Instrumental Variables

EC 607, Set 9

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Prologue

Schedule

Last time

Matching and propensity-score methods

- Conditional independence
- Overlap

Today

Instrumental variables (and two-stage least squares)

Upcoming

Assignment 2

Research designs

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Selection on observables and/or unobservables

We've been focusing on ***selection-on-observables* designs**, *i.e.*,

$$(\mathrm{Y}_{0i}, \mathrm{Y}_{1i}) \perp\!\!\!\perp \mathrm{D}_i | \mathrm{X}_i$$

for **observable** variables X_i.

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***Selection-on-unobservable* designs** replace this assumption with two new (but related) assumptions

1. $(\mathrm{Y}_{0i},\, \mathrm{Y}_{1i}) \perp \mathrm{Z}_i$
2. $\mathrm{Cov}(\mathrm{Z}_i,\, \mathrm{D}_i) \neq 0$

Selection on observables and/or unobservables

Our main goal in causal-inference minded (applied) econometrics boils down to isolating **"good" variation** in D_i (exogenous/as-good-as-random) from **"bad" variation** (the part of D_i correlated with $\mathbf{Y}_{0i}$ and $\mathbf{Y}_{1i}$).

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Which set of research designs is more palatable?

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   Seems more plausible. Possible to validate. May be underpowered.

Instrumental variables

Introduction

Instrumental variables (IV)† is the canonical selection-on-unobservables design—isolating *good variation* in $\mathbf{D}_i$ via some magical instrument $\mathbf{Z}_i$.

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Consider some model (structural equation)

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To guarantee consistent OLS estimates for β_1, want $\mathbf{Cov}(\mathbf{D}_i,\, \varepsilon_i) = \mathbf{0}$.
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Alternative: Estimate β_1 via instrumental variables.

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   i.e., $\mathbf{Z}_i$ is excludable from equation (1).

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2. $\text{Cov}(\text{Lottery}_i, \varepsilon_i) = 0$ since the lottery is randomized.

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The IV estimator

The IV estimator for our model

$$\mathrm{Y}_i = \beta_0 + \beta_1 \mathrm{D}_i + \varepsilon_i \tag{1}$$

with (valid) instrument Z_i is

$$\hat{\beta}_{\mathrm{IV}} = \left(\mathrm{Z}'\mathrm{D}\right)^{-1} \left(\mathrm{Z}'\mathrm{Y}\right)$$

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If you have no covariates, then

$$\hat{\beta}_{\mathrm{IV}} = \frac{\mathrm{Cov}(\mathrm{Z}_i,\ \mathrm{Y}_i)}{\mathrm{Cov}(\mathrm{Z}_i,\ \mathrm{D}_i)}$$

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If you have additional (exogenous) covariates $\mathbf{X}_i$, then

$$\mathbf{Z} = \begin{bmatrix} \mathrm{Z}_i & \mathbf{X}_i \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} \mathrm{D}_i & \mathbf{X}_i \end{bmatrix}$$

Proof: Consistency

With a valid instrument $\mathbf{Z}_i$, $\hat{\beta}_{\text{IV}}$ is a consistent estimator for β_1 in

$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{X}_i + \varepsilon_i \tag{1}$$

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$$\begin{aligned}
&\operatorname{plim}\left(\hat{\beta}_{IV}\right) \\
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&\quad = \beta \quad ✔
\end{aligned}$$

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First stage Estimate the effect of the instrument Z_i on our endogenous variable D_i and (predetermined) covariates X_i. Save $\widehat{\mathrm{D}}_i$.

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Second stage Estimate the model we wanted—but only using the variation in $\mathbf{D}_i$ that correlates with Z_i, *i.e.*, $\widehat{\mathrm{D}}_i$.

$$\mathbf{Y}_i = \beta_1 \widehat{\mathrm{D}}_i + \beta_2 \mathbf{X}_i + \varepsilon_i$$

Note The controls $\mathbf{X}_i$ must match in the first and second stages.

Two-stage least squares

IV estimation

This two-step procedure, with a valid instrument, produces an estimator $\hat{\beta}_1$ that is consistent for β_1.

$$\hat{\beta}_{2SLS} = \left(\mathrm{D}'\mathrm{P_Z}\mathrm{D}\right)^{-1}\left(\mathrm{D}'\mathrm{P_Z}\mathrm{Y}\right)$$

$$\mathrm{P_Z} = \mathrm{Z}\left(\mathrm{Z}'\mathrm{Z}\right)^{-1}\mathrm{Z}'$$

where $\mathbf{D}$ is a matrix of our treatment and predetermined covariates ($\mathbf{X}_i$) and Z is a matrix of our instrument and our predetermined covariates.

Two-stage least squares

IV estimation

Important notes

- The controls $(\mathbf{X}_i)$ must match in the first and second stages.
- *Related:* Nonlinear first stages can mess things up.
- If you have exactly **one instrument** and exactly **one endogenous variable**, then 2SLS and IV are identical.
- Your second-stage standard errors are not correct.

Two-stage least squares

The reduced form

In addition to the regressions within the two stages of 2SLS

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there is a third important and related regression: the reduced form.

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The **reduced form** regresses the outcome Y_i (LHS of the second stage) on our instrument Z_i and covariates X_i (RHS of the first stage).

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Thus, the reduced form provides a consistent estimate of the causal effect of our instrument on the outcome.

Two-stage least squares

The reduced form, continued

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$$\widehat{\beta}_1^{\text{2SLS}} = \frac{\widehat{\pi}_1}{\widehat{\gamma}_1}$$

when you have exactly one instrument.

Two-stage least squares

The reduced form, intuition

This expression for the 2SLS (and IV) estimator can be very helpful.

$$\widehat{\beta}_1^{\text{2SLS}} = \frac{\widehat{\pi}_1}{\widehat{\gamma}_1} = \frac{\text{Reduced-form estimate}}{\text{First-stage estimate}}$$

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$\widehat{\pi}_1$ gives the estimated causal effect of the scholarship lottery on income, but what share of lottery winners graduate? We need to rescale if $<$ 100%.

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$\widehat{\pi}_1$ gives the estimated causal effect of the scholarship lottery on income, but what share of lottery winners graduate? We need to rescale if $< 100\%$.

$\widehat{\gamma}_1$ estimates the effect of winning the scholarship lottery on graduation—the share of winners who graduated due to winning. We can scale with $\widehat{\gamma}_1$!

Two-stage least squares

The reduced form, example

To see why this scaling makes sense, imagine that 50% of lottery winners graduate from college due to the lottery, *i.e.*, $\widehat{\gamma}_1 = 0.50$.†

† Imagine none of the applicants would have graduated otherwise

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However, half of the winners did not graduate, so $\widehat{\pi}_1$ "underestimates" the effect of college graduation by combining graduates by nongraduates.

Thus, we want to double $\widehat{\pi}_1$, *i.e.*, divide by $\widehat{\gamma}_1$: $\widehat{\pi}_1/\widehat{\gamma}_1$ = \$5,000/0.5 = \$10,000.

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Two-stage least squares

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Let's push a bit deeper into IV's mechanics and intuition.

Setup

In this section, we'll use medical trials as a working example.†

† Credit/thanks go to Michael Anderson for this example—and much of these notes.

IV: Mechanics and intuition

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$\mathbf{D}_i$ indicates whether i *takes* the treatment (medication). ε_i captures all other factors that affect $\mathbf{Y}_i$. Or in potential-outcomes framework:

$$\begin{aligned} \mathbf{Y}_i &= \mathbf{Y}_{1i}\mathbf{D}_i + \mathbf{Y}_{0i}(1 - \mathbf{D}_i) \\ \mathbf{Y}_{0i} &= \beta_0 + \varepsilon_i \\ \mathbf{Y}_{1i} &= \mathbf{Y}_{0i} + \beta_1 \end{aligned}$$

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IV: Mechanics and intuition

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IV: Mechanics and intuition

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Analysis 2 **IV!** Instrument medication $\mathbf{D}_i$ with intention to treat $\mathbf{Z}_i$.

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First question: Is $\mathbf{Z}_i$ a valid instrument for $\mathbf{D}_i$?

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IV: Mechanics and intuition

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$\therefore$ $\mathbf{Z}_i$ is a valid instrument for $\mathbf{D}_i$ and IV consistently estimates β_1.

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Let's see how IV "solves" this problems.

IV: Mechanics and intuition

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Let's see how IV "solves" this problems.

First, assume noncompliance only affects treated individuals—*i.e.*, treated folks sometimes don't take their pills; control folks never take pills.

IV: Mechanics and intuition

Noncompliance, continued

The **first stage** recovers the share of treatment individuals who take the pill

$$\text{D}_i = \gamma_1 \text{Z}_i + u_i$$

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IV: Mechanics and intuition

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which we know IV rescales using the first stage

$$\widehat{\beta}_1^{\text{IV}} = \frac{\widehat{\pi}_1}{\widehat{\gamma}_1} = \frac{\widehat{\pi}_1}{\mathbf{0.50}} = \mathbf{2} \times \widehat{\pi}_1$$

Noncompliance, continued

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Further example N_{Trt} = 10; trt. compliance = 50%; ctrl. compliance = 100%.

$$\overline{\mathrm{Y}}_{\text{Trt}} = \frac{5(\beta_0 + \beta_1) + 5(\beta_0)}{10} = \beta_0 + \frac{\beta_1}{2}$$

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IV: Mechanics and intuition

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So our reduced-form estimate (the ITT) is $\widehat{\gamma}_1 = \dfrac{\beta_1}{\mathbf{2}}$ (half the true effect).

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So our reduced-form estimate (the ITT) is $\widehat{\gamma}_1 = \dfrac{\beta_1}{\mathbf{2}}$ (half the true effect).

IV consistently estimates β_1 via rescaling the ITT by the rate of compliance

$$\widehat{\beta}_1^{\text{IV}} = \frac{\pi}{\gamma} = \frac{\beta_1/\mathbf{2}}{\mathbf{1}/\mathbf{2}} = \beta_1$$

IV: Mechanics and intuition

Takeaways

Main points

1. IV **rescales** the causal effect of $\mathbf{Z}_i$ on $\mathbf{Y}_i$ by the causal effect of $\mathbf{Z}_i$ on $\mathbf{D}_i$.

IV: Mechanics and intuition

Takeaways

Main points

1. IV **rescales** the causal effect of $\mathbf{Z}_i$ on $\mathbf{Y}_i$ by the causal effect of $\mathbf{Z}_i$ on $\mathbf{D}_i$.

2. IV **does not** compare treated compliers to untreated compliers.

IV: Mechanics and intuition

Takeaways

Main points

1. IV **rescales** the causal effect of $\mathbf{Z}_i$ on $\mathbf{Y}_i$ by the causal effect of $\mathbf{Z}_i$ on $\mathbf{D}_i$.

2. IV **does not** compare treated compliers to untreated compliers.
   Such a comparison/estimator would re-introduce selection bias.

Thus far, we assumed homogeneous treatment effects.

Q What happens **when treatment effects are heterogeneous**?

A Let's recall what our instruments are doing (with Venn diagrams!).

Credit Glen Waddell introduced me to IV via Venn.

Figure 1

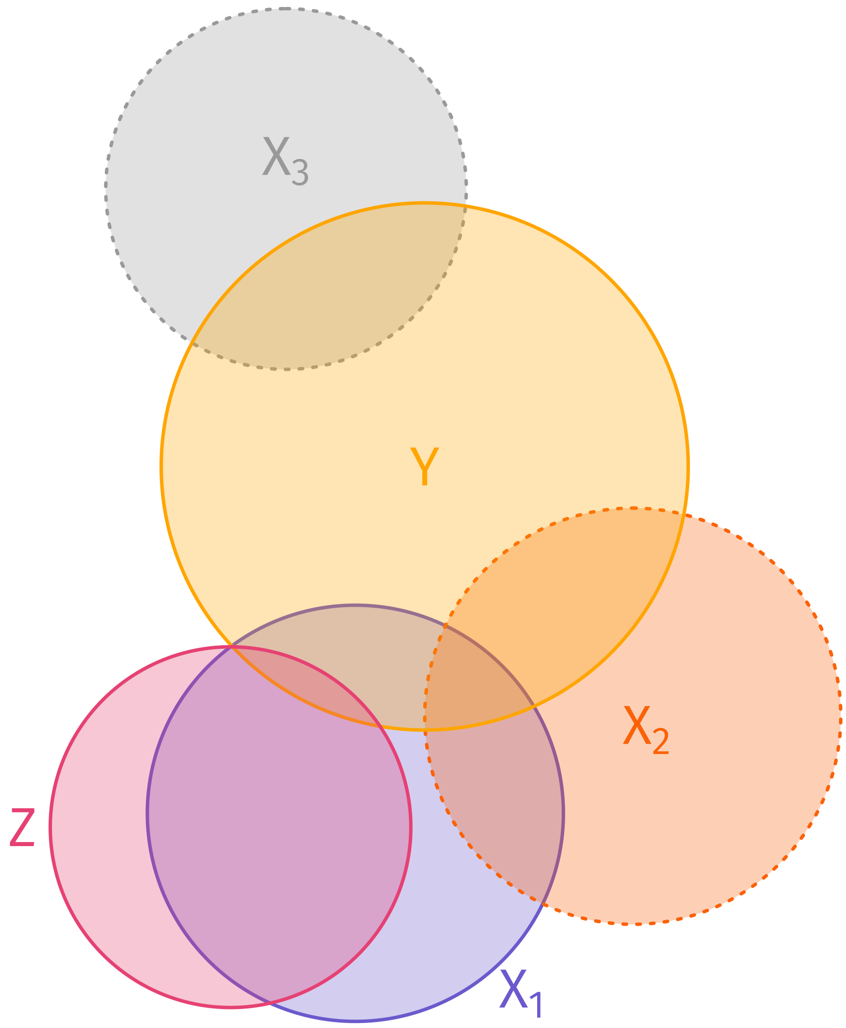


Figure 2

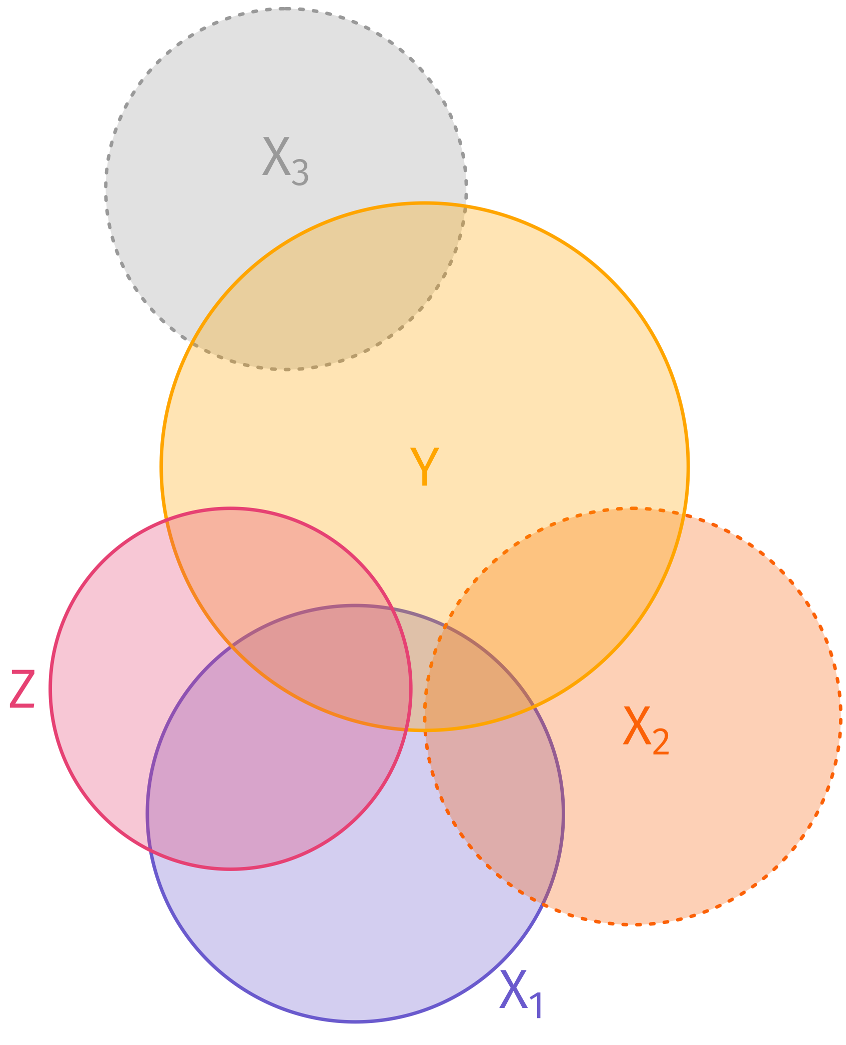


Figure 3

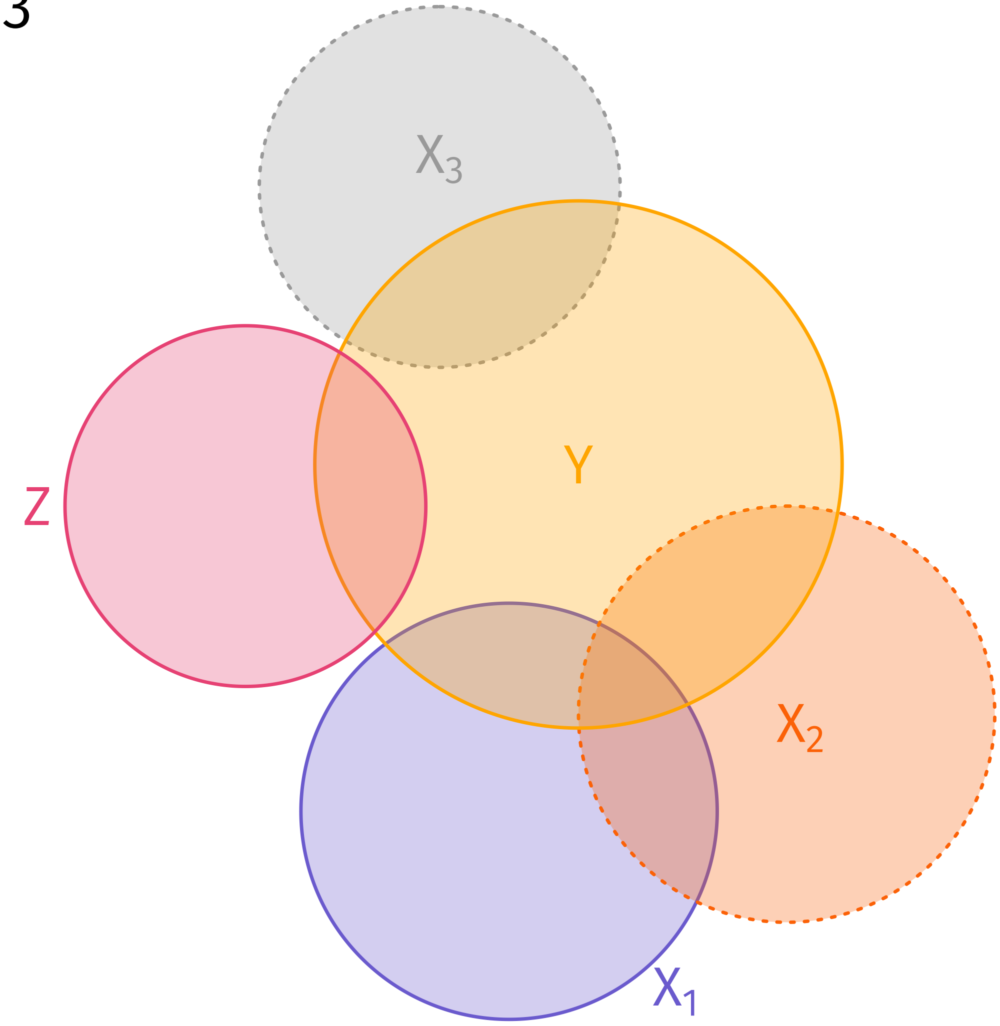
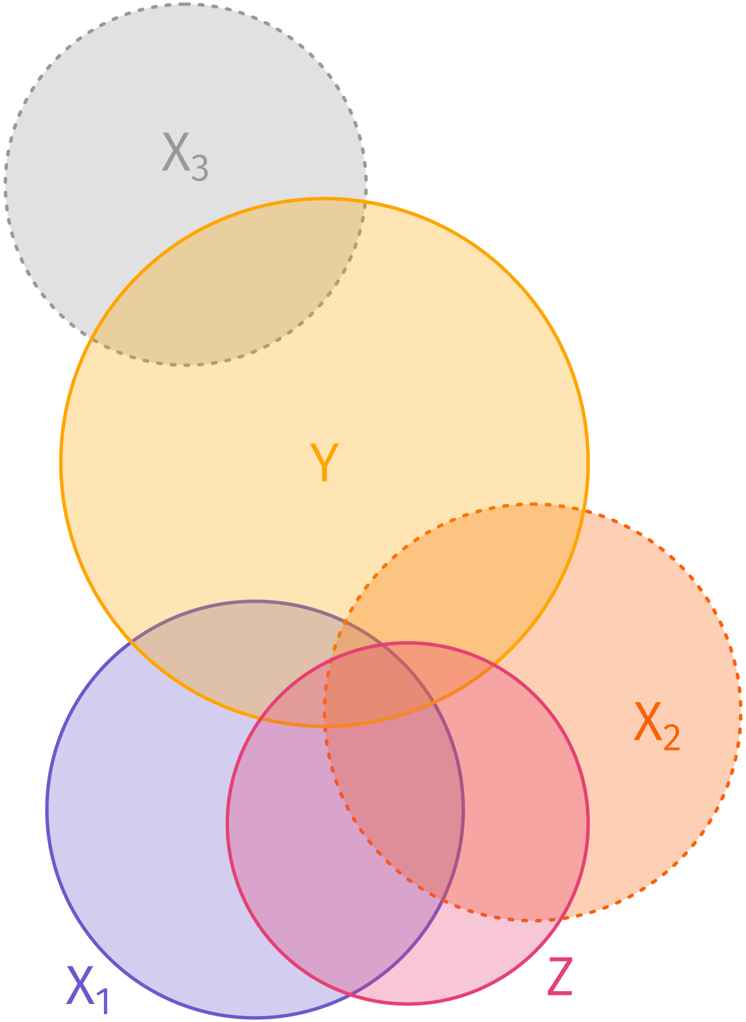


Figure 4



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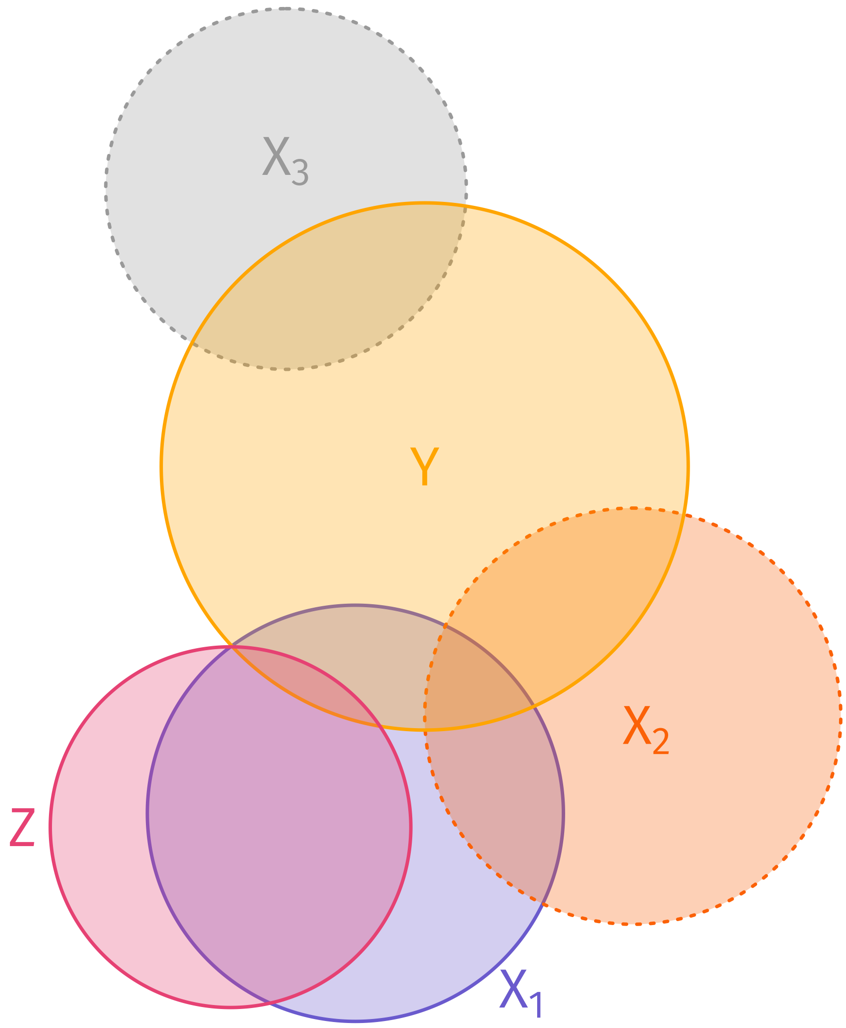

Can you draw the DAGs?

IV + heterogeneity

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Q If treatment effects vary, then what do IV and 2SLS estimate?

A Not ATE. And not TOT. They estimate the LATE.†

† See Angrist, Imbens, and Rubin (1996).

IV + heterogeneity

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IV + heterogeneity

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In other words: IV focuses on the individuals whose $\mathbf{D}_i$ changes due to $\mathbf{Z}_i$.

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However, *compliers* are only one of four possible groups.

1. **Compliers** $\mathbf{D}_i = 1$ iff $\mathbf{Z}_i = 1$.
2. **Always-takers** $\mathbf{D}_i = 1 \ \forall \mathbf{Z}_i$.
3. **Never-takers** $\mathbf{D}_i = 0 \ \forall \mathbf{Z}_i$.
4. **Defiers** $\mathbf{D}_i = 1$ iff $\mathbf{Z}_i = 0$.

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1. Compliers $\mathbf{D}_i = 1$ iff $\mathbf{Z}_i = 1$. Only take pills **when treated**.
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IV + heterogeneity

The LATE

Because IV only uses variation in $\mathbf{D}_i$ that correlates with $\mathbf{Z}_i$, IV mechanically drops *always-takers* and *never-takers*.

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Thus, IV estimates a treatment effect **using only *compliers***.

Hence the "local" in *local average treatment effect*.

IV + heterogeneity

The LATE: Medical-trial example

Imagine treatment works for some $(\beta_{1,i} < 0)$ and not for others $\left(\beta_{1,j} = 0\right)$.

Suppose individuals know their response to blood-pressure medication.

IV + heterogeneity

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Then our compliers will be individuals for whom $\beta_{1,j} = 0$.

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- $\beta_{1,i} < 0$ individuals always take the pill.
- $\beta_{1,j} = 0$ individuals only take the pill when treated.

Then our compliers will be individuals for whom $\beta_{1,j} = 0$.

Thus, IV's LATE will indicate no treatment effect $\left(\widehat{\beta}_1^{\text{IV}} = 0\right)$.

IV + heterogeneity

The LATE

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IV doesn't estimate the ATE or TOT, so it would be inconsistent for them.†

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Takeaway$_2$ Different instruments have different LATEs.

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Monotonicity

We've already written down the two classical IV/2SLS assumptions

- *First stage:* $\mathrm{Cov}(\mathrm{Z}_i,\, \mathrm{D}_i) > 0$
- *Exclusion restriction:* $\mathrm{Cov}(\mathrm{Z}_i,\, \varepsilon_i) = 0$

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but we need a third assumption to get ensure IV's complier-based LATE interpretation.

- **Monotonicity** (**Uniformity**): $\text{D}_i(z) \geq \text{D}_i(z')$ or $\text{D}_i(z) \leq \text{D}_i(z') \;\; \forall i$
  Heckman: *Uniformity* of responses *across persons.*
  Imbens and Angrist (1994): Instrument has monotone effect on D_i.

Monotonicity

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Example $\tau_c = 1$ and $\tau_d = 2$. $\Pr(\text{complier}) = 2/3$ and $\Pr(\text{defier}) = 1/3$.

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Example $\tau_c = 1$ and $\tau_d = 2$. $\mathbf{Pr}(\mathbf{complier}) = 2/3$ and $\mathbf{Pr}(\mathbf{defier}) = 1/3$.

Then the "LATE" is 0.†

† Some people would instead say that there is no LATE when you violate monotonicity.

Until now, we've focused on using a single instrument.

The 2SLS estimator accomodates multiple instruments.†

† Whether you can find multiple valid instruments is another question.

Multiple instruments

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Motivation

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Using terminology from the *system-of-equations* literature,

- one instrument for one endogenous variable: **just identified**
- multiple instruments for one endogenous variable: **over identified**

Multiple instruments

In practice

With (valid) instruments Z_{1i} and Z_{2i}, or first stage becomes

$$\mathrm{D}_i = \gamma_0 + \gamma_1 \mathrm{Z}_{1i} + \gamma_2 \mathrm{Z}_{2i} + \gamma_3 \mathrm{X}_i + u_i$$

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With (valid) instruments $\mathbf{Z}_{1i}$ and $\mathbf{Z}_{2i}$, or first stage becomes

$$\mathrm{D}_i = \gamma_0 + \gamma_1 \mathrm{Z}_{1i} + \gamma_2 \mathrm{Z}_{2i} + \gamma_3 \mathrm{X}_i + u_i$$

while our second stage is still

$$\mathrm{Y}_i = \beta_0 + \beta_1 \widehat{\mathrm{D}}_i + \beta_2 \mathrm{X}_i + v_i$$

Multiple instruments

Example: Quarter of birth

Back to our quest to estimate the returns to education.

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Multiple instruments

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Back to our quest to estimate the returns to education.

Angrist and Krueger (1991) proposed *quarter of birth* as a set of instruments for years of schooling.

Accordingly, their first stage looks something like[†]

$$\begin{aligned}\text{Schooling}_i = \gamma_0 &+ \gamma_1 \mathbb{I}(\text{Born Q1})_i + \gamma_2 \mathbb{I}(\text{Born Q2})_i \\ &+ \gamma_3 \mathbb{I}(\text{Born Q3})_i + \gamma_4 \mathbb{I}(\text{Born Q4})_i \\ &+ \gamma_5 \text{X}_i + u_i\end{aligned}$$

† We need to drop one of the quarter-of-birth indicators to avoid perfect collinearity.

Example: Quarter of birth

Q Is quarter of birth a valid instrument?

Multiple instruments

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Multiple instruments

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A1 Students cannot drop out of school until a certain age, and quarter of birth affects your age at the time you begin school.

Example Some states require students to stay in school until they are 16.

- Students who start school at age **6** drop out after **10** years of schooling.
- Students who start school at age **5** drop out after **11** years of schooling.

Multiple instruments

Example: Quarter of birth

If students must begin school in calendar year in which they turn 6

- December birthdates: begin school at 5.75; drop out with 10.25 yrs.
- January birthdates: begin school at 6.75; drop out with 9.25 yrs.

Multiple instruments

Example: Quarter of birth

If students must begin school in calendar year in which they turn 6

- December birthdates: begin school at 5.75; drop out with 10.25 yrs.
- January birthdates: begin school at 6.75; drop out with 9.25 yrs.

For some group, quarter of birth may affect the number of years in school.

Multiple instruments

Example: Quarter of birth

It turns out that the first stage is also pretty weak in this setting.

Weak instruments can cause several problems for 2SLS/IV:

Multiple instruments

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1. Our estimator is a ratio of the reduced form and the first stage, so a weak first stage can blow up reduced-form estimates (amplifying reduced-form noise/bias).

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2. Many weak instruments lead to a finite-sample issue in which 2SLS is biased toward OLS—our first stage is essentially overfitting.

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Weak instruments can cause several problems for 2SLS/IV:

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2. Many weak instruments lead to a finite-sample issue in which 2SLS is biased toward OLS—our first stage is essentially overfitting.

What about our other requirements for a valid instrument?

Multiple instruments

Example: Quarter of birth

Q2 Is quarter of birth uncorrelated with ε_i (*excludable*)?

Multiple instruments

Example: Quarter of birth

Q2 Is quarter of birth uncorrelated with ε_i (*excludable*)?

A2 While quarter of birth may be fairly arbitrary for some families, other families might time births.

If these birth timers differ from other couples along other dimensions (*e.g.*, income or education), then quarter of birth may correlate with ε_i.

Multiple instruments

Example: Quarter of birth

Q3 Is the effect monotone?

Multiple instruments

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Multiple instruments

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Consider December births.

- Original idea: December birthdates will start school at age 5.7, inducing more years of education before 16.

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Example: Quarter of birth

Q3 Is the effect monotone?

A3 Some† argue that monotonicity may be violated in this setting.

Consider December births.

- Original idea: December birthdates will start school at age 5.7, inducing more years of education before 16.
- *Redshirting* idea: Parents hold back December kids so they can be older (*i.e.*, 6.7), inducing fewer years of education before 16.

† *E.g.*, Aliprantis (2012)

2SLS and R

`estimatr`

You can implement 2SLS/IV in many ways in R.

Today: `esitmatr` and `iv_robust()`.

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You can implement 2SLS/IV in many ways in R.

Today: `esitmatr` and `iv_robust()`.

Specifically, we give `iv_robust()` the relationship that we want separted from the instrument by `|`, *e.g.*,

```
# Estimate 2SLS
iv_robust(Y ~ D | Z, data = sample_df, se_type = "classical") %>%
  tidy() %>% select(1:5)
```

```
#>          term estimate std.error statistic      p.value
#> 1 (Intercept) 5.786204 2.9744230  1.945320 0.0546020456
#> 2           D 1.107801 0.3043264  3.640173 0.0004372703
```

Now in two stages!

Of course, we can estimate 2SLS in two stages.

```r
# First stage
stage1 = lm_robust(D ~ Z, data = sample_df, se_type = "classical")
# First-stage results
stage1 %>% tidy() %>% select(1:5)
```

```
#>          term  estimate std.error statistic      p.value
#> 1 (Intercept) 8.8226148 0.3169568 27.835389 2.486413e-48
#> 2           Z 0.3257347 0.1031506  3.157857 2.112927e-03
```

Second stage

We just need to add $\widehat{\mathrm{D}}_i$ to our dataset.

```r
# Add fitted (first-stage) values to data
sample_df %<>% mutate(D_hat = stage1$fitted.values)
# Second stage
stage2 = lm_robust(Y ~ D_hat, data = sample_df, se_type = "classical")
# Second-stage results
stage2 %>% tidy() %>% select(1:5)
```

```
#>          term estimate std.error statistic    p.value
#> 1 (Intercept) 5.786204 5.4132099  1.068904 0.28773854
#> 2       D_hat 1.107801 0.5538496  2.000184 0.04824759
```

Standard errors

However, recall that our second-stage standard errors are not correct.

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Second-stage results

Term	Est.	S.E.	t stat.	p-Value
Int	5.786	5.413	1.07	0.2877
D hat	1.108	0.554	2.00	0.0482

2SLS and R

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Second-stage results

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Int	5.786	5.413	1.07	0.2877
D hat	1.108	0.554	2.00	0.0482

2SLS results

Term	Est.	S.E.	t stat.	p-Value
Int	5.786	2.974	1.95	0.0546
D	1.108	0.304	3.64	0.0004

IV and 2SLS

Conclusions

1. IV/2SLS focus on **isolating some "good" variation** in $\mathbf{D}_i$ via $\mathbf{Z}_i$.
2. Important **requirements**: strong first stage, excludability, monotonicity.
3. IV and 2SLS **rescale the reduced form** with the first stage.
4. Estimates are **LATE from compliers**.
5. Different instruments can produce **different LATEs**.
6. A **weak first stage** can lead to problems.

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