Sheet 7

Due 17.30 Tuesday 6th March

Hand in solutions to questions 1a, 2b, 4b, 4c, 5a.

Please write your student ID number on your work and staple it together.

1. Find $\phi(245)$ and calculate:

   **(a) $4^{169} \mod 245$, (2 marks)

   (b) $13^{1696968} \mod 245$.

2. Solve the congruences

   (a) $x^{101} \equiv 2 \mod 245$,

   **(b) $y^{29} \equiv 1 \mod 245$. (2 marks)

3. Solve the congruence $x^{11} \equiv 5 \mod 41$.

4. Consider the matrices:

$$A = \begin{pmatrix} 3 & 2 & 9 & 1 \\ 3 & 1 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 2 & 2 & 9 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 \\ 1 & -2 \\ 0 & 1 \\ 2 & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}.$$

   (a) Calculate AB and BC.

   **(b) Calculate $(AB)C$ and $A(BC)$. Check they are equal (associativity). (2 marks)

   **(c) Calculate C^{-1}. (2 marks)

5. Let $A \in \mathcal{M}(n, m)$ and $B \in \mathcal{M}(m, p)$. Let $\lambda \in \mathbb{R}$.

   **(a) Prove that $A(\lambda B) = (\lambda A)B$. (2 marks)

   (b) Prove that $(\lambda A)B = \lambda(AB)$.