Universidade Federal
de Campina Grande

Processamento Adaptativo de Sinais

Introdução à Filtragem Adaptativa

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Universidade Federal de Campina Grande (UFCG)

Sumário

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1. Filtros lineares FIR
2. Cálculo de gradientes com relação à parâmetros de filtros FIR lineares
3. Exemplos de aplicação
   - Identificação de sistemas lineares
   - Equalização linear

Filtros lineares FIR

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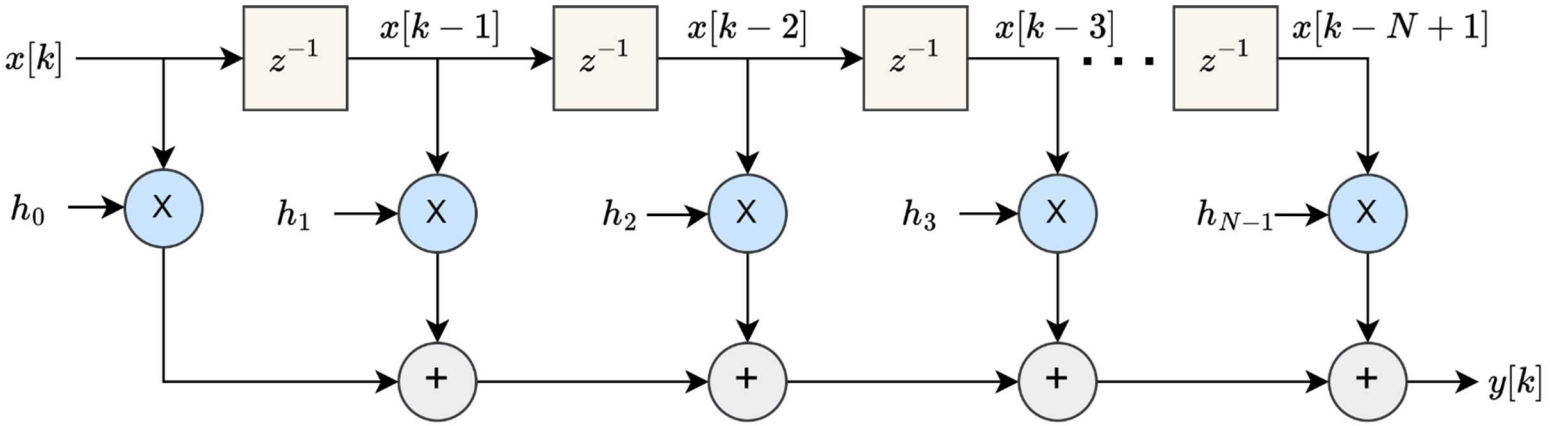
- **Filtros lineares com resposta ao impulso finita (*finite impulse response* – FIR)**:



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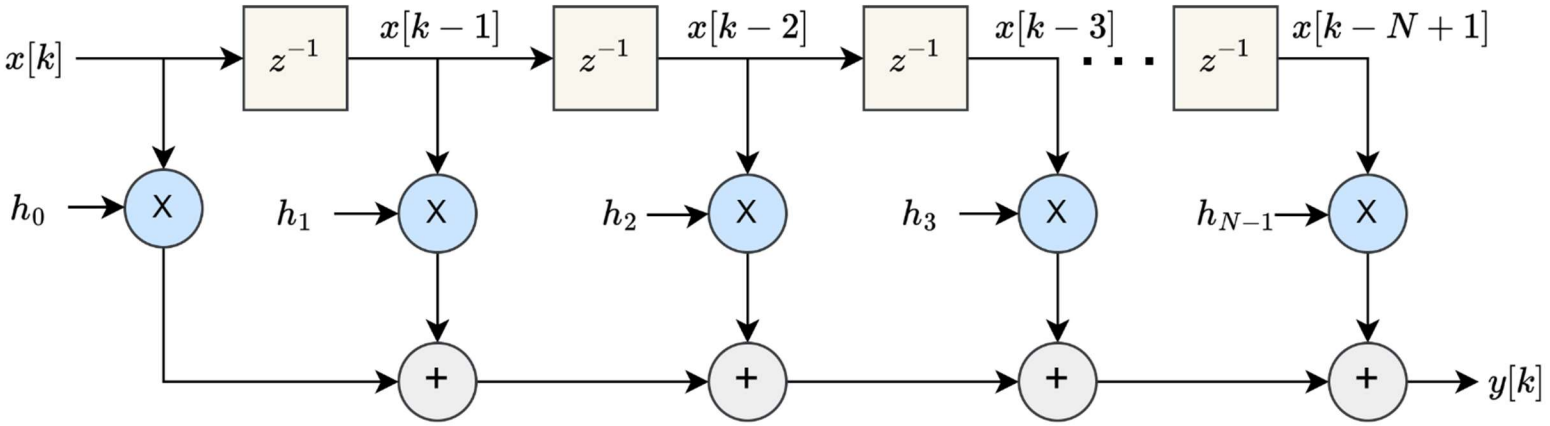

$$\boldsymbol{x}[k] = [x[k], x[k-1], \cdots, x[k-N+1]]^T$$

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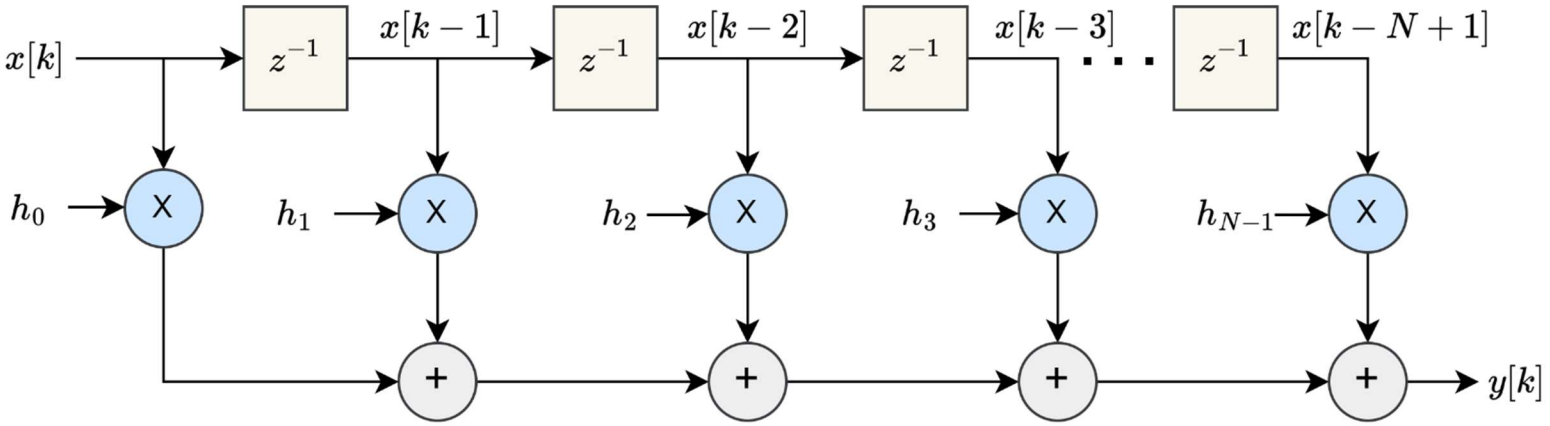

$$\boldsymbol{x}[k] = [x[k], x[k-1], \cdots, x[k-N+1]]^T$$

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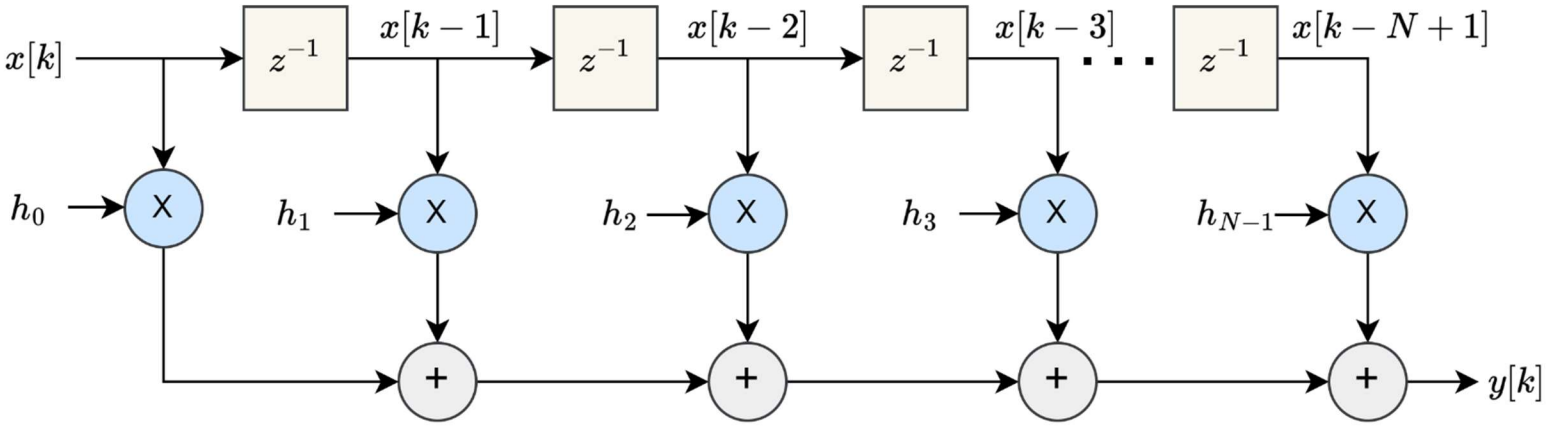

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$$\boldsymbol{h}[k] = [h_0[k], h_1[k], \cdots, h_{N-1}[k]]^T$$

$$y[k] = \sum_{i=0}^{N-1} h_i[k]x[k-i] = \boldsymbol{h}^T[k]\boldsymbol{x}[k]$$

Cálculo de gradientes com relação à parâmetros de filtros FIR lineares

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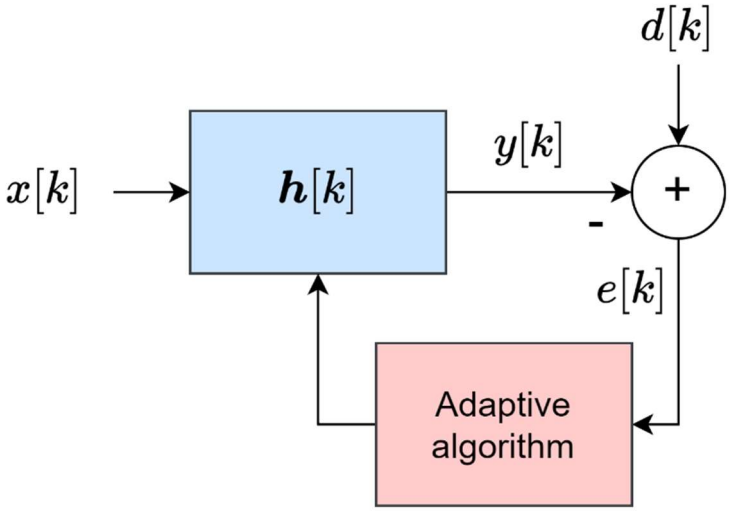
- **Gradientes da função erro quadrático instantâneo**:



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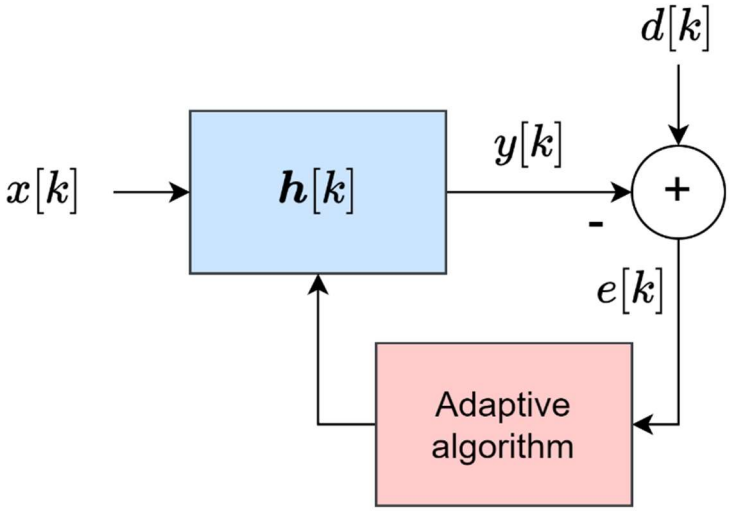
$$J[k] = |e[k]|^2 = (d[k] - y[k])^2$$



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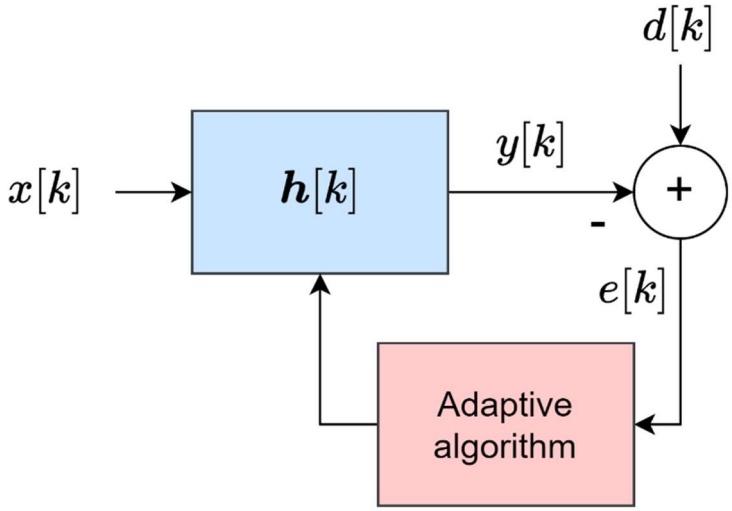

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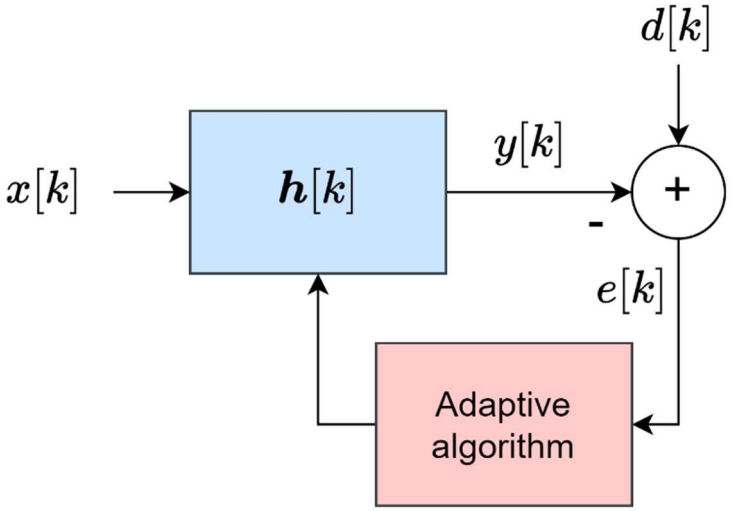

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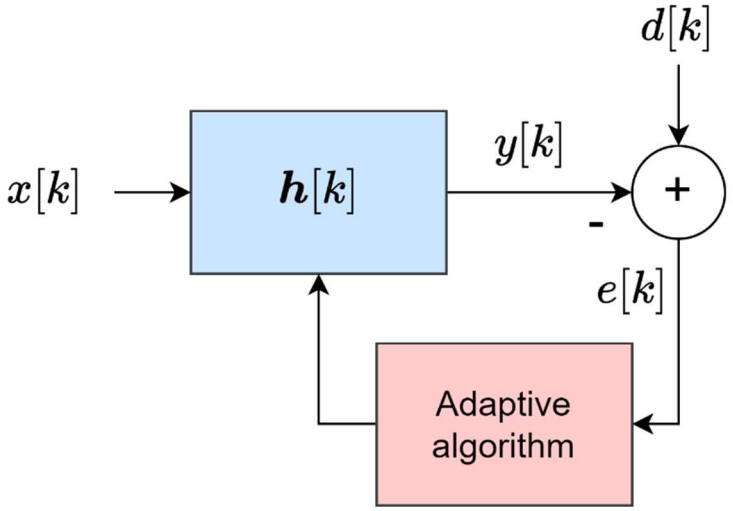

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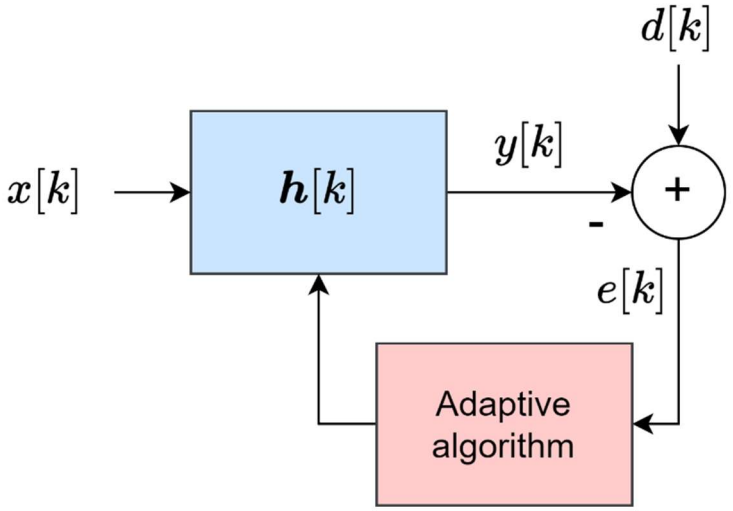

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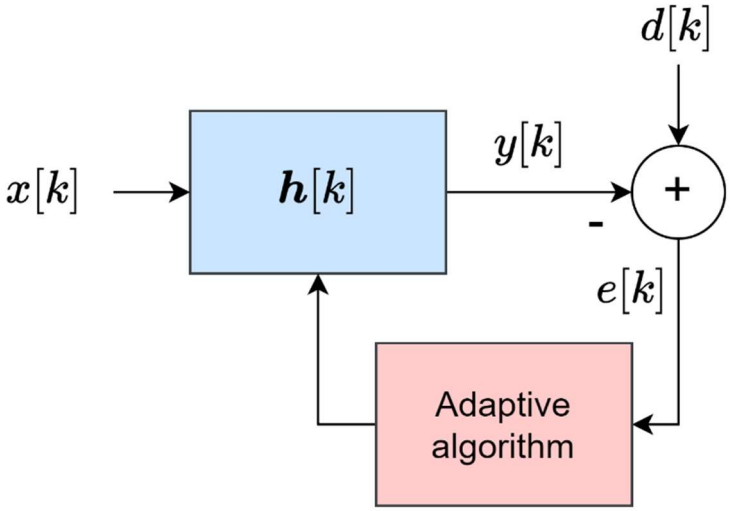

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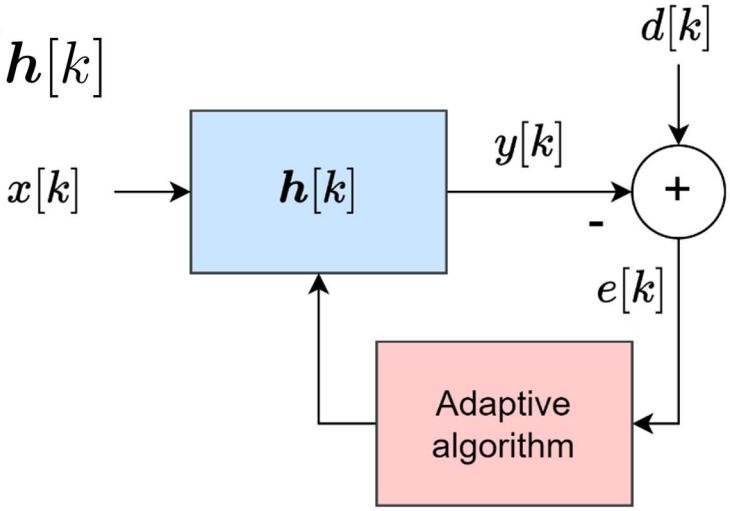

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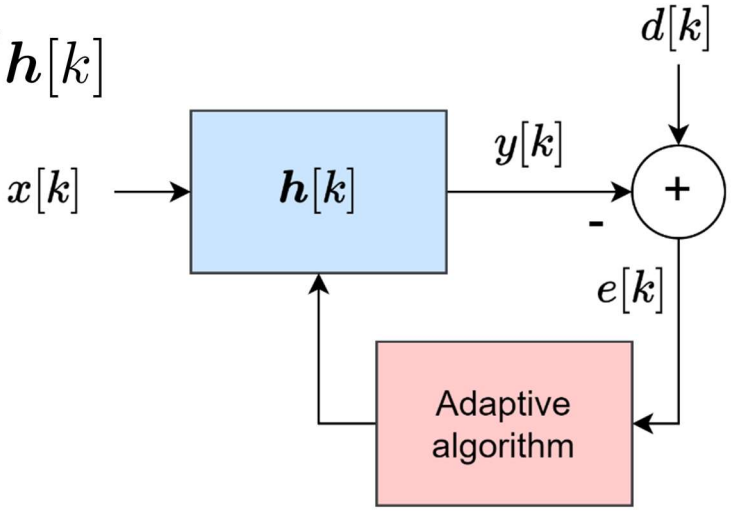

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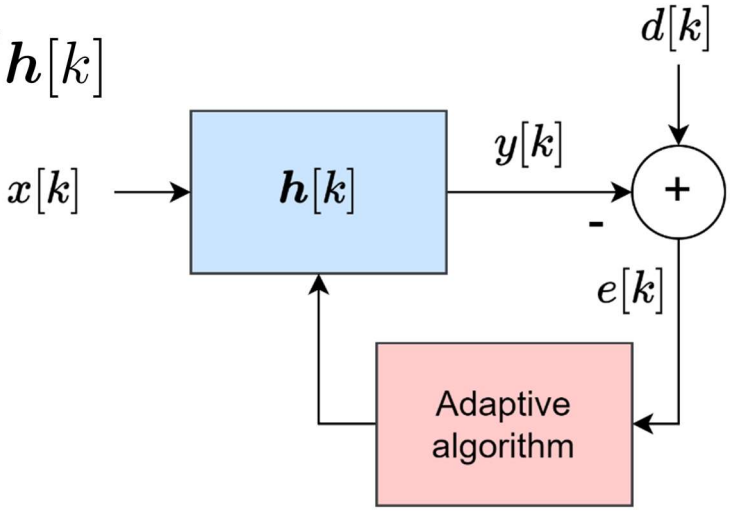

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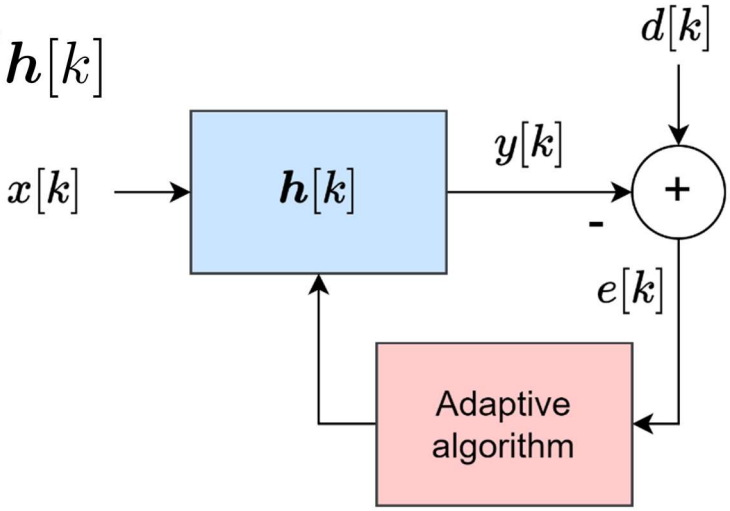
$$\frac{\partial J[k]}{\partial \boldsymbol{h}[k]} = -2(d[k] - y[k])\boldsymbol{x}[k]$$



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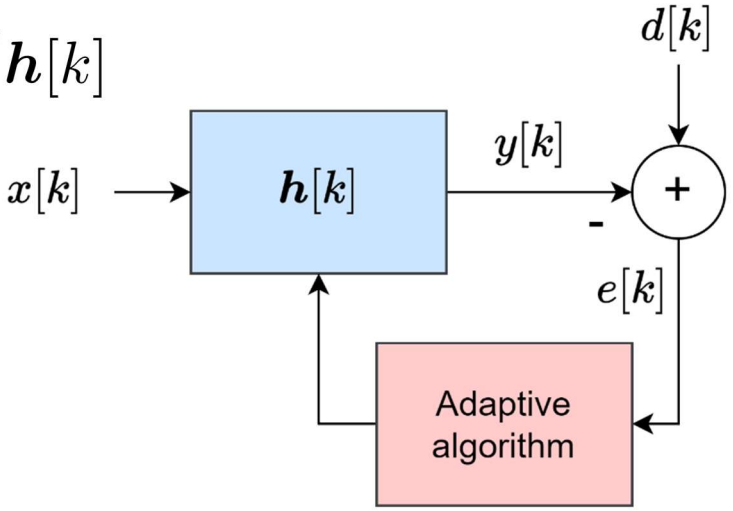

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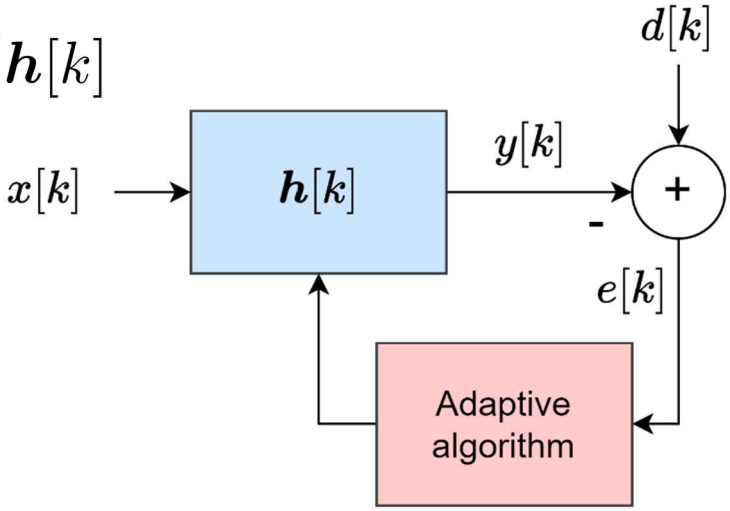

Atualização de $\boldsymbol{h}[k]$ no sentido oposto ao do vetor gradiente:

$$\Delta\boldsymbol{h}[k] = 2e[k]\boldsymbol{x}[k]$$

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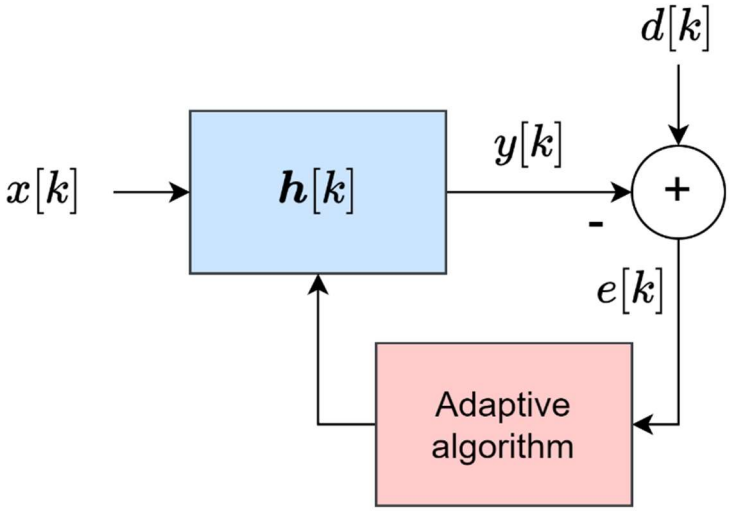
- **Algoritmo do gradiente descendente para função erro quadrático instantâneo**:

$$\boldsymbol{h}[k+1] = \boldsymbol{h}[k] + \mu \Delta \boldsymbol{h}[k]$$

$$\boldsymbol{h}[k+1] = \boldsymbol{h}[k] + \mu e[k] \boldsymbol{x}[k]$$



Exemplos de aplicação

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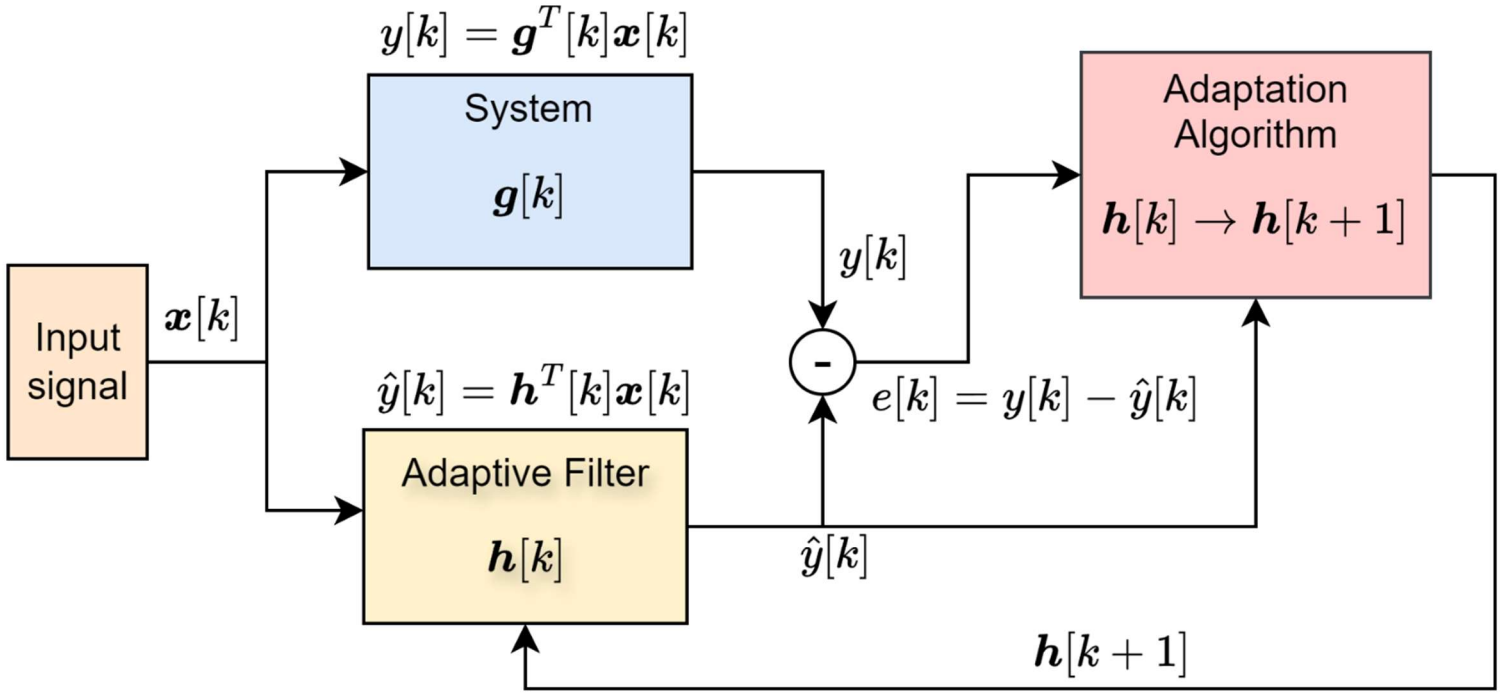
- Exemplo: identificação de sistemas LIT



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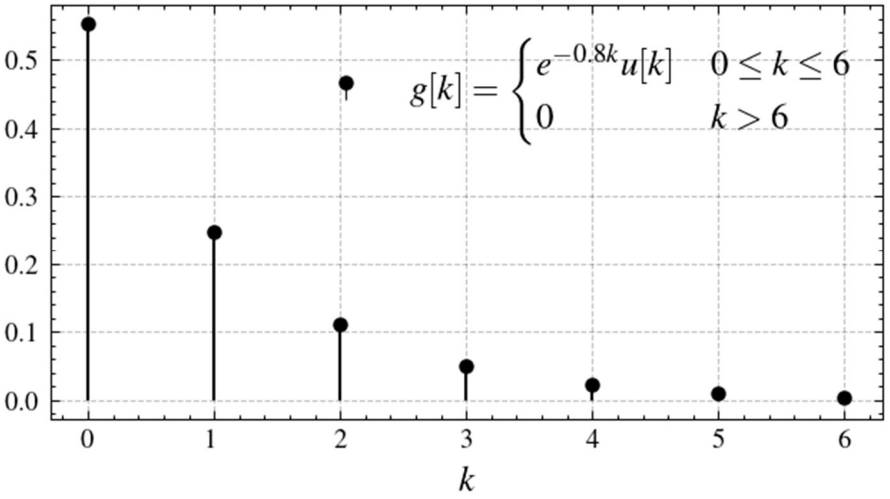

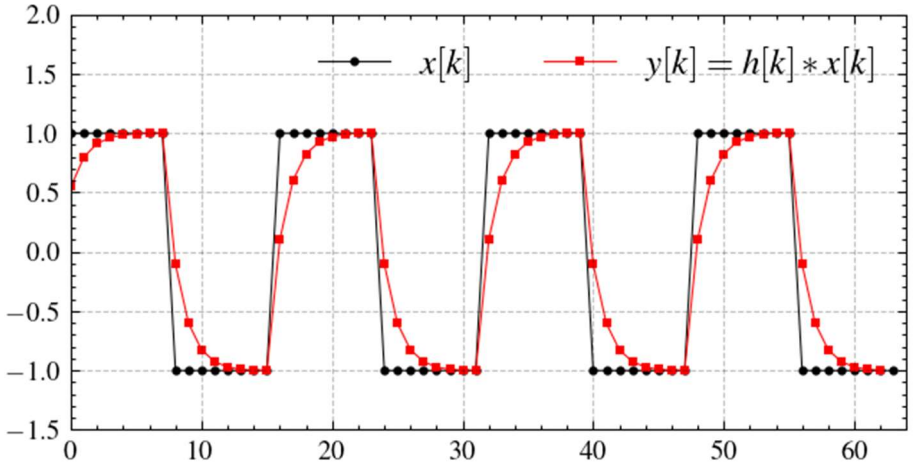

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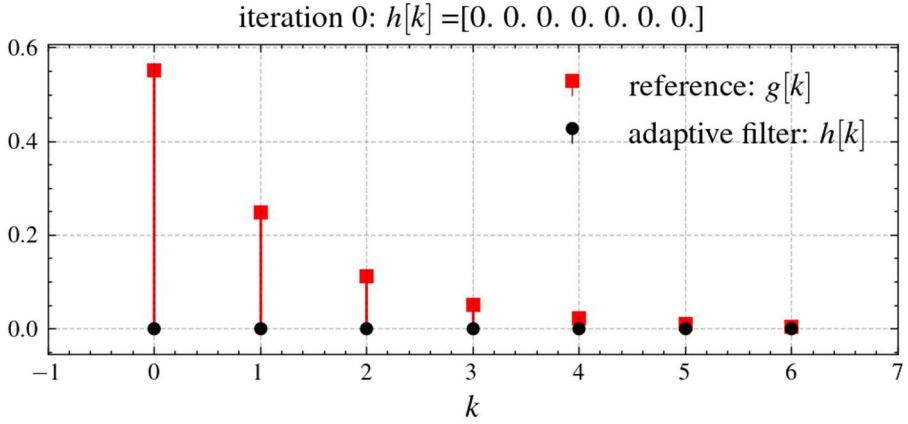

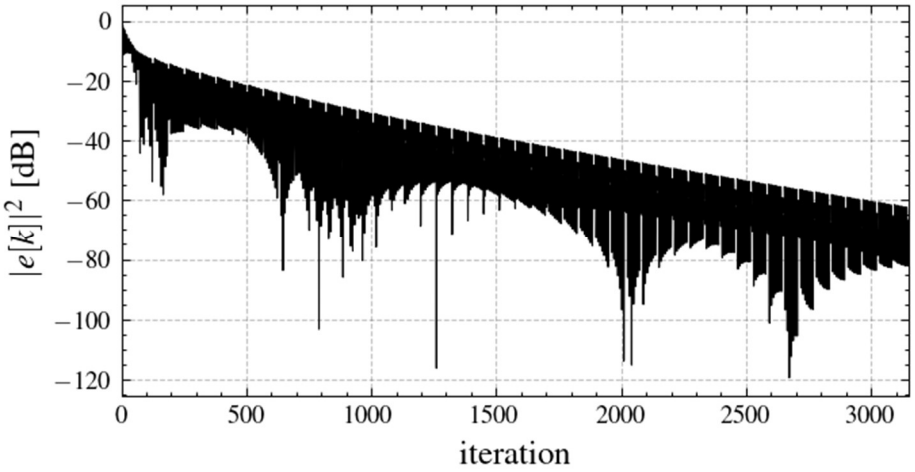

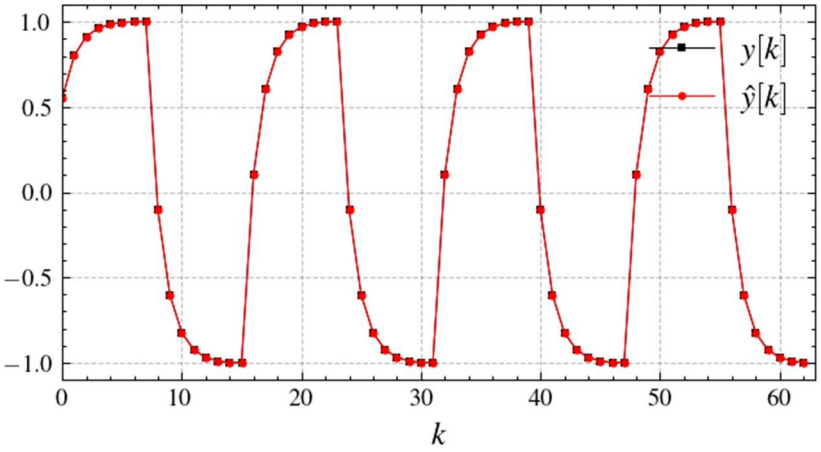

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