$$\boxed{\begin{aligned} z^2 &= \frac{1}{c^2} = \frac{c^2+s^2}{c^2} = \frac{c^2}{c^2} + \frac{s^2}{c^2} = 1+t^2 \\ \frac{dt}{d\theta} &= \frac{d}{d\theta}\frac{s}{c} = \frac{s_\theta c - s c_\theta}{c^2} = \frac{c^2+s^2}{c^2} = \frac{1}{c^2} = z^2 \\ \frac{dz}{d\theta} &= \frac{d}{d\theta}\frac{1}{c} = \frac{1_\theta c - 1 c_\theta}{c^2} = \frac{s}{c^2} = \frac{1}{c}\frac{s}{c} = zt \end{aligned}}$$

$\Downarrow$ $\Downarrow$

$$\left[\begin{aligned} t &= \tan\theta \\ \sqrt{1+t^2} &= \sec\theta \\ dt &= (\sec\theta)^2 d\theta \end{aligned}\right] \qquad \left[\begin{aligned} z &= \sec\theta \\ \sqrt{z^2-1} &= \tan\theta \\ dz &= (\sec\theta)(\tan\theta)\,d\theta \end{aligned}\right]$$

$$\left[\begin{aligned} t &= \tan\theta \\ \sqrt{1+t^2} &= \sec\theta \\ dt &= (\sec\theta)^2 d\theta \end{aligned}\right]$$

$$\begin{aligned} &\int t^\alpha \sqrt{1+t^2}^{\,\beta} dt \\ &= \int (\tan\theta)^\alpha (\sec\theta)^\beta (\sec\theta)^2 d\theta \\ &= \int (\tan\theta)^\alpha (\sec\theta)^{\beta+2} d\theta \\ &= \int \frac{(\operatorname{sen}\theta)^\alpha}{(\cos\theta)^\alpha} \frac{1}{(\cos\theta)^{\beta+2}} d\theta \\ &= \int (\operatorname{sen}\theta)^\alpha (\cos\theta)^{-\alpha-\beta-2} d\theta \end{aligned}$$

$$\left[\begin{aligned} z &= \sec\theta \\ \sqrt{z^2-1} &= \tan\theta \\ dz &= (\sec\theta)(\tan\theta)\,d\theta \end{aligned}\right]$$

$$\begin{aligned} &\int z^\alpha \sqrt{z^2-1}^{\,\beta} dz \\ &= \int (\sec\theta)^\alpha (\tan\theta)^\beta (\sec\theta)(\tan\theta)\,d\theta \\ &= \int (\sec\theta)^{\alpha+1} (\tan\theta)^{\beta+1} d\theta \\ &= \int \frac{1}{(\cos\theta)^{\alpha+1}} \frac{(\operatorname{sen}\theta)^{\beta+1}}{(\cos\theta)^{\beta+1}} d\theta \\ &= \int (\operatorname{sen}\theta)^{\beta+1} (\cos\theta)^{-\alpha-\beta-2} d\theta \end{aligned}$$