# **Controls**

EC 607, Set 06

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# Prologue

# Schedule

#### Last time

The conditional independence assumption:  $\{Y_{0i}, Y_{1i}\} \perp D_i | X_i$ I.e., conditional on some controls  $(X_i)$ , treatment is as-good-as random.

## **Today**

- Omitted variable bias
- Good vs. bad controls

## **Upcoming**

Topics: Matching estimators

## Revisiting an old friend

Let's start where we left off: Returns to schooling.

We have two linear, population models

$$Y_i = \alpha + \rho s_i + \eta_i \tag{1}$$

$$Y_i = \alpha + \rho s_i + X_i' \gamma + \nu_i \tag{2}$$

We should not interpret  $\hat{\rho}$  causally in model (1) (for fear of selection bias).

For model (2), we can interpret  $\hat{\rho}$  causally **if**  $\mathbf{Y}_{si} \perp \mathbf{I} \mathbf{s}_i | \mathbf{X}_i$  (CIA).

In other words, the CIA says that our **observable vector**  $X_i$  **must explain all** of correlation between  $s_i$  and  $\eta_i$ .

#### The OVB formula

We can use the omitted-variable bias (OVB) formula to compare regression estimates from **models with different sets of control variables**.

We're concerned about selection and want to use a set of control variables to account for ability  $(A_i)$ —family background, motivation, intelligence.

$$Y_i = \alpha + \beta s_i + v_i \tag{1}$$

$$Y_i = \pi + \rho s_i + A_i' \gamma + e_i \tag{2}$$

What happens if we can't get data on  $A_i$  and opt for (1)?

$$rac{\mathrm{Cov}(\mathrm{Y}_i,\,\mathrm{s}_i)}{\mathrm{Var}(\mathrm{s}_i)} = 
ho + \gamma' \delta_{As}$$

where  $\delta_{As}$  are coefficients from regressing  $A_i$  on  $s_i$ .

#### Interpretation

Our two regressions

$$Y_i = \alpha + \beta s_i + v_i \tag{1}$$

$$\mathbf{Y}_i = \pi + \rho \mathbf{s}_i + \mathbf{A}_i' \gamma + e_i$$
 (2)

will yield the same estimates for the returns to schooling

$$rac{ ext{Cov}( ext{Y}_i,\, ext{s}_i)}{ ext{Var}( ext{s}_i)} = 
ho + \gamma' \delta_{As}$$

if (**a**) schooling is uncorrelated with ability ( $\delta_{As} = 0$ ) or (**b**) ability is uncorrelated with earnings, conditional on schooling ( $\gamma = 0$ ).

### Example

Table 3.2.1, The returns to schooling

	•			
	1	2	3	4
Schooling	0.132	0.131	0.114	0.087
	(0.007)	(0.007)	(0.007)	(0.009)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT

Here we have four specifications of controls for a regression of log wages on years of schooling (from the NLSY).

#### Example

Table 3.2.1, The returns to schooling

	1	2	3	4
Schooling	0.132	0.131	0.114	0.087
	(0.007)	(0.007)	(0.007)	(0.009)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT

**Column 1** (no control variables) suggests a 13.2% increase in wages for an additional year of schooling.

#### Example

Table 3.2.1, The returns to schooling

	1	2	3	4	
Schooling	0.132	0.131	0.114	0.087	
	(0.007)	(0.007)	(0.007)	(0.009)	
Controls	None	Age Dum.	2 + Add'l	3 + AFQT	

**Column 2** (age dummies) suggests a 13.1% increase in wages for an additional year of schooling.

#### Example

Table 3.2.1, The returns to schooling

	1	2	3	4
Schooling	0.132	0.131	0.114	0.087
	(0.007)	(0.007)	(0.007)	(0.009)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT

**Column 3** (column 2 controls plus parents' ed. and self demographics) suggests a 11.4% increase in wages for an additional year of schooling.

#### Example

Table 3.2.1, The returns to schooling

	1	2	3	4
Schooling	0.132	0.131	0.114	0.087
	(0.007)	(0.007)	(0.007)	(0.009)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT

**Column 4** (column 3 controls plus AFQT<sup>†</sup> score) suggests a 8.7% increase in wages for an additional year of schooling.

#### Example

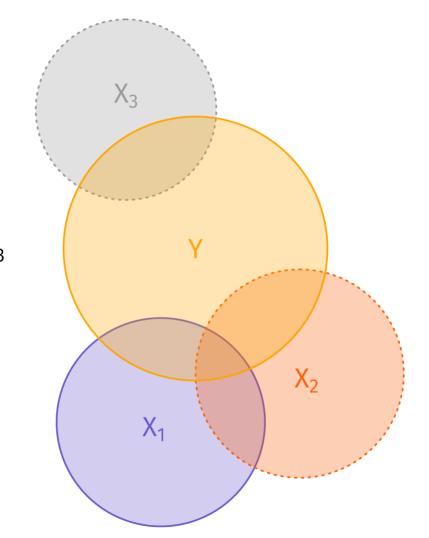
Table 3.2.1, The returns to schooling

	1	2	3	4
Schooling	0.132	0.131	0.114	0.087
	(0.007)	(0.007)	(0.007)	(0.009)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT

As we ratchet up controls, the estimated returns to schooling drop by 4.5 percentage points (34% drop in the coefficient) from **Column 1** to **Column 4**.

$$\frac{\operatorname{Cov}(\mathbf{Y}_i, \mathbf{s}_i)}{\operatorname{Var}(\mathbf{s}_i)} = \rho + \gamma' \delta_{As}$$

If we think **ability positively affects wages**, then it looks like we also have **positive selection into schooling**.



Omitted: X<sub>2</sub> and X<sub>3</sub>

#### Note

This OVB formula **does not** require either of the models to be causal.

The formula compares the regression coefficient in a **short model** to the regression coefficient on the same variable in a **long model**.<sup>†</sup>

#### The OVB formula and the CIA<sup>†</sup>

In addition to helping us think through and sign OVB, the formula

$$rac{ ext{Cov}( ext{Y}_i,\, ext{s}_i)}{ ext{Var}( ext{s}_i)} = 
ho + \gamma' \delta_{As}$$

drives home the point that we're leaning *very* hard on the conditional independence assumption to be able to interpret our coefficients as causal.

- Q When is the CIA plausible?
- A Two potential answers
  - 1. Randomized experiments
  - 2. Programs with arbitrary cutoffs/lotteries

Control variables play an enormous role in our quest for causality (the CIA).
Q Are "more controls" always better (or at least never worse)?

A No. There are such things as...

#### Defined

- Q What's a bad control—when can a control make a bad situation worse?
- A Bad controls are variables that are (also) affected by treatment.
- Q Okay, so why is it bad to control using a variable affected by treatment?

Hint It's a flavor of selection bias.

Let's consider an example...

#### Example

Suppose we want to know the effect of college graduation on wages.

- 1. There are only two types of jobs: blue collar and white collar.
- 2. White-collar jobs, on average, pay more than blue-collar jobs.
- 3. Graduating college increases the likelihood of a white-collar job.

Q Should we control for occupation type when considering the effect of college graduation on wages? (Will occupation be an omitted variable?)

A No. Imagine college degrees are randomly assigned. When we condition on occupation, we compare degree-earners who chose blue-collar jobs to non-degree-earners who chose blue-collar jobs. Our assumption of random degrees says **nothing** about random job selection.

#### Formal-ish derivation

More formally, let

- $W_i$  be a dummy for whether i has a white-collar job
- Y<sub>i</sub> denote i's earnings
- $C_i$  refer to i's randomly assigned college-graduation status

$$egin{aligned} \mathbf{Y}_{i} &= \mathbf{C}_{i} \mathbf{Y}_{1i} + (1 - \mathbf{C}_{i}) \, \mathbf{Y}_{0i} \ \mathbf{W}_{i} &= \mathbf{C}_{i} \mathbf{W}_{1i} + (1 - \mathbf{C}_{i}) \, \mathbf{W}_{0i} \end{aligned}$$

Becuase we've assumed  $C_i$  is randomly assigned, differences in means yield causal estimates, *i.e.*,

$$E[Y_i \mid C_i = 1] - E[Y_i \mid C_i = 0] = E[Y_{1i} - Y_{0i}]$$
  
 $E[W_i \mid C_i = 1] - E[W_i \mid C_i = 0] = E[W_{1i} - W_{0i}]$ 

#### Formal-ish derivation, continued

Let's see what happens when we throw in some controls—*e.g.*, focusing on the the wage-effect of college graduation for white-collar jobs.

$$egin{aligned} E[\mathbf{Y}_i \mid \mathbf{W}_i = 1, \, \mathbf{C}_i = 1] - E[\mathbf{Y}_i \mid \mathbf{W}_i = 1, \, \mathbf{C}_i = 0] \ &= E[\mathbf{Y}_{1i} \mid \mathbf{W}_{1i} = 1, \, \mathbf{C}_i = 1] - E[\mathbf{Y}_{0i} \mid \mathbf{W}_{0i} = 1, \, \mathbf{C}_i = 0] \ &= E[\mathbf{Y}_{1i} \mid \mathbf{W}_{1i} = 1] - E[\mathbf{Y}_{0i} \mid \mathbf{W}_{0i} = 1] \ &= E[\mathbf{Y}_{1i} \mid \mathbf{W}_{1i} = 1] - E[\mathbf{Y}_{0i} \mid \mathbf{W}_{1i} = 1] \ &+ E[\mathbf{Y}_{0i} \mid \mathbf{W}_{1i} = 1] - E[\mathbf{Y}_{0i} \mid \mathbf{W}_{0i} = 1] \ &= \underbrace{E[\mathbf{Y}_{1i} - \mathbf{Y}_{0i} \mid \mathbf{W}_{1i} = 1]}_{\text{Causal effect on white-collar workers} + \underbrace{E[\mathbf{Y}_{0i} \mid \mathbf{W}_{1i} = 1] - E[\mathbf{Y}_{0i} \mid \mathbf{W}_{0i} = 1]}_{\text{Selection bias}} \end{aligned}$$

#### Formal-ish derivation, continued

By introducing a bad control, we introduced selection bias into a setting that did not have selection bias without controls.

Specifically, the selection bias term

$$E[\mathrm{Y}_{0i}\mid \mathrm{W}_{1i}=1]-E[\mathrm{Y}_{0i}\mid \mathrm{W}_{0i}=1]$$

describes how college graduation changes the composition of the pool of white-class workers.

Note Even if the causal effect is zero, this selection bias need not be zero.

#### A trickier example

A timely/trickier example: Wage gaps (e.g., female-male or black-white).

- Q Should we control for occupation when we consider wage gaps?
  - What are we trying to capture?
  - If we're concerned about discrimination, it seems likely that discrimination also affects occupational choice and hiring outcomes.
  - Some motivate occupation controls with groups' differential preferences.

What's the answer?

#### Proxy variables

Angrist and Pischke bring up an interesting scenario that intersects omitted-variable bias and bad controls.

- We want to estimate the returns to education.
- Ability is omitted.
- We have a proxy for ability—a test taken after schooling finishes.

We're a bit stuck.

- 1. If we omit the test altogether, we've got omitted-variable bias.
- 2. If we include our proxy, we've got a back control.

With some math/luck, we can bound the true effect with these estimates.

#### Example

Returning to our OVB-motivated example, we control for occupation.

Table 3.2.1, The returns to schooling

		•			<u> </u>
	1	2	3	4	5
Schooling	0.132	0.131	0.114	0.087	0.066
	(0.007)	(0.007)	(0.007)	(0.009)	(0.010)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT	4 + Occupation

Schooling likely affects occupation; how do we interpret the new results?

#### Conclusion

Timing matters.

The right controls can help tremendously, but bad controls hurt.

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