# AN EVALUATION OF THE ALTMAN Z-SCORE MODEL IN PREDICTING CORPORATE BANKRUPTCY FOR CANADIAN PUBLICLY LISTED FIRMS

by

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# Approval

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# Abstract

The purpose of this study is to assess the effectiveness of Altman's Z score in predicting corporate bankruptcy for Canadian listed companies. First, we estimate Altman's original model and test the efficiency of the cut-off region, coefficients and the variables used. Secondly, we add Cash Flow from Operation (CFO) to Total Liabilities (TL) ratio as a sixth variable to improve Altman's (1968) standard Z score model. By testing and comparing these models using a sample of 70 bankrupt firms against a population sample of 1,047 non-distressed firms, this study determines which models has a higher discriminating ability. While Altman's research focuses on manufacturing companies, for the purpose of this study, we selected firms from five sectors; energy, consumer discretionary, consumer staples, industrials and materials. We use Multiple Discriminant Analysis (MDA) to compare the predictive abilities of these models. Our results show that the cut-off regions and the coefficients used by Altman should be time-varying and that our augmented model produces better results, when removing the effects of outliers.



**Keywords:** Altman Z score, MDA, Six variable model, RD-CA model, CFO to Total Liability ratio, Canada, Corporate Bankruptcy, Bankruptcy prediction.

# Dedication

I would like to dedicate this thesis equally to my mother Dr. Mahjabeen Sultana Begum and my wife Shaila Shams who have been immensely supportive of my work. I would also like to dedicate this work to my father, Dr. Farid Ahmed who has been an inspiration for me since I was little and naïve. Last but not the least, I would like to dedicate this work to the hundreds of MSc students to come in the future cohorts – May you find the same amount of enjoyment that I felt undertaking this thesis and this Master of Finance.

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I would like to dedicate this thesis to my parents, Parmeelabye Govind and Sanjai Govind without whom I would not have been able to pursue this Master degree in finance. I would also like to thank my sisters, Yajna Govind and Bhavna Govind for their unflinching support.

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# Table of Contents

# List of Figures

# List of Tables

# Glossary

| | |
|---|---|
| TSE | Toronto Stock Exchange |
| LSE | London Stock Exchange |
| NYSE | New York Stock Exchange |
| AI | Artificial Intelligence |
| NF | Non-Failed or Healthy Firms |
| EBIT | Earnings before Interest and Tax |
| NF | Non Failed Firms or Non-Bankrupt Firms or Healthy firms |
| MT Algorithm | Mersenne Twister algorithm |
| Firms | The terms **Firms** and **Companies** have been used interchangeably in the following literature |

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# 1: Introduction

## 1.1 Purpose of Study and Overview

In the wake of the 2008 financial crisis, corporate bankruptcy has become an issue of paramount importance. Despite being developed in the 1960s, Altman's Z score is still extensively used to predict corporate bankruptcy. While Altman (1968) proved the efficacy of his model using manufacturing firms during the period from 1946 to 1965, this study examines how effective his model is in predicting bankruptcy for a sample of Canadian firms for the period of 1998 to 2017. Our research question also examines whether the original Z score can be improved upon.

The purpose of this study is firstly to provide an in-depth analysis of this popular Z-score model in the perspective of the Canadian Publicly listed companies. Secondly, to extend the original model by adding CFO to Total Debt ratio as the sixth variable, hereafter referred to as the RD-CA model, and thirdly, to investigate whether the new model created has a higher bankruptcy predictability than the original model.

To arrive at the RD-CA model, data have been collected over the years 1998 to 2017. This study is broken down into 3 further sections for better understanding and to assist the reader to delve deeper into the subject matter.

Section 2 provides a literature review that summarizes the important studies in the field of Z-score and associated improvement studies. The section also undertakes a detailed analysis of past research material to ascertain the best practices in improving and determining the Z-score variables. It also approaches the different methods used in testing the precision of Z-scores as a predictor for bankruptcy.

Section 3, Data and Methodology explains in detail the source of Data and how the Data has been obtained. The section goes further to give a clear overview of the methodologies used in this study. It starts with the methodologies used to derive the coefficients of the revised Z-score model along with a revised version of the J-UK model with an added CFO to Debt ratio. Our revised coefficients and added variable are then used to derive a new model which has been called RD-CA model.

Section 4 contains the results of the study where all the four models – the original Altman's Z-score, the revised Altman's Z-score, the J-UK model and the RD-CA model are compared and contrasted.

Finally, Section 5 summarizes and concludes from the results and earlier sections to give the reader a definitive direction in terms the predictive ability of the RD-CA model versus Altman's original Z score.

## 1.2 Motivations

2018 marks the $50^{th}$ anniversary of the seminal paper by Edward I Altman, in which he presented the then revolutionary bankruptcy prediction model, widely known today as the Altman's Z-Score (Altman E. I., 1968). In his study, he pioneered the use of Multiple Discriminant Analysis (MDA) technique to build a bankruptcy prediction model powered by five financial ratios.

Prior to his paper, bankruptcies were modelled prevalently by univariate ratio analysis models, such as the classic study conducted by Beaver which established a platform for further multivariate models (Beaver, 1966). Much work has been done since, on improving the multivariate Altman Z-Score model and usage of Multiple Discriminant Analysis (MDA) has been rife. However, other technologies such as logistic regression (Logit) models have been developed in later years, the best known of them being the Ohlson's O-Score (Ohlson, 1980). More recently, increasing computing power has seen the development of a variety of AI techniques such as neural networks, genetic algorithms, case-based reasoning and recursive partitioning (Richard H.G. Jackson, 2013). Despite the advent of such newer techniques, the use of Altman's Z-score has not receded in popularity. In fact, renowned research and investment management firms such as the Morningstar, to this day, do not shy away from comparing their much-coveted structural Distance to Default model to the Z-score to boast their efficacy (Morningstar, Inc., 2009).

Interestingly, even though Altman's Z-score was based on US firms and subsequent studies have spanned over all the continents from Africa to Asia, Australia and Europe, it is evident that in-depth studies to the Canadian Market is comparatively lacking. Recently, a research was undertaken which incorporated Beaver's paper and added the use of the CFO to Total Debt ratio to the original model and tested its efficacy for British publicly listed companies (Jeehan Almamy, 2016). The paper concluded that the 6$^{th}$ variable in their "J-UK" model indeed was an improving factor on the Z-score and the companies sampled were the ones listed in the British stock markets. Such studies have also been conducted for the US market indeed by Altman himself in 1968, when he tested the efficacy of this 6$^{th}$ variable in 1968, but no such study was recently undertaken for the Canadian stock market, in spite of the TSE being comparable to the NYSE or the LSE.

## 1.3 Background

Corporate bankruptcy carries costs and negative externalities affecting stakeholders both within the company and externally. Even though established models such as Modigliani and Miller (1958) strongly proposed the benefits of debt, many authors have demonstrated later that the costs of bankruptcy often outweigh the benefits of higher debts (Myers, 1966). Several papers have since studied the costs of such events and classified such costs into different categories such as direct, indirect (or "shortfall") and loss of tax credits costs (Altman E. I., A Further Empirical Investigation of the Bankruptcy Cost Question, 1984) (James S. Ang, 1982) or costs borne by different parties (Branch, 2002). Detailed studies on the costs of bankruptcies have come up with a variety of results such as the administrative costs being a function of firm size (James S. Ang, 1982) whilst others have explored the fact that the ratio of bankruptcy costs to firm size falls as firm size increases (Warner, 1977). Some researchers have even argued that direct bankruptcy costs do exist but do not dictate the reorganization decisions *post eventum* (Timothy C G Fisher, 2005). Nonetheless, it is quite evident that, categorisation or not, corporate bankruptcies do carry costs.

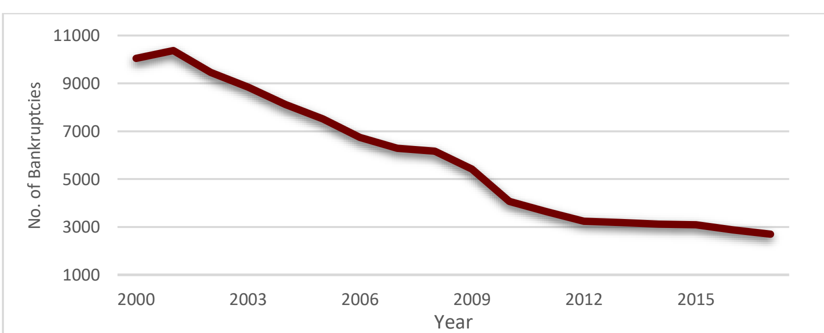



*Figure 1 - Corporate Bankruptcies in Canada 2000-2017*

Bankruptcy in Canada is also not a new phenomenon. Exploring the past 17 years of bankruptcy data from 2000-2017, it can be deduced that Canada has been enjoying a steady 7.23% average annualised fall in corporate bankruptcies since 2000 ( Figure 1) (Office of the Superintendent of Bankruptcy Canada, 2018).

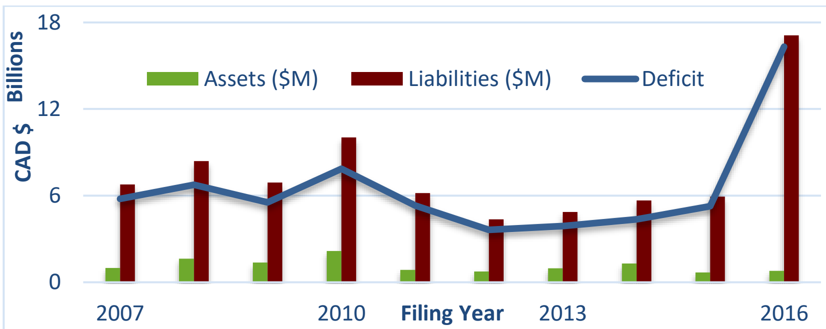



*Figure 2 - Assets and Liabilities of Insolvent Businesses in Canada*

However, a closer look at the aggregate of "shortfall or deficit" which is the difference of liabilities from the assets shows that the costs of bankruptcy have increased drastically between 2007 and 2016 ( Figure 2).

As an example, the number of corporate bankruptcies in Canada in the year of 2007 was 6,293 and the average deficit per business was 916,733 CAD dollars. In contrast, the number of corporate bankruptcies in 2016 fell by 57% since 2007 to 2,700 but the average deficit increased 6.5 times to more than 6 million CAD dollars. The net effect was a deficit increase of 182% over the 9 years or an annualized average increase of 20%. The results suggest that corporate bankruptcies have become a rarer but much more precarious phenomenon over the years.

The number of firms going bankrupt has fallen as an overall trend falling almost 12% on an annualized average rate. However, in recent years, a small number of very large business insolvency filings have had a disproportionately large effect on the average deficit of the insolvency filings. For example, in 2016, the average deficit of insolvency filings was $16 billion which is three times the deficit of the $5.2 billion figure of 2015. The lowest average deficit was in 2012 standing at $3.6 billion. The deficit is the difference between the assets and liabilities of a business at the time of bankruptcy filing.

This premise thus sets the stage for us to investigate further into corporate bankruptcy models to help improve the warning systems set in place to prevent such calamities.

Further analysis into Canadian corporate bankruptcy data also reveals that over the years, the construction has consistently been most bankrupt-prone sector while the accommodation and food services bankruptcies has increased steadily to take the second place in this infamous list (Office of the Superintendent of Bankruptcy Canada, 2019).

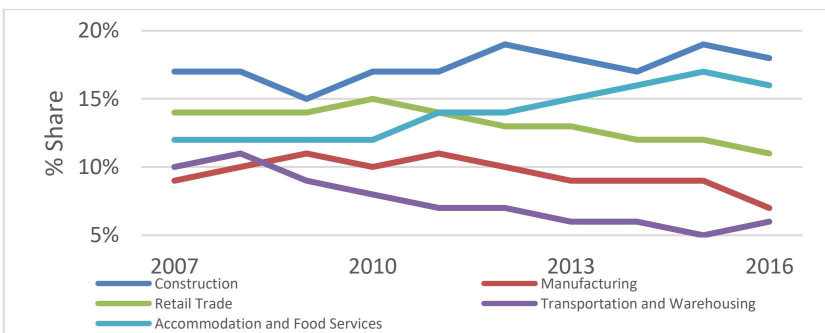



*Figure 3 - Corporate Insolvency shares by Top Five Sectors*

On the other hand, the sectors of retail trade, manufacturing and transportation and warehousing have all seen declining trends in such events. These trends also assist in identifying the sectors for which it is pertinent to carry out prediction model studies.

The 5 sectors of energy, consumer discretionary, consumer staples, industrials and materials were specifically chosen since they comprise of almost 50% of the Canadian GDP while the rest of the sectors comprise a little less than 50% of the total GDP. Moreover, other sectors such as Financial, health care and information technology have very low levels of Total Assets compared to revenue, due to the service nature of their business.

Finally, the Altman model was specifically designed for the non-service sector, which has aided in the decision of choosing the specific sectors.

## 2: Literature Review

This study is based primarily on three key pieces of literature. The first piece of literature was related to the detailed foundation of the Altman's Z-score which came from his works published in 1983 (Altman E. I., Corporate Financial Distress, 1983) and 2002 (Altman E. I., Bankruptcy,Credit Resik, and High Yield Junk Bonds, 2002). Both of these references gave detailed explanations into his methodology, results and conclusions. The second piece of primary literature pertained to the choice of financial ratios. The earliest research of note and relevancy to this study was conducted by William H. Beaver in 1966 where he utilized a univariate approach by analysing 30 accounting ratios subdivided into six groups:- Cash-flow ratios, Net-Income Ratios, Debt to Total Asset Ratios, Liquid-Asset to Total Asset Ratios, Liquid-Asset to Current Debt Ratios and Turnover Ratios (Beaver, 1966). The third piece of literature gave a more contemporary reference to the combination of Beaver's work and Altman's work to add a 6$^{th}$ variable to the original model for UK listed corporations (Jeehan Almamy, 2016). The author's in this study referred to their model as the J-UK model.

The Altman Model can be looked at as a sum of three parts. The first part is the variables themselves, the second part is the coefficients attached to each variable and the third part is the cut-off ranges that dictate the judgment on resultant Z-scores. This model has stayed exactly same since 1968 for the first two parts discussed above. The cut-off points, the third part, as originally published by Altman as 1.81-2.99 has also not changed over the years (Altman E. I., Z-Score History & Credit Market Outlook, 2017).

One reason for this lack of evolution in the model has been because of its success in producing excellent results to testing, both for differing industries and for differing regions. These results have led researchers to believe in the model to great extents. One study used Z score, as a “fact”— to identify small and medium firm health in Tuxtepec, Mexico, where 75% were identified as healthy while 25% were either in the grey zone or alerted as bankrupt (Hernandez, 2018). Similar studies conducted for NIFTY 50 index (Sanesh, 2016), Industrial Listed companies in Italy (Celli, 2015), insurance companies listed in the Amman Stock exchange (Al-Manaseer & Al-Oshaibat, 2018) and the Indonesian listed Banking Industry (Muammar Khaddafi, 2017) illustrated the reliability of the Altman Model. However, two of the above papers at some point indicated the need for a revised Z-score range for the specific regions.

This however does not mean that the model has not been challenged. In fact, previous literature has proved time and again that some or all of the three parts have room for improvement.

For example, Muminovic in his paper concluded that even though the Z-score model was prevalent in the Serbian capital market, keeping the model coefficients and variables the same, while making minor cosmetic accounting changes, produced inaccurate results- thus indicating the need for a revised model, both in terms of coefficients and in terms of cut-off ranges (Muminović, 2013).

Thai et. al used the original Altman model to study the predictability of the model for Malaysian firms. In their study, they found out that the model produced excellent results for data 5 years prior to bankruptcy with X1 being the highest discriminating variable.

However, they also improvised the cut-off scores and used four ranges instead of the original three (Thai, Goh, Teh, Wong, & Ong, 2014).

The MDA methods used in deriving the model has also been tested. In fact, one of the few recent Canadian studies on the Altman Z model was conducted in 2017, which attempted to create a hybrid Forecasted Artificial Neural Network (FANN) model to best the Altman model (Mohammad Mahbobi, 2017). The study concluded to be successful, however, it also exposed that artificial neural network (ANN) models and LOGIT models do not behave better than the Z model.

The use of Discriminant Analysis has been tested multiple times. A study on 33 failed vs 33 NF listed firms in USA, controversially concluded Factor Analysis as a better alternative to Discriminant Analysis only after ignoring the “grey zones” for both models (Chi, 2012). Logit models have also been used as alternative methods without comparing to MDA (Kim & Gu, 2010) and also compared with MDA methods (Affes & Hentati-Kaffel, 2016) to give contradicting results on superiority of either method.

Finally, the J-UK model that this study endeavors to replicate has used the same MDA methods but, as discussed before, have made changes to all the three parts, variables, coefficients and ranges in an effort to make a better model.

Finally, each paper mentioned here has tried to look at one aspect of the model, which this study has tried to change by examining all three aspects in order to evaluate the Altman Z-score model.

# 3: Methodology

## 3.1 Data

A short overview of the steps taken in this study is provided below and explained in more details in the later sections. All data were obtained from the Bloomberg LP terminal and missing data was collected from the SEDAR website (SEDAR, 2018).

## 3.2 Brief overview of Methodology

This section gives a brief description of the steps taken to arrive at the RD-CA model. The first step was to analyse the Z-scores of each year starting from 2003 to 2017 in view to evaluate the effectiveness of Altman's original cut-off region. Next, a sample of Bankrupt firms was collected and only 5 sector specific firms were chosen from the selection.

The dataset contained 70 bankrupt firms from the 5 sectors. The next step involved drawing 20 samples of healthy firms from a list of 1,047 companies. The sample of 70 bankrupt firms was then combined with each sample of 70 healthy firms. The MDA was conducted on 20 samples of 70 healthy firms and 70 bankrupt firms using Altman's original 5 variables model. We then ran the MDA for another 20 samples of 70 healthy firms and 70 bankrupt firms based on Altman's 5 variables model plus the sixth CFO to total debt variable and compared the results of both models. Of the 20 samples, the sample with the most optimum statistical characteristics was chosen to form the RD-CA model.

The three models original Altman Z-score, J-UK model and the RD-CA model were examined against the sample of 1074 NF and 70 bankrupt firms for separation and other statistical characteristics. Finally, recognizing the ineffectiveness of the J-UK model, only the original Altman Z-score model and the RD-CA model were compared to illustrate the efficacy of the RD-CA model.

## 3.3 Periodic analysis of Z-Score

To understand the effectiveness of the Z-score or rather the cut-off points, an analysis is necessary. This intuition was backed by Altman himself, when he claimed that the Z-scores have been changing year over year on a Z-score golden jubilee interview event (Larry Cao, CFA, CFA Institute, 2019). To conduct the analysis, the Z-scores for each year starting from 2003 to 2017 were compiled and statistical operations were performed to find the mean, median, standard deviation and kurtosis. The kurtosis was particularly used instead of skew since the "tailedness" of the data was given more importance due to outliers. To remove the effects of outliers, the highest 10% and the lowest 10% results were omitted. Frequency distribution tables were drawn for each list and finally frequency distribution graphs were produced to illustratively understand the movement of the Z-scores over the years mentioned above. **Error! Reference source not found.** in the Appendix depicts all 15 years.

## 3.4 Sample Selection of Bankrupt Firms

To obtain the sample of bankrupt firms, data points were collected from the Bloomberg databases and examined from the years of 1998 to 2017. A query for the field of companies that filed for bankruptcy for the mentioned years resulted in a total 174 firms.

From the sample of 174 companies, only the companies relevant to the sectors of energy, consumer discretionary, consumer staples, industrials and materials were taken into consideration. Moreover, companies which had not started operations, and which did have notable operating activities prior to going bankrupt were disregarded. This left only 70 bankrupt companies in the sample.

## 3.5 Construction of Non-Bankrupt Strata Sample

To construct the sample of Non-Bankrupt (NF) companies for the random sampling process, for each sector and each year, firm data was queried on Bloomberg to receive separate lists of companies. Within these lists, for specific bankrupt firms, the corresponding prior year of bankruptcy filing was referenced in the collected NF firm lists and Total Asset sizes were filtered at a range of +/- 50% of the referred bankrupt firm. To construct its strata of sample firms, the NF list of 2007 was looked at and only firms with an asset size in the range of 2.75- 6.18 Billion were chosen. Finally, duplicate firms within the same year and firms that filed for bankruptcy in succeeding years up to 2017 were omitted.

## 3.6 Multiple Discriminant Analysis (MDA)

As mentioned earlier, different models before Altman`s 1968 paper used univariate models to predict corporate failures. The superiority of Altman`s model lay not only in his multivariate model of prediction but also in the approach that resulted into the multivariate model. Statistical tools are useful in different contexts and often times using the wrong statistical methods result in undesirable results. As a rule of thumb, when dependent variables are qualitative and independent variables are quantitative then discriminant analysis should be used. The table below gives a guidance on the different methodologies that should be utilized in different situations.

*Table 1 - Statistical method matrix*

| **Dependent Variables** | | | |
|---|---|---|---|
| **Independent Variables** | | **Quantitative** | **Qualitative** |
| | **Quantitative** | Regression | Discriminant Analysis/ Binary Regression |
| | **Qualitative** | Hypothesis Testing/ Regression with dummy variables | Cross-tab/ Chi-squared test |

For the purpose of this paper, we will use the multiple discriminant analysis (MDA) to assess the effectiveness of Altman Z score in classifying firms, based on their characteristics, into bankruptcy and non-bankruptcy. The MDA is a dimension reduction technique which attempts to draw a cut off line between the two group classifications. It does so by simultaneously maximizing the distance between the mean of both groups and minimizing the spread within each group (Brighman & Daves, 2012).

## 3.7 MDA result robustness checks

To test the robustness of the MDA results, several checks were performed. These are explained below.

After obtaining the coefficient from the MDA, the first test that we will carry out is the Wilks' Lambda which is a measure of means differences which assesses the ability of the discriminant function in classify firms into bankruptcy and non-bankruptcy. The lambda value calculated will range between 0 and 1. The higher the lambda value, the lower is the function's ability to discriminate.

The next test that we will analyse is the Chi squared test, the formula for which is as below.

$$\chi^2 = \frac{\sum(O-E)^2}{E}$$

Where

$\chi^2$ = The symbol of Chi-squared

O = Observed Value & E=Expected Value

What the above formula means is that firstly, the chi-squared value, owing to it being a sum of squares, would be a positive number. Secondly, the a higher chi-squared value would mean that the data is further away from the expected value derived from regression or MDA (or for that fact any statistical formula generating method) (Brooks, 2014) (Light, 2019).

The data produced by the MDA is subject to this test for an initial robustness check. However, more interestingly is the p-value which is fed by the chi-squared results along with the degrees of freedom (in this case there were 21 degrees of Freedom). The p-value or probability value denotes the statistical probability of accepting the null (or base) hypothesis or $H_0$. A low p-value would indicate a rejection of the null hypothesis while a high p-value (usually higher than 0.1) would indicate an acceptance of the hypothesis. Simply put, a p value of say 0.12 would mean a 12% chance that the $H_0$ is correct. For this test, the null and alternative hypothesis were as follows: -

$H_0$ = The within class covariance matrices are equal

$H_1$ = The within class covariance matrices are different

This test is pertinent since the within class covariance matrices being the same would mean that the independent variables X1 to X6 perform in identical manners in both failed and NF firms. This in return would mean that no amount of analysis would produce discriminating results (Brooks, 2014).

## 3.8 Choice of Accounting Ratios

Having decided on the statistical method to adopt, Altman (1968) considered five ratios from the following categories; liquidity, profitability, leverage, solvency and activity. The ratios were selected mainly because of their relevance in literature. Since this paper attempts to replicate the model used by Altman, the following discriminant function has been used:

$$Z = V_1X_1 + V_2X_2 + V_3X_3 + V_4X_4 + V_5X_5 + V_6X_6$$

Where,

Z = single discriminant score

$V_j$ = discriminant coefficients, where j= 1, 2, 3, 4, 5 and 6

$$X_1 = \frac{Working\ Capital}{Total\ Assets}$$

$$X_2 = \frac{Retained\ Earnings}{Total\ Assets}$$

$$X_3 = \frac{Earnings\ Before\ Interest\ \&\ Taxes\ (EBIT)}{Total\ Assets}$$

$$X_4 = \frac{Market\ Value\ of\ Equity}{Book\ Value\ of\ Total\ Liabilities}$$

$$X_5 = \frac{Sales}{Total\ Assets}$$

$$X_6 = \frac{Cash\ Flow\ from\ Operations\ (CFO)}{Total\ Assets}$$

$X_1 = \frac{Working\ Capital}{Total\ Assets}$ The denominator consists of the net working capital which is current assets minus the current liabilities. This ratio indicates the liquidity position of the firm. It helps stakeholders to understand how much assets are tied up in working capital or how much fund is available to meet short term financial obligations. A low working capital to total assets ratio would normally imply financial difficulties experienced by companies. It could also be interpreted as an early signal to bankruptcy where companies are unable to pay creditors and suppliers and as a result of which production slows down and sales contract. (Charles, 1942) is a strong proponent of this ratio and uses it as a leading indicator for firm cessation.

$X_2 = \frac{Retained\ Earnings}{Total\ Assets}$ This ratio uses "cumulative profitability over time" to indicate a firm's ability to generate income for reinvestment rather than using debt or equity financing. One of the disadvantages of using this ratio, however, is that it is in favour of well-established firms at the expense of newly-formed firms. The rationale is that since newly-established firms have just started operations, they did not have enough time to generate high earnings. This would result in low retained earnings to total assets which would imply that those firms are more likely to be classified as bankrupt. The argument brought forward by Altman is that the probability of failure is much higher in a firm's start-up phase.

$X_3 = \frac{Earnings\ Before\ Interest\ \&\ Taxes\ (EBIT)}{Total\ Assets}$ Given that the firm's financial health depends on its ability to utilize assets to generate return, this measure is deemed important in determining corporate failure.

$X_4 = \frac{Market\ Value\ of\ Equity}{Book\ Value\ of\ Total\ Liabilities}$ The numerator takes into consideration both common and preferred equity shares. This ratio measures the extent to which a firm can decrease in value before it faces bankruptcy. The more the company is exposed to debt, the higher the likelihood of it becoming bankrupt. For instance, let us assume that the market value of equity of a firm is $1500 whilst the book value of total liabilities is $500. In this situation, the firm can absorb a three-quarter decrease in value before becoming insolvent. On the other hand, if the firm has a market value of equity of $500 and a book value of liabilities of $1500, the equity cushion being lower, the firm can only absorb a quarter decrease in value before insolvency. Interestingly, other research prior to Altman made on corporate failure do not incorporate the market value aspect in their studies.

$X_5 = \frac{Sales}{Total\ Assets}$ This is a common measure to assess the firm's efficacy in utilizing asset to generate revenue. The asset turnover ratio has been added by Altman it has significant discriminating ability on the overall Z score model.

$X_6 = \frac{Cash\ Flow\ from\ Operations\ (CFO)}{Total\ Assets}$ This ratio has been added to the standard Altman Z score model which is in line with the J UK model. Being less subjective to manipulation, the CFO to total assets ratio provides a better metric for evaluating a company's financial health. Further, it assesses the company's ability to generate cash from its operations as opposed to other sources of funding. The CFO to total assets ratio is hence considered to be of significant importance predicting corporate bankruptcy.

## 3.9 Selection between 5 or 6 variables

The MDA described in Section 3.6 was undertaken 40 times. 20 times for the sample with 6 variables and 20 times with 5 variables. After conducting the checks presented in Section 3.7, one special check, the Wilk's lambda, was taken as a measure of explicability of the models. The Wilk's lambda was then plotted concurrently for the respective 5 and 6 variable MDA and compared to derive the final choice of the number of variables to choose in the final sample.

## 3.10 Selection of optimum sample

Once the MDA analysis was undertaken for all 20 samples, the samples were then compared on all the robustness checks explained earlier. On top of these robustness checks, the formulae generated from the MDA was applied on the whole population of 1074 NF firms plus the 70 failed firms to produce their respective RD-CA scores. Once these scores were produced, frequency distribution exercises were undertaken, and frequency distribution curves were drawn for each of the 1144 Z scores separately for each sample. Within each sample (produced by the original MDA sample formulae), the RD-CA score frequency distribution curve of the 70 Failed firms were superimposed on the 1074 NF score distributions. The peak separations were spotted for each category and the a "separation of peaks" was calculated as the following formula: -

$$Peak\ Separation = (Peak\ of\ NF\ Firms) - (Peak\ of\ Failed\ Firms)$$

Following this exercise, the means, standard deviations and kurtosis of each category were calculated. A similar separation of means was also calculated. Once these results were tabulated, (please see index), the manual choice-by-observation was processed.

For the choice, initially all sample formulae were rejected which either had a negative peak separation or a negative mean separation. This was done since a negative separation in either case indicates strongly that the expected RD-CA scores have resulted in higher values for failed firms when compared with NF firms.

## 3.11 Comparison of the three models

The tests described in Section 3.8 were also conducted on the Altman Z-score and the J-UK model by applying the respective formulae to the sample of 1074+70 firms. The resultant score data were then fed into separate Frequency distribution graphs with similar superimpositions as 3.9. Similar tests of separation, mean, standard deviation and kurtosis were also conducted to arrive at the two finalist models between the three models.

## 3.12 Comparison of Altman Z-score to RD-CA model

Once the two finalist models were confirmed as the Altman Z-score and RD-CA models, a final robustness check was conducted — the Type I and Type II error tests. It is at this conjecture that the current study deviates greatly from the existing literature, in the sense that, these tests were not conducted on the original 70+70 MDA sample but rather on the whole population of 1074+70 firms. The Type I and Type II errors are fondly known as the "false positive" and "false negative" errors. In this case the Type I error is classified as firms that are bankrupt but are falsely marked as "Successful" while the Type II error is misclassifying the NF firms as bankrupt (Altman E. I., Predicting Corporate Bankruptcy: The Z Score Model - Empirical Results, 1983). The following table depicts the Type I and Type II error

| Actual Group Membership | Predicted Group Membership | |
|---|---|---|
| | Bankrupt | Non-Bankrupt |
| Bankrupt Firms | Correct | Misclassified $_{\text{Type I}}$ |
| NF Firms | Misclassified $_{\text{Type II}}$ | Correct |

To find the values for the Altman Z-model, the following original cut-off scores were used

$$Distress\ Zone\ (Failed\ Firm) < 1.81$$

$$1.81 < Grey\ Area\ (Co - existence\ no\ misclassification\ possible) < 2.99$$

$$Safe\ Zone\ (NF\ Firm) > 2.99$$

The cut-off regions stated above were specifically prescribed by Altman. The logic behind this choice was three-fold. Firstly, the “Grey Area” or “zone of ignorance” was chosen as the region was believed to host an overlapping population frequency of both the Failed and Non-Failed Firms. Secondly, the Distress Zone was empirically found out to have minimum number of Non-Failed firms present, while finally, the Safe Zone which falls above a Z-score of 2.99 had no existence of distressed firms. This meant that these ranges allowed very little chance of Type 1 Error while also proving effective in minimizing Type 2 Errors.

To find the values of the RD-CA model, a series of 4 ranges were used. The first range was the centroid range as produced by the MDA. The second was the peak separation ranges of the failed and NF firms. Since the Failed firms in the 70+70 sample produced 2 peaks a third range was also used in a similar fashion. Lastly, the fourth range was the peak separation ranges produced by the 1074+70 population. The results were tabulated and compared to arrive at the conclusion.

# 4: Results

## 4.1 The changes in Altman Z-Score

As explained earlier, a survey of the Altman Z-score model produced a set of interesting results. The mean Z-score of the whole TSX index (years 2003-21018) did not actually rise but fluctuated widely between a range of 3 to 16. As anticipated, the Z-score faced a major drop in the financial crisis of 2008. However, the market value of the index did not seem to dictate the Z score.

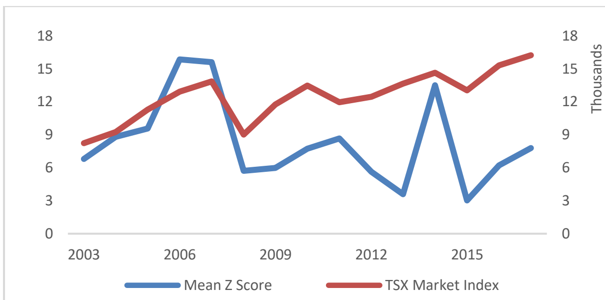



*Figure 4 - Mean Z Score 2003-2018*

The standard deviation of the survey data followed the same pattern as the mean. A closer investigation revealed the standard deviation of the Z score to be very close to 2 times the mean over the 15 years.

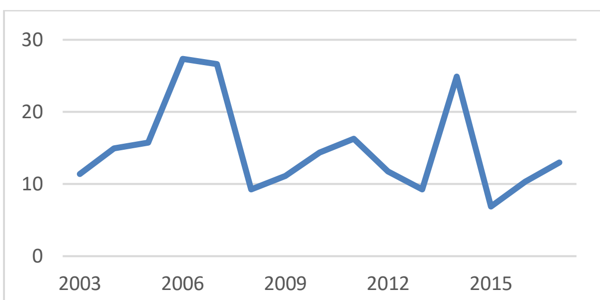



*Figure 5 - Standard Deviation of Z-Score 2003-2018*

The kurtosis of the Z-score was quite high at ranging from 4-39 with the most prevalent numbers being close to 10. This meant that even with the reduction of the index data by 10% in each tail, the remaining data was quite dispersed. There was also a consisted positive skew of close to 3 indicating that the dispersion occurred mostly in the positive direction. However, a closer look at the **Error! Reference source not found.**, it can be seen that the distribution was at the same time very densely populated around the peak while the rest of the population was sparsely populated to make the tails longer than usual.

## 4.2 MDA and comparison of the 5-variable and 6-variable models

Following the multiple discriminant analysis of the 20 samples, the Wilks' lambda of each sample was compared. Table 7 (in the Appendix) illustrates the Wilks' lambda across the 20 samples while **Error! Reference source not found.**6 displays the results graphically. As depicted from the graph, the 5-variable model lags the 6-variable model by a small margin and adding a sixth variable to the model decreases the Wilks' lambda of respective MDA.

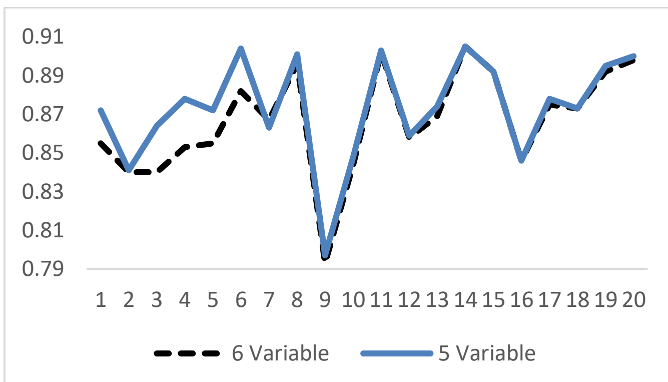



*Figure 6 - Wilks Lambda for the 20 Samples*

Thus, the resultant 6-variable formulae were taken to derive the resultant optimum RD-CA model. The statistical summary of the scores from the 1074+70 population are depicted in Table 7. As a comparison, the Wilks' lambda of the finally chosen sample twenty was 0.898 for the 6-variable model when compared with 0.900 for the 5-variable model.

The resultant RD-CA score distribution peaks, the NF-Failed firm score means, standard deviations and kurtosis were recorded in Table 8, with the 8 samples that passed the basic statistical tests highlighted. Sample 20 was chosen from the 8 samples since it had the maximum peak and mean separations along with optimum deviation and kurtosis characteristics.

$$Sample\ 20\ Score = 0.495\ X_1 - 0.595\ X_2 + 0.636\ X_3 + 0.480\ X_4 + 0.287\ X_5 - 0.120\ X_6$$

This equation is then transformed for the $X_5$ component (dividing by 100) to form the pre-final RD-CA pre-final (pf) model as a comparative to the Altman Z model.

$$\boldsymbol{RD - CA\ (Sample\ 20)\ score}$$
$$\boldsymbol{= 0.495\ X_1 - 0.6\ X_2 + 0.64\ X_3 + 0.48\ X_4 + 0.003\ X_5 - 0.12\ X_6}$$

*Table 2- MDA Robustness check tables*

| Sample Number | Wilks' Lambda | P-value | $\chi^2$ (Chi-squared) |
|---|---|---|---|
| 20 (RD-CA Model) | 0.898 | 0.029 | 14.023 |

As shown in table 2 the Wilks' lambda for the RD-CA model comes at a lower level than the J-UK model lambda of 0.983 reported by Almamy (Jeehan Almamy, 2016). The p-value also comes at 0.029 which is lower than 0.05 and thus helps us reject the null hypothesis of within-class covariance matrices being equal. With such a high number of observations chi-squared value is at acceptable level of 14.023. In comparison, the J-UK model had a chi-squared value of 93.240.

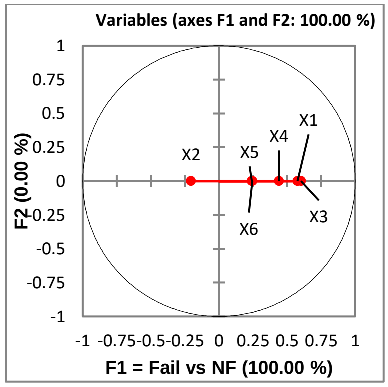
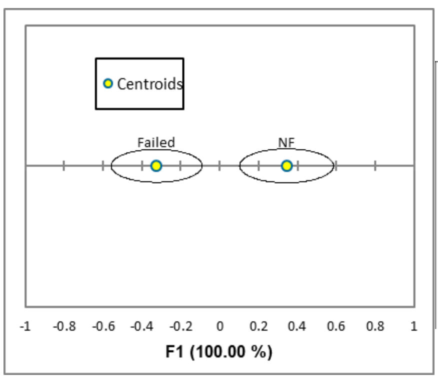



*Figure 7- Variable factor correlations*

The discriminating power of each variable of sample 20 was also plotted on a graph depicted in Figure 7. As can be seen the ratios $X_3$, $X_1$, $X_2$, are the most explanatory variables and thus have the highest coefficients in the RD-CA (pf) model too.




**Functions at Group Centroids**

| Failed/Passed | Function 1 |
|---|---|
| 1 | .344 |
| 2 | -.324 |

*Figure 8 - RD-CA model Discriminant Analysis centroids*

The centroids of the sample were found out and plotted on a linear scale. Other data robustness checks also came back as quite positive such as the Type I and Type II error tests as shown in Table 3

*Table 3 - Type I and Type II error on sample*

**Classification Results[a,c]**

| | | Failed/Passed | Predicted Group Membership: NF | Predicted Group Membership: Failed | Total |
|---|---|---|---|---|---|
| Original | Count | NF | 48 | 22 | 70 |
| | | Failed | 18 | 52 | 70 |
| | % | NF | 66.7 | 33.3 | 100.0 |
| | | Failed | 25.7 | 74.3 | 100.0 |
| Cross-validated[b] | Count | NF | 45 | 25 | 70 |
| | | Failed | 19 | 51 | 70 |
| | % | NF | 68.5 | 37.9 | 100.0 |
| | | Failed | 27.1 | 72.9 | 100.0 |

a. 71.4% of original grouped cases correctly classified.
b. Cross validation is done only for those cases in the analysis. In cross validation, each case is classified by the functions derived from all cases other than that case.
c. 69.6% of cross-validated grouped cases correctly classified.

As can be seen the model had accuracy of 70% which was quite high when compared to the J-UK model which subdivided its results to before and after crisis results and had varying ranges of accuracy between 67% to 81%.

## 4.3 Initial comparison between the three models

Once the pre-final model was chosen, it was used to receive the scores of all the 1074+70 sample of firms. The same was done for the Altman Z-score and the J-UK model. The summary statistics of the resultant score are given in Table 4

*Table 4 - Statistical Summary and comparison of the three models*

| Model | Score Distribution Peaks | | | Score Means | | | Score Standard Deviation | | Score Kurtosis | |
|---|---|---|---|---|---|---|---|---|---|---|
| | NF | Failed | Separation | NF | Failed | Separation | NF | Failed | NF | Failed |
| Altman Z Score model | 2.689 | 0.643 | 2.045 | 8.509 | 0.600 | 7.909 | 65.03 | 11.09 | 23.0 | -0.93 |
| J-UK Model | -0.047 | 0.021 | -0.068 | -0.187 | -0.500 | 0.313 | 2.58 | 1.10 | -6.3 | -2.95 |
| Sample 20 (RD-CA) | 1.224 | 0.689 | 0.534 | 7.098 | 1.893 | 5.204 | 51.95 | 6.73 | 23.9 | 5.96 |

Table 2 and the figures given above clearly depict that while the Altman Z model produces a better separation than the RD-CA model, it fails to produce a comparatively better discrimination between each group. The Altman Z model's score standard deviation for bankrupt firms is 11.09 which is twice that of the RD-CA model's standard deviation of 6.73. On the other hand, the respective standard deviation for the NF firms in case of Altman is 65.03 which is a higher number than the RD-CA number of 51.95. The J-UK model on the other hand is much lower in both cases with 1.1 for the Bankrupt and 2.58 for the NF firms. One would have the notion that this means the J-UK model is a better predictor, but having a look at the frequency graph, shows clearly that there is not peak separation in the J-UK model, if anything, the J-UK model produces slightly higher scores for the bankrupt firms when compared to the NF firms.

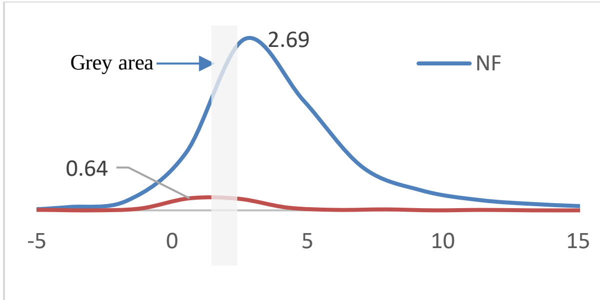



*Figure 9 - Frequency Distribution of Altman Z-Score for Canadian Firms*

The frequency distribution curves drawn from the 3 models, the Altman Z-score (Figure 9), the J-UK model (Figure 10) and the RD-CA (sample 20) model (Figure 11) depicts that the J-UK model to be the least effective.

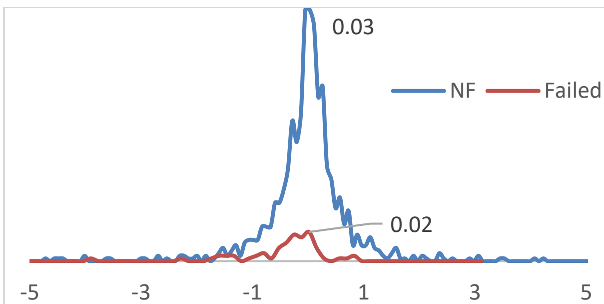



*Figure 10 - Frequency Distribution of the J-UK Model for Canadian Firms*

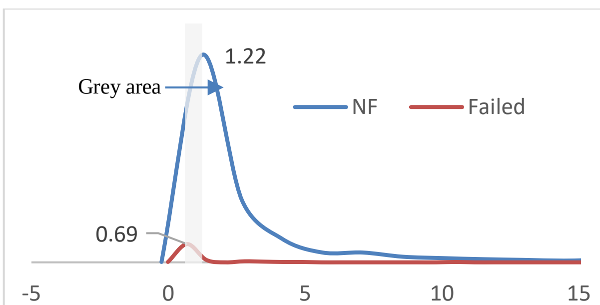



*Figure 11 - Frequency Dstribution of the RD-CA Model for Canadian Firms*

As can be seen from the above figures, all three models fail to produce a clear separation of the data points, but whilst the Altman Z model and the RD-CA (pf) model produce failed peaks lower than the NF peaks, the J-UK model has both its peaks at the same point. All three models produce a "peak within a peak" for the firms indicating that clear separation is not possible. Rather, the predictability scores of the bankrupt firms act as a subset of the NF firm scores. The "grey area" mentioned by Altman in his study is rather the separation “width” within the peaks of each group. This helps us reject the J-UK model as a viable alternative for the Altman Z model.

It is also quite interesting to see how the RD-CA (pf) model produces peaks with much less standard deviation which in return results in a “narrower” grey area. This grey area has been defined as the “zone of ignorance” by Altman (Altman E. I., Establishing a practical cutoff point, 1983).

In the final step, however it can be seen that the RD-CA (pf) does not fair well against the Altman model with results of the Type I and Type II error being inconclusive.

*Table 5 - Type I and II error comparison RD-CA (pf) vs Altman whole population*

| | Lower bound | Upper Bound | Original Group Membership | Predicted Group Membership | | | Total | Inaccuracy % | Accuracy % |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Failed | NF Firm | Grey | | | |
| Sample 20 (RD-CA) | -0.324 | 0.324 | Failed | 0 | 33 | 37 | 70 | 47.1% | 0% |
| | | | NF Firm | 0 | 867 | 180 | 1047 | 0% | 82.8% |
| Sample 20 (RD-CA) | 0.69 | 1.22 | Failed | 53 | 12 | 5 | 70 | 17.1% | 75.7% |
| | | | NF Firm | 400 | 455 | 192 | 1047 | 38.2% | 43.5% |
| Altman Z | 1.81 | 2.99 | Failed | 43 | 10 | 17 | 70 | 14.3% | 61.4% |
| | | | NF Firm | 376 | 472 | 199 | 1047 | 24.5% | 45.1% |

As can be seen from Table 5, with the centroids extracted from the discriminant analysis, the Sample 20 (RD-CA) model does not fair well, however with the use of frequency peaks, its accuracy increases dramatically for the Failed firms to 75.7% when compared to the Altman model 61.4%. On the other hand, its accuracy for the NF firms falls sharply to 43.5% when compared to Altman model NF accuracy of 45.1%.

## 4.4 The RD-CA final model

The results in the above validation tests inspired a second revision of sample 20 but this time with some outliers taken out. The results were quite promising both in case of

Type I and II error checking and in terms of Wilks' lambda which came to a much lower figure of 0.799.

*Table 6 - Type I and II error comparison RD-CA vs Altman whole population*

| | Lower bound | Upper Bound | Original Group Membership | Predicted Group Membership | | | Total | Inaccuracy % | Accuracy % |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Failed | NF Firm | Grey | | | |
| RD-CA | -0.851 | 0.061 | Failed | 4 | 42 | 24 | 70 | 60.0% | 6% |
| | | | NF Firm | 4 | 969 | 74 | 1047 | 0% | 92.6% |
| RD-CA | 0.40 | 0.70 | Failed | 53 | 9 | 8 | 70 | 12.9% | 75.7% |
| | | | NF Firm | 259 | 598 | 190 | 1047 | 24.7% | 57.1% |
| Altman Z | 1.81 | 2.99 | Failed | 43 | 10 | 17 | 70 | 14.3% | 61.4% |
| | | | NF Firm | 376 | 472 | 199 | 1047 | 35.9% | 45.1% |

Table 6 clearly depicts that the RD-CA version after excluding 2 outlying bankrupt firms, data produced robust results. These results were better in both cases of accuracy vs inaccuracy and Failed vs NF firms. In case of accuracy, both the figures of Failed (75.7%) and NF firms (57.1%) were considerably higher than the Altman model figures of 61.4% and 45.1%. The RD-CA model used was as follows: -

$$\boldsymbol{RD - CA\ (unstandardized) score = 0.431\ X_1 + 0.125\ X_2 + 0.58\ X_3 + 0.39\ X_4 - 0.31\ X_5 + 0.2\ X_6}$$

which when transformed to divide $X_5$ by 100 became the final model as: -

$$\boldsymbol{RD - CA\ score = 0.431\ X_1 + 0.125\ X_2 + 0.58\ X_3 + 0.39\ X_4 - 0.003\ X_5 + 0.2\ X_6}$$

<u>Where</u>

$$X_1 = \frac{Working\ Capital}{Total\ Assets}$$

$$X_2 = \frac{Retained\ Earnings}{Total\ Assets}$$

$$X_3 = \frac{Earnings\ Before\ Interest\ \&\ Taxes\ (EBIT)}{Total\ Assets}$$

$$X_4 = \frac{Market\ Value\ of\ Equity}{Book\ Value\ of\ Total\ Liabilities}$$

$$X_5 = \frac{Sales}{Total\ Assets}$$

$$X_6 = \frac{Cash\ Flow\ from\ Operations\ (CFO)}{Total\ Assets}$$

As noted, the value of $X_5$ Sales to Total Assets should have positively affected the RD-CA score since increasing sales ratios should positively impact a firm's health.

# 5: Conclusion and Further Studies

## 5.1 Conclusion

The first conclusion of the study is that the Altman-Z score cut-off range $1.81 < Z < 2.99$ should be updated regularly instead of being used as a fixed range. This is especially true since the Z score of market index for Canada is a dynamic variable and the numerators and denominators of the equation do not move in the same proportion from one year to the next. This results in the prevalent Z-scores (which is in fact a very small range) to move much higher than the cut-off ranges. This phenomenon in-turn could render the model useless in its predictability usage. Hence, extreme care must be exercised when assessing firm health on this metric and performance measurement must be relative to the levels persistent in the market at the time of assessment.

Secondly, it is observed that adding a sixth variable to the model produces modest improvements through the robustness checks such as Wilks' lambda, which could indicate that a 5 variable model might suffice.

It has also been clarified through this study that, the cut-off scores indeed are not ranges that should not be viewed as having supreme conclusive power since these ranges or rather their peaks overlap each other, and no "real" discrimination happens with the use of MDA.

However, it has been proved beyond doubt that the coefficients of the model do need to be changed since the discriminating power of the original Altman model seems to be fading. This change, along with a more dynamic computation of cut-off ranges has produced the RD-CA model which exceptionally outmatches the Altman Z-score model in precision.

## 5.2 Further Studies

This study, though detailed, is not free from limitations, of which there are several. The first remarkable limitation is the use of 5 sectors. Since the advent of the Altman Z-score, it has been used extensively for all firms across all sectors and has been a staple name in the financial and investment analysis industry. It is with this notion that the study was intended. The intention was to produce a generalized scoring model to predict corporate failure. However, after all the analysis and as already explained in the concluding remarks, it seems that when it comes to scoring, there is no one-size-fits-all equation. Therefore, further studies could be conducted with individual sectors to produce a series of scoring models. It is the belief of the researchers of this study that such methodology would most definitely produce robust separation in the MDA process.

It is a strong recommendation of the researchers that further research be focused not only on MDA but also on Factor Analysis since such processed might be more suited for the categorical classification and separation activities.

Secondly, this study focuses on only six variables. The foundations of this belief have been Altman himself and Beaver, but numerous other researchers have also had influence on such decision. This study does not in itself test the efficacy of other ratios to any extent. It is the belief of the researchers in this study that, due to changing times, the importance of other financial ratios might very well have increased, and further separate univariate and multivariate studies could be conducted to produce excellent models.

Thirdly, the production of other ratios should also focus on a decreasing reliance on Total Assets as the dominant denominator in the equation. It is the belief of the researchers that due to this fact alone, the asset size would drastically affect any prediction model scores, be it Z-scores or RD-CA scores.

Fourthly, the final RD-CA model provided an unexpected result in having a negative impact of X5, the sales to total assets ratio. This is in-line with the studies for the J-UK model and should be further investigated to unearth the reasoning behind such peculiarities.

Finally, it has been proven through this study that the “centroid theory” does not hold and the presence of the “peak-within-the-peak” phenomenon of the failed firms within the non-failed firms. It is thus of great importance that further research should focus on the frequency distribution curves to derive robust results.

# Appendices

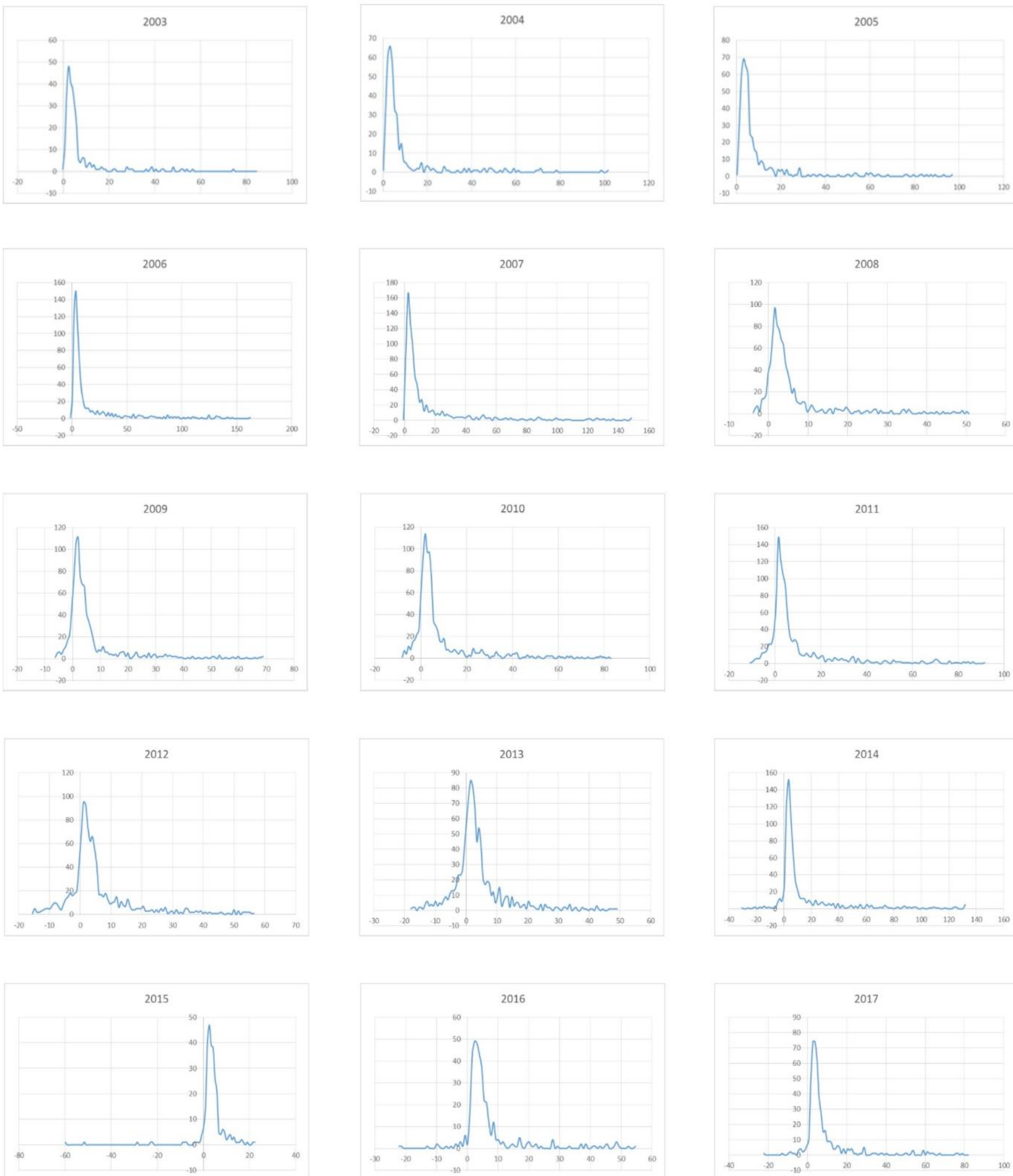



*Figure 12- Z Score Frequency Distribution from 2003 to 2017*

*Table 7 - Wilks' Lambda of 5 variable and 6 variable RD-CA model samples*

| Sample | 5 Variable | 6 Variable |
|---|---|---|
| 1 | 0.872 | 0.855 |
| 2 | 0.841 | 0.84 |
| 3 | 0.864 | 0.84 |
| 4 | 0.878 | 0.853 |
| 5 | 0.872 | 0.855 |
| 6 | 0.904 | 0.882 |
| 7 | 0.863 | 0.867 |
| 8 | 0.901 | 0.896 |
| 9 | 0.797 | 0.793 |
| 10 | 0.848 | 0.844 |
| 11 | 0.903 | 0.902 |
| 12 | 0.859 | 0.858 |
| 13 | 0.874 | 0.869 |
| 14 | 0.905 | 0.905 |
| 15 | 0.892 | 0.892 |
| 16 | 0.846 | 0.846 |
| 17 | 0.878 | 0.875 |
| 18 | 0.873 | 0.873 |
| 19 | 0.895 | 0.892 |
| 20 | 0.9 | 0.898 |

*Table 8 - Statistical Summary of 6-variable models*

| Sample Number | Coefficients of Variables | | | | | | Score Distribution Peaks | | | Score Means | | | Score Standard Deviation | | Score Kurtosis | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | NF | Failed | Separation | NF | Failed | Separation | NF | Failed | NF | Failed |
| **1** | **0.54** | **-0.70** | **0.81** | **0.418** | **0.172** | **0.44** | **0.98** | **0.64** | **0.35** | **6.25** | **1.80** | **4.46** | **43.63** | **6.38** | **628** | **34** |
| **2** | **0.68** | **-1.92** | **2.03** | **0.259** | **0.257** | **-0.04** | **0.86** | **0.63** | **0.23** | **4.84** | **2.45** | **2.39** | **32.41** | **9.21** | **421** | **49** |
| **3** | **0.68** | **-1.92** | **2.03** | **0.259** | **0.257** | **-0.04** | **0.86** | **0.63** | **0.23** | **4.84** | **2.45** | **2.39** | **32.41** | **9.21** | **421** | **49** |
| 4 | 0.78 | -1.81 | 1.98 | 0.127 | 0.048 | -0.37 | 0.29 | 0.58 | -0.29 | 2.97 | 1.96 | 1.01 | 21.47 | 8.03 | 395 | 59 |
| 5 | **0.54** | **-0.70** | **0.81** | **0.418** | **0.172** | **0.44** | **0.98** | **0.64** | **0.35** | **6.25** | **1.80** | **4.46** | **43.63** | **6.38** | **628** | **34** |
| 6 | **0.61** | **-0.61** | **0.76** | **0.097** | **-0.183** | **0.47** | **0.36** | **0.20** | **0.15** | **1.74** | **0.78** | **0.96** | **10.23** | **2.90** | **380** | **40** |
| 7 | 0.87 | -1.46 | 1.53 | -0.086 | -0.081 | 0.05 | -0.09 | 0.55 | -0.64 | -0.23 | 1.04 | -1.27 | 15.36 | 6.05 | 463 | 62 |
| **8** | **0.57** | **-0.95** | **1.15** | **0.166** | **0.010** | **0.25** | **0.67** | **0.38** | **0.29** | **2.93** | **1.29** | **1.64** | **18.80** | **4.69** | **453** | **41** |
| 9 | 0.92 | -1.60 | 1.59 | 0.035 | 0.178 | -0.17 | 0.05 | 0.42 | -0.37 | 1.57 | 1.53 | 0.04 | 14.29 | 6.89 | 735 | 64 |
| 10 | 0.88 | -1.01 | 1.09 | 0.004 | -0.136 | -0.23 | 0.25 | 0.11 | 0.15 | 0.79 | 0.90 | -0.11 | 8.65 | 4.23 | 856 | 65 |
| 11 | 0.75 | -2.18 | 2.36 | 0.025 | -0.004 | -0.13 | 0.31 | 0.44 | -0.13 | 1.75 | 2.00 | -0.24 | 18.78 | 9.28 | 843 | 64 |
| 12 | 0.87 | -1.47 | 1.56 | 0.149 | -0.002 | -0.07 | 0.25 | 0.60 | -0.35 | 3.06 | 1.72 | 1.34 | 20.54 | 6.73 | 382 | 54 |
| 13 | 0.65 | -1.57 | 1.50 | -0.206 | 0.032 | 0.24 | -0.07 | 0.96 | -1.03 | -1.87 | 0.82 | -2.70 | 26.20 | 6.75 | 450 | 51 |
| 14 | 0.87 | -3.17 | 3.01 | 0.223 | 0.029 | -0.05 | 0.77 | 0.85 | -0.08 | 5.10 | 3.51 | 1.59 | 36.30 | 14.38 | 402 | 59 |
| 15 | 0.61 | -2.13 | 2.32 | 0.064 | 0.118 | 0.03 | 0.26 | 0.45 | -0.19 | 2.24 | 2.06 | 0.17 | 19.35 | 9.15 | 693 | 63 |
| 16 | 0.98 | -1.22 | 1.24 | 0.020 | -0.109 | -0.05 | 0.10 | 0.35 | -0.25 | 1.13 | 1.14 | -0.01 | 10.62 | 5.18 | 813 | 65 |
| 17 | 0.81 | -0.89 | 0.99 | -0.037 | -0.084 | 0.16 | 0.17 | 0.22 | -0.05 | 0.09 | 0.67 | -0.57 | 8.71 | 3.60 | 550 | 65 |
| **18** | **0.92** | **-1.01** | **1.05** | **0.174** | **-0.124** | **0.01** | **0.49** | **0.46** | **0.04** | **3.13** | **1.38** | **1.75** | **20.58** | **5.01** | **475** | **43** |
| 19 | 1.04 | -2.61 | 2.46 | 0.267 | 0.054 | -0.20 | 0.50 | 0.92 | -0.43 | 5.40 | 3.12 | 2.28 | 36.87 | 12.19 | 384 | 55 |
| **20** | **0.50** | **-0.60** | **0.64** | **0.480** | **0.29** | **-0.12** | **1.22** | **0.69** | **0.53** | **7.10** | **1.89** | **5.20** | **51.95** | **6.73** | **658** | **38** |

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