Appendix A VBA Code

The following pages contain the source code that was used to simulate the model as outlined above.

I have categorised the functions into the following subchapters:

- Functions in the section "Simple Luhmann Economy Model" are the core of the model. I have
 to apologise that some error messages are in German. The meaning should be obvious by the
 if-then-else clauses. I tried to keep comments (although few) in English.
- Functions in the section "Fuzzy clustering" refer to the implementation of the fuzzy-c-means
 clustering algorithm. Sepcial care regarding underflow (as a likely event) had to be taken in
 the modules.
- Functions of "Exploration" refer to functions that were used in the simulation and exploration part (e.g. the identification of cycles).
- Auxiliary functions like seeking minima, or taking care of the torus are contained in the last section.

Some debugging messages have not been erased but commented out because I regarded them as helpful in understanding the code.

The Beta distribution is given as follows:

$$f(x) = \frac{1}{\beta(a,b)} x^{a-1} (1-x)^{b-1} 1_{(0,1)}(x)$$
$$\beta(a,b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx$$
$$a,b > 0$$

Formula 1: density function of Beta probability distribution, Beta function

A.1 Simple Luhmann Economy Model

A.1.1 Displays of Wealth (Show Off)

A.1.2 Making Proposals

Making proposals includes the following functions:

- The function propose is the main loop through all agents
- The function proposeij takes care about the actions of an individual agent identified by row and column index i and j
- The functions mmprice, pricerule and showoffrule contain the respective rules to find price minima and maxima, create proposals following the price rule and for creating proposals according to the showoff rule
- The function convertproposals and convertijproposals are used to ease implementation and exchange (absolute and relative) addresses of proposer and proposee.

```
Sub propose(step, n, m, g, ni, mj, ab, ohorizon, o, xy, p, op, sp)
Dim i, j, k As Integer
ReDim own (1 To g) As Long
op = 0
sp = 0
For i = 1 To n
    For j = 1 To m
        For k = 1 To g
           own(k) = ab(i, j, k)
        Call proposeij(step, n, m, g, ni, mj, i, j, own, ohorizon, o, xy, p, op, sp)
    Next j
Next i
End Sub
Sub proposeij(step, n, m, g, oi, oj, i, j, own, ohorizon, o, xy, p, op, sp)
'n,m rows,columns
'g goods
'i,j proposing entity
'own portfolio owned
'o observations of prior deals
'xy showoffs
'p result
Dim k, l, omade As Integer
Dim smax, bmin As Double
Dim smaxii, smaxjj As Integer
Dim bminii, bminjj As Integer
If ohorizon <= 0 Then GoTo pre_bye
omade = 0
For k = 1 To q
   For l = k + 1 To g
        Call mmprice(n, m, g, oi, oj, i, j, k, l, o, bmin, bminii, bminjj, smax, smaxii,
smaxjj, omade)
        'prices found for goods combination k, l
        Call pricerule(step, n, m, g, i, j, k, l, own, bmin, bminii, bminjj, smax, smaxii,
smaxjj, p, op, sp, omade)
         'proposals made
```

```
Next 1
Next k
pre_bye:
If omade = 0 Then
    MsgBox ("proposeij> no observations made")
    Call showoffrule(n, m, g, oi, oj, i, j, own, xy, p, sp)
End If
bve:
End Sub
Sub mmprice(n, m, g, oi, oj, i, j, k, l, o, bmin, bminii, bminjj, smax, smaxii, smaxjj, omade)
Dim ii, jj As Integer
Dim oA, oB As Long
smax = 0

bmin = 9E+15
For ii = -oi To oi
   For jj = -oj To oj
exclude self, law of the first distinction
If ii = 0 And jj = 0 Then GoTo jjloop
          For r = 1 To 2
              r = 1 role accepter, 2 role proposer
s = 1, a sale of (k) - i.e. a chance for buying - was observed
              oA = o(i, j, ii, jj, r, 1, k) 
oB = o(i, j, ii, jj, r, 1, 1) 
If oA = 0 Or oB = 0 Then GoTo rcontinue
               omade = omade + 1
              If Abs(oB / oA) < bmin Then
                   bmin = Abs(oB / oA)
                   bminii = ii
                  bminjj = jj
              End If
rcontinue:
              s = 2, a purchase of (k) - i.e. a chance for selling - was observed oA = o(i, j, ii, jj, r, 2, k)
              oB = o(i, j, ii, jj, r, 2, 1)
              If oA = 0 Or oB = 0 Then GoTo rloop
              omade = omade + 1
              If Abs(oB / oA) > smax Then
                   smax = Abs(oB / oA)
                   smaxii = ii
                   smaxjj = jj
              End If
rloop:
         Next r
jjloop:
    Next jj
Next ii
' smax and bmin identified
End Sub
```

```
Sub pricerule(step, n, m, g, i, j, k, l, own, bmin, bminii, bminij, smax, smaxii, smaxjj, p,
op, sp, omade)
Dim buyA, sellA, B2buyA, B2sellA As Long
If smax > bmin And omade <> 0 Then
    B2buyA = own(1)
    buyA = Int(B2buyA / bmin)
    While B2buyA <> bmin * buyA And B2buyA > 0
        B2buyA = B2buyA - 1
       buyA = Int(B2buyA / bmin)
    Wend
'propose to buy A
    If B2buyA > 0 Then
        p(i, j, bminii, bminjj, k) = buyA
       p(i, j, bminii, bminjj, l) = -B2buyA
        op = op + 1
        own(1) = own(1) - B2buyA
    End If
    sellA = own(k)
    B2sellA = Int(smax * sellA)
    While B2sellA <> smax * sellA And sellA > 0
       sellA = sellA - 1
       B2sellA = Int(smax * sellA)
    Wend
'propose to sell A
    If sellA > 0 Then
        p(i, j, smaxii, smaxjj, k) = -sellA
        p(i, j, smaxii, smaxjj, 1) = B2sellA
        op = op + 1
        own(k) = own(k) - sellA
    End If
End If
End Sub
```

```
Sub showoffrule(n, m, g, oi, oj, i, j, own, xy, p, sp)
Dim ii, jj, k, l As Integer
Dim a, pA, pB As Long
ReDim maxg(1 To g) As Long
ReDim maxwhor(1 To g) As Integer
ReDim maxwhoc(1 To g) As Integer
Dim maxM As Long
Dim maxMg As Integer
'ReDim minG(1 To g) As Long
'ReDim minwhor(1 To g) As Integer
'ReDim minwhoc(1 To g) As Integer
For k = 1 To g
    minG(k) = 10000000000#
    For ii = -oi To oi
       For jj = -oj To oj

If ii = 0 And jj = 0 Then GoTo jjloop
            a = xy(torus(i + ii, n), torus(j + jj, m), k)
            If a > maxg(k) Then
                maxg(k) = a
                maxwhor(k) = ii
                maxwhoc(k) = jj
            End If
             If a <> 0 And a < minG(k) Then
                 minG(k) = a
                 minwhor(k) = ii
                 minwhoc(k) = jj
             End If
jjloop:
       Next jj
    Next ii
Next k
For k = 1 To g
    If maxg(k) - own(k) > 2 Then
        maxM = 0
        For l = 1 To g
            If own(1) > maxM And 1 <> k Then
               maxM = own(1)
                maxMg = 1
            End If
        Next 1
        If maxM > 0 Then
            pA = Int((maxg(k) - own(k)) / 2)
            p(i, j, maxwhor(k), maxwhoc(k), k) = pA
            pB = Int((minG(1) - own(1)) / 2)
pB = -1
' greedy ... no more than 1 or other
            p(i, j, maxwhor(k), maxwhoc(k), maxMg) = pB
' do adapt ownership of {\tt maxMg} not {\tt k}
            own(maxMg) = own(maxMg) + pB
            sp = sp + 1
        End If
    End If
kloop:
Next k
End Sub
```

```
Sub convertproposals(n, m, g, ni, mj, z, d)
Dim i, j As Integer
For i = 1 To n
   For j = 1 To m
      Call convertijproposals(n, m, g, ni, mj, i, j, z, d)
   Next j
Next i
End Sub
Sub convertijproposals(n, m, g, ni, mj, i, j, z, ByRef d)
Dim ii, jj, k, l As Integer
Dim v1, v2 As Long
For ii = -ni To ni
   For jj = -mj To mj
       For k = 1 To g
          v1 = z(i, j, ii, jj, k)

If v1 = 0 Then GoTo kloop
           For l = k + 1 To g
              v2 = z(i, j, ii, jj, l)
If v2 = 0 Then GoTo lloop
'MsgBox (s)
lloop:
          Next 1
kloop:
      Next k
   Next jj
Next ii
End Sub
```

A.1.3 Accepting Deals

Accepting Deals includes the following functions:

- The function acceptijdeals is the main function. It loops through all combinations
 of tradeable goods.
- The functions bestprice and acceptbestdeal contain the identification of the best prices and the accepting of deals

```
Sub acceptijdeals(step, g, i, j, ni, mj, own, d, dd)
Dim ii, jj, k, l As Integer
Dim dA, dB As Long
Dim smax, bmin As Double
Dim smaxii, smaxjj As Integer
Dim bminii, bminjj As Integer
For k = 1 To g
    For l = k + 1 To g
         Call bestprice(i, j, ni, mj, k, l, own, d, smax, smaxii, smaxjj, bmin, bminii, bminjj)
         'best price selected
         Call acceptbestdeal(step, i, j, k, l, own, d, smax, smaxii, smaxjj, bmin, bminii,
bminjj, dd)
         'best deal accepted
    Next 1
Next k
End Sub
Sub bestprice(i, j, ni, mj, k, l, own, d, smax, smaxii, smaxjj, bmin, bminii, bminjj)
Dim ii, jj As Integer
Dim dA, dB As Long
smax = 0
bmin = 9E+15
For ii = -ni To ni
    For jj = -mj To mj
        dA = d(i, j, ii, jj, k)

dB = d(i, j, ii, jj, l)
         If dA = 0 Or dB = 0 Then GoTo jjloop
         If dA < 0 And dB > 0 Then
             If own(k) + dA > 0 And Abs(dB / dA) > smax Then
                 smax = Abs(dB / dA)
                 smaxii = ii
                 smaxjj = jj
             End If
             GoTo jjloop
        End If
         If dA > 0 And dB < 0 Then
             If own(1) + dB > 0 And Abs(dB / dA) < bmin Then bmin = Abs(dB / dA)
                 bminii = ii
                 bminjj = jj
             End If
        End If
jjloop:
   Next ii
Next ii
End Sub
```

```
Sub acceptbestdeal(step, i, j, k, l, own, d, smax, smaxii, smaxjj, bmin, bminii, bminjj, dd)
Dim qK, qL As Long
If smax = 0 Then
    If bmin < 9E+15 Then
         qK = d(i, j, bminii, bminjj, k)
         qL = d(i, j, bminii, bminjj, l)
         dd(i, j, bminii, bminjj, k) = qK
         dd(i, j, bminii, bminjj, l) = qL
         own(k) = own(k) + qK
own(1) = own(1) + qL
    End If
    GoTo bye
End If
If bmin \geq= 9E+15 Then
    If smax > 0 Then
         qK = d(i, j, smaxii, smaxjj, k)

qL = d(i, j, smaxii, smaxjj, l)
         dd(i, j, smaxii, smaxjj, k) = qK
         dd(i, j, smaxii, smaxjj, l) = qL

own(k) = own(k) + qK
         own(1) = own(1) + qL
    End If
    GoTo bye
End If
If smax > bmin Then
'If ni > 1 Then MsgBox ("double deal accepted")
     qK = d(i, j, bminii, bminjj, k)
     qL = d(i, j, bminii, bminjj, l)
    dd(i, j, bminii, bminjj, k) = qK
    dd(i, j, bminii, bminjj, 1) = qL
    own(k) = own(k) + qK
    own(1) = own(1) + qL
    qK = d(i, j, smaxii, smaxjj, k)
    qL = d(i, j, smaxii, smaxjj, l)
    \begin{array}{lll} \text{dd(i, j, smaxii, smaxjj, k)} = \text{qK} \\ \text{dd(i, j, smaxii, smaxjj, l)} = \text{qL} \end{array}
    own(k) = own(k) + qK
    own(1) = own(1) + qL
End If
'accepted deals filled to dd
'ownership adaped
bye:
End Sub
```

A.1.4 Observing Deals

```
Sub observedeals(n, m, g, oii, ojj, dd, o)
' ReDim o(1 To n, 1 To m, -1 To 1, -1 To 1, 1 To 2, 1 To 2, 1 To 2) As Integer
' observations ( observer (row, column), observee (relrow, relcol),
                role(accepter, proposer), signA(sellA, buyA), scarce good) qty
Dim i, j, ii, jj, k, l, ni, nj, role, saleA As Integer
Dim ddA, ddB As Long
For i = 1 To n
    For j = 1 To m
        dd(i,j,...) has accepted deal
        For ii = -oii To oii
            For jj = -ojj To ojj
                For k = 1 To g
                    ddA = dd(i, j, ii, jj, k)
                    If ddA = 0 Then GoTo kloop
                    For l = k + 1 To g
                        ddB = dd(i, j, ii, jj, l)
If ddB = 0 Then GoTo lloop
                        inner loop
                         saleA = 1
                        If ddA > 0 Then saleA = 2
                        saleA=1 it's a sale of A from the viewpoint of the accepter (and
proposer as *-1)
                         saleA=2 it's a purchase of A from the viewpoint of the accepter (and
proposer as * -1)
                        walk through all neighbours of accepter
                         For ni = -oii To oii
                             For nj = -ojj To ojj
                                o(torus(i + ni, n), torus(j + nj, m), -ni, -nj, 1, saleA, k) =
ddA
                                 o(torus(i + ni, n), torus(j + nj, m), -ni, -nj, 1, saleA, 1) =
ddB
                            Next nj
                        Next ni
                         walk through all neighbours of proposer
                         For ni = -oii To oii
                            For nj = -ojj To ojj
                                 o(torus(i + ii + ni, n), torus(j + jj + nj, m), -ni, -nj, 2,
torus(saleA + 1, 2), k) = -ddA
                                 o(torus(i + ii + ni, n), torus(j + jj + nj, m), -ni, -nj, 2,
torus(saleA + 1, 2), 1) = -ddB
                            Next nj
                        Next ni
lloop:
                    Next 1
kloop:
                Next k
jjloop:
           Next jj
       Next ii
   Next j
Next i
End Sub
```

A.1.5 Clearing Deals

The function for clearing (additionally) verifies the bookkeeping (no short selling) rules.

```
Sub cleardeals(step, n, m, g, oii, ojj, ab, dd, ndeals, flow, fstat, delta)
Dim i, j, k, l, ii, jj As Integer
Dim oA, oB As Long
Dim fA, fB As Double
delta = 0
ndeals = 0
For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                If ii = 0 And jj = 0 Then GoTo jjloop
                 For k = 1 To g
                    oA = dd(i, j, ii, jj, k)
If oA <> 0 Then GoTo kbreak
                 Next k
kbreak:
                If oA = 0 Then GoTo jjloop
                 For l = k + 1 To g
                    oB = dd(i, j, ii, jj, 1)
                     If oB <> 0 Then GoTo lbreak
lbreak:
                If oB = 0 Then GoTo jjloop
' Assertions
                If ab(i, j, k) + oA < 0 Then
                    GoTo jjloop
                 End If
                 If ab(torus(i + ii, n), torus(j + jj, m), k) - oA < 0 Then
                    GoTo jjloop
                 End If
                 If ab(i, j, l) + oB < 0 Then
                    GoTo jjloop
                 End If
                 If ab(torus(i + ii, n), torus(j + jj, m), 1) - oB < 0 Then
                    GoTo jjloop
                 End If
' Inner loop
                 delta = delta + Abs(oA) + Abs(oB)
                ndeals = ndeals + 1
                fA = Abs(oA)
                fB = Abs(oB)
                 ab(i, j, k) = ab(i, j, k) + oA
                 If fstat = 1 Or (fstat = 2 And oA > 0) Or (fstat = 3 And oA < 0) Then
                     flow(i, j, k) = flow(i, j, k) + fA
                 ab(torus(i + ii, n), torus(j + jj, m), k) = ab(torus(i + ii, n), torus(j + jj, m), k)
m), k) - oA
                 If fstat = 1 Or (fstat = 2 And oA < 0) Or (fstat = 3 And oA > 0) Then
                    flow(torus(i + ii, n), torus(j + jj, m), k) = flow(torus(i + ii, n),
torus(j + jj, m), k) + fA
                End If
                 ab(i, j, l) = ab(i, j, l) + oB
                 If fstat = 1 Or (fstat = 2 \text{ And oB} > 0) Or (fstat = 3 \text{ And oB} < 0) Then
                     flow(i, j, l) = flow(i, j, l) + fB
                 End If
                 ab(torus(i + ii, n), torus(j + jj, m), 1) = ab(torus(i + ii, n), torus(j + jj, m), 1)
m), 1) - oB
                 If fstat = 1 Or (fstat = 2 And oB < 0) Or (fstat = 3 And oB > 0) Then
                     flow(torus(i + ii, n), torus(j + jj, m), l) = flow(torus(i + ii, n), l)
torus(j + jj, m), l) + fB
```

A.1.6 Trade Runs

To compute trade runs, the following functions are used:

- The function ngoodsrun takes care of the interface to an .xls spreadsheet. The
 Output depends on the last two parameters. By setting them appropriately the function
 either delivers stocks, flows, or trade run statistics
- The function dorun is the main module that guides the calculation of traderuns. The
 main loop through all iterations given by an input parameter is located in that function.
 To be able to extract detailed information or statistics runs at a later stage (with a later
 call of the same function) the method of static variables is used.
- The function iteration takes care about one single trade run

```
Function ngoodsrun(g, numbiter, ohorizon, pshowoff, a As Range, Optional fstat = 0, Optional
showstat = 0)
If pshowoff < 0 Or pshowoff > 1 Then
    MsgBox ("wrong showoff probability")
    Exit Function
    End If
Dim i, j, k, l, ii, jj, n, m As Integer
n = a.Rows.Count
m = a.Columns.Count
If g < 2 Then
    MsgBox ("Anzahl Güter < 2 oder nicht ganzzahlig")
    Exit Function
End If
If Int(n / g) < 1 Then
    MsgBox ("Anzahl Zeilen zu klein")
    Exit Function
If Int(n / g) * Int(g) <> n Then
    MsgBox ("Anzahl Güter " + Str(Int(n / g)) + " inkonsistent zu Zeilenzahl")
    Exit Function
End If
n = Int(n / q)
ReDim ab(1 \text{ To } n, 1 \text{ To } m, 1 \text{ To } g) As Long
                                                                                    'stock
ReDim flow(1 To n, 1 To m, 1 To g) As Double
                                                                                   'flow
For k = 1 To g
    For i = 1 To n
       For j = 1 To m
            ab(i, j, k) = Int(a((k-1) * n + i, j).Cells.Value)
        Next j
    Next i
Next k
ii = oh(ohorizon, n)
jj = oh(ohorizon, m)
```

```
ReDim z(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long 'proposals

ReDim d(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long 'proposed deals (converted)

ReDim dd(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long 'deals

ReDim o(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To 2, 1 To 2, 1 To g) As Long 'observation
                                                                                             'observations
If showstat <> 0 Then GoTo statistics
ReDim xs(1 To 2, 1 To 18) As Double
Call dorun(1, numbiter, n, m, g, ii, jj, pshowoff, ohorizon, ab, z, d, dd, o, flow, fstat, xs)
' display
ReDim s(1 \text{ To } g * n, 1 \text{ To } m) As Double
If fstat = 0 Then
    For k = 1 To g
         For i = 1 To n
              For j = 1 To m
                  s((k-1) * n + i, j) = ab(i, j, k)
             Next j
         Next i
     Next k
Else
     For k = 1 To g
         For i = 1 To n
For j = 1 To m
                  s((k-1) * n + i, j) = flow(i, j, k)
             Next j
         Next i
     Next. k
End If
GoTo bye
statistics:
ReDim s(1 To minlong(numbiter, 5000), 1 To 18) As Double
Call dorun(0, numbiter, n, m, g, ii, jj, pshowoff, ohorizon, ab, z, d, dd, o, flow, fstat, s)
bve:
ngoodsrun = s
End Function
Sub dorun(do_show, numbiter, n, m, g, ii, jj, pshowoff, ohorizon, ab, zz, d, dd, o, flow,
fstat, xstat)
Dim ps As Double 'percentage showoff proposals
Dim op, sp, ndeals, lnod, nnod As Long
Dim delta, deltaold, mm, x, y, z As Double
Dim i, j, k, l As Long
Static s(1 To 5000, 1 To 18) As Double
If do_show = 1 Then
     lnod = 0
     mm = 0
     ReDim mab(1 To g) As Double
     For k = 1 To g
        x = gmean(n, m, k, ab)
         mab(k) = x
        mm = mm + x
    Next k
     Call inits(5000, 18, s)
     Dim c, cc As Integer 'cycle detection
     c = 5
                                 'depth of comparison
     ReDim abold(0 To c, 1 To n, 1 To m, 1 To g) As Double
     ReDim citer(0 To c) As Long
     Call saveab(n, m, g, ab, abold, cc)
     citer(cc) = 0
```

```
y = dobserveablex(n, m, g, ohorizon, ab, mab)
          z = dobserveablea(n, m, g, ohorizon, ab)
z = dfuzzycluster(n, m, g, 2, 1.5, ab)
         For i = 1 To numbiter
                 Call iteration(i, n, m, g, ii, jj, pshowoff, ohorizon, ab, zz, d, dd, o, op, sp,
ndeals, flow, fstat, delta)
                  If Abs(delta) < 0.00001 And Abs(deltaold) < 0.00001 Then
                          MsgBox ("dorun> 2 phase no delta, step: " + Str(i) + " last no deals: " +
Str(lnod))
                          GoTo ibreak
                 End If
                  deltaold = delta
                  If ndeals = 0 Then lnod = i
' cycle check when no deals are made
                  If ndeals = 0 Then
                          nnod = nnod + 1
                           j = cyclefound(n, m, g, ab, abold, citer, c)
                           If j <> 0 Then
                                   MsgBox ("dorun> cycle found at " + Str(i + 1) + " same as " + Str(citer(j)) +
" last no deals: " + Str(lnod))
                                  GoTo ibreak
                          End If
                           cc = cc + 1
                          If cc > c Then
                                  cc = 1
                           End If
                          Call saveab(n, m, g, ab, abold, cc)
                          citer(cc) = i
                  End If
                    j = i
                    If numbiter > 500 Then
                            If i <= numbiter - 500 Then
                                     GoTo iloop
                             Else
                                    j = i - numbiter + 500
                             End If
                    End If
' statistics collection
' only zero deals
                   If ndeals <> 0 Then GoTo iloop
                  j = torus(i, 5000)
                  s(j, 1) = i
                  s(j, 2) = dgmean(n, m, ab, mab(1), 1)
                  s(j, 3) = dgmean(n, m, ab, mab(2), 2)
                  s(j, 4) = dgmean(n, m, ab, mab(3), 3)
                   s(j, 5) = dall(n, m, g, ab, mab)
                   s(j, 6) = dweighted(n, m, g, ab, mab, mm)
                  s(j, 5) = dobserveablex(n, m, g, ohorizon, ab, mab)
                  s(j, 6) = dobserveablea(n, m, g, ohorizon, ab)
                  s(j, 7) = op
                  s(j, 8) = sp
                  s(j, 9) = ndeals
                  For k = 1 To minlong(3, g)
                           For l = k + 1 To minlong(3, g)
                                    If k = 1 Then GoTo lloop
                                            x = maxAprice(n, m, oh(ohorizon, n), oh(ohorizon, m), k, l, dd)
y = \text{minAprice}(n, \ m, \ oh(ohorizon, \ n), \ oh(ohorizon, \ m), \ k, \ l, \ dd) z = \text{avgAprice}(n, \ m, \ oh(ohorizon, \ n), \ oh(ohorizon, \ m), \ k, \ l, \ dd) 'MsgBox ("dorun maxminstat> k " + Str(k) + " l " + Str(l) + " max(" + Str(9 + (k - 1) * 2 + l) * (k - 1) 
- k) + " x " + Str(x)
                                             s(i, 9 + (k - 1) * 2 + 1 - k) = x
```

End Sub

```
s(j, 12 + (k - 1) * 2 + 1 - k) = y

s(j, 15 + (k - 1) * 2 + 1 - k) = z
lloop:
            Next l
       Next k
iloop:
    Next i
ibreak:
   MsgBox ("dorun> No deals encountered " + Str(nnod) + " times, last at " + Str(citer(cc)))
^{\mbox{\scriptsize I}} if any inbetween step was reached call last one back
    If citer(cc) <> 0 Then Call abback(n, m, g, ab, abold, cc)
Else
    For k = 1 To minlong (5000, numbiter)
       If s(k, 1) = 0 Then GoTo kbreak
kbreak:
    If k \le minlong(5000, numbiter) Then
        For i = 1 To k - 1
For j = 1 To 18
               xstat(i, j) = s(k - i, j)
            Next j
        Next i
    Else
         For i = 1 To minlong (5000, numbiter)
            For j = 1 To 18
                 xstat(i, j) = s(minlong(numbiter, 5000) - i + 1, j)
        Next i
    End If
End If
```

```
Sub iteration(i, n, m, g, ni, mj, pshowoff, ohorizon, ByRef ab, ByRef z, ByRef d, ByRef dd, ByRef o, op, sp, ndeals, flow, fstat, delta)
ReDim xy(1 To n, 1 To m, 1 To g) As Integer
Call showoff(n, m, g, pshowoff, ab, xy)
Call initz(n, m, g, ni, mj, z)
Call propose(i, n, m, g, ni, mj, ab, ohorizon, o, xy, z, op, sp)
Call initz(n, m, g, ni, mj, d)
Call convertproposals(n, m, g, ni, mj, z, d)
Call initz(n, m, g, ni, mj, dd)
Call acceptdeals(i, n, m, g, ni, mj, ab, z, d, dd)
Call inito(n, m, g, ni, mj, o)
Call observedeals(n, m, g, ni, mj, dd, o)
Call cleardeals(i, n, m, g, ni, mj, ab, dd, ndeals, flow, fstat, delta)
'If i > 171 Then MsgBox ("iteration Z> " + Str(i))
End Sub
Sub acceptdeals(step, n, m, g, ni, mj, ab, z, d, dd)
Dim i, j, k, ii, jj As Integer
Dim oA As Long
ReDim own(1 To g) As Long
For i = 1 To n
   For j = 1 To m
       For k = 1 To g
Next k
        reduce for proposals made
        For ii = -ni To ni
           For jj = -mj To mj
               For k = 1 To g
                   oA = z(i, j, ii, jj, k)
If oA < 0 Then own(k) = own(k) + oA
           Next jj
       Next ii
For k = 1 To g
   If own(k) < 0 Then MsgBox ("acceptdeals B step" + Str(step) + "> i=" + Str(i) + " j=" +
Str(j) + "own(" + Str(k) + ") = " + Str(own(k)))
Next k
       Call acceptijdeals(step, g, i, j, ni, mj, own, d, dd)
For k = 1 To g
    If own(k) < 0 Then MsgBox ("acceptdeals C step" + Str(step) + "> i=" + Str(i) + " j=" +
Str(j) + "own(" + Str(k) + ") = " + Str(own(k)))
Next k
   Next j
Next i
End Sub
```

A.2 Fuzzy Clustering

The following modules implement the fuzzy-c-means clustering algorithm.

A.2.1 Clustering

For clustering the following four functions are used:

- The function showgfuzzycluster takes care of the interface to an .xls spreadsheet
- The function fuzzycluster is the main module that guides the calculation
- The functions nextu, nextv calculate next approximation steps

```
Function showgfuzzycluster(g, ww, nnc, a As Range)
Dim n, m, i, j, k, nstocks As Integer
nstocks = Int(g)
n = a.Rows.Count
m = a.Columns.Count
If g < 2 Or nstocks \iff g Then
    MsgBox ("Anzahl Güter < 2 oder nicht ganzzahlig")
    Exit Function
End If
If Int(n / g) < 1 Then
    MsgBox ("Anzahl Zeilen zu klein")
    Exit Function
End If
If Int(n / g) * Int(g) <> n Then
    MsgBox ("Anzahl Güter " + Str(Int(n / g)) + " inkonsistent zu Zeilenzahl")
    Exit Function
End If
n = Int(n / g)
Dim nc As Integer
nc = Int(nnc.Cells.Value)
If nnc.Cells.Value <> nc Then
    MsgBox ("Anzahl Cluster nicht ganzzahlig")
    Exit Function
End If
If nc < 1 \text{ Or } nc > n * m \text{ Then}
    MsgBox ("Anzahl Cluster < 1 oder >" + Str(n * m))
    Exit Function
End If
Dim w As Double
w = ww.Cells.Value
If w \le 1 Then
    MsgBox ("w muss > 1")
    Exit Function
ReDim abc(1 To n, 1 To m, 1 To nstocks) As Double
For k = 1 To nstocks
    For i = 1 To n
        For j = 1 To m
            abc(i, j, k) = a((k-1) * n + i, j).Cells.Value
        Next j
    Next i
```

```
Sub fuzzycluster(ByVal w, n, m, nstocks, nc, ByRef ab, ByRef c)
Dim i, j, k, l As Integer
Dim x, xvi As Double
Dim delta As Double
delta = 0.0001
'initialize u(ij)k=c(ij)k
For i = 1 To n
    For j = 1 To m
'no zero begin
        xvi = 0
        For l = 1 To nstocks
           xvi = xvi + Abs(ab(i, j, l))
        Next 1
        If xvi = 0 Then GoTo jloop
'no zero end
        xvi = 0
        For k = 1 To nc - 1
            x = Rnd()
            If xvi + x < 1 Then
               c(i, j, k) = x
            Else
                c(i, j, k) = 1 - xvi
            End If
            xvi = xvi + c(i, j, k)
        Next k
        c(i, j, nc) = 1 - xvi
'smooth
        xvi = 0
        For k = 1 To nc
           If c(i, j, k) < 1 / (1.5 * nc) Then c(i, j, k) = 1 / (1.5 * nc) xvi = xvi + c(i, j, k)
        Next k
        For k = 1 To nc
          c(i, j, k) = c(i, j, k) / xvi
        Next k
jloop:
   Next i
Next i
'cluster centers
ReDim v(1 To nc, 1 To nstocks) As Double
Dim deltav As Double
deltav = 0
1 = 0
Call nextv(w, n, m, nc, nstocks, ab, c, v, deltav)
While deltav > delta And 1 < 300
    1 = 1 + 1
    Call nextu(w, n, m, nc, nstocks, ab, c, v)
    Call nextv(w, n, m, nc, nstocks, ab, c, v, deltav)
If deltav > delta Then
    MsgBox ("fuzzycluster> bad convergence, deltav: " + Str(deltav))
    GoTo bye
End If
For i = 1 To n
    For j = 1 To m
        xvi = 0
        For k = 1 To nc
            xvi = xvi + c(i, j, k)
       Next k
'if zeroes excluded
        If xvi \iff 0 And Abs(xvi - 1) \implies delta Then
           MsgBox ("fuzzycluster> restriction violated xvi: " + Str(xvi) + " i: " + Str(i) +
" j: " + Str(j))
       End If
    Next j
Next i
bye:
End Sub
```

```
Sub nextv(ByVal w, n, m, nc, nstocks, ByRef ab, ByRef c, ByRef v, ByRef deltav)
On Error GoTo Sorry
Dim eps As Double
eps = 0.00000000000001
Dim k, l, ll As Integer
Dim vold, vnew, sux, su, xvi As Double
Dim i, j As Integer
deltav = 0
For k = 1 To nc
    For l = 1 To nstocks
        MsgBox ("nextv> cluster" + Str(k) + " stock" + Str(l))
        sux = 0
        su = 0
         MsgBox ("nextv> cluster" + Str(k) + " stock" + Str(l) + " sux, su init")
        MsgBox ("nextv> again n" + Str(n) + " m" + Str(m))
        For i = 1 To n
            MsgBox ("nextv cluster" * Str(k) + " stock" + Str(l) + " i" + Str(i) + " i pre
xvi")
            For j = 1 To m
                 MsgBox ("nextv cluster" * Str(k) + " stock" + Str(l) + " i" + Str(i) + " j" +
Str(j) + " pre xvi")
                xvi = 0
                For ll = 1 To nstocks
                   xvi = xvi + ab(i, j, ll)
                Next. 11
                If Abs(xvi) > eps Then
                     MsgBox ("nextv cluster" * Str(k) + " stock" + Str(1) + " i" + Str(i) + "
j" + Str(j))
                    sux = sux + c(i, j, k) ^ w * ab(i, j, l)
                    su = su + c(i, j, k) ^ w
                     MsgBox ("nextv cluster" * Str(k) + " stock" + Str(1) + " i" + Str(i) + "
j" + Str(j) + " completed")
                End If
            Next j
        Next i
        MsgBox ("nextv> cluster" + Str(k) + " stock" + Str(l) + " pre completion")
        If Abs(su) > eps Then
            vnew = sux / su
            vold = v(k, 1)
            deltav = deltav + Abs(vold - vnew)
            v(k, 1) = vnew
            MsgBox ("nextv> su=0")
             GoTo bye
        End If
   Next. 1
Next k
bye:
Exit Sub
Sorrv:
If Err.Number = 6 Then
    Resume Next
   MsgBox "nextv> " & Err.Number & vbCrLf & vbCrLf & Err.Description
End If
End Sub
```

```
Sub nextu(ByVal w, n, m, nc, nstocks, ByRef ab, ByRef c, ByRef v)
On Error GoTo Sorry
Dim eps As Double
eps = 0.00000000000001
Dim i, j, k, l, ll As Integer
Dim xv, xvi, xvj, x, e As Double
ReDim singularity(1 To n, 1 To m) As Boolean
For i = 1 To n
   For j = 1 To m
       singularity(i, j) = False
   Next j
Next i
Dim nsing As Integer
For ll = 1 To nc
   for all i<=>ll among clusters seek uik
    For i = 1 To n
        For j = 1 To m
'no zero begin
            xvi = 0
            For l = 1 To nstocks
               xvi = xvi + Abs(ab(i, j, l))
            Next 1
            If xvi = 0 Then GoTo jloop
'no zero end
            for all k \le i,j among data seek uik
            xvi = 0
            For l = 1 To nstocks
                x = 0
                x = ab(i, j, l) - v(ll, l)
                xvi = xvi + x * x
                xvi = xvi + ((ab(i, j, l) - v(ll, l)) ^ 2)
            Next 1
            For k = 1 To nc
                xvj = 0
                For l = 1 To nstocks
                    x = ab(i, j, l) - v(k, l)
                    xvj = xvj + x * x
                    xvj = xvj + ((ab(i, j, 1) - v(k, 1)) ^ 2)
                Next 1
                If Abs(xvj) > eps Then
                    x = xvi / xvj
                    e = 1 / (2 * (w - 1))
                    If Abs(x) > eps Then xv = xv + Exp(e * Log(x))
                    xv = xv + (xvi / xvj) ^ (1 / (2 * (w - 1)))
                Else
                    MsgBox ("nextu> xvj = 0")
                     GoTo bye
                End If
            Next k
            new uik computed
            If Abs(xv) > eps Then
               c(i, j, 11) = 1 / xv
                MsgBox ("nextu> xv <> 0, ll" + Str(ll) + " i" + Str(i) + " j" + Str(j))
                c(i, j, 11) = 0
                singularity(i, j) = True
            End If
jloop:
       Next j
   Next i
Next 11
```

End Sub

```
'repair singularities
For i = 1 To n
    For j = 1 To m
        If singularity(i, j) Then
             MsgBox ("repair i" + Str(i) + " j" + Str(j))
             xvi = 0
             nsing = 0
For k = 1 To nc
                If Abs(c(i, j, k)) \le eps Then
                     nsing = nsing + 1
                     xvi = xvi + c(i, j, k)
                 End If
             Next k
             k = 1
             1 = 1
             While k < nsing
                 If c(i, j, l) = 0 Then
                      k = k + 1
                      x = Rnd()
x = 2 * Rnd() / nc
If xvi + x < 1 Then
c(i, j, 1) = x
                          c(i, j, 1) = 1 - xvi
                      End If
                     xvi = xvi + c(i, j, l)
                 End If
                 1 = 1 + 1
             Wend
             While 1 <= nc
                 If c(i, j, 1) = 0 Then c(i, j, 1) = 1 - xvi
                      GoTo jloop2
                 End If
                 1 = 1 + 1
             Wend
        End If
jloop2:
   Next j
Next i
bye:
Exit Sub
Sorry:
If Err.Number = 6 Then
   Resume Next
Else
   MsgBox "nextu> " & Err.Number & vbCrLf & vbCrLf & Err.Description
End If
```

A.2.2 Visualisation (Conditional Formatting)

To aid the visualisation of clusters the following functions are used:

- The function showgcenter takes care of the interface to an .xls spreadsheet
- The function fuzzyclustercenter computes the cluster centres

```
Function showgcenter (nstocks, ww, nnc, d As Range, a As Range)
Dim n, n2, m, i, j, k, l, g As Integer
Dim eps As Double
eps = 0.00001
g = Int(nstocks.Cells.Value)
n = a.Rows.Count
n2 = d.Rows.Count
m = a.Columns.Count
If m <> d.Columns.Count Then
    MsgBox ("showgcenter> Anzahl Spalten stimmen nicht überein")
    Exit Function
If Int(n2 / g) < 1 Then
    MsgBox ("Anzahl (Daten) Zeilen zu klein")
    Exit Function
If Int(n2 / g) * Int(g) <> n2 Then
    MsgBox ("Anzahl Güter " + Str(Int(n / g)) + " inkonsistent zu Zeilenzahl Daten")
    Exit Function
End If
n2 = Int(n2 / g)
If g < 2 Then
    MsgBox ("Anzahl Güter < 2 oder nicht ganzzahlig")
    Exit Function
Dim nc As Integer
nc = Int(nnc.Cells.Value)
If nnc.Cells.Value <> nc Then
    MsgBox ("Anzahl Cluster nicht ganzzahlig")
    Exit Function
End If
If nc < 1 \ Or \ nc > n \ * m \ Then
    MsgBox ("Anzahl Cluster < 1 oder >" + Str(n * m))
    Exit Function
End If
If Int(n / nc) < 1 Then
   MsgBox ("Anzahl (Cluster) Zeilen zu klein")
    Exit Function
If Int(n / nc) * Int(nc) <> n Then
   MsgBox ("Anzahl Cluster " + Str(Int(n / nc)) + " inkonsistent zu Zeilenzahl Cluster")
    Exit Function
n = Int(n / nc)
Dim w As Double
w = ww.Cells.Value
If w \le 1 Then
    MsgBox ("w muss > 1")
    Exit Function
End If
```

```
If n \iff n2 Then
    MsgBox ("showgcenter> Zeilen Daten und Zeilen Cluster inkonsistent")
    Exit Function
End If
'MsgBox ("showgcenter> n=" + Str(n) + " m=" + Str(m) + " nc=" + Str(nc) + " g=" + Str(g))
ReDim c(1 To n, 1 To m, 1 To nc) As Double
For k = 1 To nc
   For i = 1 To n
       For j = 1 To m
           c(i, j, k) = a((k - 1) * n + i, j).Cells.Value
    Next i
Next k
ReDim abc(1 To n, 1 To m, 1 To g) As Double
For k = 1 To g
   For i = 1 To n
       For j = 1 To m
'MsgBox ("showgcenter> step A.(" + Str(i) + "," + Str(j) + ")")
          abc(i, j, k) = d((k-1) * n + i, j).Cells.Value
       Next j
   Next i
Next k
ReDim cc(1 To nc, 1 To g) As Double
Call fuzzyclustercenter(w, n, m, g, nc, abc, c, cc)
showgcenter = cc
Exit Function
End Function
Sub fuzzyclustercenter(w, n, m, g, nc, abc, c, cc)
Dim i, j, k As Integer
Dim x, mu, mm, eps As Double
eps = 0.00001
For i = 1 To n
   For j = 1 To m
       x = 0
        For k = 1 To nc
           x = x + c(i, j, k)
       Next k
       If Abs(x - 1) > eps Then
            MsgBox ("showqcenter> exclusion at x(" + Str(i) + "," + Str(j) + ") = " + Str(x))
            Exit Function
       End If
    Next j
Next i
For k = 1 To nc
   For l = 1 To g
       mu = 0
       mm = 0
        For i = 1 To n
            For j = 1 To m
               x = c(i, j, k) ^ w
               mu = mu + x * abc(i, j, l)
               mm = mm + x
           Next j
       Next i
       If Abs(mm) > eps Then cc(k, 1) = mu / mm
    Next 1
Next k
End Sub
```

A.3 Exploration

The following modules were used for the exploration of the model

A.3.1 Simulation Runs

The following functions are used to generate initial distributions of wealth and to obtain statistics regarding the overall behaviour.

In the main function nsimrun the same technique (as in dorun) of using static variables to extract further details by a second call is applied.

```
Function nsimrun(Optional r = 10, Optional g = 3, Optional n = 6, Optional m = 6, Optional a = 6)
7, Optional b = 11, Optional c = 5, Optional a1 = 1, Optional a2 = 1, Optional b1 = 2,
Optional b2 = 1, Optional c1 = 1, Optional c2 = 2, Optional save = 0)
Dim i, i2, j, k, l, ii, jj As Integer
Dim pshowoff As Double
pshowoff = 1
Dim ohorizon As Integer
ohorizon = 1
ii = oh(ohorizon, n)
jj = oh(ohorizon, m)
ReDim z(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long ReDim d(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long
                                                                                          'proposals
                                                                                         'proposed
deals (converted)
                                                                                         'deals
ReDim dd(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To g) As Long
ReDim o(1 To n, 1 To m, -ii To ii, -jj To jj, 1 To 2, 1 To 2, 1 To g) As Long
                                                                                         'observations
ReDim ab(1 \text{ To } n, 1 \text{ To } m, 1 \text{ To } g) As Long
                                                                                          'stock
Static absave(1 To 3, 1 To 6, 1 To 6, 1 To 3) As Long
                                                                                          'stock save
                                                                                          'flow
ReDim flow(1 To n, 1 To m, 1 To g) As Double
Dim ns As Integer
ns = 10
ReDim s(1 To r, 1 To ns) As Double
                                                                                         'simulation
statistics
ReDim xs(1 To ns) As Double
Dim x As Double
If save <> 0 Then
    ReDim s(1 To g * n, 1 To m) As Double
    For k = 1 To g
For i = 1 To n
            For j = 1 To m
                 s((k-1) * n + i, j) = absave(1, i, j, k)
                 If save = 2 Then s((k-1) * n + i, j) = absave(2, i, j, k)
             Next j
        Next i
    Next k
    GoTo bye
End If
For i = 1 To r
    Call newab(g, n, m, a, b, c, a1, a2, b1, b2, c1, c2, ab)
    Call saveab(n, m, g, ab, absave, 3)
    For k = 1 To g
       x = 0
        For i2 = 1 To n
            For j = 1 To m
                x = x + ab(i2, j, k)
            Next j
        Next i2
        If k \le ns - 7 Then s(i, 7 + k) = x
    Call simrun(2000, n, m, q, ii, jj, pshowoff, ohorizon, ab, z, d, dd, o, flow, xs)
    s(i, 1) = i
    For j = 2 To 7
        s(i, j) = xs(j)
    Next j
    If s(i, 2) > 2000 Then
        Call moveab(n, m, g, absave, 3, 1)
MsgBox ">2000 ..." + Str(absave(1, 6, 6, 3)) + "..2.." + Str(absave(3, 6, 6, 3))
    End If
    If s(i, 5) = 2 Then
        Call moveab(n, m, g, absave, 3, 2)
MsgBox ">cycle 2..." + Str(absave(2, 6, 6, 3)) + "..2.." + Str(absave(3, 6, 6, 3))
Next i
bye:
nsimrun = s
End Function
```

```
Sub simrun(numbiter, n, m, g, ii, jj, pshowoff, ohorizon, ab, zz, d, dd, o, flow, s)
Dim ps As Double 'percentage showoff proposals Dim op, sp, ndeals, lnod, nnod As Long
Dim delta, deltaold, mm, x, y, z As Double
Dim i, j, k, l As Long
lnod = 0
mm = 0
ReDim mab(1 To g) As Double
For k = 1 To g
   x = gmean(n, m, k, ab)
   mab(k) = x
   mm = mm + x
Next k
                         'cycle detection
Dim c, cc As Integer
                        'depth of comparison
c = 20
ReDim abold(0 To c, 1 To n, 1 To m, 1 To g) As Double
ReDim citer (0 To c) As Long
cc = 0
Call saveab(n, m, g, ab, abold, cc)
citer(cc) = 0
For i = 1 To numbiter
    Call iteration(i, n, m, g, ii, jj, pshowoff, ohorizon, ab, zz, d, dd, o, op, sp, ndeals,
flow, fstat, delta)
    If Abs(delta) < 0.00001 And Abs(deltaold) < 0.00001 Then
        s(2) = i
        s(3) = lnod
        s(4) = 0
        s(5) = 0
        GoTo ibreak
    End If
    deltaold = delta
    If ndeals = 0 Then lnod = i
' cycle check when no deals are made
    If ndeals = 0 Then
        nnod = nnod + 1
        j = cyclefound(n, m, g, ab, abold, citer, c)
        If j <> 0 Then
            s(2) = i
            s(3) = lnod
            s(4) = citer(j)
            If j <= cc Then
                s(5) = cc - j
                s(5) = c + cc - j
            End If
            GoTo ibreak
        End If
        cc = cc + 1
        If cc > c Then
            cc = 1
        End If
        Call saveab(n, m, g, ab, abold, cc)
        citer(cc) = i
    End If
iloop:
Next i
ibreak:
s(6) = nnod
s(7) = citer(cc)
If i > numbiter Then
    s(2) = i
    s(3) = lnod
    s(4) = 0
    s(5) = 0
End If
End Sub
```

A.3.2 Model (Trade Run) Statistics

The following functions are used during trade runs, to compute and collect various statistics, e.g. average prices, number of deals, exchanged quantities etc.

```
Function qtyijAbuys(oii, ojj, i, j, dd)
Dim ii, jj As Integer
Dim noA As Double
For ii = -oii To oii
    For jj = -ojj To ojj
       If dd(i, j, ii, jj, 1) > 0 Then noA = noA + dd(i, j, ii, jj, 1)
Next ii
qtyijAbuys = noA
End Function
Function qtyijAsales(oii, ojj, i, j, dd)
Dim ii, jj As Integer
Dim noA As Double
'MsgBox ("qtyAijsales> i: " + Str(i) + " j: " + Str(j))
For ii = -oii To oii
    For jj = -ojj To ojj
If dd(i, j, ii, jj, 1) < 0 Then
'MsgBox ("qtyAijsales> qty: " + Str(dd(i, j, ii, jj, 1)))
            noA = noA - dd(i, j, ii, jj, 1)
         End If
    Next jj
Next ii
qtyijAsales = noA
End Function
Function qtyAsales(n, m, oii, ojj, dd)
Dim i, j, ii, jj As Integer
Dim noA As Double
For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                 If dd(i, j, ii, jj, 1) < 0 Then
    noA = noA - dd(i, j, ii, jj, 1)</pre>
            Next jj
        Next ii
    Next j
Next i
qtyAsales = noA
End Function
```

End Function

```
Function qtyAbuys(n, m, oii, ojj, dd)
Dim i, j, ii, jj As Integer
Dim noA As Double
For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
                 If dd(i, j, ii, jj, 1) > 0 Then
    noA = noA + dd(i, j, ii, jj, 1)
                End If
            Next jj
        Next ii
    Next j
Next i
qtyAbuys = noA
End Function
Function noAbuys(n, m, oii, ojj, dd)
Dim i, j, ii, jj, noA As Integer
For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj
If dd(i, j, ii, jj, 1) > 0 Then noA = noA + 1
            Next jj
        Next ii
   Next j
Next i
noAbuys = noA
End Function
Function noAsales(n, m, oii, ojj, dd)
Dim i, j, ii, jj, k, noA As Integer
For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
            For jj = -ojj To ojj

If dd(i, j, ii, jj, 1) < 0 Then noA = noA + 1
            Next jj
        Next ii
   Next j
Next i
noAsales = noA
```

```
Function minAprice(n, m, oii, ojj, k, l, dd)
Dim i, j, ii, jj, a As Integer
Dim b, bmin As Double
bmin = 9E+15
For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
             For jj = -ojj To ojj
                  a = dd(i, j, ii, jj, k)
                  If a <> 0 Then
                      b = Abs(dd(i, j, ii, jj, l) / a)
                      If b <> 0 And b < bmin Then bmin = b
             Next jj
        Next ii
   Next j
Next i
If bmin >= 9E+15 Then bmin = 0
minAprice = bmin
End Function
Function maxAprice(n, m, oii, ojj, k, l, dd)
Dim i, j, ii, jj, a As Integer
Dim b, bmax As Double
bmax = 0
For i = 1 To n
    For j = 1 To m
        For ii = -oii To oii
             For jj = -ojj To ojj
                  a = dd(i, j, ii, jj, k)
If a <> 0 Then
                      b = Abs(dd(i, j, ii, jj, l) / a)
If b <> 0 And b > bmax Then bmax = b
                  End If
             Next jj
        Next ii
    Next j
Next i
maxAprice = bmax
End Function
Function avgAprice(n, m, oii, ojj, k, l, dd)
Dim i, j, ii, jj As Integer
Dim a, ak, b, bl, aprice As Double
For i = 1 To n
    For j = 1 To m
         For ii = -oii To oii
             For jj = -ojj To ojj
                  ak = Abs(dd(i, j, ii, jj, k))
bl = Abs(dd(i, j, ii, jj, l))
If ak <> 0 And bl <> 0 Then
                      a = a + ak
                      b = b + bl
                  End If
             Next jj
        Next ii
    Next j
Next i
If a <> 0 Then aprice = b / a
avgAprice = aprice
End Function
```

A.3.3 Finding Cycles

```
Function cyclefound(n, m, g, ab, abold, citer, c)
Dim cc, j As Integer
'MsgBox ("cyclefound> start")
j = 0
For cc = 0 To c
    If citer(cc) <> 0 Then
        If cyclefoundcc(n, m, g, ab, abold, citer, c, cc) Then
            j = cc
            GoTo ccbreak
       End If
    End If
Next cc
ccbreak:
cyclefound = j
'MsgBox ("cyclefound> passed")
End Function
Function cyclefoundcc(n, m, g, ab, abold, citer, c, cc)
Dim i, j, k As Integer
Dim starti, startj As Integer
Dim startok As Boolean
Dim a, maxg As Long
'MsgBox ("cyclefoundcc> start")
starti = 0
startj = 0
startok = False
For i = 1 To n
    For j = 1 To m
        startok = True
        maxg = 0
        For k = 1 To g
            a = ab(i, j, k)
            If a > maxg Then maxg = a
            If a \iff abold(cc, i, j, k) Then startok = False
        If startok Then
            If maxg = 0 Then GoTo jcontinue
            starti = i - 1
            startj = j - 1
            GoTo ibreak
        End If
jcontinue:
   Next j
'MsgBox ("cyclefoundcc> no startij identified")
GoTo bye
ibreak:
'assert.debug startok = True
'MsgBox ("cyclefoundcc> startij identified" + Str(starti) + ", " + Str(startj))
For i = 1 To n
    For j = 1 To m
        For k = 1 To g
'MsgBox ("cyclefoundcc> run" + Str(i) + ", " + Str(j) + ", " + Str(k))
If ab(torus(starti + i, n), torus(startj + j, m), k) <> abold(cc, i, j, k) Then 'MsgBox ("cyclefoundcc> inequality identified")
                startok = False
                GoTo bye
            End If
```

```
Next k
Next j
Next i
bye:
cyclefoundcc = startok
'MsgBox ("cyclefoundcc> passed")
End Function
```

A.3.4 Computing Distances

```
Function dobserveablex(n, m, g, ohorizon, ab, mab)
Dim i, ii, j, jj, k As Integer
Dim x, y, z, mk, mm As Double
ReDim xy(1 \text{ To } n, 1 \text{ To } m, 1 \text{ To } g) As Long
Call showoff(n, m, g, 1, ab, xy)
mm = 0
For k = 1 To g
   mm = mm + mab(k)
Next k
y = 0
For i = 1 To n
     For j = 1 To m
           For k = 1 To g
                z = ab(i, j, k)

mk = mab(k)
                For ii = -oh(ohorizon, n) To oh(ohorizon, n)
                      For jj = -oh(ohorizon, m) To oh(ohorizon, m)

If ii = 0 And jj = 0 Then GoTo jjloop

x = ab(torus(i + ii, n), torus(j + jj, m), k)

If x <> 0 Then y = y + Abs(z - x) * mk
jjloop:
                     Next jj
               Next ii
          Next k
    Next j
Next i
x = (2 * oh(ohorizon, n) + 1) * (2 * oh(ohorizon, m) + 1) - 1
dobserveablex = y / (n * m * x * mm)
End Function
```

```
Function dfuzzycluster(n, m, g, nc, w, ab)
ReDim c(1 To n, 1 To m, 1 To nc) As Double ReDim cc(1 To nc, 1 To g) As Double
ReDim x(1 \text{ To nc}) As Double
Dim i, j As Integer
Dim y As Double
Call fuzzycluster(w, n, m, g, nc, ab, c)
Call fuzzyclustercenter(w, n, m, g, nc, ab, c, cc)
For i = 1 To no
    x(i) = 0
    For j = 1 To g
     y = y + cc(i, j) ^ 2
    Next j
   x(i) = y ^0.5
Next i
y = 0
For i = 1 To nc - 1
y = y + Abs(x(i) - x(i + 1))
Next i
dfuzzycluster = y
End Function
Function gmean(n, m, k, ab)
Dim i, j, a As Integer
Dim x, y As Double
x = 0
a = 0
For i = 1 To n
    For j = 1 To m
      y = ab(i, j, k)
If y \le 0 Then GoTo jloop
       x = x + y
        a = a + 1
jloop:
   Next j
Next i
If a > 0 Then gmean = x / a
End Function
Function dgmean(n, m, ab, mm, k)
Dim i, j, a As Integer
Dim x, y As Double
x = 0
y = 0
For i = 1 To n
   For j = 1 To m
      x = ab(i, j, k)
If x \le 0 Then GoTo jloop
       y = y + Abs(mm - x)
        a = a + 1
jloop:
   Next j
Next i
If a > 0 Then dgmean = y / a
End Function
```

```
Function dall(n, m, g, ab, mab)
Dim i, j, k As Integer
Dim x, y As Double
x = 0
For i = 1 To n
    For j = 1 To m
        y = 0
         For k = 1 To g
          y = y + Abs(ab(i, j, k) - mab(k))
        Next k
        x = x + y
    Next j
Next i
dall = x / (n * m)
End Function
Function dweighted(n, m, g, ab, mab, mm)
Dim i, j, k As Integer
Dim x, y As Double
x = 0
For i = 1 To n
    For j = 1 To m
        y = y + Abs(ab(i, j, k) - mab(k)) * mab(k)
Next k
        x = x + y / mm
   Next j
Next i
dweighted = x / (n * m)
End Function
Function dobserveablea(n, m, g, ohorizon, ab)
Dim i, ii, j, jj, k As Integer
Dim x, y, z As Double
For i = 1 To n
   For j = 1 To m
        For k = 1 To g
             z = ab(i, j, k)
For ii = -oh(ohorizon, n) To oh(ohorizon, n)
                 For jj = -oh(ohorizon, m) To oh(ohorizon, m)

If ii = 0 And jj = 0 Then GoTo jjloop
                     y = y + Abs(z - ab(torus(i + ii, n), torus(j + jj, m), k))
jjloop:
                 Next jj
            Next ii
        Next k
   Next j
Next i
x = (2 * oh(ohorizon, n) + 1) * (2 * oh(ohorizon, m) + 1) - 1 dobserveablea = y / (n * m * g * x)
End Function
```

A.4 Auxiliary Functions

```
Function wealth(x, n)
Dim y As Double
y = x * n
wealth = Int(y + 0.49)
End Function
Sub saveab(n, m, g, ab, abold, cc)
Dim i, j, k As Integer
For i = 1 To n
   For j = 1 To m
       For k = 1 To g
           abold(cc, i, j, k) = ab(i, j, k)
      Next k
   Next j
Next i
End Sub
Sub abback(n, m, g, ab, abold, cc)
Dim i, j, k As Integer
For i = 1 To n
   For j = 1 To m
      For k = 1 To g
      ab(i, j, k) = abold(cc, i, j, k)
Next k
   Next j
Next i
End Sub
Function ldistg(g, p)
Dim x As Double
Dim i, j As Integer
i = p
x = Log(g)
While j > 1 And i > 1
   j = j - 1
i = i - 1
   x = x + Log(j)
Wend
ldistg = x
End Function
Function torus (ByVal x, n)
While x < 1
x = x + n
Wend
While x > n
   x = x - n
Wend
torus = x
End Function
```

```
Function oh (ByVal x, n)
If x < 1 Then x = 1
While (2 * x) > (n - 1) And x > 1
x = x - 1
oh = x
End Function
Sub initz(n, m, g, ni, mj, ByRef z)
Dim i, j, ii, jj, k As Integer For i = 1 To n
    For j = 1 To m
        For ii = -ni To ni
For jj = -mj To mj
                 For k = 1 To g
                   z(i, j, ii, jj, k) = 0
                 Next k
             Next jj
        Next ii
    Next j
Next i
End Sub
Sub inito(n, m, g, ni, mj, ByRef o)
Dim i, j, ii, jj, r, s, k As Integer For i = 1 To n  
For j = 1 To m
        For ii = -ni To ni
For jj = -mj To mj
For r = 1 To 2
                       For s = 1 To 2
                          For k = 1 To g
                               o(i, j, ii, jj, r, s, k) = 0
                           Next k
                       Next s
                  Next r
             Next jj
         Next ii
    Next j
Next i
End Sub
Sub inits(a, b, s)
Dim i, j As Integer
For i = 1 To a
   For j = 1 To b
       s(i, j) = 0
    Next j
Next i
End Sub
Function minlong(a, b)
Dim x As Long
x = a
If b < x Then x = b
minlong = x
End Function
```

```
Sub moveab(n, m, g, abold, f, t)
Dim i, j, k As Integer

For i = 1 To n
    For j = 1 To m
    For k = 1 To g
        abold(t, i, j, k) = abold(f, i, j, k)
    Next k
    Next j
Next i
End Sub
```