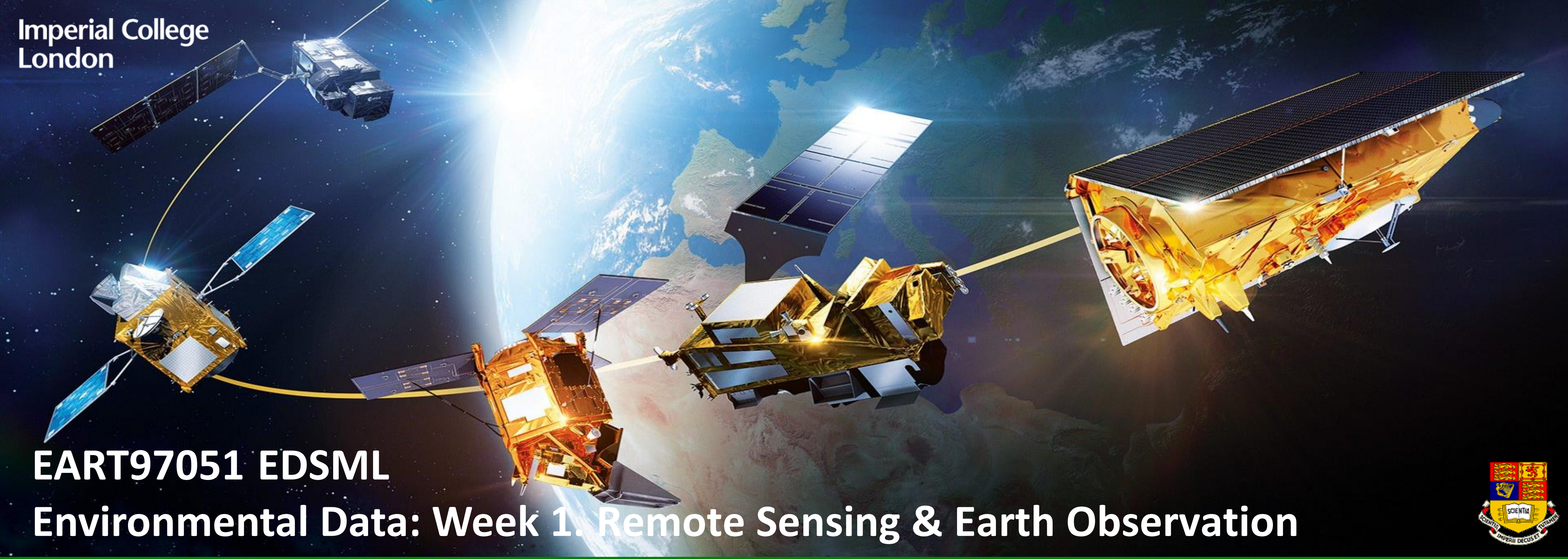

Imperial College London

EART97051 EDSML

Environmental Data: Week 1. Remote Sensing & Earth Observation

3. Colour coordinate transformations & Principal Components Analysis

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Lecture plan

3A Colour coordinate transformations

1. Definition of HSI transformation from RGB
2. Spectral enhancement & decorrelation stretch
3. Data fusion methods
    a) Intensity substitution fusion for Pan-sharpening
    b) Smoothing Filter based Modulation for Pan-sharpening

3B PCA and its uses

1. Principles of PCA, geometric & math definitions
2. Feature oriented PC selection - Understanding the statistics
3. Tasselled Cap Transform

3A. Colour coordinate transformations & applications

3A.1 RGB-HSI transformation

A colour is expressed as a composite of three primaries: Red, Green and Blue, according to tristimulus theory. For RGB additive colour display of digital images, a simple RGB colour cube is the most appropriate model.

But for colour perception, a colour is more intuitively and more quantitatively described in terms of three variables.

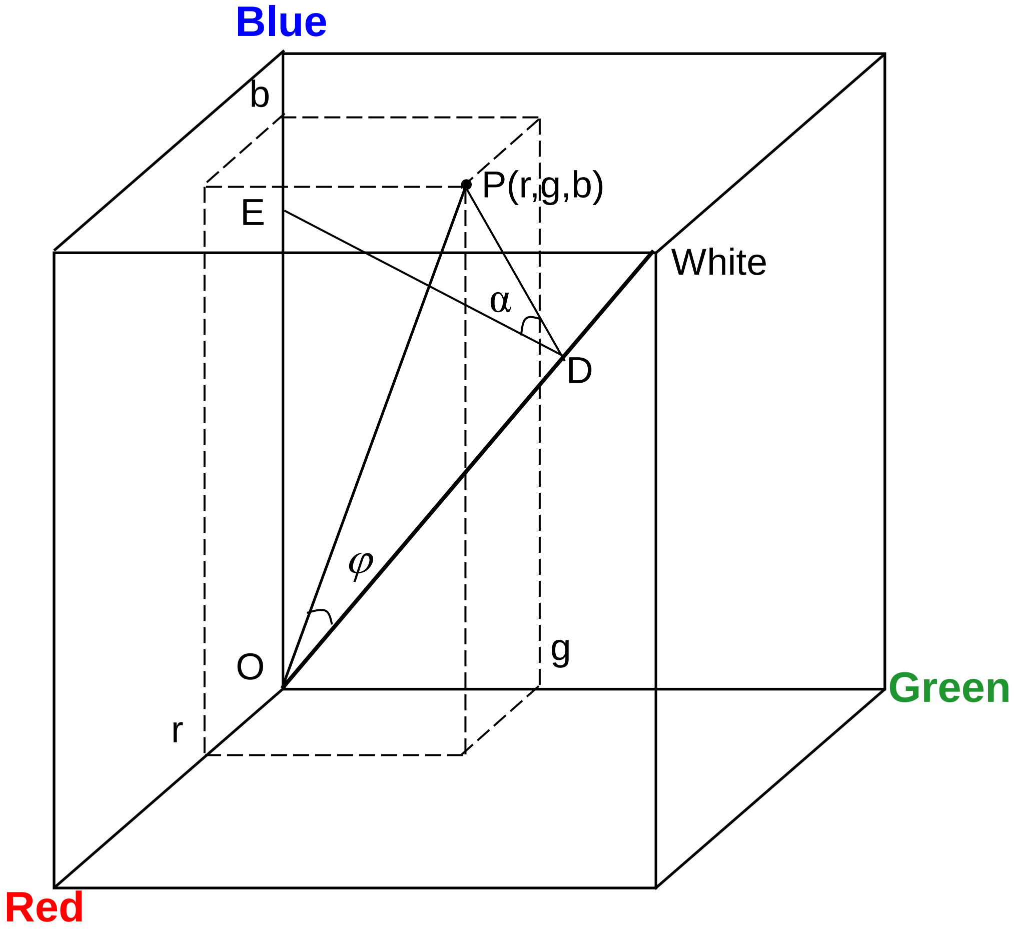
- **H**ue: colour spectral range
- **S**aturation: colour purity
- **I**ntensity: colour brightness (energy level)

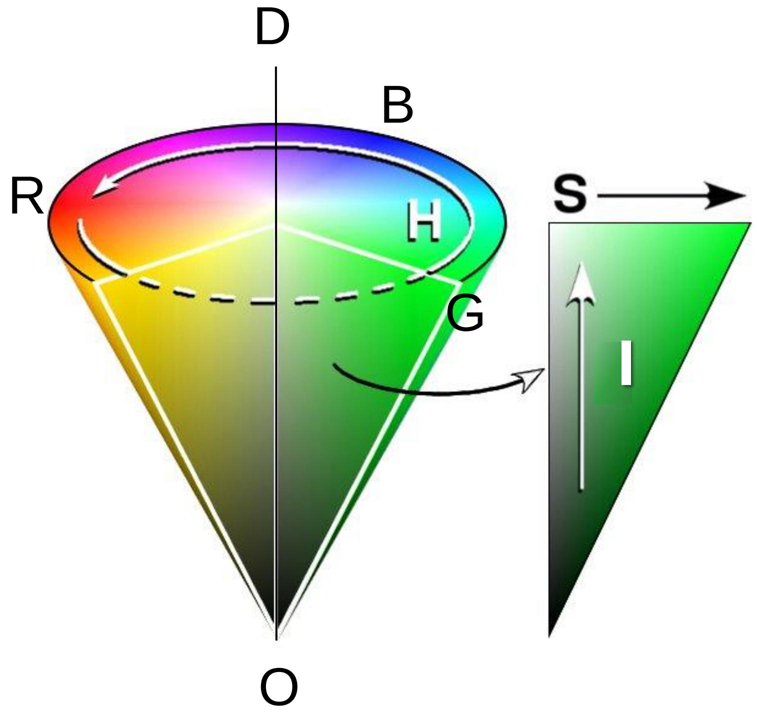
The RGB-HSI colour coordinate transformation within a colour cube is similar to a transformation from three-dimensional Cartesian to Conical (or 3D polar) coordinates.

We can exploit this relationship between RGB and HSI with two key benefits:

1. **Decorrelation Stretch** – for image spectral enhancement (2 common methods)
2. **Data fusion** – for sharpening image texture (3 common methods)

Colour coordinate transformation & definition of H, S & I





- Any colour in a 3-band colour composite is a vector $\mathbf{P}(r,g,b)$ within a colour cube with edge length of 255 (for 24 bits RGB colour display).
- The major diagonal line connecting the origin and the furthest vertex is called **grey line**.
- Intensity of a colour vector P is defined as the length of its projection on the grey line (*OD*),
- The hue is the azimuth angle around the grey line (α), and
- The saturation is the angle between the colour vector P and the grey line (φ).

RGB-HSI colour coordinate transformation is based on the colour cube:

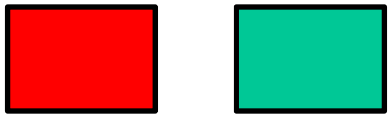
		Value ranges
Intensity	$I(r,g,b) = \frac{1}{\sqrt{3}}(r+g+b)$	0 - 255
Hue	$H(r,g,b) = \arccos\frac{2b-g-r}{2V}$ $where \quad V = \sqrt{(r^2+g^2+b^2)-(rg+rb+gb)}$	0 – 2π or 0 - 360 deg
Saturation	$S(r,g,b) = \arccos\frac{r+g+b}{\sqrt{3(r^2+g^2+b^2)}}$	0 - 1

$$\boxed{S(r,g,b) = \frac{Max(r,g,b) - Min(r,g,b)}{Max(r,g,b)}}$$

The greater difference between min & max, the more saturated the colour

The last formula for saturation implies that a colour vector reaches **full saturation if at least one of its *r, g* and *b* components is equal to 0**, while not all of them are 0.

For instance, colour P(r, g, b) = (255, 0, 0) is pure red with full saturation and P(r, g, b)=(0, 200, 150) is a greenish cyan with full saturation.



So, what is the HSI-RGB transformation?

Given intensity I, hue angle α and saturation angle φ, we can also derive the HSI-RGB transformation, trigonometrically, using the same 3D polar geometry depicted in the colour cube:

$$B(\alpha, \varphi, I) = \frac{I}{\sqrt{3}}(1 + \sqrt{2}\tan\phi\cos\alpha)$$

$$G(\alpha, \varphi, I) = \frac{I}{\sqrt{3}}[1 - \sqrt{2}\tan\phi\cos(\frac{\pi}{3} + \alpha)]$$

$$R(\alpha, \varphi, I) = \frac{I}{\sqrt{3}}[1 + \sqrt{2}\tan\phi\cos(\frac{2\pi}{3} + \alpha)]$$

…you do not need to remember these formulae only why/how this is transformation useful?

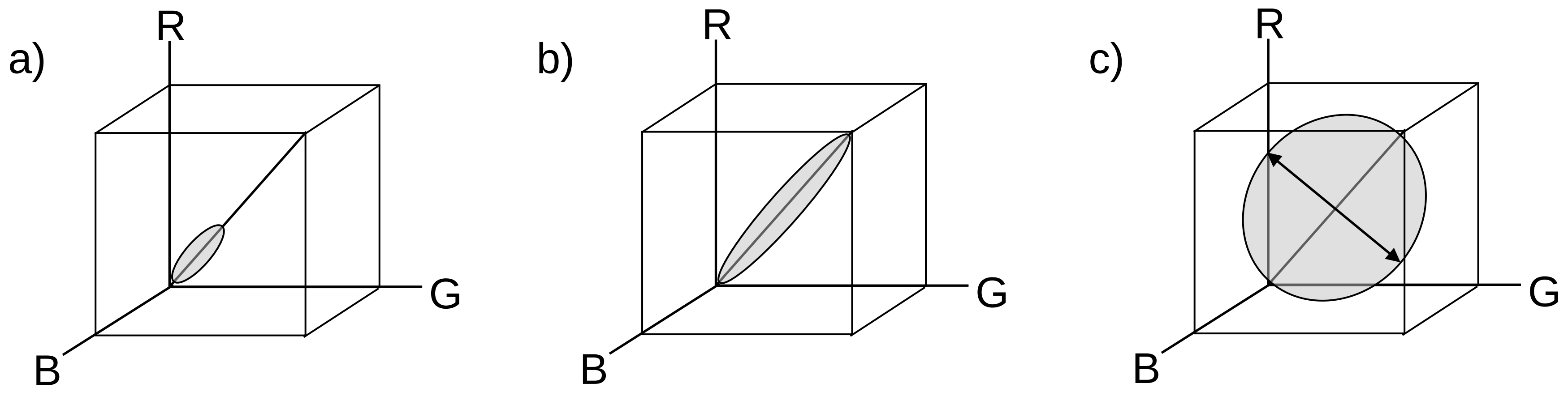
3A.2 Spectral enhancement via **Decorrelation Stretch**

High correlation generally exists among spectral bands of all multi-spectral images.

a) So the original (raw) image band information (displayed in RGB) forms a slim cluster along the grey line and occupies only a very small part of the colour cube space (a)

b) Contrast enhancement on individual bands can elongate the cluster in the colour cube but cannot increase the volume of the cluster because such a stretch changes only the intensity and only makes the cluster longer (as in b).

c) To increase the volume of data cluster in the colour cube, it should be expanded in both directions (along and perpendicular to the grey line). This is equivalent to stretching both I and S components (as in c).

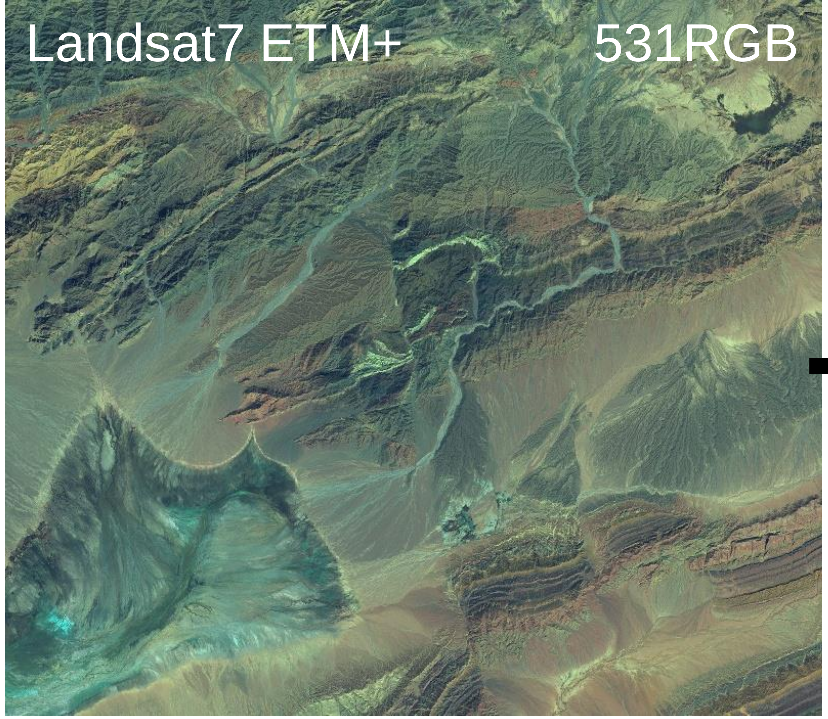


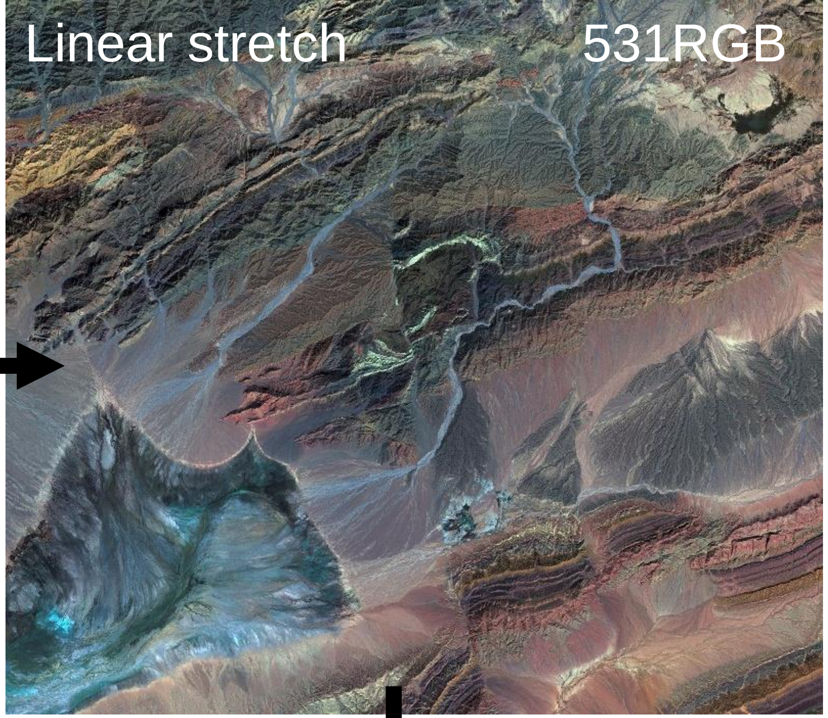
a) HSI based Decorrelation Stretch (HSIDS)

The HISDS technique involves three steps (preceded by linear stretch or scaling):

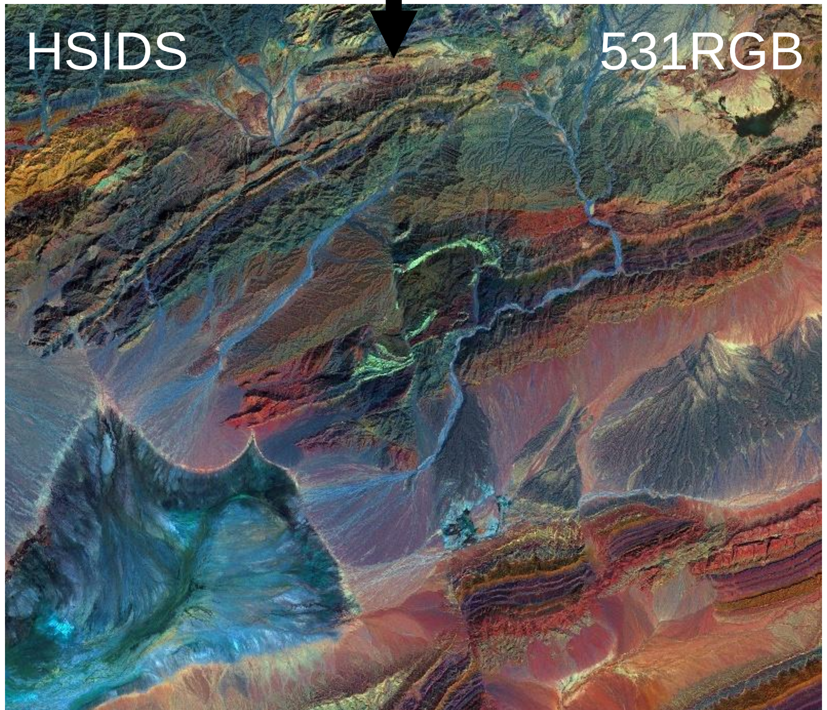
1. Scaling/Linear stretch/clipping
2. Transformation from RGB >> HSI
3. **Saturation component stretching**
4. Transformation from HSI >> RGB

- In the third step, Hue could be also stretched….
  - **But**, in the inverse transform (back to RGB display), the output colours may not be the same as those in the original image = spectral properties will have been changed!
  - This ambiguity can make image interpretation very difficult.
- The limited H range of a colour composite image is mainly caused by colour bias.
  - If the average brightness of one band is significantly higher than those of the other two bands, the colour composite will have an obvious colour ‘cast’ of the primary colour assigned to the band of highest brightness.
  - It is therefore better to use linear stretch to remove inter-band colour bias and thus to increase the H variation of a colour composite.
- The **wider H value range achieved via linear stretch** maximises the spectral information shown in H (rather than in S and I) this is fundamentally different from achieving a wide H range by stretching the H component itself.





Stretched & colour bias removed



‘Decorrelation stretched’ & colours enhanced

- HSIDS **enhances the colour saturation** of a colour composite image and thus effectively improves the visual quality on image spectral information **without significant distortion of image spectral characteristics**.
- The resulting images are therefore easy to understand and interpret.
- Statistically, the processing **reduces the inter band correlation** as shown in the table below.

The correlation coefficients before and after decorrelation stretch of the Landsat-7 ETM+ Bands 5, 3 and 1 RGB colour composite shown in the previous slide.

Correlation Matrix **before** HSIDS			
	Band 1	Band 3	Band 5
Band 1	1.00	0.945	0.760
Band 3	0.945	1.00	0.881
Band 5	0.760	0.881	1.00

Correlation Matrix **after** HSIDS			
	Band 1	Band 3	Band 5
Band 1	1.00	**0.842**	**0.390**
Band 3	0.842	1.00	**0.695**
Band 5	0.390	0.695	1.00

Bands 1, 2 & 3 are less well correlated after

b) Direct Decorrelation Stretch technique (DDS)

- Performs a saturation stretch without using RGB-HSI and HSI-RGB transformations.
- DDS achieves the same effect as the HSIDS but involves **only simple arithmetic operations** and can be **controlled quantitatively**.
- DDS is much faster, more flexible and more effective than HSIDS

  ……………..Sounds good but how does it work?
- A colour vector P and the grey line define a plane or a slice of the RGB cube. If we remove this slice (as shown in next figure), the grey line, full saturation line and maximum intensity line form a triangle that includes all the colours with the same hue but various intensities and saturations.
- P is between the grey (achromatic) line and the maximum saturation (chromatic) line and it can be considered as a sum of two vectors: vector ***a*** representing the **achromatic (zero saturation)** component or the white light in the colour, and vector ***c*** representing **chromatic (full saturation)** component that is relevant to the pure colour of the hue.

Given $\mathbf{P} = (r, g, b)$ Let $a = Min(r, g, b)$

Then $\mathbf{a} = (a, a, a)$

$$\mathbf{c} = (r - a, g - a, b - a)$$

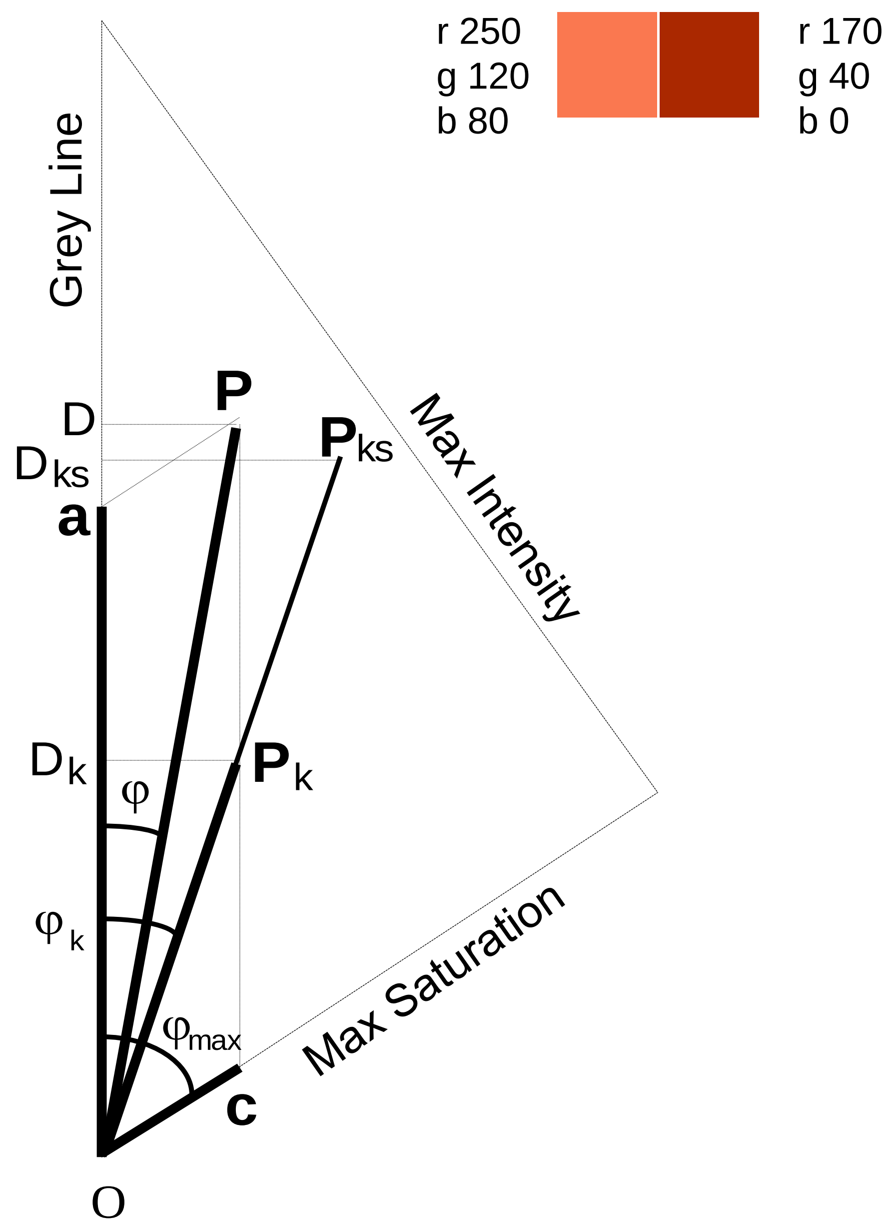
$$= \mathbf{P} - \mathbf{a}$$

Or $\mathbf{P} = \mathbf{a} + \mathbf{c}$

A direct decorrelation stretch is achieved by reducing the achromatic component (a) of the colour vector P, as defined below:

$$\mathbf{P}_k = \mathbf{P} - k\mathbf{a}$$

where k is an achromatic factor and $0<k<1$.



Liu & Moore, 1996. DDS technique for RGB colour composition, IJRS, 17, 5, p1005

DDS cont'd

- So this operation shifts the colour vector $\mathbf{P}$ away from the achromatic line to form a new colour vector $\mathbf{P}_k$ with increased saturation ($\varphi_k > \varphi$) but with decreased intensity ($OD_k < OD$).
- To restore the intensity to a desired level, linear stretch can then be applied to each image in red, green and blue layers. This will elongate P_k to P_{ks} , with the same hue and saturation as P_k but with increased intensity ($OD_{ks} > OD_k$).
- DDS operation does not affect hue since it only reduces the achromatism of the colour and leaves the hue (represented by c) unchanged. This is easy to understand if we re-write the DDS formula as:

$$\mathbf{P}_k = \mathbf{P} - k\mathbf{a} = \mathbf{c} + \mathbf{a} - k\mathbf{a} = \mathbf{c} + (1-k)\mathbf{a}$$

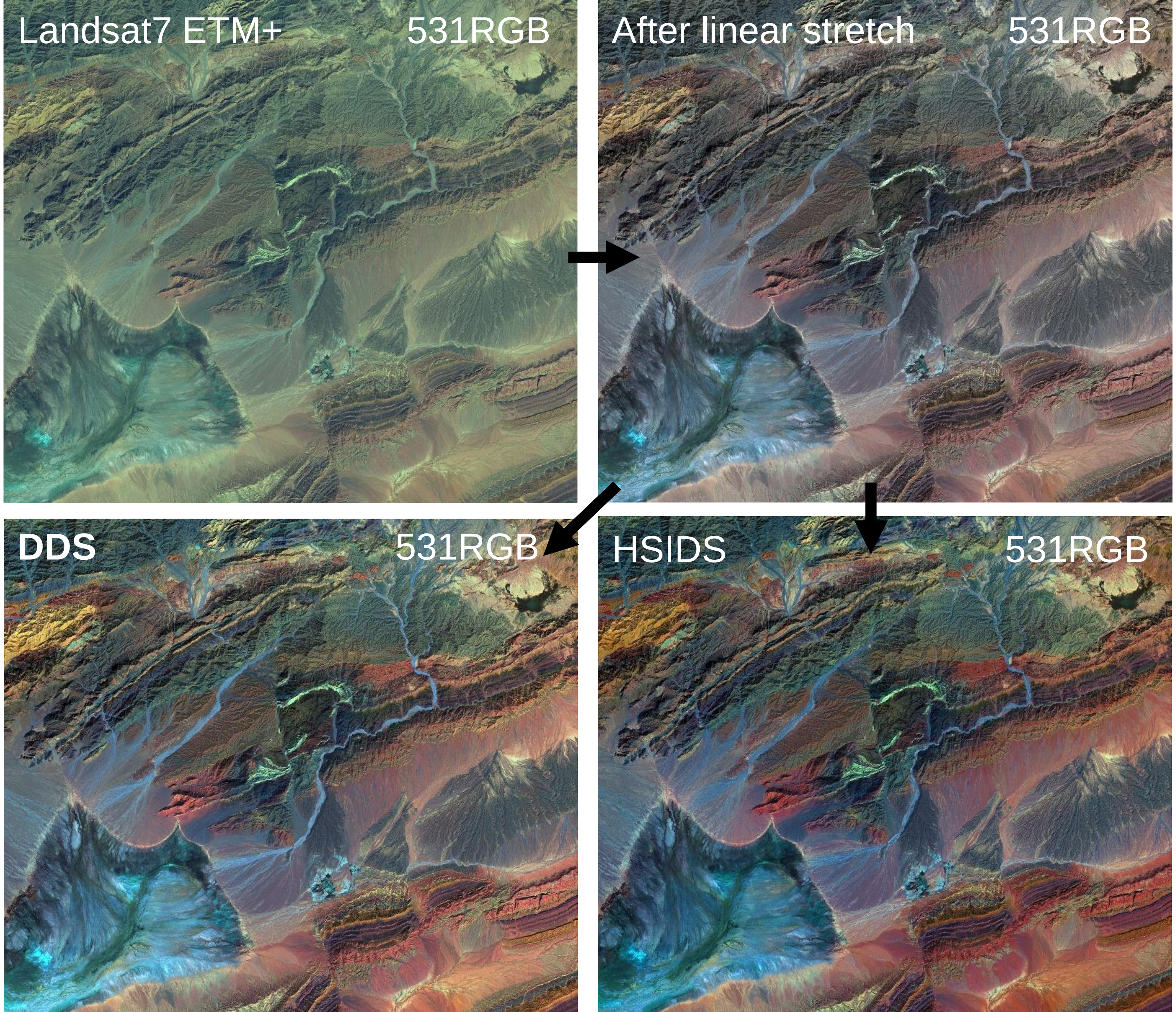
- The numerical operations for the DDS are:

$$r_k = r - ka = \boldsymbol{r} - \boldsymbol{k}\,\mathbf{min}(\boldsymbol{r},\boldsymbol{g},\boldsymbol{b})$$
$$g_k = g - ka = \boldsymbol{g} - \boldsymbol{k}\,\mathbf{min}(\boldsymbol{r},\boldsymbol{g},\boldsymbol{b})$$
$$b_k = b - ka = \boldsymbol{b} - \boldsymbol{k}\,\mathbf{min}(\boldsymbol{r},\boldsymbol{g},\boldsymbol{b})$$

The final colour saturation of the output image is controlled (scaled) by the achromatic factor ***k***.

Original

Stretched &
colour bias
removed



Direct
'Decorrelation
stretched'
(DDS) &
colours
enhanced

'Decorrelation
stretched' &
colours
enhanced

DDS cont'd

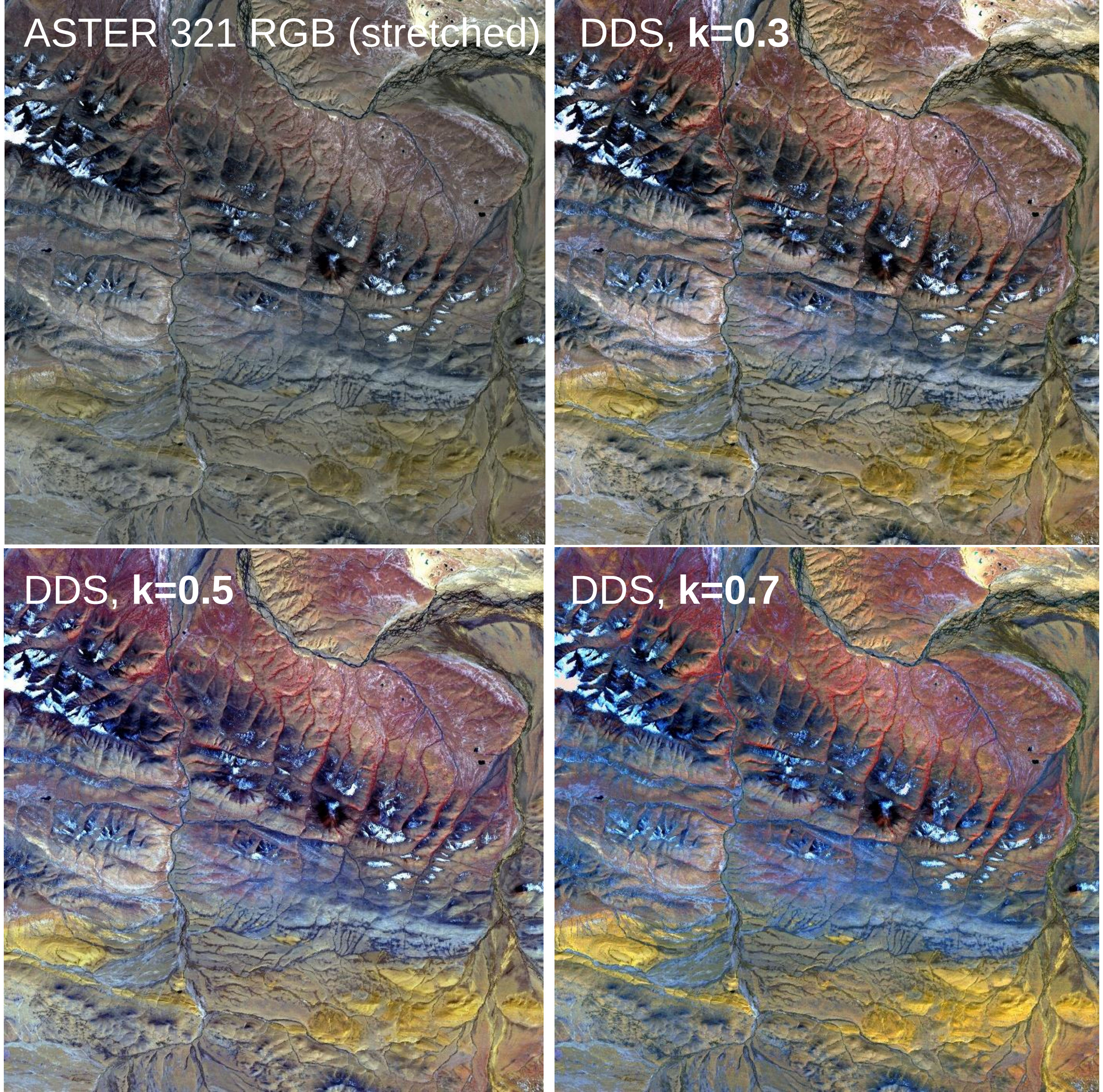
- The final colour saturation of the output image is controlled (scaled) by the achromatic factor *k*.
- As already stated in the HSIDS, the 3 bands for colour composition must be well stretched (e.g. BCET or linear stretch with appropriate clipping) <u>before the DDS</u> is applied.
- In DDS, the scaling value *k* can be specified by the user. It should be based on the saturation level of the original colour composite. The lower the saturation of an image is, the greater the *k* value should be (within the range of *0 - 1*)
- *k = 0.5* is generally good for most cases.
- Let's compare an initial BCET colour composite with the DDS colour composites using *k* = 0.3, *k* = 0.5 and *k* = 0.7.

Effect of varying k

All DDS images show increased saturation without distortion of hue, in comparison with the BCET colour composite

Their saturation and intensity increases with the size of k.

The merits of simplicity and quantitative control of DDS are obvious



3A.3 **Data fusion techniques**

e.g. For resolution fusion or ‘**Pan-sharpening**’ (& for fusing multi-source data)

Can be achieved by fusing a low- and a high-resolution image of the same area (*e.g.* a Landsat 30 m with a SPOT panchromatic 10 m)

Low resolution CC (30m) | High resolution Pan (15m) | Pan-sharpened CC (15m)

The resultant fused (pan-sharpened) image is a mixture of **low-resolution spectral information** (from the low-resolution colour composite (cc)) and **high-resolution spatial and textural detail** (from the Panchromatic image).

There are several commonly used methods – some are better than others..

a) Intensity substitution for pan-sharpening

Resolution fusion (or ‘Pan-sharpening’) achieved via the following steps:

1. Geo- or co-reference a low- and a high-resolution image of the same area (*e.g.* a Landsat 30 m with a SPOT panchromatic 10 m)
2. RGB-HSI transformation on the low-res colour composite image
3. **Replace low-res Intensity component with high-res image**
4. HSI-RGB transformation for display

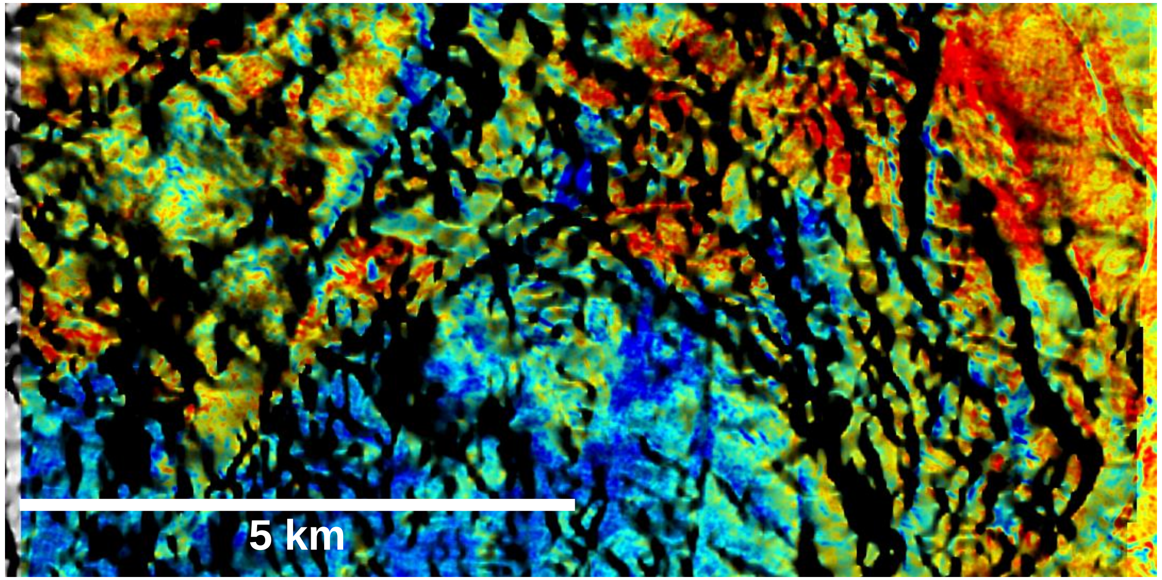
- The fused or pan-sharpened image is a mixture of low-resolution spectral information and high-resolution spatial and textural detail
- Overall resolution is apparently improved but colour distortion is unavoidably introduced.
- The **colour distortion may be significant if the spectral range of the three MS colour composite bands is very different** from that of the Panchromatic band, i.e. the intensity component (as the summation of the three MS bands) is different from the Pan that is used for intensity replacement. NB look carefully at band widths first

b) RGB-HSI multi-source data integration

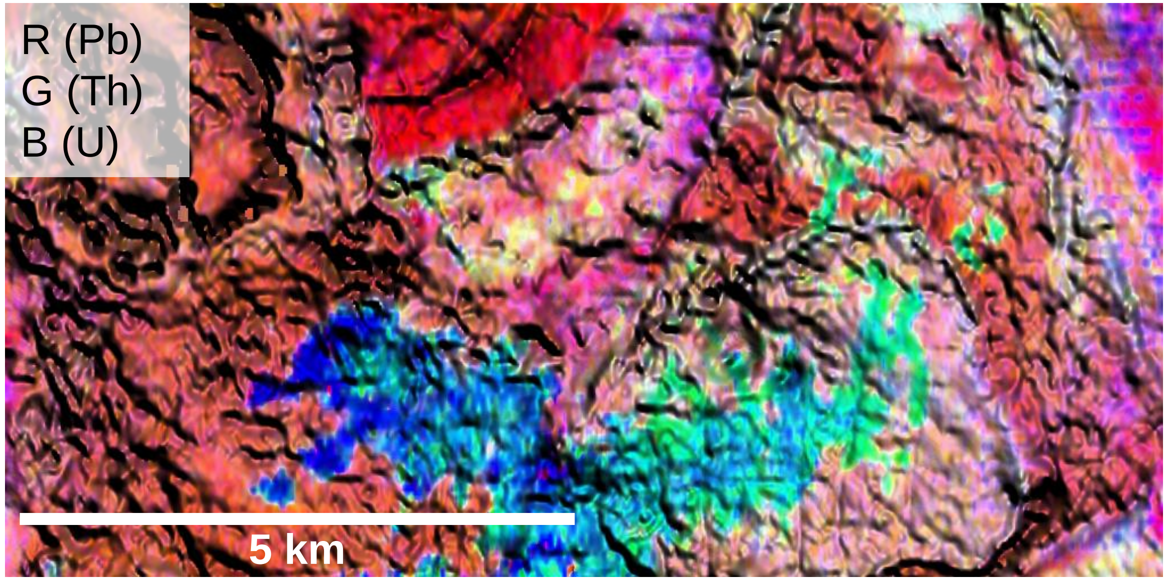
e.g. fusing low-res geophysical data with high-res DEM, Pan image or other high-res data, achieved by the following steps:

1. Geo- or co-reference the low- & high- resolution datasets to be fused
2. RGB-HSI transformation on the low-res colour composite image
3. **Replace intensity component by geophysical or geochemical dataset**
4. HSI-RGB transformation for display

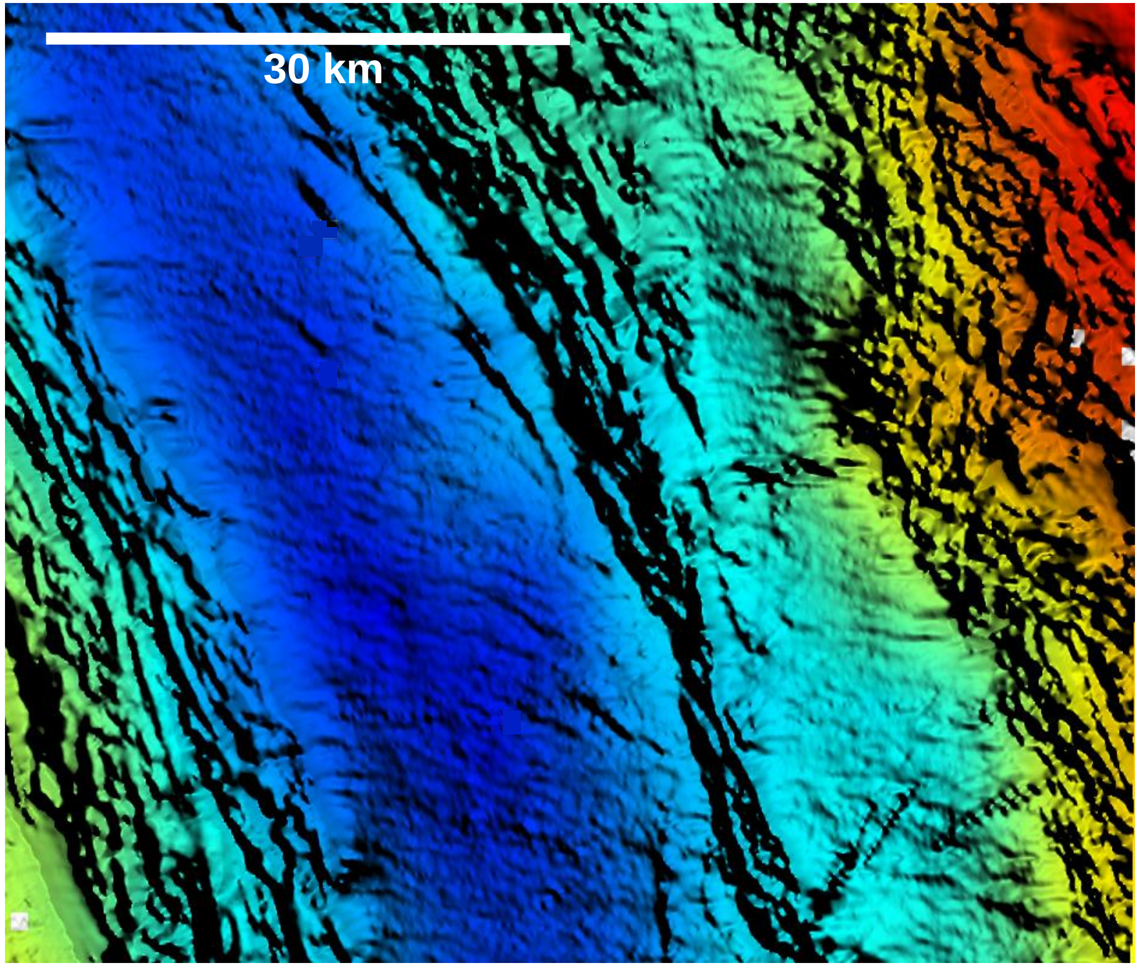
- The resultant image contains both spectral information of the original image bands and geophysical or geochemical information as intensity variation.
- The interpretation of such images needs experience on the datasets used - and may or may not be practically useful!
- A more productive method is to use the 'colour drape' technique in which a geophysical or geochemistry dataset is used as if it were a DEM (digital elevation model) with a colour composite image draped onto it in 3D perspective view.



L8 iron-oxide spectral index as pseudocolour (H and S), and magnetic susceptibility as Intensity (I)



Radiometric data (U, Th & Pb colour composite) as H and S, and magnetic susceptibility as Intensity (I)



Gravity as pseudocolour (H and S)
Magnetic susceptibility as Intensity (I)

c) Brovey transform data fusion (intensity modulation)

$$R_b = \frac{3RP}{R+G+B}$$

$$G_b = \frac{3GP}{R+G+B}$$

$$B_b = \frac{3BP}{R+G+B}$$

We can re-write the formulae on the left in a simpler format as follows:

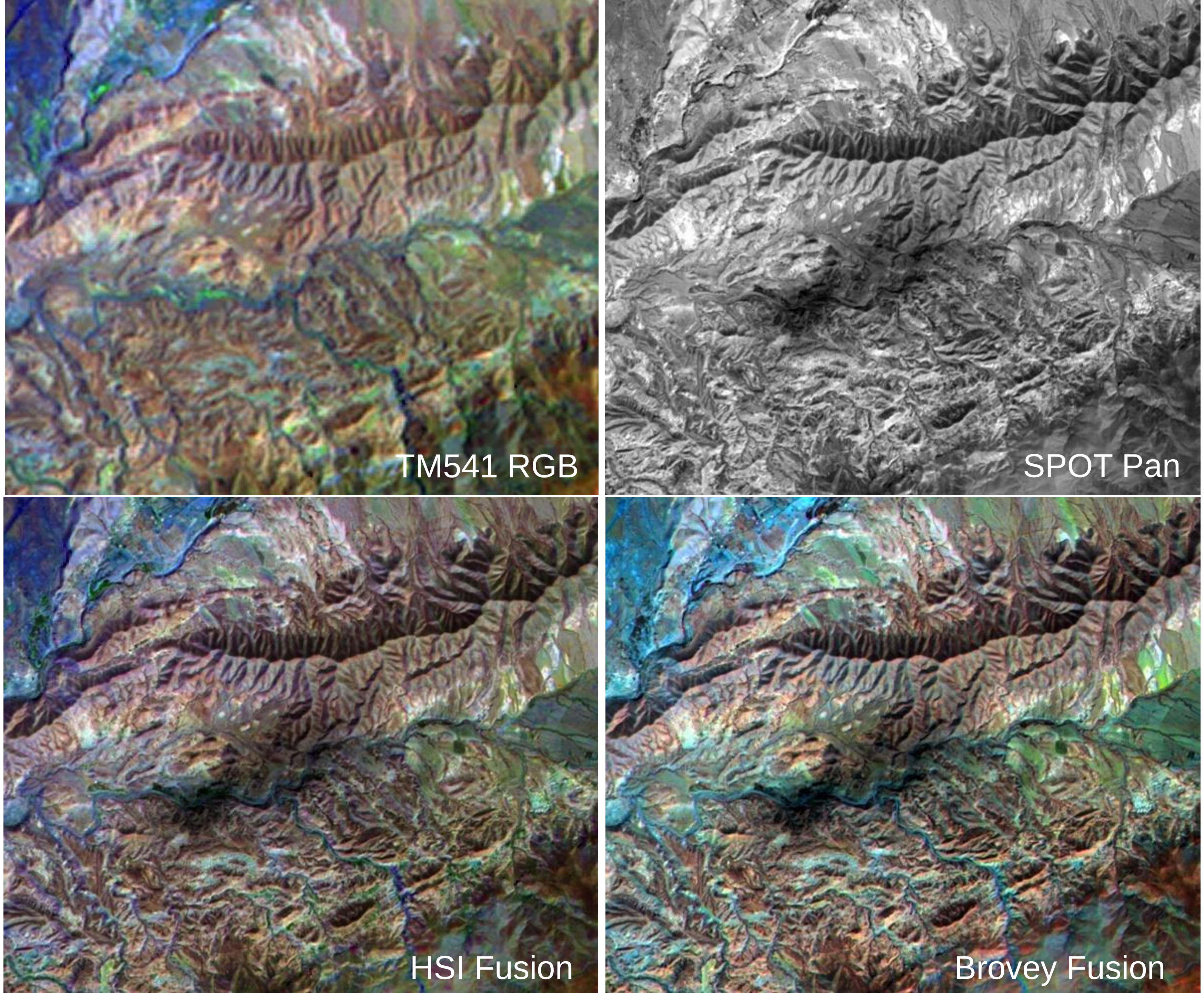
$$R_b = R \times P / I$$

$$G_b = G \times P / I$$

$$B_b = B \times P / I$$

If image *P* is a higher resolution image, then above formulae performs image fusion and improves spatial resolution, or if *P* is a raster dataset of different source, multi-source data integration is performed.

The Brovey transform achieves a <u>similar result to the HSI fusion technique but</u> without carrying out the RGB-HSI and HSI-RGB transformations and thus **simpler and faster.** However, it introduces **<u>colour distortion</u>** as well.


TM541 RGB
SPOT Pan
HSI Fusion
Brovey Fusion

A word of warning about HSI and Brovey methods

- Both HSI and Brovey transform image fusion techniques can cause colour distortion **if the spectral range of the intensity replacement (or modulation) image is different from the spectral range covered by the three bands used in a colour composite.**
  a) Inevitable in colour composites that do not use consecutive spectral bands.
  b) Can be severe in vegetated areas if the images are acquired in different growth seasons.
- **Preserving the original spectral properties is very important** for applications based on spectral signatures, such as lithology, soil and vegetation.
- The spectral distortion introduced by these fusion techniques is uncontrolled and unquantified because the images for fusion are often taken by different sensor systems, on different dates, or in different seasons.
  – Such fusion cannot really be regarded as spectral '*enhancement*' and should be avoided to prevent unreliable interpretation.

d) Spectral-conservation data fusion technique:

Smoothing Filter based Intensity Modulation (SFIM)

The **spectral distortion problem is only avoided if colour composites that use consecutive spectral bands which occupy the same spectral range as the panchromatic band used for the fusion**

- Limits their use to pan-sharpening and one band combination, e.g. True Colour or Standard False Colour composites (depending on the spectral range of the Panchromatic band)

However, the **Smoothing Filter based Intensity Modulation** (SFIM) technique (Liu, 1999) is a genuine spectral-conservation image fusion technique which is applicable to co-registered multi-resolution images.

How does it work?

Liu 2000. SFIM: a spectral preserve image fusion technique for improving spatial details, IJRS, 21, 18

The DN value of a daytime optical image of reflective spectral band λ is mainly decided by two factors: the solar radiation impinging on the land surface or irradiance $E(\lambda)$, and the spectral reflectance of the land surface $\rho(\lambda)$… $DN(\lambda) = \rho(\lambda)E(\lambda)$

Low resolution image: $DN(\lambda)_{low} = \rho(\lambda)_{low}E(\lambda)_{low}$

High resolution image: $DN(\gamma)_{high} = \rho(\gamma)_{high}E(\gamma)_{high}$

SFIM formula:
$$DN(\lambda)_{SFIM} = \frac{DN(\lambda)_{low} \times DN(\gamma)_{high}}{DN(\gamma)_{mean}}$$

$$= \frac{\rho(\lambda)_{low}\cancel{E(\lambda)_{low}} \times \cancel{\rho(\gamma)_{high}}E(\gamma)_{high}}{\cancel{\rho(\gamma)_{low}}\cancel{E(\gamma)_{low}}}$$

$$\approx \rho(\lambda)_{low}E(\lambda)_{high}$$

where $DN(\lambda)_{SFIM}$ is the fused higher resolution pixel corresponding to $DN(\lambda)_{low}$ and $DN(\gamma)_{mean}$, which is the **local mean of** $DN(\gamma)_{high}$ **over a neighbourhood equivalent to the resolution of** $DN(\lambda)_{low}$

General form of the SFIM algorithm is:

$$IMAGE_{SFIM} = \frac{IMAGE_{low} \times IMAGE_{high}}{IMAGE_{mean}}$$

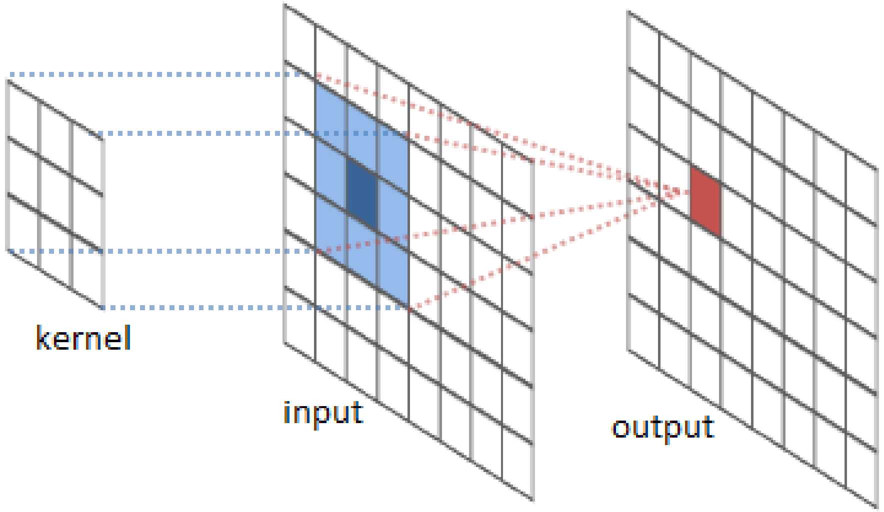
** For deriving the mean over a local neighbourhood – look-up convolution theory and spatial filtering using a kernel window (also RS/Image_Filtering.html

- The ratio between $IMAGE_{high}$ and $IMAGE_{mean}$ in the formula cancels the spectral and topographical contrast of the high resolution image but **retains the high resolution textures**.

- The **SFIM image** of $DN(\lambda)_{SFIM}$ is a product of the **higher resolution spatial texture** introduced from the higher resolution image, $E(\gamma)_{high}$, **and the lower resolution spectral reflectance** of the original lower resolution image, $\rho(\lambda)_{low}$. It is therefore **independent of the spectral properties of the high resolution image** used for intensity modulation.

- SFIM can thus be understood as *a low resolution image directly modulated by high resolution textures* and the result is independent of the contrast and spectral variation of the higher resolution image.

Image convolution or spatial filtering



Filtering is a **neighbourhood operation** - a DN value in the output image is determined by applying an expression to the input pixel DN values within a neighbourhood around that pixel position. A concept also known as ‘box filtering’ in the spatial domain

Filtering can also be done by Fourier Transform (FT) in the frequency domain

Examples include **Smoothing** (moving average filter) and **Sharpening & edge detection**

Smoothing: The neighbourhood window (or **convolution kernel**) moves across the image, calculating the average of the input DN values within the window around the central pixel. The average DN value is then assigned to the central pixel in the output image.

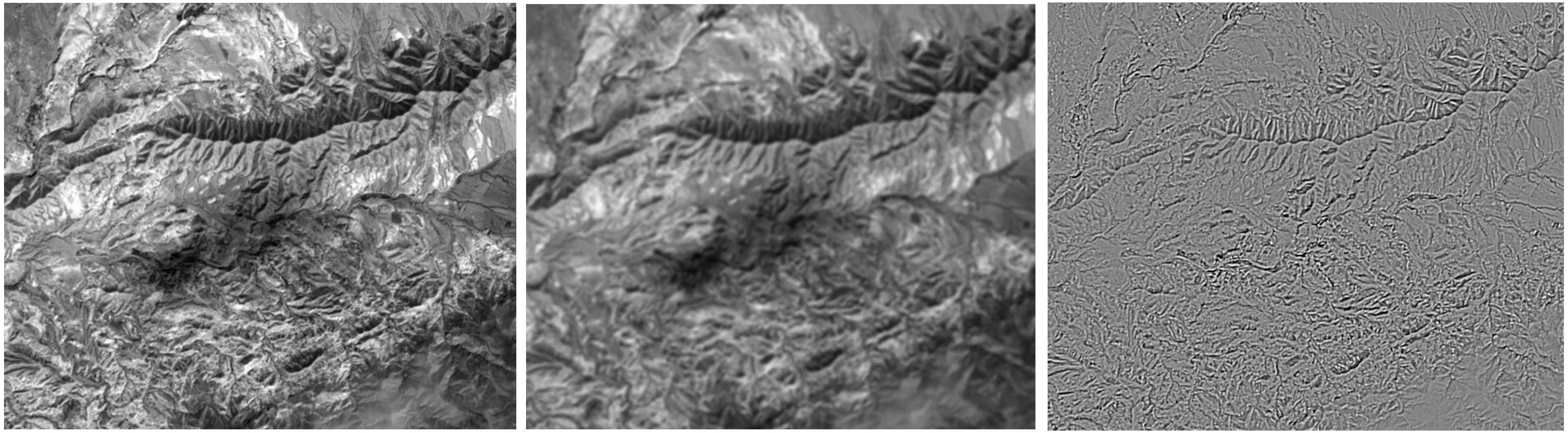
The **larger the kernel, the greater the effect**, e.g. 3x3, 5x5, 9x9 etc

0	0	0						
0	199	192	158	111	110	123	130	130
0	189	149	108	111	113	120	126	125
	130	100	98	108	113	113	114	120
	85	100	96	104	108	107	101	94
	85	95	98	96	100	103	100	96
	79	94	87	77	69	70	87	84
	77	80	72	71	60	52	59	64
	68	67	63	58	53	51	54	52

Output pixel x is the weighted sum of n neighbouring input pixels. The matrix of weights is called the **convolution kernel**, e.g. below a 3×3 kernel used for a moving average filter:

$$K = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

https://primer-computational-mathematics.github.io/book/d_geosciences/Remote%20Sensing/Image_Filtering.html



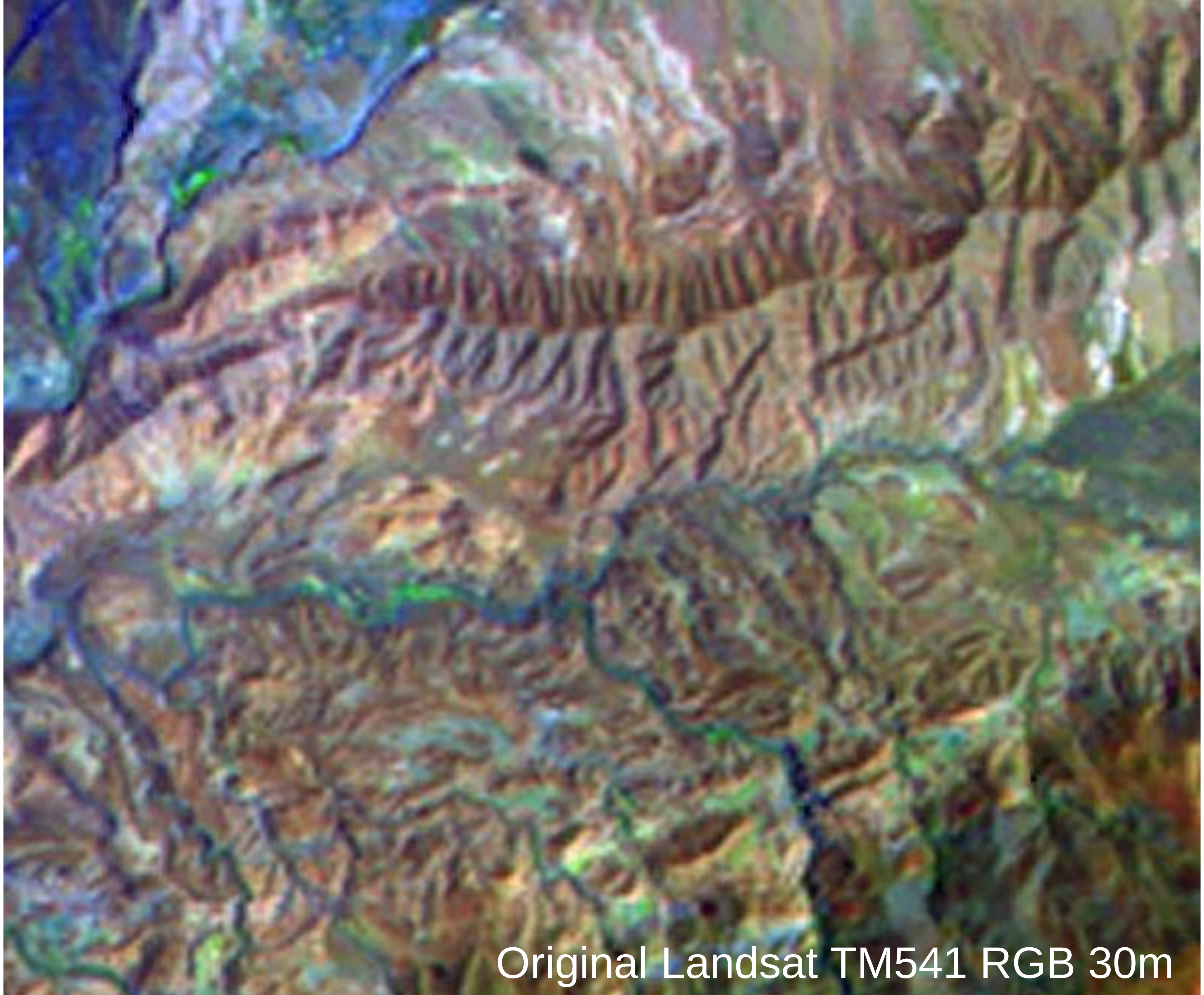
(a) (b) (c)

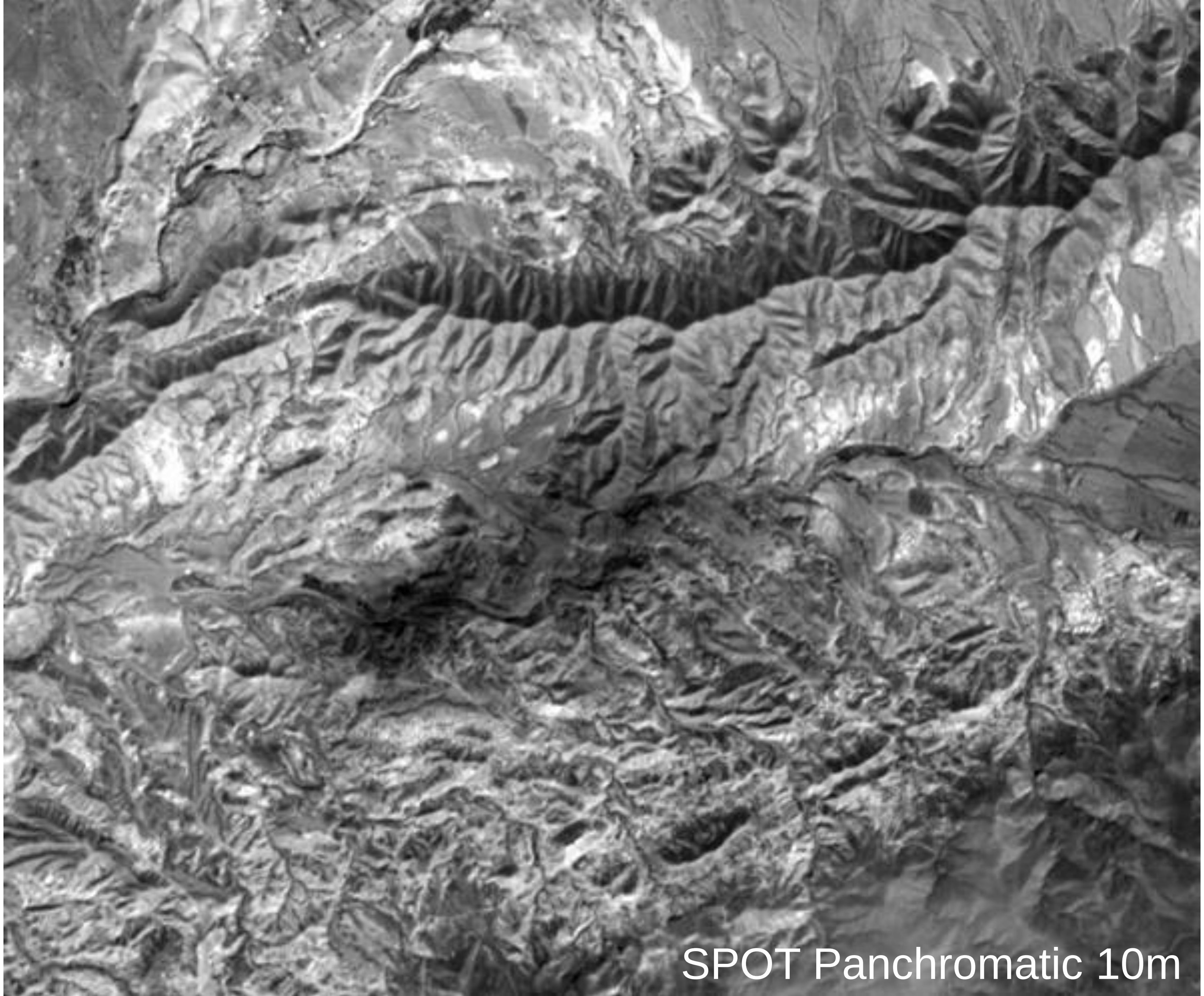
(a) Original SPOT Pan image ($Image_{High}$)

(b) Smoothed SPOT Pan image with a 5×5 smoothing filter ($Image_{Mean}$)

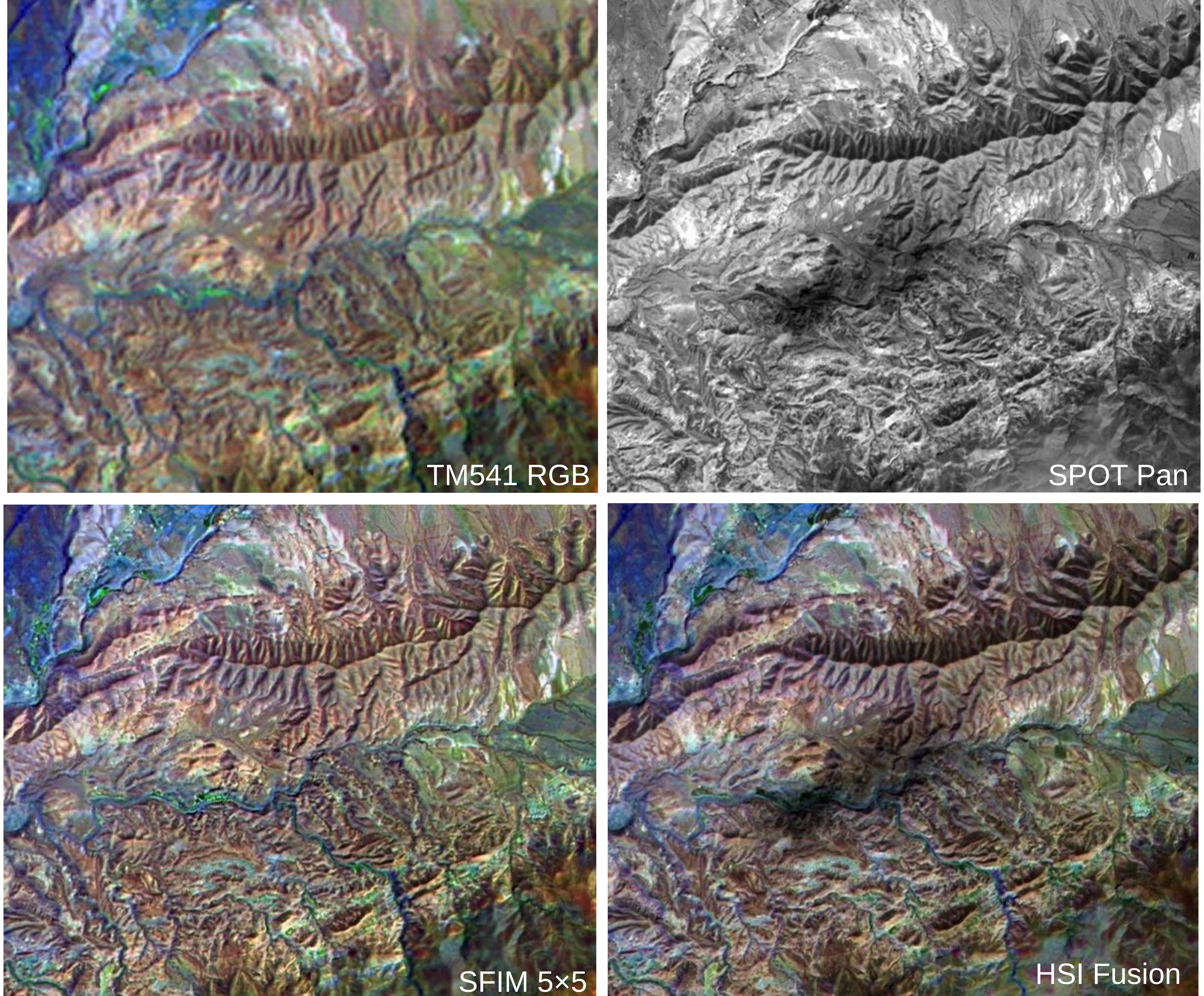
(c) The ratio image between (a) and (b) (= high-res textural information)

- SFIM can thus be understood as *a **low resolution image directly modulated by high resolution textures*** and the result is independent of the contrast, shadows and spectral variation of the higher resolution image.
- SFIM therefore honours the spectral properties as well as contrast of the original low resolution image


Original Landsat TM541 RGB 30m

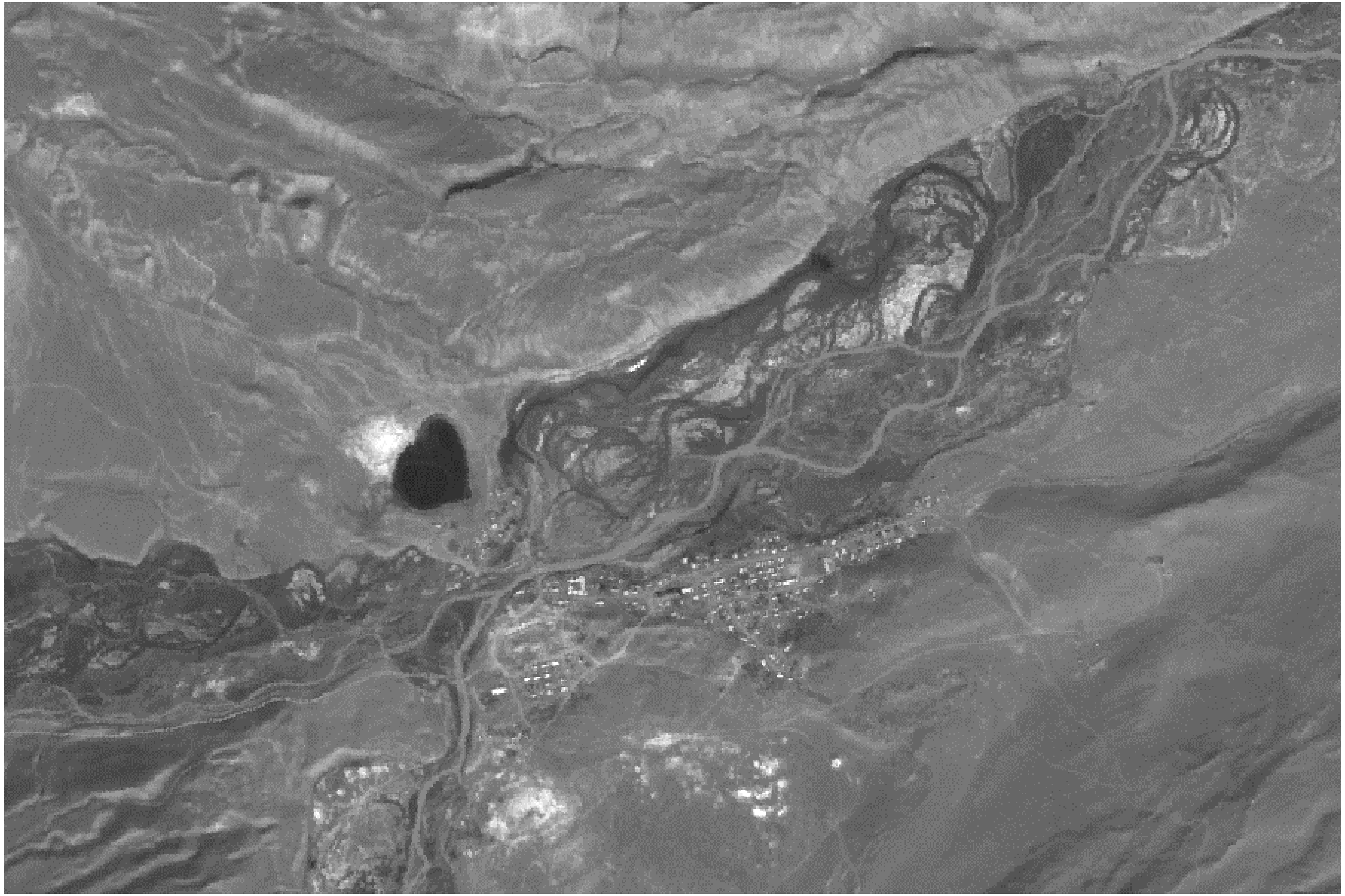

SPOT Panchromatic 10m


SFIM 3x3


TM541 RGB
SPOT Pan
SFIM 5×5
HSI Fusion

Landsat 8 OLI 432 RGB 30m Resolution

SPOT 5 Pan 5m Resolution



Landsat 8 OLI 432 RGB – SPOT 5 Pan Fusion Image

Summary of the SFIM

- For SFIM operations, the lower resolution image **must be interpolated to the same pixel size** as the higher resolution image by bilinear or cubic interpolation in the co-registration process.
- The advantage of the SFIM over the HSI and Brovey transform fusion technique is that it improves spatial details with the added benefit of **preserving image spectral properties** and contrast.
- However, The SFIM is **more sensitive to image co-registration accuracy** than the HSI and Brovey transform.
  - Inaccurate co-registration may cause blurring to edges in the fused images.
  - This problem can be alleviated using a smoothing filter with a kernel larger than the resolution ratio between the higher and lower resolution images. For instance, for TM (30m) and SPOT Pan (5m) fusion 5×5 and 7×7 filters will produce better results than a 3×3 filter.
  - In ESE we have developed an advanced pixel-wise image co-registration algorithm that resolves that problem.

3A.3 Colour coordinate transformation summary

- Colour quality can be described in Hue, Saturation and Intensity,
- The RGB-HSI transformation and the inverse transformation HSI-RGB can be derived based on either 3D geometry (or matrix operations) for coordinates rotation of the RGB colour cube.
- The RGB-HSI and HSI-RGB transformations allow manipulation of colour intensity, hue and saturation components separately with great flexibility.
- One major application is the saturation stretch based HSIDS technique that **enhance image colour saturation without altering the hues of the colours**.
- DDS is a clever short-cut based on colour vector decomposition in achromatic and chromatic components, the **DDS performs saturation stretch directly** in RGB domain without involving the RGB-HSI and HSI-RGB transformations.
- Another application of RGB-HSI and HSI-RGB transformations is **data fusion to improve image spatial resolution** and **integrate raster datasets** from different sources via intensity replacement.
- The Brovey transform is an intensity modulation based technique achieving similar data fusion results though the principle is different.
- The SFIM is a truly **spectral preservation pan-sharpening** technique which requires precise image co-registration.

3A.4 Revision Questions

1. Using a diagram of RGB colour cube to explain the mathematic definition and physical meaning of intensity, hue and saturation.
2. What are the value ranges of intensity, hue and saturation according to the RGB colour cube model of RGB-HSI transformation?
3. Why RGB-HSI is a useful image processing technique?
4. Describe the principle of decorrelation stretch (DS) with the aid of diagrams.
5. Describe the major steps of HSI DS and explain how the image inter-band correlation is reduced and why.
6. What is the drawback of stretching the hue component in the HSI decorrelation stretch? How can we expand the value range of the hue component without stretching the hue component directly?
7. Using a diagram to explain the principle of DDS. In what senses are DDS and HSI DS are similar, and different?
8. How to improve the spatial resolution of a 30m resolution TM colour composite with a 10m resolution SPOT panchromatic image using RGB-HSI transformation and Brovey transform?
9. Explain the major problem of image fusion using RGB-HSI transformation and Brovey transform.
10. How does SFIM achieve pan-sharpening which preserves spectral properties?

3B. Principal Component Analysis (PCA)

- The Principal Component Analysis (PCA) is a general method to analyse **correlated multi-variable datasets**.
  - Remotely sensed multi-spectral imagery is a typical kind of dataset for which PCA is an effective technique for spectral enhancement and information manipulation.
- PCA is based on linear algebraic matrix operations and multi-variable statistics.
- PCA can effectively **concentrate the most important information from many correlated image spectral bands into fewer uncorrelated principal components** and therefore has a functionality of reducing the size of dataset and enables effective image RGB display of information.
- Based on the concept of PCA as a coordinate rotation, we can expand it to the general concept physical property orientated image coordinate transformation.
  - This leads to the widely used **Tasselled Cap** transformation in derivation of multi-spectral indices of brightness, greenness and wetness.

3B.1 Principles of PCA

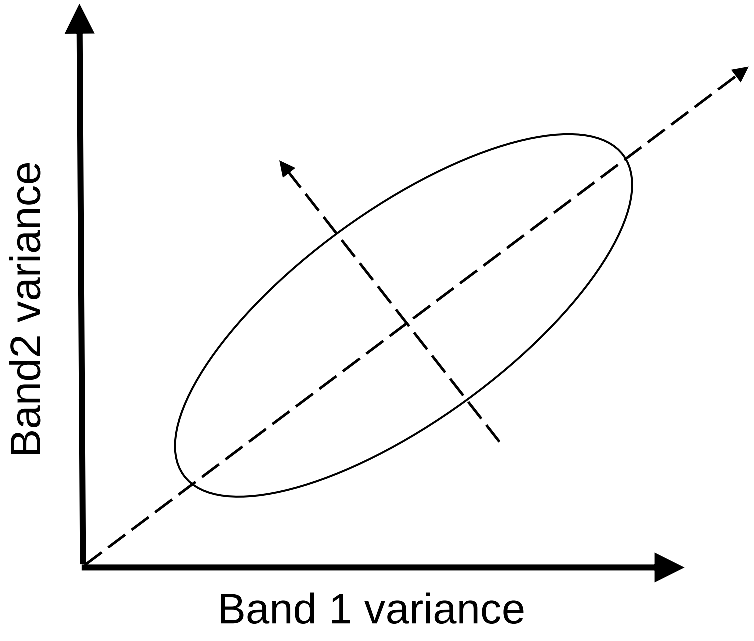
The table below shows that the six reflective spectral bands of a Landsat TM image are highly correlated.

e.g. the correlation between band 5 and 7 is 0.993. This indicates 99.3% information redundancy between these two bands with only 0.07% of unique information! As such, multi-spectral imagery is not efficient for information storage……

Correlation matrix of bands 1-5, 7 of a TM sub-scene

Correlation	TM1	TM2	TM3	TM4	TM5	TM7
TM1	1.000	0.962	0.936	0.881	0.839	0.850
TM2	0.962	1.000	0.991	0.965	0.933	0.941
TM3	0.936	0.991	1.000	0.979	0.955	0.964
TM4	0.881	0.965	0.979	1.000	0.980	0.979
TM5	0.839	0.933	0.955	0.980	1.000	0.993
TM7	0.850	0.941	0.964	0.979	0.993	1.000

Consider an $\boldsymbol{m}$ band multi-spectral image as an $\boldsymbol{m}$ dimensional raster dataset in an $\boldsymbol{m}$ dimension orthogonal coordinate system, forming an $\boldsymbol{m}$ dimensional ellipsoid cluster.

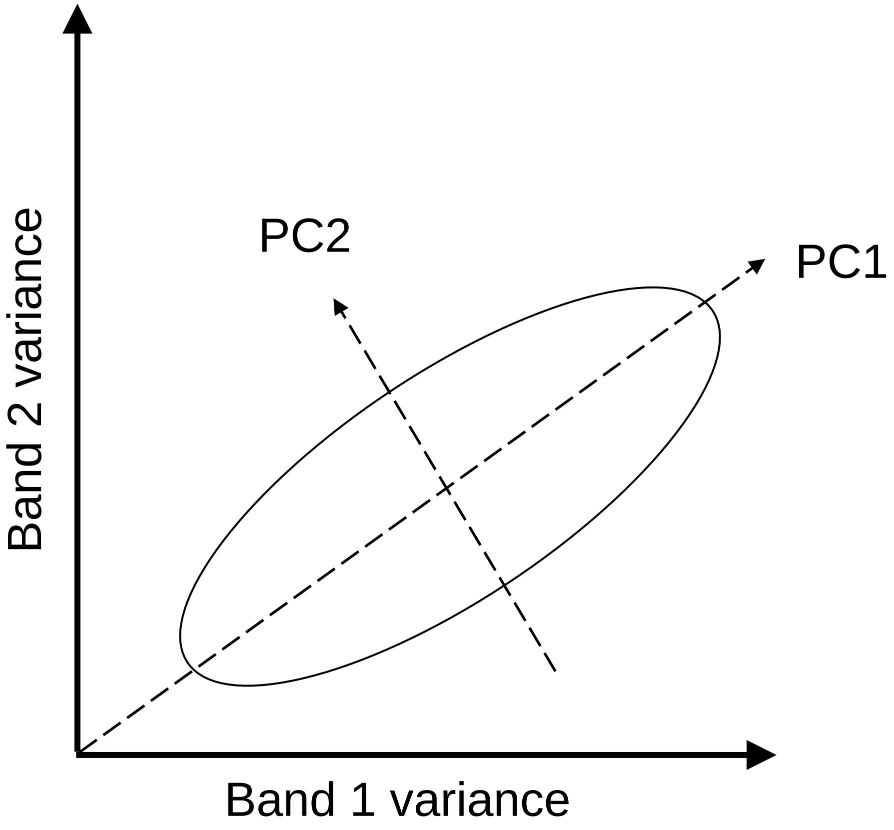


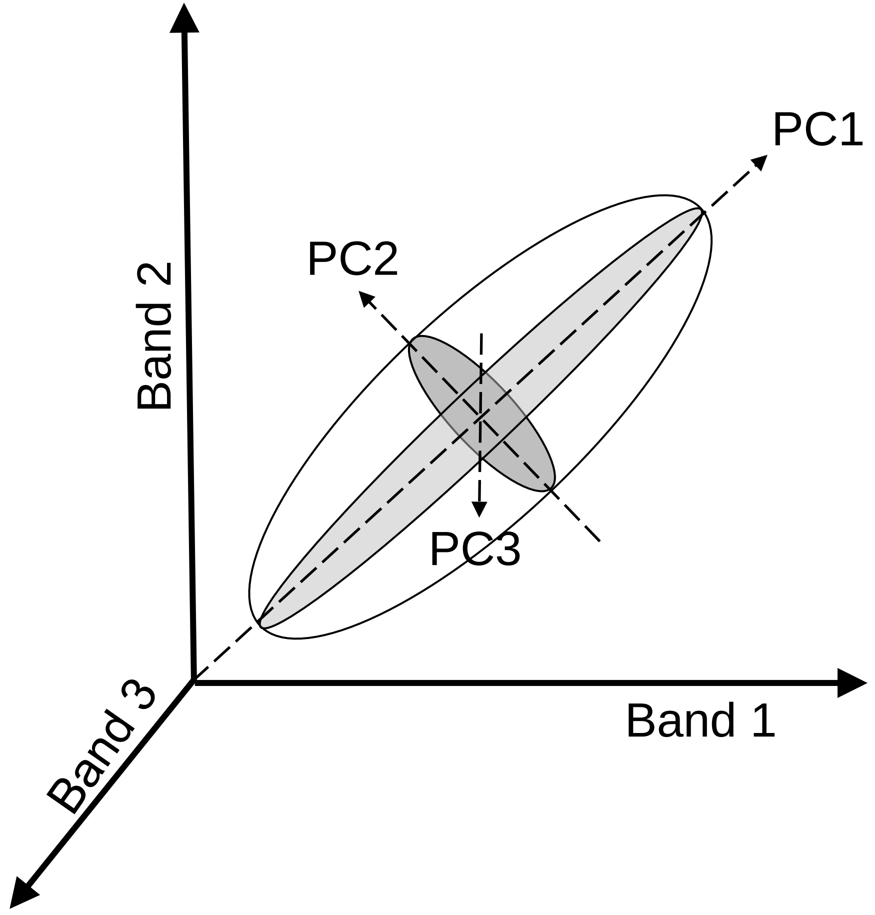
- Then the **m-band coordinate system is oblique to the ellipsoid data cluster axes**, if the $\boldsymbol{m}$-bands are correlated.
- The data ellipsoid cluster also has axes which form an orthogonal coordinate system and, in this system, the same image data are represented by $\boldsymbol{n}$ independent components (where $\boldsymbol{n}$<=$\boldsymbol{m}$) called **principal components (PCs)**. Thus, the PCs are the image data represented in the ellipsoid data cluster coordinates
- **PCA = coordinate rotational operation to transform the coordinates of the original image bands to match the ellipsoid of the image data cluster axes**.

- The 1st PC represents the longest axis of the data cluster and the 2nd PC the second longest, and so on. The axes of the high order PCs may be so short that they contain no substantial information…..then the apparent $\boldsymbol{m}$ dimensional ellipsoid is effectively degraded to $\boldsymbol{n}$ independent dimensions (where $\boldsymbol{n}$<=$\boldsymbol{m}$).
- Thus PCA reduces image dimensionality and represent nearly the same image information, with fewer independent dimensions, in a smaller dataset, and without redundancy (since the new, rotated PCs are independent).

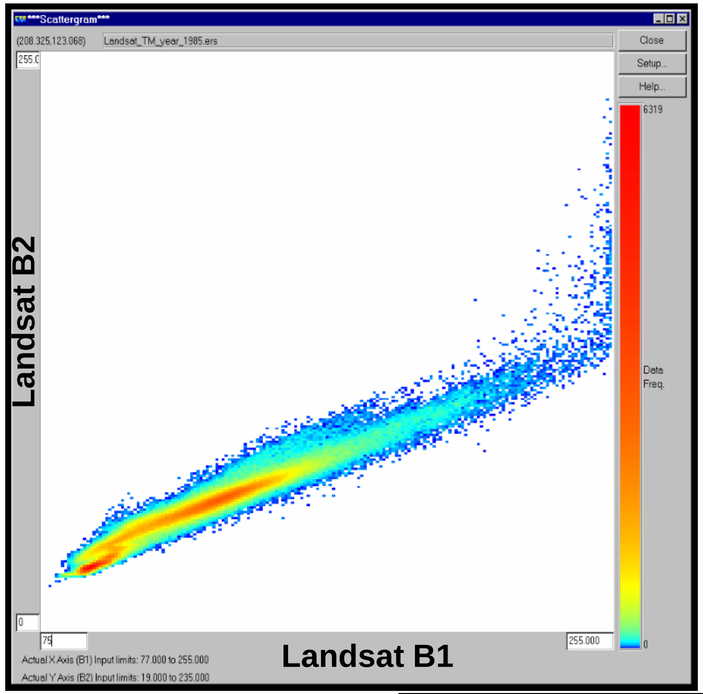
Geometric definition summarised

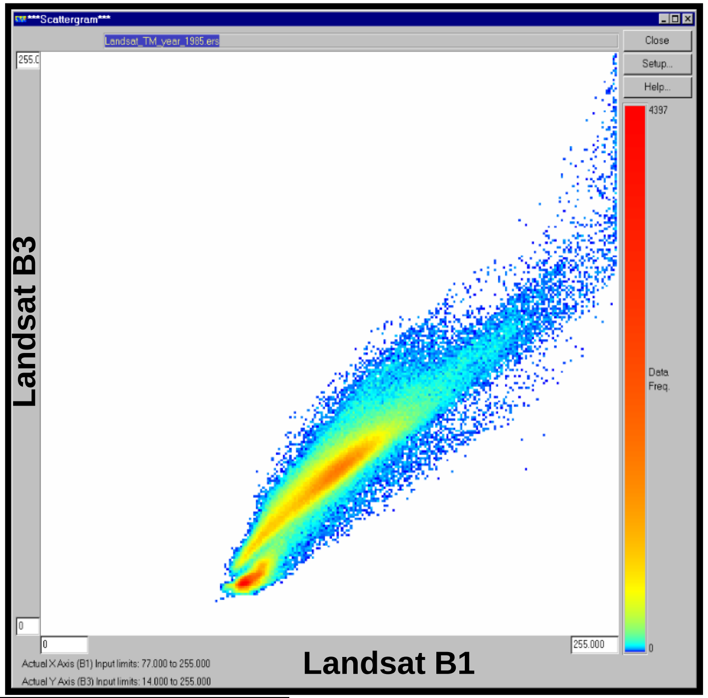
In summary, the PCA is a **linear transformation converting *m* correlated dimensions to *n* (*n*<=*m*) independent (uncorrelated) dimensions***.* This transformation is a **coordinate rotation** operation which rotates the original image band coordinate system to match the image ellipsoid cluster axes.



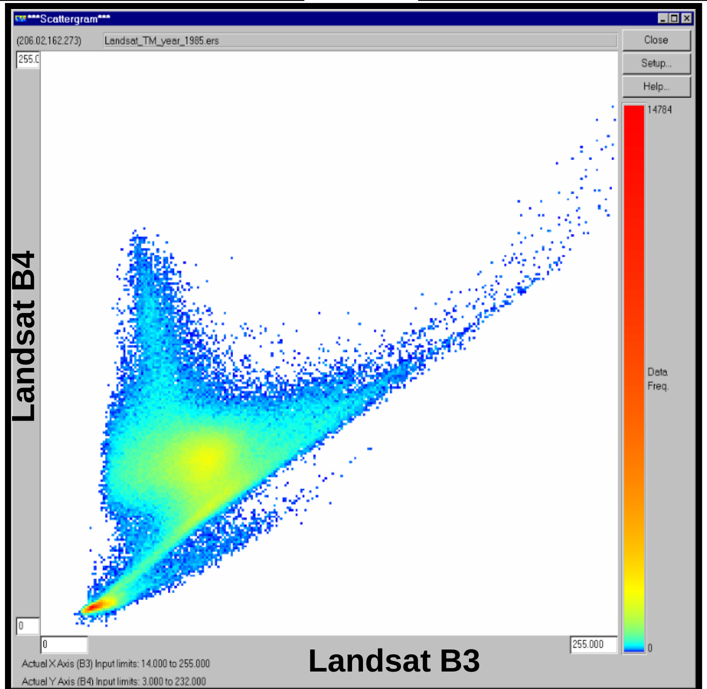


Examples of inter-band correlation, between different Landsat band pairs





The best way to visualise and understand an *m* dimensional cluster is via its **covariance matrix**



Let $\mathbf{X}$ represent an m band multi-spectral image, its **covariance matrix** Σ_x **,** is the full representation of the m dimensional ellipsoid cluster of the multi-band image.

$$\Sigma_x = \varepsilon\{(\mathbf{x}-\mathbf{m}_x)(\mathbf{x}-\mathbf{m}_x)^t\} \approx \frac{1}{N-1}\sum_{j=1}^{N}(\mathbf{x}_j-\mathbf{m}_x)(\mathbf{x}_j-\mathbf{m}_x)^t$$

Where the sample $\mathbf{x}_j = (x_{j1}, x_{j2}, \cdots x_{jm})^t$ $(\mathbf{x}_j \in \mathbf{x}, j = 1, 2, \ldots, N)$ is any $\mathbf{m}$ dimensional pixel vector of an $\mathbf{m}$ bands image $\mathbf{X}$, N is the total number of pixels and $\mathbf{m}_x$ the mean vector of the image

$$\mathbf{m}_x = \varepsilon\{\mathbf{x}\} = \frac{1}{N-1}\sum_{j=1}^{N}\mathbf{x}_j$$

A **covariance matrix** of a 4-band image: $\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{pmatrix}$

The elements on the major diagonal of the covariance matrix are the ***variance*** of each image band, while the symmetrical elements off the major diagonal are the ***covariance*** between two different bands.

The covariance matrix is a non-negative definite matrix, symmetrical along its major diagonal. Such a matrix can be converted into a diagonal matrix via basic matrix operations.

For independent variables in a multi-dimensional space, $\sigma_{ij} = \sigma_{ji} = 0$, and thus they have a diagonal covariance matrix.

In mathematics, PCA is simply a process to **find a transformation $\mathbf{G}$ that diagonalizes the covariance matrix** Σ_x of the m bands image $\mathbf{X}$ to produce an ***n*** principal components image $\mathbf{Y}$ with a **diagonal covariance matrix** Σ_y

Let $\mathbf{y} = \mathbf{G}\mathbf{x}$ then $\Sigma_y = \varepsilon\{(\mathbf{y} - \mathbf{m}_y)(\mathbf{y} - \mathbf{m}_y)^t\}$

$$\mathbf{m}_y = \varepsilon\{\mathbf{y}\} = \varepsilon\{\mathbf{G}\mathbf{x}\} = \mathbf{G}\varepsilon\{\mathbf{x}\} = \mathbf{G}\mathbf{m}_x$$

$$\begin{aligned}\Sigma_y &= \varepsilon\{(\mathbf{G}\mathbf{x} - \mathbf{G}\mathbf{m}_x)(\mathbf{G}\mathbf{x} - \mathbf{G}\mathbf{m}_x)^t\} \\ &= \mathbf{G}\varepsilon\{(\mathbf{x} - \mathbf{m}_x)(\mathbf{x} - \mathbf{m}_x)^t\}\mathbf{G}^t \\ &= \mathbf{G}\Sigma_x\mathbf{G}^t\end{aligned}$$

According to the rules of matrix operations we can prove that the transformation $\mathbf{G}$ is the $\sum_x$ **transposed matrix of the *eigenvectors*** of $n \times m$.

$$\mathbf{G} = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1m} \\ g_{21} & g_{22} & \cdots & g_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1} & g_{n2} & \cdots & g_{nm} \end{pmatrix} = \begin{pmatrix} \mathbf{g}_1^t \\ \mathbf{g}_2^t \\ \cdots \\ \mathbf{g}_n^t \end{pmatrix}$$

Σ_y is a diagonal matrix with eigenvalues of Σ_x as non-zero elements along the major diagonal:

The ***eigenvalue*** $\boldsymbol{\lambda_i}$ is the variance of PC_i image, and it is proportional to the information contained in PC_i. **The information content decreases with the increment of the *PC* rank from 1 to n**.

$$\Sigma_y = \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix}$$

$$\lambda_1 > \lambda_2 > \cdots > \lambda_n$$

Any eigenvector of matrix Σ_x is defined as a vector: $\mathbf{g}$ $(\mathbf{g} \in \mathbf{G})$

So we can establish this equation: $\Sigma_x \mathbf{g} = \lambda \mathbf{g}$ or $(\Sigma_x - \lambda \mathbf{I})\mathbf{g} = 0$

This formula is called ***characteristic polynomial*** of Σ_x . Thus once the i^{th} eigenvalue is known then the i^{th} eigenvector $\mathbf{g}_i$ is determined.

Eigenvalues of Σ_x can be calculated from its ***characteristic equation***:

$$|\Sigma_x - \lambda \mathbf{I}| = 0$$

Eigenvector matrix $\mathbf{G}$ decides how each principal component is composed from the original $\boldsymbol{m}$ image bands.

$$PC_i = \mathbf{g}_i^t \mathbf{X} = \sum_{k=1}^{m} \mathbf{g}_{ik} Band_k$$

where g_{ik} is the element of $\mathbf{G}$ at the i^{th} row and k^{th} column or the k^{th} element of the i^{th} eigenvector $\mathbf{g}_i^t = (\mathbf{g}_{i1}, \mathbf{g}_{i2}, \cdots \mathbf{g}_{ik} \cdots, \mathbf{g}_{im})$

3B.2 Feature oriented PC Selection

Covariance Matrix of b1 to b5 & b7 of a Landsat multispectral image and its meaning

Covariance	B1	B2	B3	B4	B5	B7
B1	232.202	196.203	305.763	348.550	677.117	345.508
B2	196.203	178.980	284.415	335.185	660.570	335.997
B3	305.763	284.415	460.022	545.336	1083.993	551.367
B4	348.550	335.185	545.336	674.455	1347.927	678.275
B5	677.117	660.570	1083.993	1347.927	2802.914	1402.409
B7	345.508	335.997	551.367	678.275	1402.409	711.647

High information content is defined by degree of correlation between bands
(with high correlation variance decreases)

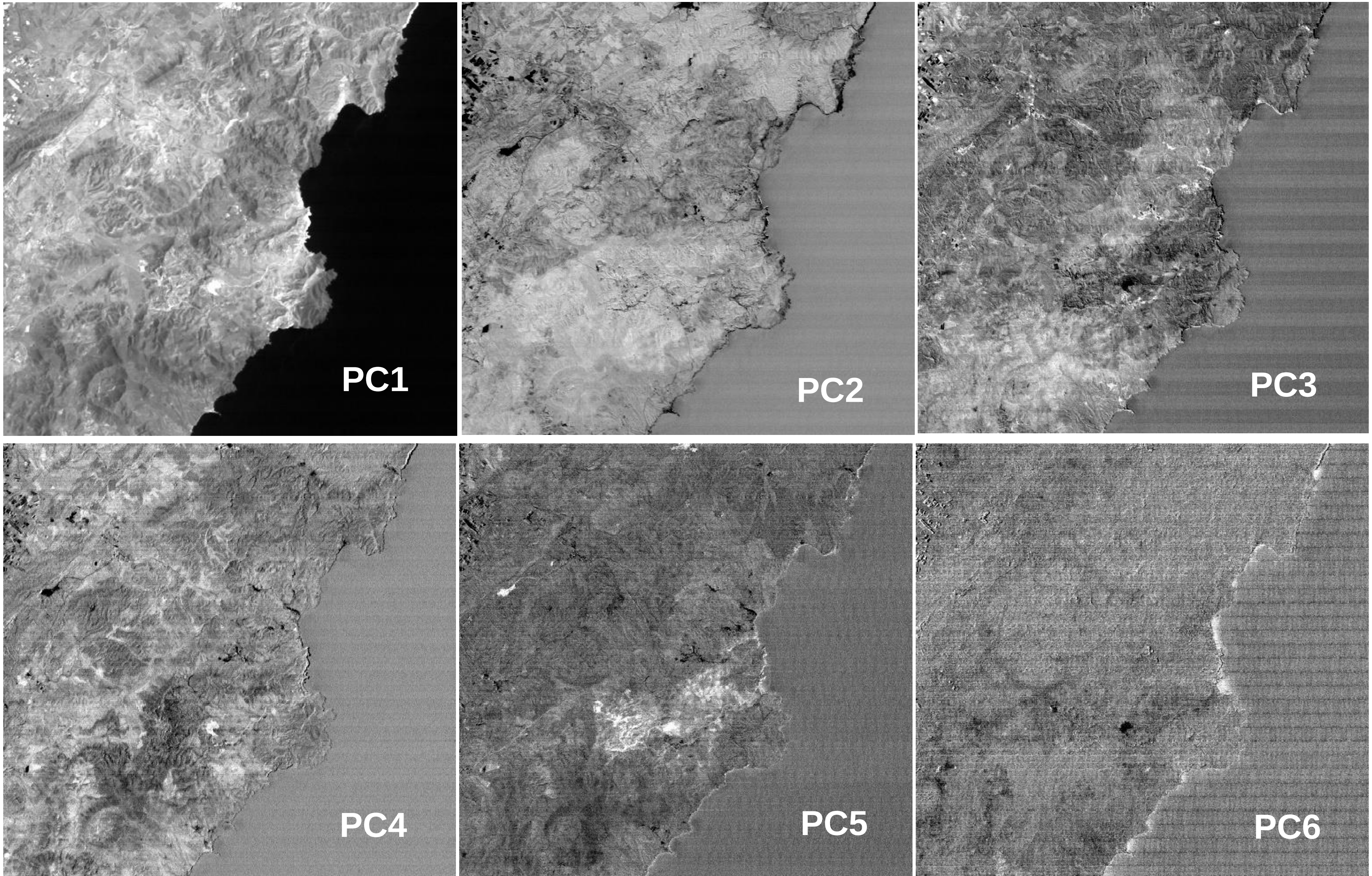
How do we use this information?

Eigenvectors (g_i) & eigenvalues (λ) of covariance matrix of Landsat b1 to b5 & b7

For an example sub-scene. From this, we can begin to understand the connection between eigenvectors and spectral signatures in bands (equivalent to band differencing)

Bands: \ Eigenvalues:	*PC1*	*PC2*	*PC3*	*PC4*	*PC5*	*PC6*
B1 (B)	0.190	-0.688	-0.515	**-0.260**	**-0.320**	-0.233
B2 (G)	0.183	-0.362	0.032	0.050	0.136	0.902
B3 (R)	0.298	-0.418	**0.237**	**0.385**	**0.638**	-0.354
B4 (NIR)	0.366	-0.136	**0.762**	-0.330	-0.389	-0.079
B5 (SWIR1)	0.751	0.433	-0.296	**-0.318**	**0.242**	0.013
B7 (SWIR2)	0.378	0.122	-0.093	**0.756**	**-0.511**	0.011
Eigenvalue variance (λ):	4928.73	102.31	15.58	9.01	3.57	1.01
Information:	97.4%	2.02%	0.31%	0.18%	0.07%	0.02%
			Vegetation bright	Fe-oxide bright, hydrated-mins dark	Fe-oxide bright & hydrated-mins bright	

PC Images of 6 TM Reflective Spectral Bands



Summary of table on slide 15

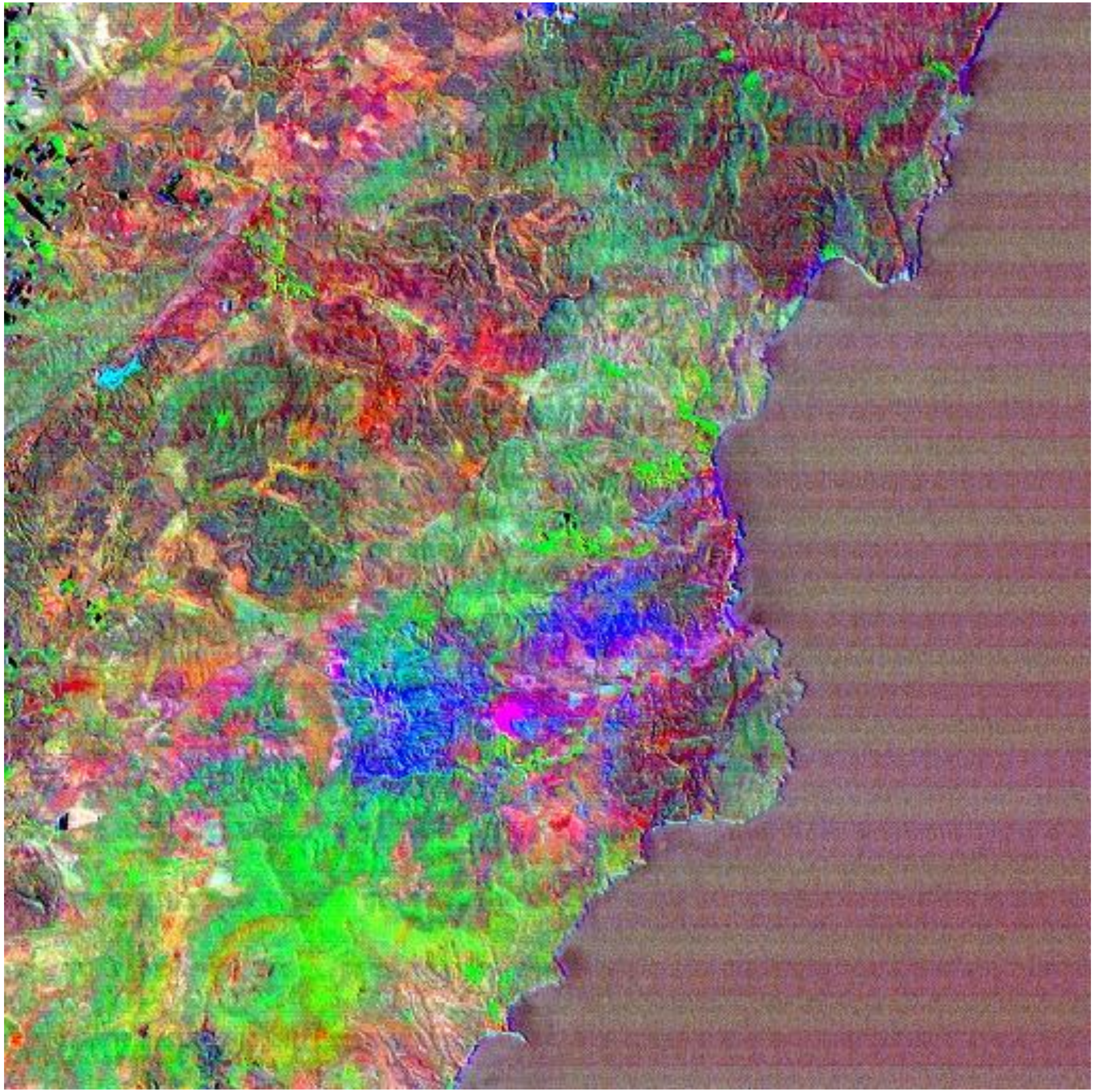
1. The elements of $\mathbf{g}_1$ are all **positive** and therefore ***PC1* is a weighted average of all the original image bands**. In this sense, it resembles a panchromatic image with broad spectral range. It has a very large eigenvalue 4928.731 (*PC1* variance) accounting for 97.4% information of the whole six bands. With fixed DN range, more information means a higher SNR.
2. *PC1* image concentrates information common to all the six bands. For Earth observation satellite images, this common information is usually **topography & albedo**.
3. The elements of $\mathbf{g}_i$ (i>1) are usually a mixture of positive and negative values and thus a PC image is a linear combination of **positively** and **negatively weighted** bands from the original input dataset.
4. Higher rank *PC*s lack topographic features and show **mainly spectral variation**. They have significantly smaller eigenvalues (*PC* variances) than *PC1*. The eigenvalues decrease rapidly with increasing PC rank and thus have lower and lower SNR, as demonstrated by increasingly noisy appearance. *PC6* image is nearly entirely noise and contains little information, as indicated by very small variance 1.012. In this sense, *PC6* can be disregarded from the dataset and thus the effective dimensionality is reduced to 5 from the original 6 with negligible information loss of 0.02%.

Feature oriented PC Selection (FPCS) Colour composite of:

PC4 (R)
PC3 (G)
PC5 (B)

- PC1 omitted – it has high SNR but contains no spectral information
- PC4 contains both fe-ox (bright) and clay-minerals (dark)
- PC3 contains vegetation information
- PC5 contains hydrated mineral information (bright)
- Higher order PCs have low SNR

(Reference the table of eigenvectors in slide 15 for the PC selection).



3B.3 Other (physical property-based) coordinate transformation

Once again we use PCA as a rotational operation of an m dimension orthogonal coordinate system of an m band multi-spectral image $\mathbf{X}$. The rotation is scene dependent and determined by the eigenvector matrix $\mathbf{G}$ of the covariance matrix Σ_x. So the PCs are simply weighted linear combinations of the input bands

If we consider this transformation in the general sense, we can arbitrarily rotate the m dimensional orthogonal coordinate system in any direction defined by a transformation $\mathbf{R}$.

$$\mathbf{y} = \mathbf{Rx}$$

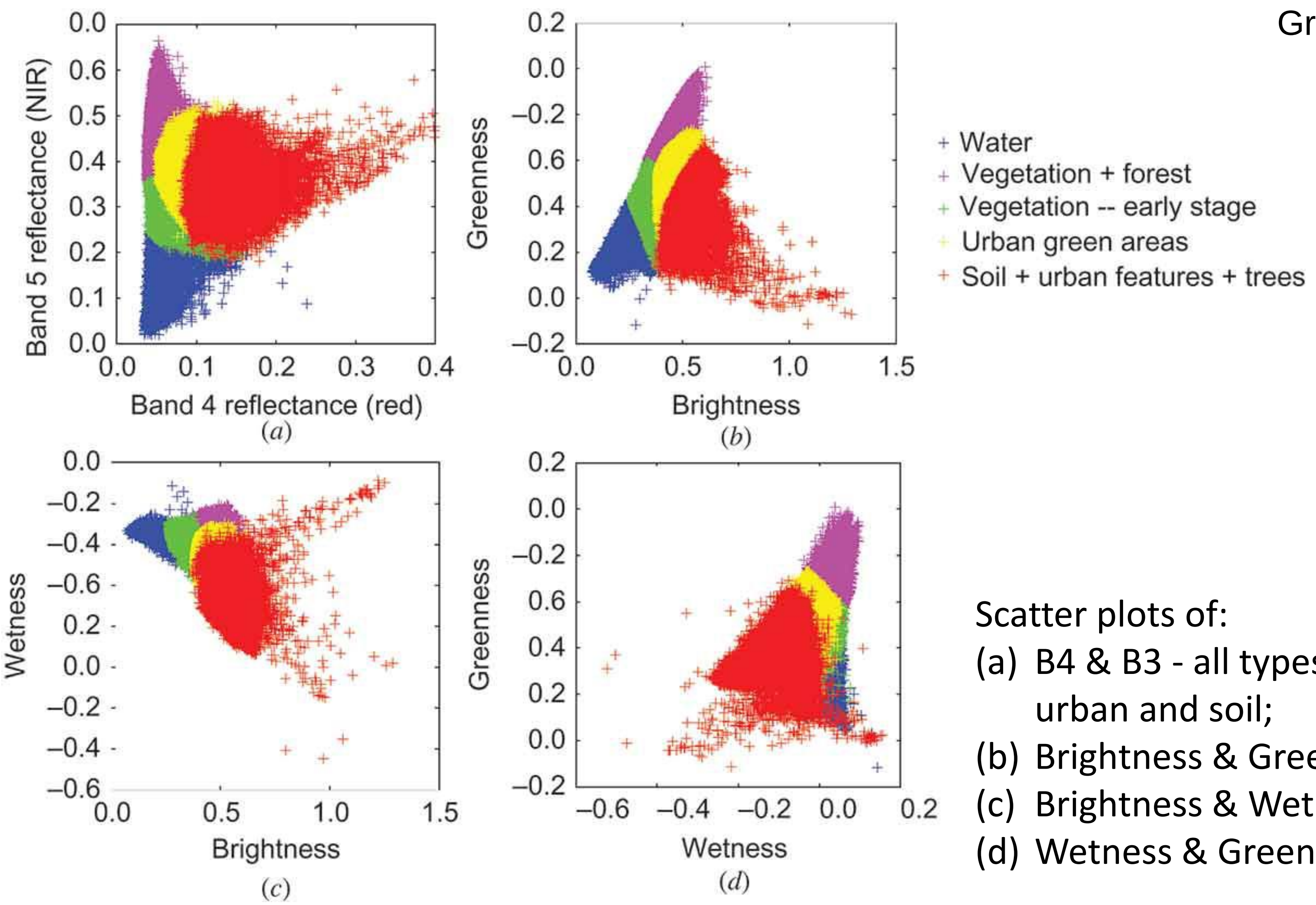
Here $\mathbf{y}$ is a linear combination of $\mathbf{x}$ specified by the coefficients (weights) in $\mathbf{R}$.

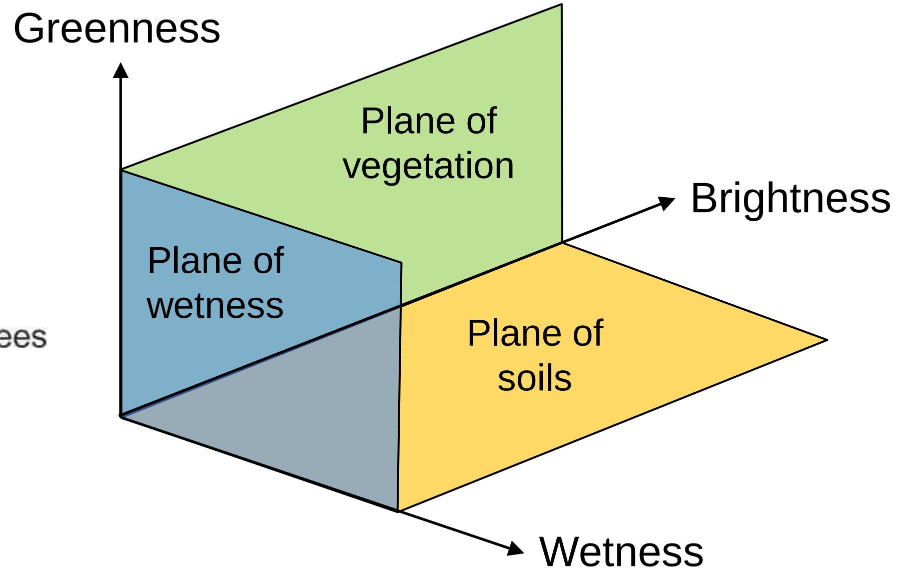
For example, a 3D rotation from $\mathbf{x}$ to $\mathbf{y}$ is defined as:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \cos\alpha_1 & \cos\beta_1 & \cos\gamma_1 \\ \cos\alpha_2 & \cos\beta_2 & \cos\gamma_2 \\ \cos\alpha_3 & \cos\beta_3 & \cos\gamma_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Tasselled Cap Transformation (TCT)

One of the most effective examples of the physical property-based coordinated transformation – uses **weights derived from the spectral responses of water, chlorophyll and dry soils**





Tasselled Cap Transform

Scatter plots of:

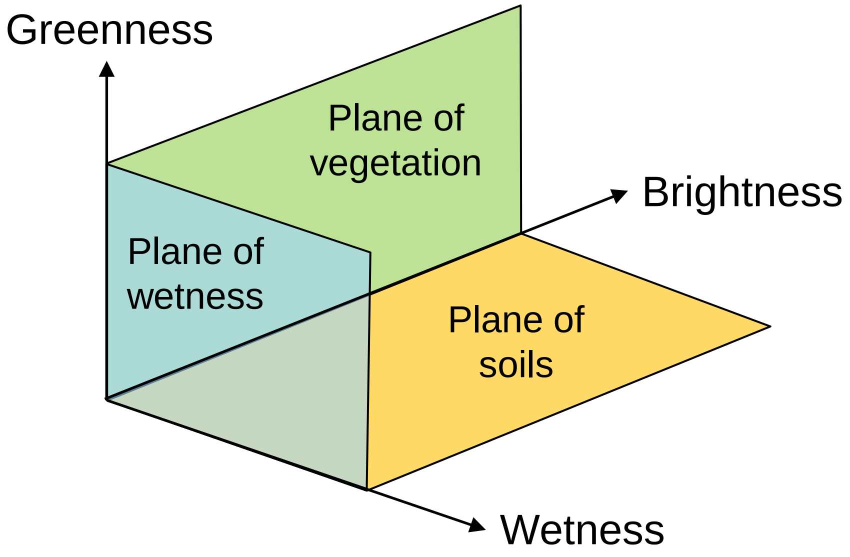
(a) B4 & B3 - all types - water, vegetation (forest & early growth), urban and soil;
(b) Brightness & Greenness - 'plane of vegetation';
(c) Brightness & Wetness - 'plane of soil'; and
(d) Wetness & Greenness – or 'transition zone'.

Ali Baig et al., 2014

Tasselled cap transformation

The goal of the tasselled cap transformation is to transform the six reflective spectral bands (1~5, 7) of TM/ETM+ (in VNIR and SWIR spectral ranges) into three orthogonal components orientated to three key properties of the land surface:

- **Brightness**
- **Greenness** (vigour of green vegetation)
- **Wetness**



$$\begin{pmatrix} Brightness, b \\ Greenness, g \\ Wetness, w \end{pmatrix} = \begin{pmatrix} bw1 & bw2 & bw3 & bw4 & bw5 & bw6 \\ gw1 & gw2 & gw3 & gw4 & gw5 & gw6 \\ ww1 & ww2 & ww3 & ww4 & ww5 & ww6 \end{pmatrix} \begin{pmatrix} TM1 \\ TM2 \\ TM3 \\ TM4 \\ TM5 \\ TM7 \end{pmatrix}$$

Kauth & Thomas (1976)

Example: Landsat5 TM tasselled cap transformation coefficients

TM band	1	2	3	4	5	7
Brightness	0.3037	0.2793	0.4343	0.5585	0.5082	0.1863
Greenness	-0.2828	-0.2435	-0.5436	0.7243	0.0840	-0.1800
Wetness	0.1509	0.1793	0.3299	0.3406	-0.7112	-0.4572

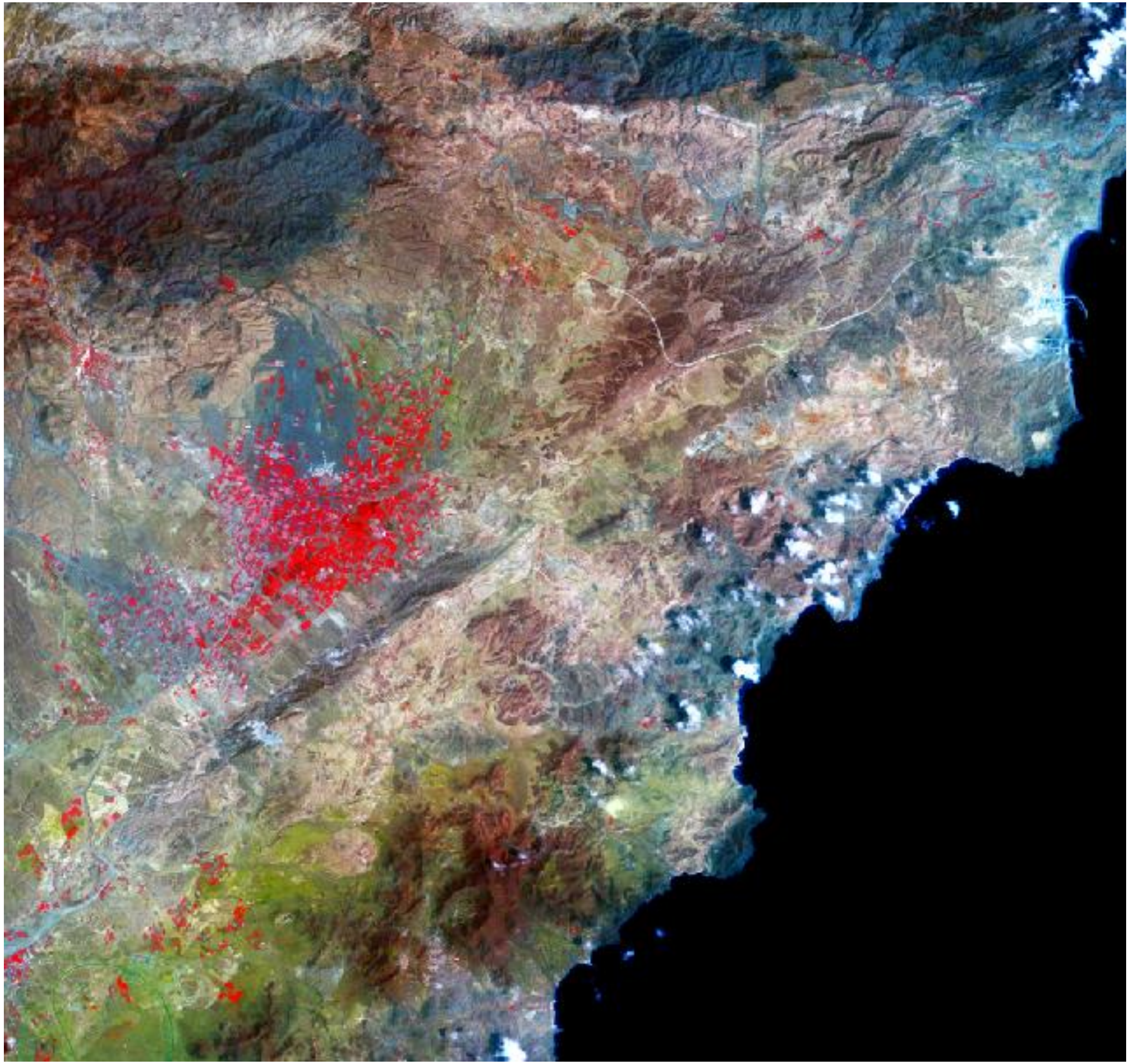
Kauth & Thomas (1976), Crist and Cicone (1984)

Example: Landsat7 ETM+ at-satellite reflectance tasselled cap transformation coefficients

ETM+ band	1	2	3	4	5	7
Brightness	0.3561	0.3972	0.3904	0.6966	0.2286	0.1596
Greenness	-0.3344	-0.3544	-0.4556	0.6966	-0.0242	-0.2630
Wetness	0.2626	0.2141	0.09926	0.0656	-0.7629	-0.5388

Crist, 1985

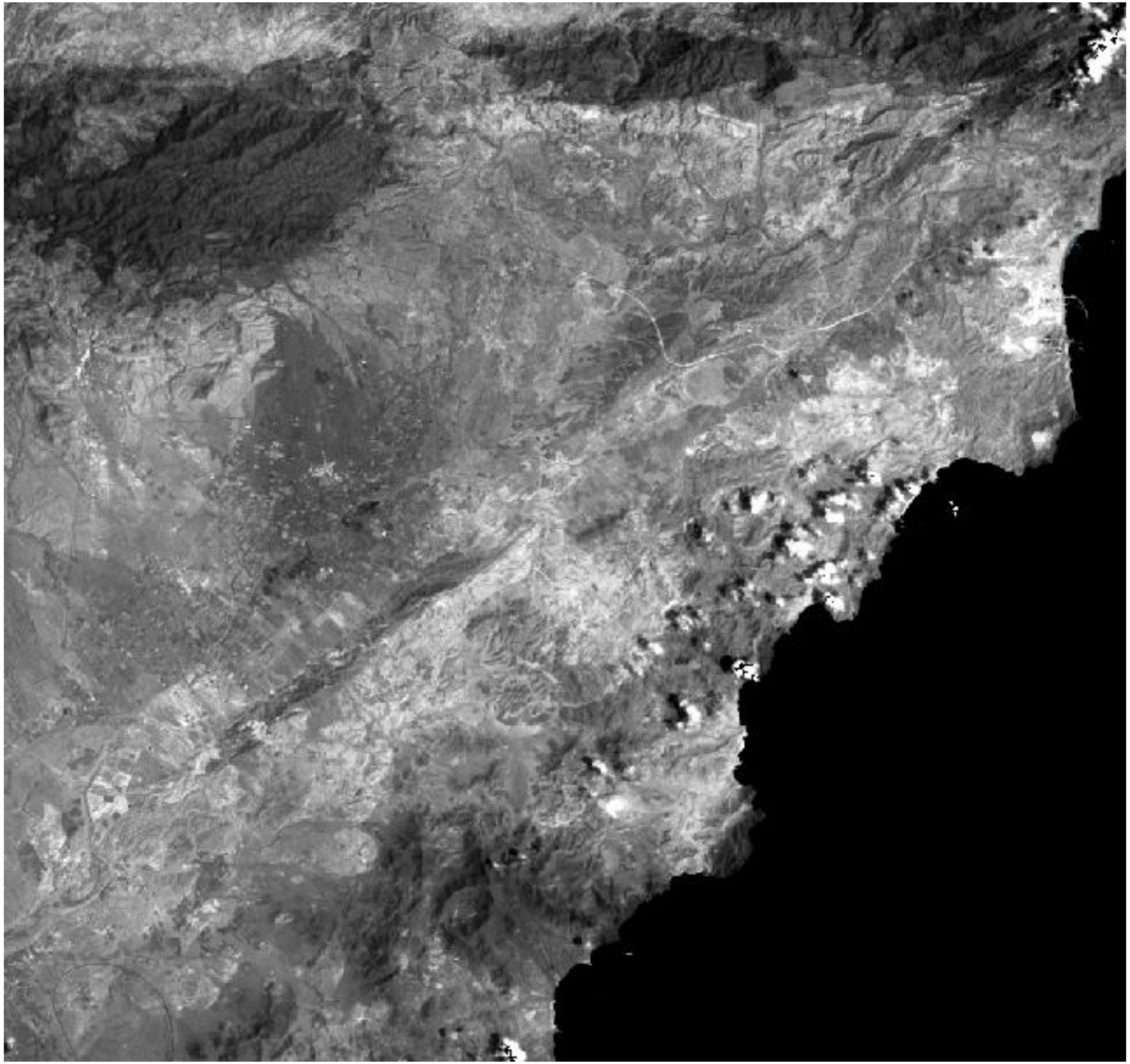
Standard False Colour Composite



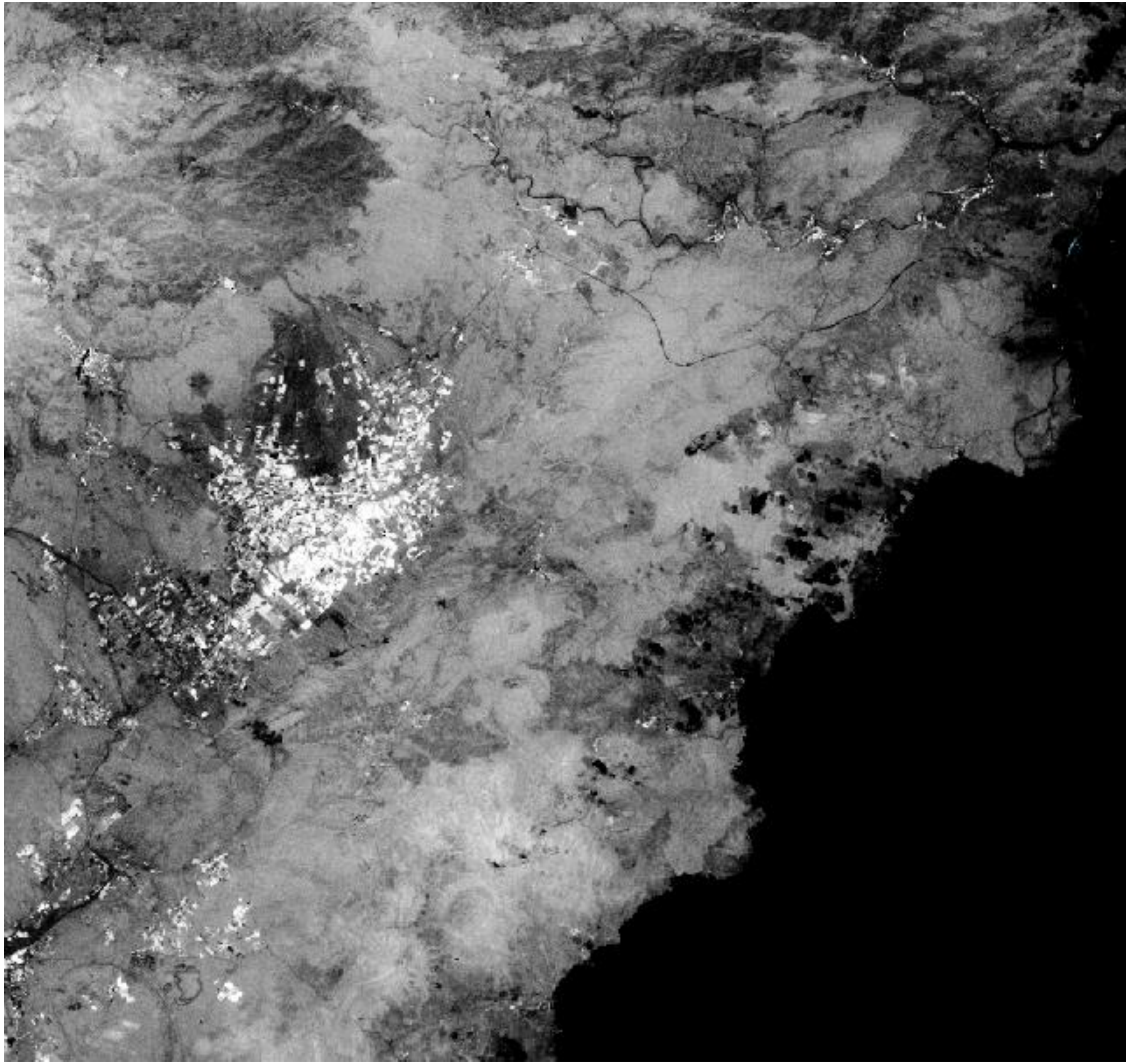
Spain 1984 (with open vegetation i.e. pre-plasticulture)

Brightness

Looks very similar to a b1 image

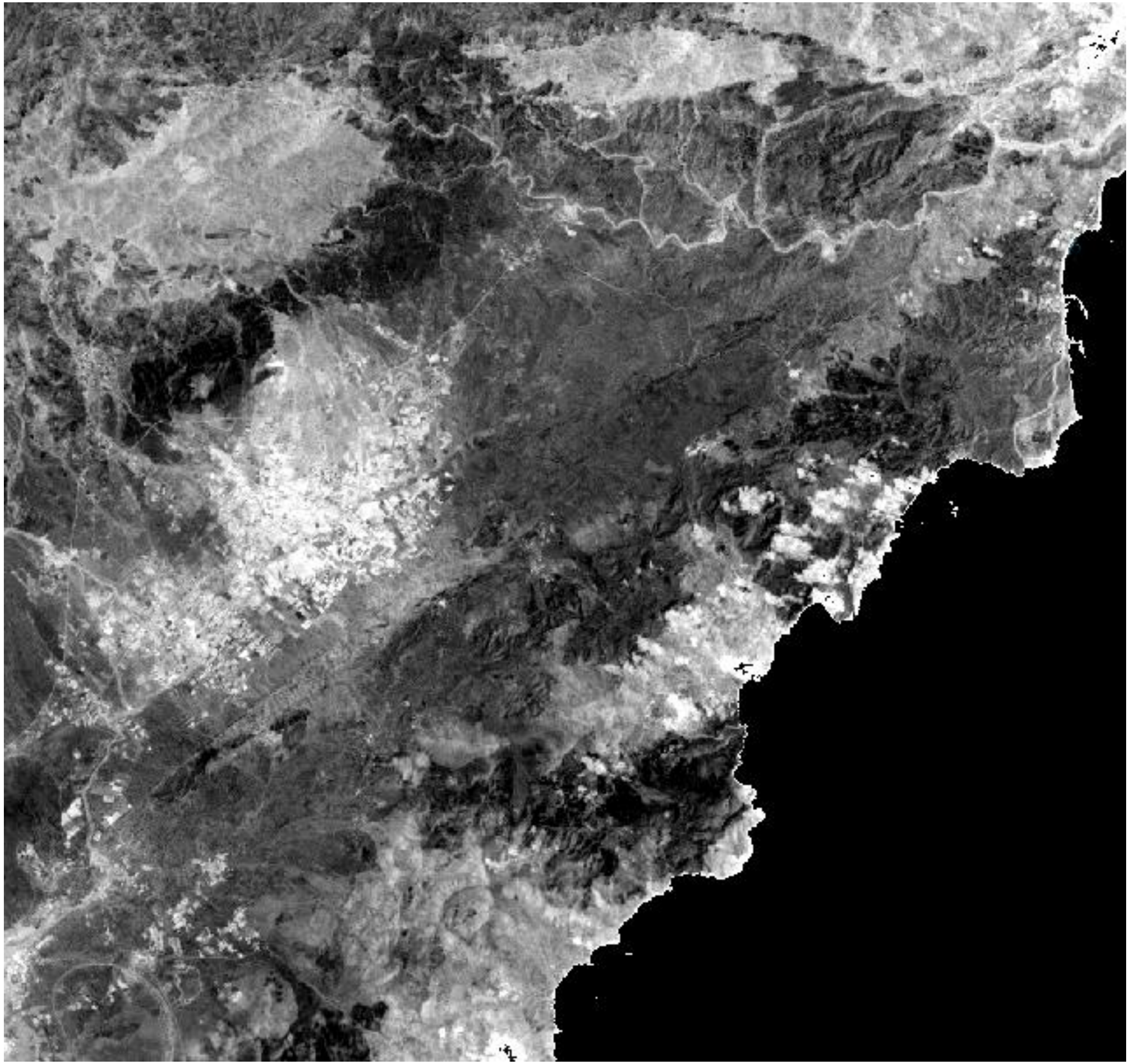


Greenness



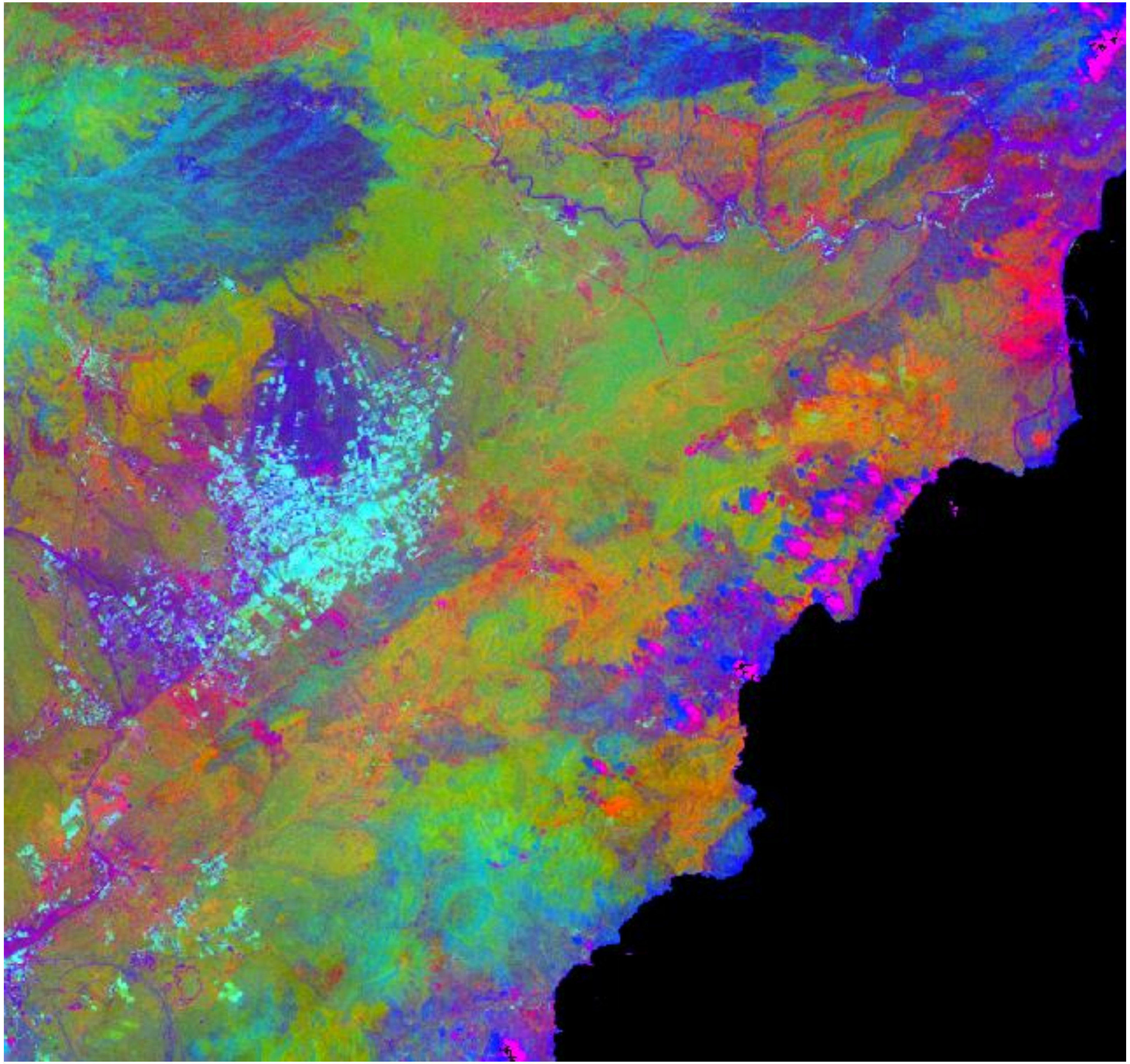
Spain 1984 (with open vegetation i.e. pre-plasticulture)

Wetness



Tasselled Cap CC:

- Brightness (R)
- Greenness (G)
- Wetness (B)



3B Remarks

PCA and differencing

- Though PCA is based on quite complex operations of covariance matrix, in the end, a *PC* image is simply formulated as a **weighted linear combination** of the original image bands.
- When analysing the eigenvectors of a *PC* image in the FPCS technique, we are essentially composing a *PC* based on image differencing: **High order PCs are nothing more than compound differencing images** however, these images are independent from each other.
- **PCA ensures use of orthogonal (independent) PCs** which are based on the data distribution and statistics, whilst **image differencing allows the user to target specific spectral signatures of interest** (although the resulting difference images are not orthogonal so not independent).
- FPCS combines the merits of the both but it may not always able to address the specific, diagnostic spectral features that we are interested in.

PC1 and Average and Intensity

- Average image, intensity image (of HSI) and *PC1* image are largely identical. All three are equivalent to a summation of the spectral bands and they all increase the image SNR. *Average* is an equally weighted summation of a *n* image bands; *Intensity* (as in HSI) is an average of three bands used for RGB-HSI transformation; and ***PC1* is a weighted summation of all the image bands based on the eigenvectors and represents the 1st eigenvalue of the image covariance matrix**.

PCADS and Saturation DS

- The concept of decorrelation stretch is rooted in PCA. But the PCA DS is not as efficient, nor as widely used, as the saturation (decorrelation) stretch techniques because it involves complex matrix operations for inverse PC transformation. Though the two types of decorrelation stretch techniques are based on different principles, the effects on the data of a band triplet of RGB colour composite are equivalent: **increasing the three dimensionality of the data cluster**.