}\text{Schooling}_i = & \gamma_0 + \gamma_1 \mathbb{I}(\text{Born Q1})_i + \gamma_2 \mathbb{I}(\text{Born Q2})_i \\ & + \gamma_3 \mathbb{I}(\text{Born Q3})_i + \gamma_4 \mathbb{I}(\text{Born Q4})_i \\ & + \gamma_5 \mathbf{X}_i + u_i\end{aligned}$$

<sup>†</sup> We need to drop one of the quarter-of-birth indicators to avoid perfect collinearity.

# Multiple instruments

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## Example: Quarter of birth

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**Q1** Why would quarter of birth affect schooling? (*First stage*)

**A1** Students cannot drop out of school until a certain age, and quarter of birth affects your age at the time you begin school.

**Example** Some states require students to stay in school until they are 16.

- Students who start school at age **6** drop out after **10** years of schooling.
- Students who start school at age **5** drop out after **11** years of schooling.



# Multiple instruments

## Example: Quarter of birth

If students must begin school in calendar year in which they turn 6

- December birthdates: begin school at 5.75; drop out with 10.25 yrs.
- January birthdates: begin school at 6.75; drop out with 9.25 yrs.

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## Example: Quarter of birth

If students must begin school in calendar year in which they turn 6

- December birthdates: begin school at 5.75; drop out with 10.25 yrs.
- January birthdates: begin school at 6.75; drop out with 9.25 yrs.

For some group, quarter of birth may affect the number of years in school.

# Multiple instruments

## Example: Quarter of birth

It turns out that the first stage is also pretty weak in this setting.

**Weak instruments** can cause several problems for 2SLS/IV:

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What about our other requirements for a valid instrument?

# Multiple instruments

## Example: Quarter of birth

Q2 Is quarter of birth uncorrelated with  $\varepsilon_i$  (*excludable*)?

# Multiple instruments

## Example: Quarter of birth

**Q2** Is quarter of birth uncorrelated with  $\varepsilon_i$  (*excludable*)?

**A2** While quarter of birth may be fairly arbitrary for some families, other families might time births.

If these birth timers differ from other couples along other dimensions (*e.g.*, income or education), then quarter of birth may correlate with  $\varepsilon_i$ .



# Multiple instruments

## Example: Quarter of birth

Q3 Is the effect monotone?

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## Example: Quarter of birth

**Q3** Is the effect monotone?

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Consider December births.

- Original idea: December birthdates will start school at age 5.7, inducing more years of education before 16.
- *Redshirting* idea: Parents hold back December kids so they can be older (*i.e.*, 6.7), inducing fewer years of education before 16.

<sup>†</sup> *E.g.*, Aliprantis (2012)

# 2SLS and R

`estimatr`

You can implement 2SLS/IV in many ways in R.

Today: `esitmatr` and `iv_robust()`.

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# 2SLS and R

## estimatr

You can implement 2SLS/IV in many ways in R.

Today: `esitmatr` and `iv_robust()`.

Specifically, we give `iv_robust()` the relationship that we want separated from the instrument by `|`, e.g.,

```
# Estimate 2SLS
iv_robust(Y ~ D | Z, data = sample_df, se_type = "classical") %>%
  tidy() %>% select(1:5)
```

```
#>           term estimate std.error statistic      p.value
#> 1 (Intercept)  5.786204  2.9744230   1.945320 0.0546020456
#> 2           D  1.107801  0.3043264   3.640173 0.0004372703
```



# 2SLS and R

## Now in two stages!

Of course, we can estimate 2SLS in two stages.

```
# First stage
stage1 = lm_robust(D ~ Z, data = sample_df, se_type = "classical")
# First-stage results
stage1 %>% tidy() %>% select(1:5)
```

```
#>           term  estimate std.error statistic      p.value
#> 1 (Intercept) 8.8226148 0.3169568 27.835389 2.486413e-48
#> 2           Z 0.3257347 0.1031506  3.157857 2.112927e-03
```

# 2SLS and R

## Second stage

We just need to add  $\hat{D}_i$  to our dataset.

```
# Add fitted (first-stage) values to data
sample_df %<>% mutate(D_hat = stage1$fitted.values)
# Second stage
stage2 = lm_robust(Y ~ D_hat, data = sample_df, se_type = "classical")
# Second-stage results
stage2 %>% tidy() %>% select(1:5)
```

```
#>           term estimate std.error statistic    p.value
#> 1 (Intercept)  5.786204  5.4132099   1.068904 0.28773854
#> 2          D_hat  1.107801  0.5538496   2.000184 0.04824759
```

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## Standard errors

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### Second-stage results

Term	Est.	S.E.	t stat.	p-Value
Int	5.786	5.413	1.07	0.2877
D hat	1.108	0.554	2.00	0.0482

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### 2SLS results

Term	Est.	S.E.	t stat.	p-Value
Int	5.786	2.974	1.95	0.0546
D	1.108	0.304	3.64	0.0004

# IV and 2SLS

## Conclusions

1. IV/2SLS focus on **isolating some "good" variation** in  $D_i$  via  $Z_i$ .
2. Important **requirements**: strong first stage, excludability, monotonicity.
3. IV and 2SLS **rescale the reduced form** with the first stage.
4. Estimates are **LATE from compliers**.
5. Different instruments can produce **different LATEs**.
6. A **weak first stage** can lead to problems.

# Table of contents

## Admin

1. Schedule

## Instrumental variables

1. Research designs
2. Introduction
3. Definition
4. DAG
5. Example
6. IV estimator

## Two-stage least squares

1. Setup
2. The reduced form
  - Defined
  - Intuition
  - Example
  - Derivation
3. Intuition and mechanics
  - Noncompliance
  - Rescaling
4. Heterogeneous treatment effects
  - Venn diagram
  - LATE
  - Example
  - Monotonicity
5. Multiple instruments
  - Example
6. 2SLS and  $\mathbb{R}$
7. Conclusions