

## Sheet 5

Due 17.30 Tuesday 20th February

Hand in solutions to questions 1, 2b, 2d.

Please write your student ID number on your work and staple it together.

- \*\*1. Using Euclid's algorithm, prove that 1876 and 365 are coprime and find integers  $k$  and  $h$  such that  $1 = 1876h + 365k$ . (6 marks)

*Solution.* Write

$$1876 = 5 \times 365 + 51$$

$$365 = 7 \times 51 + 8$$

$$51 = 6 \times 8 + 3$$

$$8 = 2 \times 3 + 2$$

$$3 = 2 + 1.$$

Therefore

$$\begin{aligned} 1 &= 3 - 2 \\ &= 3 - (8 - 2 \times 3) &= 3 \times 3 - 8 \\ &= 3 \times (51 - 6 \times 8) - 8 &= 3 \times 51 - 19 \times 8 \\ &= 3 \times 51 - 19 \times (365 - 7 \times 51) &= 136 \times 51 - 19 \times 365 \\ &= 136 \times (1876 - 5 \times 365) - 19 \times 365 &= 136 \times 1876 - 699 \times 365 \end{aligned}$$

so  $k = 136$ ,  $h = -699$ .

2. For each of the following pairs  $a, b$  find the greatest common divisor  $(a, b)$  and integers  $k$  and  $h$  such that  $(a, b) = ak + bh$ .

- (a) 267, 112

*Solution.*

$$\begin{aligned} 267 &= 2 \times 112 + 43, & 112 &= 2 \times 43 + 26, \\ 43 &= 26 + 17 & 26 &= 17 + 9 & 17 &= 9 + 8 \\ 9 &= 8 + 1; \end{aligned}$$

and

$$\begin{aligned} 1 &= 9 - 8 \\ &= 9 - (17 - 9) &= -17 + 2 \times 9 \\ &= -17 + 2 \times (26 - 17) &= -3 \times 17 + 2 \times 26 \\ &= -3 \times (43 - 26) + 2 \times 26 &= -3 \times 43 + 5 \times 26 \\ &= -3 \times 43 + 5 \times (112 - 2 \times 43) &= 5 \times 112 - 13 \times 43 \\ &= 5 \times 112 - 13 \times (267 - 2 \times 112) &= 31 \times 112 - 13 \times 267, \end{aligned}$$

so  $k = -12$ ,  $h = 31$ .

\*\* (b) 242, 1870

(2 marks)

*Solution.*

$$1870 = 7 \times 242 + 176,$$

$$242 = 176 + 66,$$

$$176 = 2 \times 66 + 44,$$

$$66 = 44 + 22$$

$$44 = 2 \times 22 + 0;$$

$$22 = 66 - 44$$

$$= 66 - (176 - 2 \times 66) = 3 \times 66 - 176$$

$$= 3 \times (242 - 176) - 176 = 3 \times 242 - 4 \times 176$$

$$= 3 \times 242 - 4 \times (1870 - 7 \times 242) = -4 \times 1870 + 31 \times 242,$$

so  $k = 31, h = -4$ .

(c) 600, 11312

*Solution.*

$$11312 = 18 \times 600 + 512,$$

$$600 = 512 + 88,$$

$$512 = 5 \times 88 + 72,$$

$$88 = 72 + 16$$

$$72 = 4 \times 16 + 8;$$

$$16 = 2 \times 8$$

so

$$8 = 72 - 4 \times 16$$

$$= 72 - 4 \times (88 - 72) = 5 \times 72 - 4 \times 88$$

$$= 5 \times (512 - 5 \times 88) - 4 \times 88 = 5 \times 512 - 29 \times 88$$

$$= 5 \times 512 - 29 \times (600 - 512) = 34 \times 512 - 29 \times 600$$

$$= 34 \times (11312 - 18 \times 600) - 29 \times 600 = 34 \times 11312 - 641 \times 600,$$

so  $k = -641, h = 34$ .

\*\* (d) 408, 126

(2 marks)

*Solution.*

$$408 = 3 \times 126 + 30,$$

$$126 = 4 \times 30 + 6,$$

$$30 = 5 \times 6;$$

so

$$6 = 126 - 4 \times 30 = 126 - 4 \times (408 - 3 \times 126) = 13 \times 126 - 4 \times 408,$$

so  $k = -4, h = 13$ .