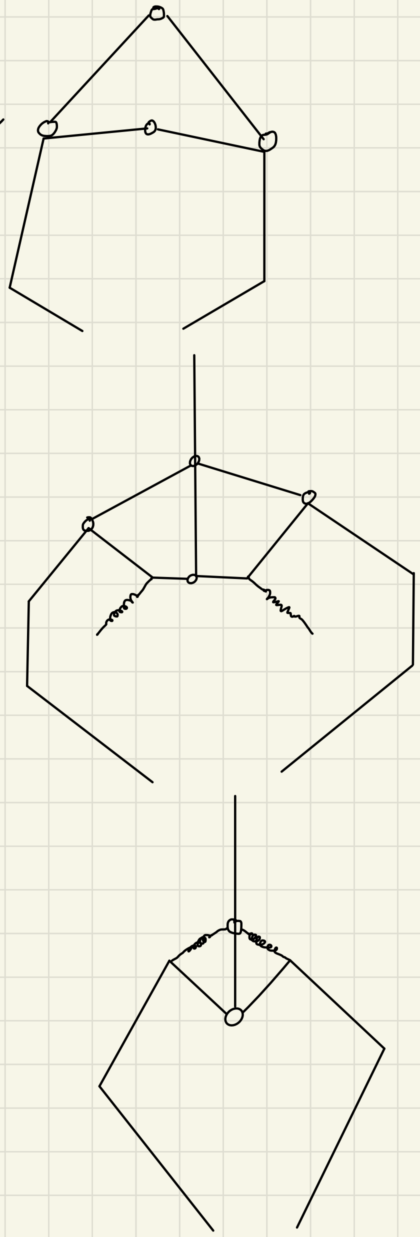
Grabber

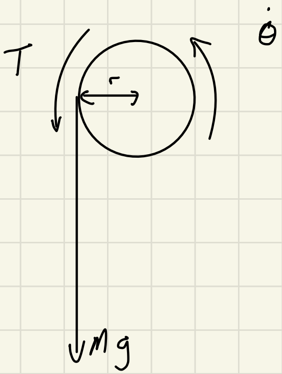


Finding the radius of the coil as a function of ϑ i.e as it is unwound.

$$r(\theta) = r_0\left(1 - \frac{\theta}{\theta_0}\right)$$

Total length of line

$$\int_0^{\theta_0} \theta r \, d\theta = l$$



g is for moon

$$r_0 \int_0^{\theta_0} \theta - \frac{\theta^2}{\theta_0} \, d\theta = l$$

$$r_0 \left[\frac{\theta^2}{2} - \frac{\theta^3}{3\theta_0} \right]_0^{\theta_0} = l$$

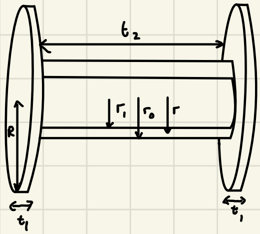
$$r_0 \frac{\theta_0^2}{6} = l$$

$$\theta_0 = \sqrt{\frac{6l}{r_0}}$$

$$r(\theta) = r_0\left(1 - \frac{\sqrt{r_0}}{\sqrt{6l}}\theta\right)$$

we want $\quad r\dot{\theta} \approx \text{constant}$

$$r_0\left(1 - \frac{\sqrt{r_0}}{\sqrt{6l}}\theta\right)\dot{\theta} = \text{constant}$$



ρ_1 : dencity of reel

ρ_2 = density of line

$t_2 : 40\,mm$
$r_1 = 15\,mm$
$R : 17.5\,mm$
$t_1 = 3\,mm$

Moment of inertia

$$I(r) = \frac{m_1(r^2 - r_1^2)}{2} + m_2 R + \frac{m_3 r_1^2}{2}$$

$$= \frac{\rho_2 t_2 \pi (r^4 - r_1^4)}{2} + \rho_1 \pi R^4 t_1 + \frac{\rho_1 \pi r_1^4 t_2}{2}$$

Let $A = \frac{\rho_2 t_2 \pi}{2}$ $\quad B = \rho_1 \pi R^4 t_1 + \frac{\rho_1 \pi r_1^4 t_2}{2}$

$$I(r) = A(r^4 - r_1^4) + B$$

$r = r(\theta) \quad \theta = \theta(t)$

$$\dot{r} = \frac{dr}{d\theta}\frac{d\theta}{dt}$$

$$= -r_0\sqrt{\frac{6L}{r_0}}\,\dot{\theta}$$

$$= -\sqrt{6Lr_0}\,\dot{\theta}$$

T is torque and will be used by us to control the descent velocity and thus is a function of time.

$$Mgr + T = \frac{d}{dt}(I\dot{\theta})$$

$$Mgr + T = \dot{I}\dot{\theta} + I\ddot{\theta}$$

$$= 4A\dot{r}r^3\dot{\theta} + \left(A(r^4 - r_1^4) + B\right)\ddot{\theta}$$

$$= \left(A(r^4 - r_1^4) + B\right)\ddot{\theta} - 4A\sqrt{6Lr_0}\,r^3\dot{\theta}^2$$

I don’t think we want to use this.

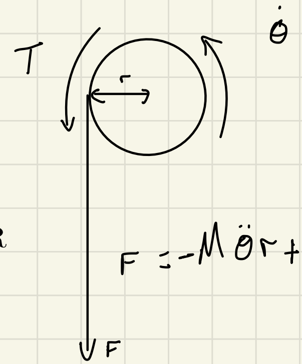
Assume moment if line is thin thus r is a constant.

$$r = \frac{r_0 + r_1}{2}$$

Thus

$$I = \frac{\rho_2 t_2 \pi \left(\left(\frac{r_0+r_1}{2}\right)^4 - r_1^4 \right)}{2} + \rho_1 \pi R^4 t_1 + \frac{\rho_1 \pi r_1^4 t_2}{2}$$

$$I \ddot{\theta} = Mgr + T(t)$$



For the mechanical idea taking torque as a constant -μNR

$$F = -M\ddot{\theta} r + Mg$$

$$I \ddot{\theta} = Fr - \mu NR$$

$$I \ddot{\theta} = M r^2 \ddot{\theta} + Mgr - \mu N r$$

$$\dot{\theta} = \left(\frac{Mgr - \mu NR}{I + Mr^2} \right) t + C$$

$$F - Mg = Ma$$

$$F - Mg = -M r \ddot{\theta}$$

At $t=0$ $\dot{\theta} = 0$ $\Rightarrow C = 0$

$$\dot{\theta} = \left(\frac{Mgr - \mu NR}{I + Mr^2} \right) t$$

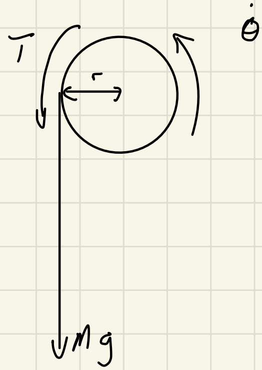
Thus the velocity increases linearly with time which is undesirable.

Add a damper such that T= -μRN- $\kappa\dot{\theta}$

$$I \ddot{\theta} = -M\ddot{\theta} r^2 + Mgr - \mu RN - \kappa \dot{\theta}$$

$$\ddot{\theta} + \frac{\kappa}{I + Mr^2} \dot{\theta} = \frac{Mgr - \mu RN}{I + Mr^2} \qquad \dot{\theta} = \frac{Mgr - \mu RN}{\kappa}$$

Electronic idea



For natural system.

$$I\ddot{\theta} = Mgr + T$$

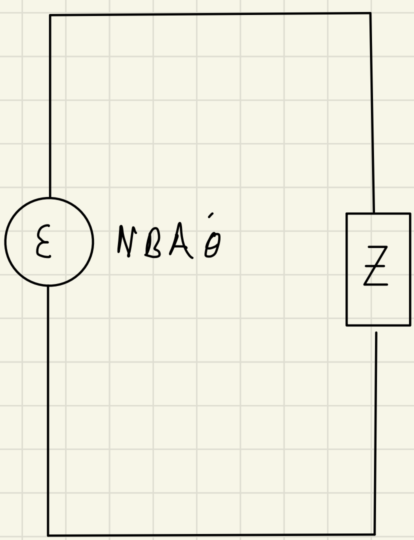
$$I\left(\dot{\bar{\theta}}(s)s - \dot{\theta}(0)\right) = \frac{Mgr}{s}$$

$$T = iBr$$

$$\varepsilon = NBA \sin\theta \; \dot{\theta}$$

$$I\dot{\bar{\theta}} s = \frac{Mgr}{s}$$

$$\bar{\theta}(s) = \frac{Mgr}{Is^2}$$



$$X(s) = C(s)\,\theta(s)$$

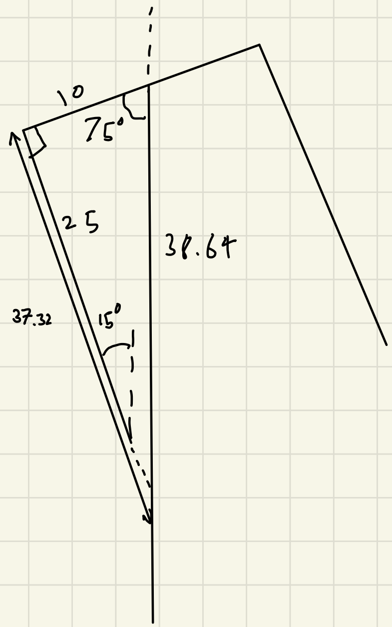
$$= k\left(\frac{sT+1}{sT}\right)\frac{Mgr}{Is^3}$$

$$i = \frac{NBA\dot{\theta}}{Z}$$

$$= \frac{kMgr}{Is^3} + \frac{kMgr}{ITs^4}$$

$$x(t) = \frac{kMgr}{2I}t^2 + \frac{kMgr}{6IT}t^3$$

Making sure the rope does not contact the side given a 15 degree tilt.



4. A constant field electric motor, when spun mechanically at 1000 rpm, generates 24 V at its armature terminals. The motor has an armature resistance of 6 Ω, an armature inductance of 0.001 H, and a rotor mechanical inertia of 0.01 kg m^2. The motor's output shaft is connected to the input of a reduction gearbox with a gear ratio of $100 : 1$, the case being rigidly attached to the ground. The output shaft of the gearbox is connected to a large rotating mass with moment of inertia 100 kg m^2. The motor is driven by a variable-voltage power supply with zero output impedance (it is a voltage source). The input to the system is the voltage of the power supply and the output is the position of the motor.

   (a) Derive an appropriate model for this system. Show that there are two feedback loops in the model. Describe the physical origin of each feedback signal.

   (b) Draw a block diagram of the system and explain why the inertia term is equal to 0.02 kg m^2.

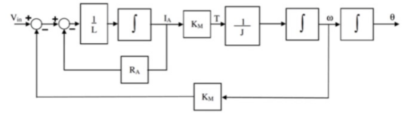
   (c) Why does the system have 3 states and not 5? There are, after all, 2 inertias and an inductance.

   (d) Derive the transfer function between the input voltage and motor output angular speed. Why does this model have only 2 states?

4. (a) The two feedback loops relate to the armature resistance that reduces applied voltage in proportion to the armature current in the motor, and to the back emf generated by the motor that reduces the effective voltage driving the armature current.

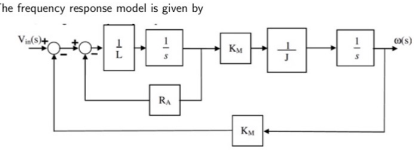
(b) The gearbox has a reduction ratio of $100 : 1$. The reflected inertia of the load inertia at the gearbox input is multiplied by the square of the reduction ratio. Thus the load inertia appears as $100 \times 1/100^2 =$ 0.01 kg m^2. The total rotational inertia at the input side is the motor inertia plus the reflected load inertia = 0.02 kg m^2. The model has a number of parameters:

- $L =$ 0.001 H.
- $R_A =$ 6 Ω.
- $K_M = (24 \times 60)/(1000 \times 2\pi) =$ 0.229 V s/rad or N m/amp
- $J =$ 0.02 kg m^2.

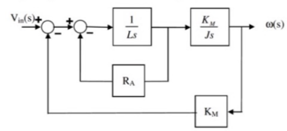


(c) The load inertia is connected to the motor inertia via a gearbox that provides a kinematic constraint. As the gearbox is considered to be ideal, the two inertias are not independent and act as a single rotational inertia. This is a kinematic constraint leading to a geometric boundary condition reducing the dimensionality of the state space (which would not be the case if the gears were elastic).

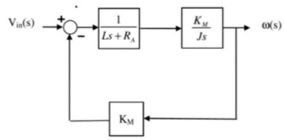
(d) The frequency response model is given by

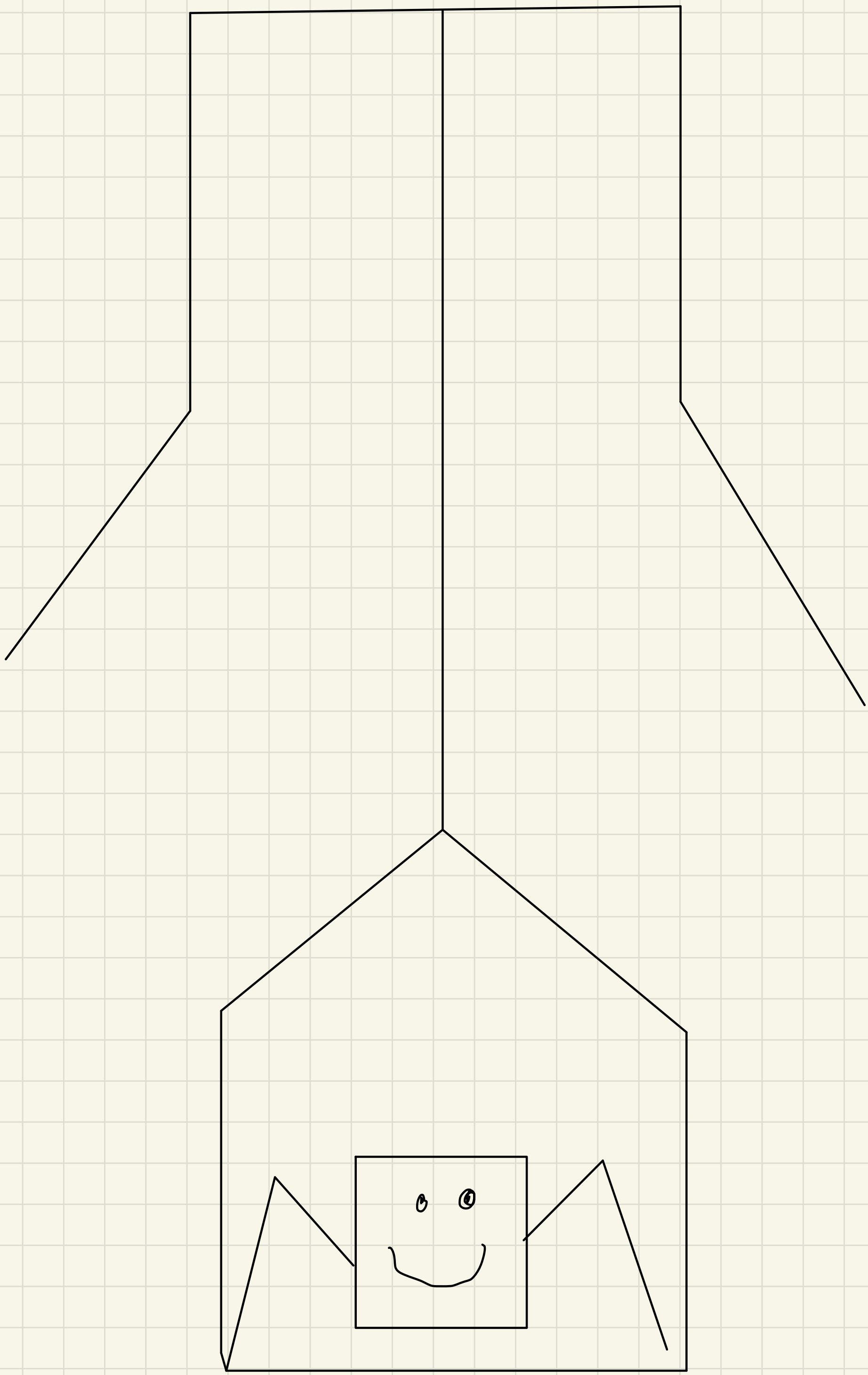


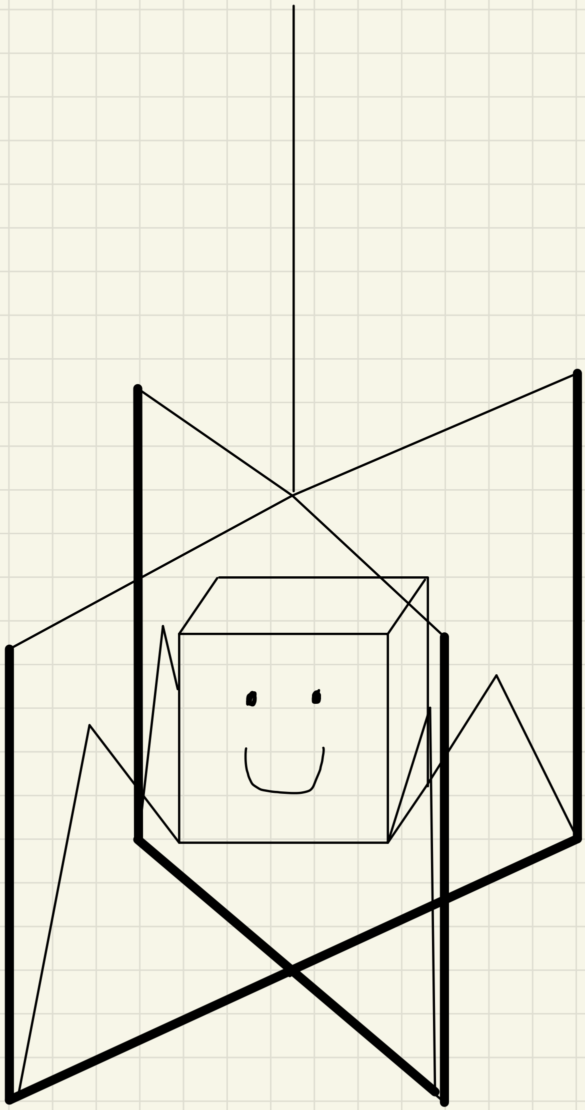
Combining blocks gives (this is no more than the graphical reduction of equations):



Removing the inner loop:





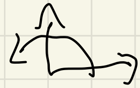


$$F - Mg = Ma$$

$$N - mg - F = 0$$

$$N - mg = F$$

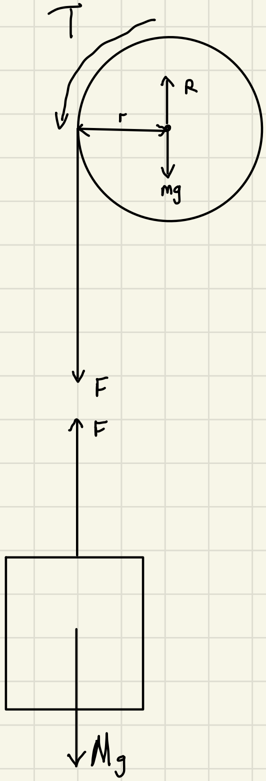
$$N - mg = Ma$$



① $I\ddot{\theta} = Fr + T$

② $0 = R - mg - F$

③ $Ma = F - Mg$



④ $a = -r\ddot{\theta}$

Sub ④ in ① $-I\frac{a}{r} = Fr + T$

$$-\frac{T}{r} - I\frac{a}{r^2} = F$$

Sub in ③ $Ma = -\frac{T}{r} - I\frac{a}{r^2} - Mg$

$$\left(M - \frac{I}{r^2}\right)a = -\frac{T}{r} - Mg$$

$I \approx 0.07$

$$a = \frac{\frac{T}{r} + Mg}{\frac{I}{r^2} - M}$$

For small I

$$M\left(\frac{\frac{T}{r} + Mg}{\frac{I}{r^2} - M}\right) + Mg = F \approx \frac{T}{r}$$

Materials.

Box: Aluminium 2024 (4.5% Cu)

Reel: Aluminium 2024

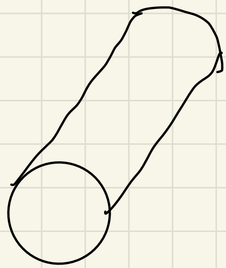
Fibre: Kevlar K49 cable 1mm diameter very strong good thermals.

https://www.dupont.com/content/dam/dupont/amer/us/en/safety/public/documents/en/Kevlar_Technical_Guide_0319.pdf

Rollers: Synthetic silicon Rubber

Springs: Steel?

Friction contact:



$$\varepsilon_{rr} = \frac{1}{E}\left(\sigma_{rr} - \nu\sigma_{\theta\theta}\right) + \alpha\Delta T$$