Resolución de los Ejercicios Propuestos

① $\displaystyle\int \frac{x-2}{\sqrt{x^2-4x+1}}\,dx$

Solución

$$\int \frac{x-2}{\sqrt{x^2-4x+1}}\,dx = \int \frac{\frac{du}{2}}{\sqrt{u}}$$

$$\boxed{\begin{aligned} u &= x^2-4x+1 \\ du &= 2x-4\,dx \\ du &= 2(x-2)\,dx \\ \frac{du}{2} &= (x-2)\,dx \end{aligned}}$$

$$= \frac{1}{2}\int u^{-1/2}\,du$$

$$= \frac{1}{2}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right) + C = u^{1/2} + C$$

$$= \sqrt{u} + C = \sqrt{x^2-4x+1} + C$$

② $\displaystyle\int \frac{1}{x^4+x^3-4x^2-4x}\,dx$

Solución

$$\int \frac{1}{x^4+x^3-4x^2-4x}\,dx$$

$$\boxed{\begin{aligned} x^4+x^3-4x^2-4x &= x\left[x^3+x^2-4x-4\right] \\ &= x\left[x^2(x+1)-4(x+1)\right] \\ &= x\left[(x+1)(x^2-4)\right] \\ &= x\left[(x+1)(x+2)(x-2)\right] \end{aligned}}$$

$$= \int \frac{1}{x(x+1)(x+2)(x-2)}\,dx$$

$$= \int \left(\frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+2)} + \frac{D}{(x-2)}\right) dx$$

$$\frac{1}{x(x+1)(x+2)(x-2)} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+2)} + \frac{D}{(x-2)}$$

$$1 = \frac{Ax+A+Bx}{(x^2+x)} + \frac{Cx-2C+Dx+2D}{(x^2-4)}$$

$$1 = (Ax+A)(x^2-4) + Bx(x^2-4) + (Cx-2C)(x^2+x) + (Dx+2D)(x^2+x)$$

$$1 = Ax^3-4Ax+Ax^2-4A+Bx^3-4Bx+Cx^3+Cx^2-2Cx^2-2Cx + Dx^3+Dx^2+2Dx^2+2Dx$$

$$1 = Ax^3+Ax^2-4Ax-4A+Bx^3-4Bx+Cx^3-Cx^2-2Cx+Dx^3+3Dx^2+2Dx$$

$$1 = (A+B+C+D)x^3 + (A-C+3D)x^2 + (-4A-4B-2C+2D)x - 4A$$

$$\begin{cases} A+B+C+D=0 \\ A-C+3D=0 \\ -4A-4B-2C+2D=0 \\ -4A=1 \end{cases}$$

$-8A-3B=1$

$-8\left(-\frac{1}{4}\right)-3B=1$

$2-3B=1$

$B=\frac{1}{3}$

$A=-\frac{1}{4}$

$A-C+3D=0$

$D=\frac{C-A}{3}$

$A+B+C+D=0$

$\left(-\frac{1}{4}\right)+\left(\frac{2}{3}\right)+C+\left(\frac{C-A}{3}\right)=0$

$-3+8+12C+4C-4A=0$

$5+16C-4\left(-\frac{1}{4}\right)=0$

$6+16C=0$

$C=-\frac{6}{16}$

$C=-\frac{3}{8}$

$D=\frac{C-A}{3}$

$D=\frac{-\frac{3}{8}+\frac{1}{4}}{3}$

$D=\frac{-3+2}{24}$

$D=-\frac{1}{24}$

$$= \int \frac{A}{x}\,dx + \int \frac{B}{x+1}\,dx + \int \frac{C}{x+2}\,dx + \int \frac{D}{x-2}\,dx$$

$$= \int \frac{-\frac{1}{4}}{x}\,dx + \int \frac{\frac{2}{3}}{x+1}\,dx + \int \frac{-\frac{3}{8}}{x+2}\,dx + \int \frac{-\frac{1}{24}}{x-2}\,dx$$

$$= -\frac{1}{4}\int \frac{dx}{x} + \frac{2}{3}\int \frac{dx}{x+1} - \frac{3}{8}\int \frac{dx}{x+2} - \frac{1}{24}\int \frac{dx}{x-2}.$$

$$= -\frac{1}{4}\ln|x| + \frac{2}{3}\ln|x+1| - \frac{3}{8}\ln|x+2| - \frac{1}{24}\ln|x-2| + C'$$

$\ln x - \ln y$

$\ln\left(\frac{x}{y}\right)$

$$= \ln\left(\frac{\sqrt[3]{(x+1)^2}}{\sqrt[4]{x}}\right) - \ln\left(\frac{\sqrt[8]{(x+2)^3}}{\sqrt[24]{x-2}}\right) + C'$$

$$= \ln\left(\frac{\frac{\sqrt[3]{(x+1)^2}}{\sqrt[4]{x}}}{\frac{\sqrt[8]{(x+2)^3}}{\sqrt[24]{x-2}}}\right) + C'$$

$$= \left(\frac{\sqrt[3]{(x+1)^2}\cdot\sqrt[24]{x-2}}{\sqrt[4]{x}\cdot\sqrt[8]{(x+2)^3}}\right)$$

③ $\int \cos^3 x\, dx$

Solución

$$\int \cos^3 x\, dx = \int \cos x \cdot \cos^2 x\, dx$$

$u = \operatorname{sen} x$
$du = \cos x\, dx$

$$= \int \cos x\,(1 - \operatorname{sen}^2 x)\, dx$$
$$= \int (\cos x - \cos x \operatorname{sen}^2 x)\, dx$$
$$= \int \cos x\, dx - \int \cos x \operatorname{sen}^2 x\, dx$$
$$= \operatorname{sen} x - \int u^2\, du$$
$$= \operatorname{sen} x - \frac{1}{3} u^3 + C$$
$$= \operatorname{sen} x - \frac{1}{3} \operatorname{sen}^3 x + C'$$

④ $\int \cos^6 x\, dx$

Solución

$$\int \cos^6 x\, dx$$

Reducción de integrales $\Rightarrow$ $\boxed{\int \cos^n x\, dx = \frac{\operatorname{sen} x \cos^{n-1} x}{n} + \left(\frac{n-1}{n}\right) \int \cos^{n-2} x\, dx}$

$$= \int \cos^6 x\, dx = \frac{\operatorname{sen} x \cos^5 x}{6} + \frac{5}{6} \int \cos^4 x\, dx$$

$$= \frac{1}{6} \operatorname{sen} x \cos^5 x + \frac{5}{6}\left[\frac{\operatorname{sen} x \cos^3 x}{4} + \frac{3}{4} \int \cos^2 x\, dx \right]$$

$$= \frac{1}{6} \operatorname{sen} x \cos^5 x + \frac{5}{24} \operatorname{sen} x \cos^3 x + \frac{15}{24} \int \cos^2 x\, dx$$

$$= \frac{1}{6} \operatorname{sen} x \cos^5 x + \frac{5}{24} \operatorname{sen} x \cos^3 x + \frac{15}{24}\left[\frac{\operatorname{sen} x \cos x}{2} + \frac{1}{2} \int \cos^0 x\, dx \right]$$

$$= \frac{1}{6} \operatorname{sen} x \cos^5 x + \frac{5}{24} \operatorname{sen} x \cos^3 x + \frac{15}{48} \operatorname{sen} x \cos x + \frac{15}{48} x + C'$$

$= \frac{1}{6}\cos^5 x \, \text{sen}\, x + \frac{5}{24}\cos^3 \text{sen}\, x + \frac{5}{16}\cos x \,\text{sen}\, x + \frac{5}{16}x + C$

⑤ $\int tg^3 x \, dx$

Solución

$\int tg^3 x \, dx = \int tg x . tg^2 x \, dx = \int tg x (sec^2 x - 1) dx$

$u = tg x$
$du = sec^2 x \, dx$

$= \int (tg x \, sec^2 x - tg x) \, dx$

$= \int tg x \, sec^2 x - \int tg x \, dx$

$= \int u \, du - Ln|sec x|$

$= \frac{1}{2} u^2 - Ln|sec x| + C$

$= \frac{1}{2} tg^2 x - Ln|sec x| + C$

⑥ $\int \frac{\sqrt{x}}{1+x} dx$

Solución

$u = \sqrt{x}$
$u^2 = x$
$2u\,du = dx$

$\int \frac{\sqrt{x}}{1+x} dx = \int \frac{u(2u\,du)}{1+u^2} = 2\int \frac{u^2 \, du}{1+u^2}$

$= 2\int \left(1 - \frac{1}{1+u^2}\right) du = 2\int du - 2\int \frac{du}{1+u^2}$

$= 2u - 2 \, arctg \, u + C$

$= 2\sqrt{u} - 2 \, arctg \sqrt{x} + C$

⑦ $\int \frac{1}{x - \sqrt[3]{x}} dx$

Solución

$\int \frac{1}{x - \sqrt[3]{x}} dx$

⑧ $\int \cos x \ln(\operatorname{sen} x)\, dx$

Solución

$$\int \cos x \ln(\operatorname{sen} x)\, dx = \ln(\operatorname{sen} x)\cdot \operatorname{sen} x - \int \cancel{\operatorname{sen} x} \cdot \frac{\cos x}{\cancel{\operatorname{sen} x}}\, dx$$

$u = \ln(\operatorname{sen} x)$

$du = \frac{\cos x}{\operatorname{sen} x}\, dx$

$\int dv = \int \cos x\, dx$

$v = \operatorname{sen} x$

$$= \operatorname{sen} x \ln(\operatorname{sen} x) - \int \cos x\, dx$$

$$= \operatorname{sen} x \ln(\operatorname{sen} x) - \operatorname{sen} x + C$$

$$= \operatorname{sen} x\left[\ln(\operatorname{sen} x) - 1\right] + C$$

⑨ $\int \frac{\sqrt{25-x^2}}{x}\, dx$

Solución

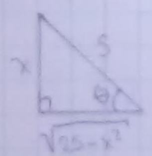
$$\int \frac{\sqrt{25-x^2}}{x}\, dx = \int \frac{\left(\sqrt{25-x^2}\right)\left(\sqrt{25-x^2}\right)}{x\left(\sqrt{25-x^2}\right)}\, dx = \int \frac{25-x^2}{x\sqrt{25-x^2}}\, dx$$

$\operatorname{sen}\theta = \frac{x}{5}$

$x = 5\operatorname{sen}\theta$

$dx = 5\cos\theta\, d\theta$

$\theta = \operatorname{arcsen}\left(\frac{x}{5}\right)$



$$= \int \left(\frac{25}{x\sqrt{25-x^2}} - \frac{x^2}{x\sqrt{25-x^2}}\right) dx$$

$$= \int \frac{25}{x\sqrt{25-x^2}}\, dx - \int \frac{x^{\cancel{2}}}{\cancel{x}\sqrt{25-x^2}}\, dx$$

$$= 25\int \frac{dx}{x\sqrt{25-x^2}} - \int \frac{x\, dx}{\sqrt{25-x^2}}$$

$$= 25\int \frac{\cancel{5}\cos\theta\, d\theta}{\cancel{5}\operatorname{sen}\theta\sqrt{25-25\operatorname{sen}^2\theta}} - \int \frac{5\operatorname{sen}\theta\, 5\cos\theta\, d\theta}{\sqrt{25-25\operatorname{sen}^2\theta}}$$

$$= 25\int \frac{\cos\theta\, d\theta}{\operatorname{sen}\theta\sqrt{25(1-\operatorname{sen}^2\theta)}} - \int \frac{25\operatorname{sen}\theta\cos\theta\, d\theta}{\sqrt{25(1-\operatorname{sen}^2\theta)}}$$

$$= \cancel{25}\int \frac{\cos\theta\, d\theta}{\cancel{5}\operatorname{sen}\theta\sqrt{1-\operatorname{sen}^2\theta}} - \int \frac{\cancel{25}\operatorname{sen}\theta\cos\theta\, d\theta}{\cancel{5}\sqrt{1-\operatorname{sen}^2\theta}}$$

$$= 5\int \frac{\cos\theta\, d\theta}{\cancel{\cos\theta}\operatorname{sen}\theta} - \int \frac{5\operatorname{sen}\theta\cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}}$$

$$= 5\int \csc\theta\, d\theta - 5\int \operatorname{sen}\theta\, d\theta$$

$$= 5 \ln|\csc\theta - \cot\theta| + 5\cos\theta + C$$

$$= 5 \ln\left|\frac{5}{x} - \frac{\sqrt{25-x^2}}{x}\right| + \cancel{5}\cdot\frac{\sqrt{25-x^2}}{\cancel{5}} + C$$

$$= 5 \ln\left|\frac{5-\sqrt{25-x^2}}{x}\right| + \sqrt{25-x^2} + C$$

(10)

$$\int \frac{x^2+3x-4}{x^2-2x-8}\,dx$$

Solución

$$\int \frac{x^2+3x-4}{x^2-2x-8} = \int\left(1 + \frac{5x+4}{x^2-2x-8}\right)dx$$

$$= \int dx + \int \frac{5x+4}{x^2-2x-8}\,dx = x + \int \frac{5x+4}{(x-\ )(x\ \ )}\,dx + C$$

$$= x + \int \frac{A}{(x+2)}\,dx + \int \frac{B}{(x-4)}\,dx + C$$

$$\frac{5x+4}{(x+2)(x-4)} = \frac{A}{(x+2)} + \frac{B}{(x-4)}$$

$$5x+4 = A(x-4) + B(x+2)$$

$$5x+4 = Ax - 4A + Bx + 2B$$

$$(A+B)x + (-4A+2B)$$

$$A+B = 5$$
$$-4A+2B = 4$$

$$4A + 4B = 20$$
$$-4A + 2B = 4$$
$$6B = 24$$
$$B = 4$$

$A+B=5 \quad A=1$

$A = 5-B$

$A = 5-4 = 1$

$$= x + \int \frac{1}{x+2}\,dx + \int \frac{4}{x-4}\,dx + C$$

$$= x + \ln|x+2| + 4\ln|x-4| + C$$

$$= x + \ln|(x+2)\cdot(x-4)^4| + C$$

(13) $\int e^{2x}\cos 3x\,dx$

solución

$$\int e^{2x}\cos 3x\,dx = e^{2x}\cdot\frac{1}{3}\operatorname{sen}3x - \int \frac{1}{3}\operatorname{sen}3x\cdot 2e^{2x}dx$$

$u = e^{2x}$
$du = 2e^{2x}dx$

$dv = \cos 3x\,dx$
$\int dv = \int \cos 3x\,dx$
$v = \frac{1}{3}\operatorname{sen}3x$

$t = e^{2x}$
$dt = 2e^{2x}dx$

$dv = \operatorname{sen}3x\,dx$
$v = -\frac{1}{3}\cos 3x$

$$= \frac{1}{3}e^{2x}\operatorname{sen}3x - \frac{2}{3}\int e^{2x}\operatorname{sen}3x\,dx$$

$$= \frac{1}{3}e^{2x}\operatorname{sen}3x - \frac{2}{3}\left[e^{2x}\cdot\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x\,2e^{2x}dx\right.$$

$$= \frac{1}{3}e^{2x}\operatorname{sen}3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\int e^{2x}\cos 3x\,dx$$

$$\int e^{2x}\cos 3x\,dx + \frac{4}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\operatorname{sen}3x + \frac{2}{9}e^{2x}\cos 3x + C$$

$$\frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\operatorname{sen}3x + \frac{2}{9}e^{2x}\cos 3x + C$$

$$13\int e^{2x}\cos 3x\,dx = \overset{3}{\cancel{9}}\left(\frac{1}{\cancel{3}}e^{2x}\operatorname{sen}3x + \frac{2}{\cancel{9}}e^{2x}\cos 3x\right) + C$$

$$\int e^{2x}\cos 3x\,dx = \frac{1}{13}\,e^{2x}3\operatorname{sen}3x + e^{2x}2\cos 3x + C$$

$$= \frac{1}{3}e^{2x}(3\operatorname{sen}3x + 2\cos 3x) + C$$

(14)

(15) $\int x \arcsen(x^2)\,dx$

solución

$$\int x \arcsen(x^2)\,dx = \arcsen(x^2)\cdot\frac{x^2}{2} - \int \frac{1}{2}x^2\cdot\frac{2x}{\sqrt{1-x^4}}\,dx$$

$u = \arcsen(x^2)$

$du = \frac{2x}{\sqrt{1-x^4}}\,dx$

$\int dv = \int x\,dx$

$v = \frac{x^2}{2}$

$t = 1-x^4$

$dt = -4x^3\,dx$

$-\frac{dt}{4} = x^3\,dx$

$$= \frac{1}{2}x^2\arcsen(x^2) - \frac{1}{2}\int\frac{2x^3}{\sqrt{1-x^4}}\,dx$$

$$= \frac{1}{2}x^2\arcsen(x^2) - \int\frac{\frac{-dt}{4}}{\sqrt{t}}$$

$$= \frac{1}{2}x^2\arcsen(x^2) + \frac{1}{4}\int t^{-1/2}\,dt$$

$$= \frac{1}{2}x^2\arcsen(x^2) + \frac{1}{4}\left(\frac{t^{1/2}}{\frac{1}{2}}\right) + C$$

$$= \frac{1}{2}x^2\arcsen(x^2) + \frac{1}{2}\sqrt{t} + C$$

$$= \frac{1}{2}x^2\arcsen(x^2) + \frac{1}{2}\sqrt{1-x^4} + C$$

(17) $\displaystyle\int \frac{x^3+x^2+x+3}{(x^2+1)(x^2+3)}\,dx$

Solución

$$\int \frac{x^3+x^2+x+3}{(x^2+1)(x^2+3)} = \int \frac{Ax+B}{(x^2+1)}\,dx + \int \frac{Cx+D}{(x^2+3)}\,dx$$

$$\frac{x^3+x^2+x+3}{(x^2+1)(x^2+3)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+3)}$$

$$x^3+x^2+x+3 = (Ax+B)(x^2+3)+(Cx+D)(x^2+1)$$

$$x^3+x^2+x+3 = Ax^3+3Ax+Bx^2+3B+Cx^3+Cx+Dx^2+D.$$

$$x^3+x^2+x+3 = (A+C)x^3+(B+D)x^2+(3A+C)x+(3B+D)$$

$$\begin{array}{l|l|l}
A+C=1 & -3A-3C=-3 \ (-3) & -3B-3D=-3 \ (-3) \\
B+D=1 & \underline{3A+C=1} & \underline{3B+D=3} \\
3A+C=1 & -2C=-2 & -2D=0 \\
3B+D=3 & C=1 & D=0 \\
 & A=0 & B=1.
\end{array}$$

$$=\int \frac{0x+1}{x^2+1}\,dx + \int \frac{x+0}{x^2+3} = \int \frac{dx}{x^2+1} + \int \frac{x\,dx}{x^2+3}$$

$u = x^2+3$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$= \operatorname{arctg} \quad + \int \frac{\frac{du}{2}}{u}$$

$$= \operatorname{arctg} x + \frac{1}{2}\int \frac{du}{u} = \operatorname{arctg} x + \frac{1}{2}\ln(u) + C$$

$$= \operatorname{arctg} x + \frac{1}{2}\ln|x^2+3| + C$$

$$= \operatorname{arctg} x + \ln\left|\sqrt{x^2+3}\right| + C$$

(19) $\int tg^4 x \sec^4 dx$

Solución

$$\int tg^4 x \sec^4 dx = \int [tg^4 x\ (tg^2 x + 1)\ \sec^2 x]\, dx$$

$$= \int (tg^6 x + tg^4 x)\ \sec^2 x\ dx$$

$$= \int (tg^6 x \cdot \sec^2 x\ +\ tg^4 x \sec^2 x)\, dx$$

$u = tg\, x$
$du = \sec^2 x\, dx$

$$= \int u^6 \cdot du\ +\ \int u^4 \cdot du$$

$$= \frac{1}{7} u^7 + \frac{1}{5} u^5 + C$$

$$= \frac{1}{7} tg^7 x + \frac{1}{5} tg^5 x + C$$

(21) $\int \ln(x^2+1)\,dx$

solución

$$\int \ln(x^2+1)\,dx = \ln(x^2+1)\cdot x - \int x\cdot\frac{2x}{x^2+1}\,dx$$

$u = \ln(x^2+1)$

$du = \frac{2x}{x^2+1}\,dx$

$dv = dx$

$v = x$

$$= x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}\,dx$$

$$= x\ln(x^2+1) - \int \left(\frac{-2}{x^2+1} + 2\right)dx$$

$$= x\ln(x^2+1) + 2\int \frac{dx}{x^2+1} - 2\int dx$$

$$= x\ln(x^2+1) + 2\,\text{arctg}\,x - 2x + C$$

(22) $\int \text{tg}x\sqrt{\sec x}\,dx$

solución

$$\int \text{tg}x\sqrt{\sec x}\,dx = \int \left(\frac{\text{tg}x\,\sqrt{\sec x}\,\sqrt{\sec x}}{\sqrt{\sec x}}\right)dx$$

$$= \int \frac{\text{tg}x\,\sec x}{\sqrt{\sec x}}\,dx = \int \frac{du}{\sqrt{u}} = \int u^{-1/2}\,du$$

$u = \sec x$

$du = \sec x\cdot \text{tg}x\,dx$

$$= 2u^{1/2} + C' = 2\sqrt{u} + C$$

$$= 2\sqrt{\sec x} + C$$

(28) $\displaystyle\int \frac{\operatorname{sen}^2 x}{\cos^4 x}\,dx$

Solución

$$\int \frac{\operatorname{sen}^2 x}{\cos^4 x}\,dx = \int \left(\frac{\operatorname{sen}^2 x}{\cos^2 x} \cdot \frac{1}{\cos^2 x}\right) dx$$

$$= \int \operatorname{tg}^2 x \cdot \sec^2 x\,dx$$

$$\boxed{\begin{array}{l} u = \operatorname{tg} x \\ du = \sec^2 x\,dx \end{array}} \quad = \int u^2 \cdot du = \frac{1}{3} u^3 + C$$

$$= \frac{1}{3} \operatorname{tg}^3 x + C$$

(27) $\displaystyle\int \frac{1+tg\,x}{1-tg\,x}\,dx$

Solución

$$\int \frac{1+tg\,x}{1-tg\,x} = \int \frac{1+\frac{sen\,x}{cos\,x}}{1-\frac{sen\,x}{cos\,x}}\,dx = \int \frac{\frac{cos\,x+sen\,x}{\cancel{cos\,x}}}{\frac{cos\,x-sen\,x}{\cancel{cos\,x}}}\,dx$$

$$= \int \frac{cos\,x+sen\,x}{cos\,x-sen\,x}\,dx$$

$u = cos\,x - sen\,x$
$du = -sen\,x - cos\,x\,dx$
$-du = sen\,x + cos\,x\,dx$

$$= \int \frac{-du}{u} = -\int \frac{du}{u}$$

$$= -Ln\,|u| + C$$

$$= -Ln\,|cos\,x - sen\,x| + C$$

(33) $\int x^2 \operatorname{arctg} x\, dx$

solución

$$\int x^2 \operatorname{arctg} x\, dx = \operatorname{arctg} x \cdot \frac{1}{3}x^3 - \int \frac{1}{3}x^3 \cdot \frac{1}{1+x^2}\, dx$$

$u = \operatorname{arctg} x$

$du = \frac{1}{1+x^2}$

$dv = x^2 dx$

$v = \frac{1}{3}x^3$

$$= \frac{1}{3}x^3 \operatorname{arctg} x - \frac{1}{3}\int \frac{x^3}{1+x^2}\, dx$$

$$= \frac{x^3}{3} \operatorname{arctg} x - \frac{1}{3}\int \left(\frac{-x}{1+x^2} + x\right) dx$$

$$= \frac{x^3}{3} \operatorname{arctg} x + \frac{1}{3}\int \frac{x\, dx}{1+x^2} - \int x\, dx$$

$t = 1+x^2$

$dt = 2x\, dx$

$\frac{dt}{2} = x\, dx$

$$= \frac{x^3}{3} \operatorname{arctg} x + \frac{1}{3}\int \frac{\frac{dt}{2}}{t} - \frac{x^2}{2} + C$$

$$= \frac{x^3}{3} \operatorname{arctg} x + \frac{1}{6}\int \frac{dt}{t} - \frac{x^2}{2} + C'$$

$$= \frac{x^3}{3} \operatorname{arctg} x + \frac{1}{6} \ln|t| - \frac{x^2}{2} + C'$$

$$= \frac{x^3}{3} \operatorname{arctg} x + \frac{1}{6} \ln(1+x^2) - \frac{x^2}{2} + C$$

(36) $\int \frac{\arcsin\sqrt{x}}{\sqrt{x}}\,dx$

solución

$$\int \frac{\arcsin\sqrt{x}}{\sqrt{x}}\,dx = \int \arcsin\sqrt{x} \cdot x^{-1/2}\,dx$$

$u = \arcsin\sqrt{x}$

$du = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$

$du = \frac{1}{2\sqrt{x}\cdot\sqrt{1-x}}\,dx$

$dv = x^{-1/2}\,dx$

$v = 2x^{1/2}$

$v = 2\sqrt{x}$

$$= \arcsin\sqrt{x} \cdot 2\sqrt{x} - \int \frac{\cancel{2\sqrt{x}}}{\cancel{2\sqrt{x}}\sqrt{1-x}}\,dx$$

$$= \arcsin\sqrt{x}\cdot 2\sqrt{x} - \int \frac{1}{\sqrt{1-x}}\,dx$$

$$= 2\sqrt{x}\arcsin\sqrt{x} - \int (1-x)^{-1/2}\,dx$$

$$= 2\sqrt{x}\arcsin\sqrt{x} - 2(1-x)^{1/2} + C$$

$$= 2\sqrt{x}\arcsin\sqrt{x} - 2\sqrt{1-x} + C$$

(37) $\int x \operatorname{arctg}\left(\sqrt{x^2-1}\right)dx$

solución

$$\int x \operatorname{arctg}\left(\sqrt{x^2-1}\right)dx = \operatorname{arctg}\left(\sqrt{x^2-1}\right)\cdot\frac{1}{2}x^2 - \int \frac{x^2}{2}\cdot\frac{1}{x\sqrt{x^2-1}}\,dx$$

$u = \operatorname{arctg}\left(\sqrt{x^2-1}\right)$

$du = \frac{1}{x\sqrt{x^2-1}}\,dx$

$\int dv = \int x\,dx$

$v = \frac{x^2}{2}$

$t = x^2-1$

$dt = 2x\,dx$

$\frac{dt}{2} = x\,dx$

$$= \frac{1}{2}x^2\operatorname{arctg}\left(\sqrt{x^2-1}\right) - \frac{1}{2}\int \frac{x}{\sqrt{x^2-1}}\,dx$$

$$= \frac{1}{2}x^2\operatorname{arctg}\left(\sqrt{x^2-1}\right) - \frac{1}{2}\int \frac{\frac{dt}{2}}{\sqrt{t}}$$

$$= \frac{1}{2}x^2\operatorname{arctg}\left(\sqrt{x^2-1}\right) - \frac{1}{4}\int t^{-1/2}\,dt$$

$$= \frac{1}{2}x^2\operatorname{arctg}\left(\sqrt{x^2-1}\right) - \frac{1}{\cancel{4}}\left(\cancel{2}t^{1/2}\right) + C$$

$$= \frac{1}{2}x^2\operatorname{arctg}\left(\sqrt{x^2-1}\right) - \frac{1}{2}\sqrt{t} + C$$

$$= \frac{1}{2}x^2\operatorname{arctg}\left(\sqrt{x^2-1}\right) - \frac{1}{2}\sqrt{x^2-1} + C$$

(40) $\int \frac{1}{\cos^4 x}\,dx$

Solución

$$\int \frac{1}{\cos^4 x}\,dx = \int \sec^4 x\,dx = \int \sec^2 x \cdot \sec^2 x\,dx$$

$u = tg\,x$

$du = \sec^2 x\,dx$

$$= \int \sec^2 x\,(1 + tg^2 x)\,dx = \int (\sec^2 x + tg^2 x \sec^2 x)\,dx$$

$$= \int \sec^2 x\,dx + \int tg^2 x \sec^2 x\,dx$$

$$= tg\,x + \int u^2 \cdot du = tg\,x + \frac{1}{3}u^3 + C$$

$$= tg\,x + \frac{1}{3} tg^3 x + C$$