深度学习

卷积神经网络（3）

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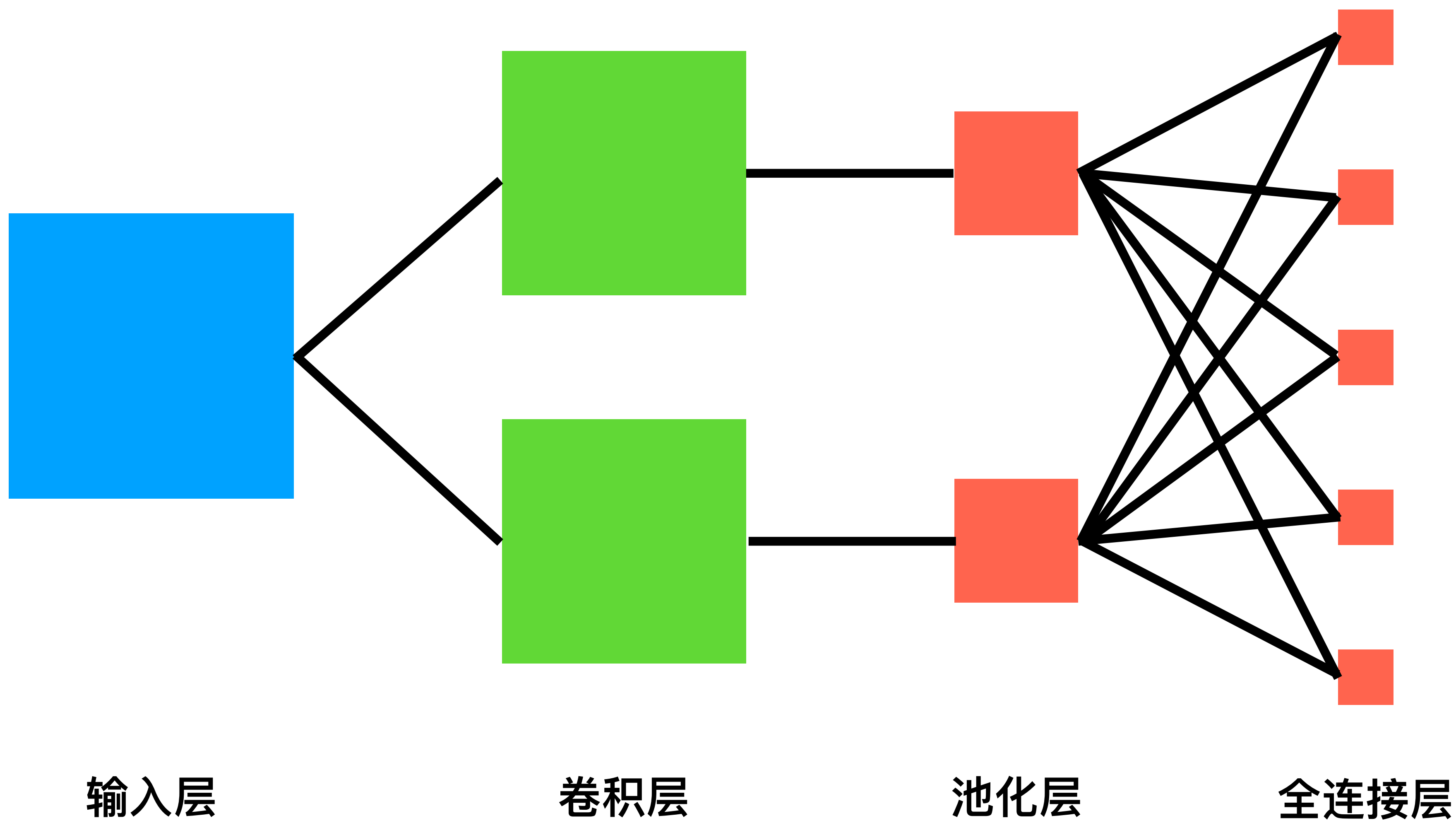
卷积的前向传播



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卷积核来源

1. 人工提取。

2. 算法习得。

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卷积核来源

1. 人工提取。

例如Sobel算子， Prewitt算子等。

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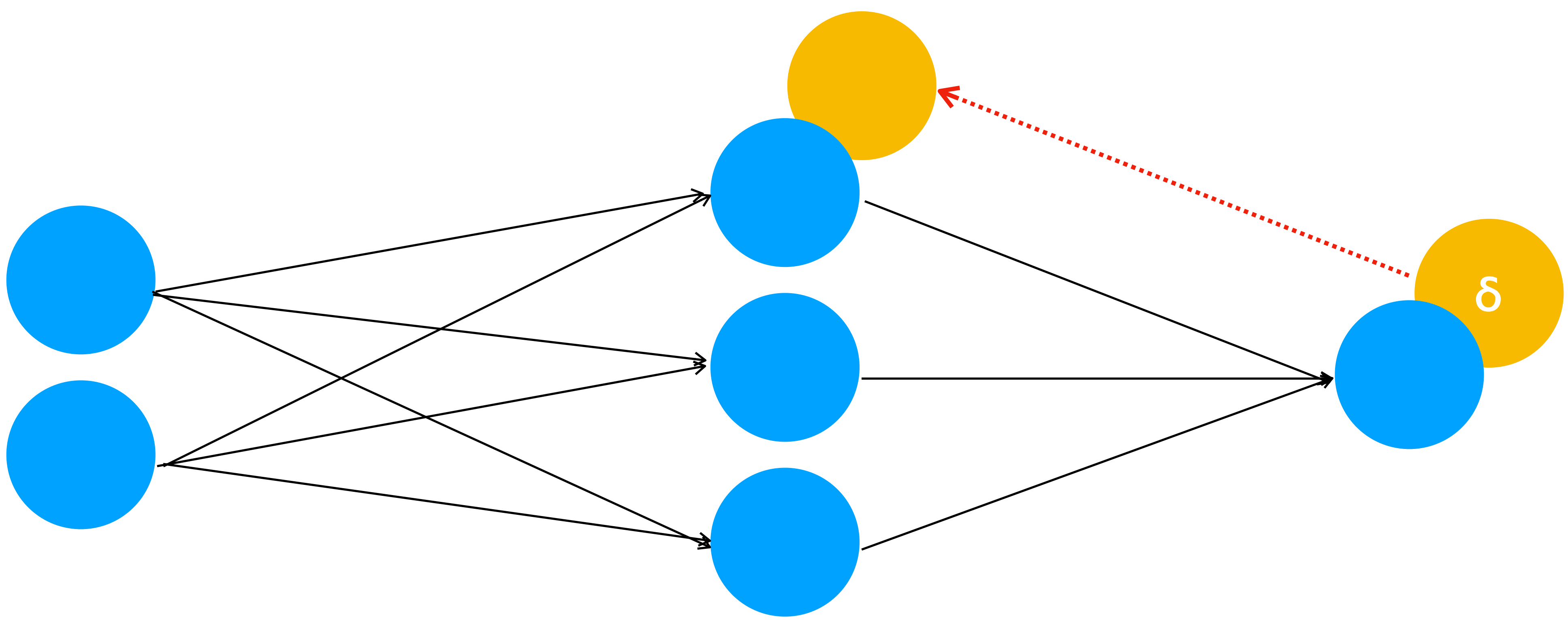
卷积核来源

2. 算法习得。

反向传播算法



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卷积核的来源

思考：两种卷积核来源的优缺点?

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卷积的参数是什么

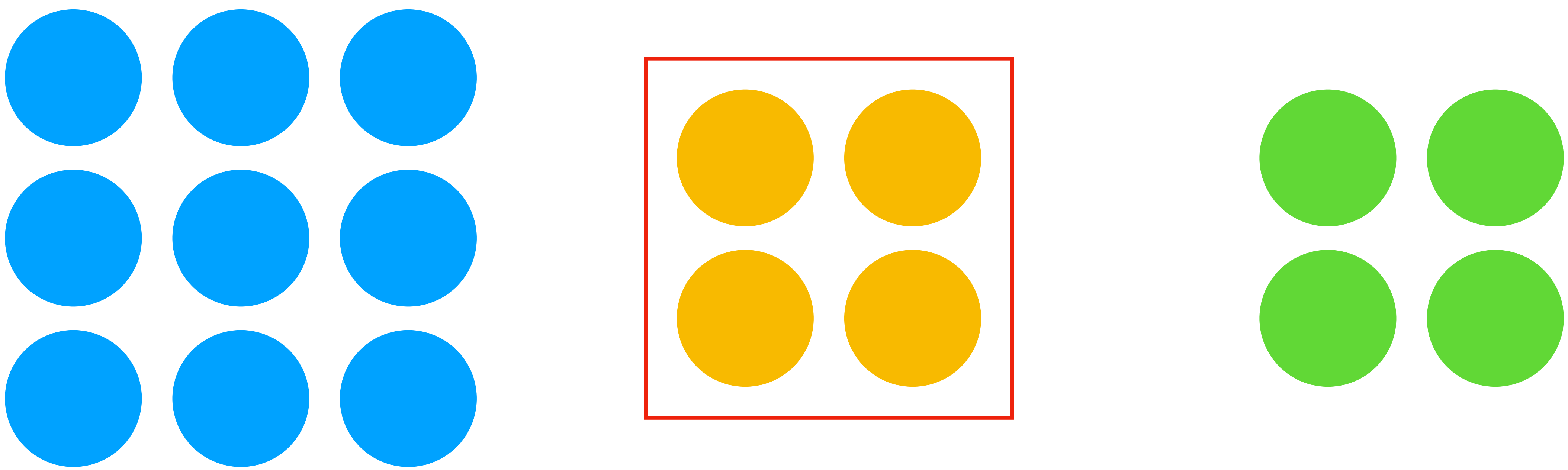

图 卷积核 生成特征图

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卷积运算

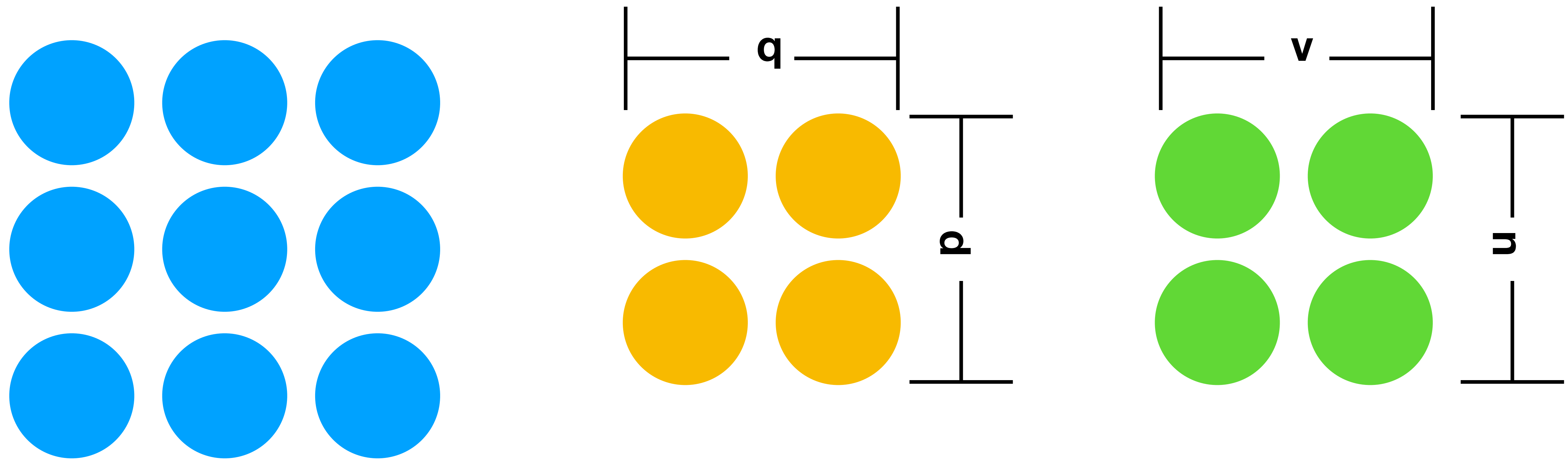

图　　卷积核　　生成特征图

$$z_{u,v}^{(l+1)} = \sum_{p} \sum_{q} w_{p,q}^{(l+1)} a_{u+p-1,v+q-1}^{(l)} + b^{(l+1)}$$

$$a_{u,v}^{(l+1)} = \sigma(z_{u,v}^{(l+1)})$$

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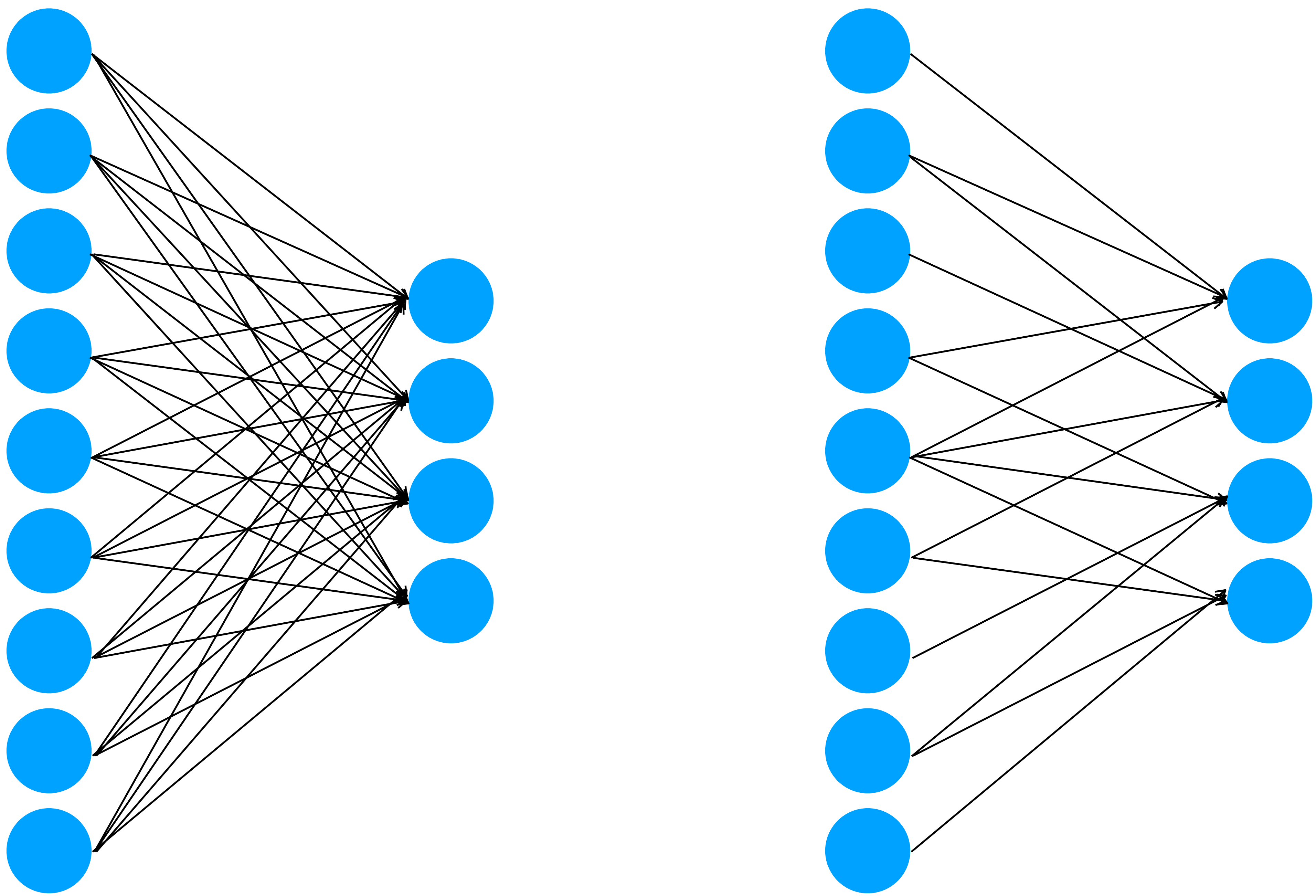
局部连接与权值共享



全连接

2*2卷积

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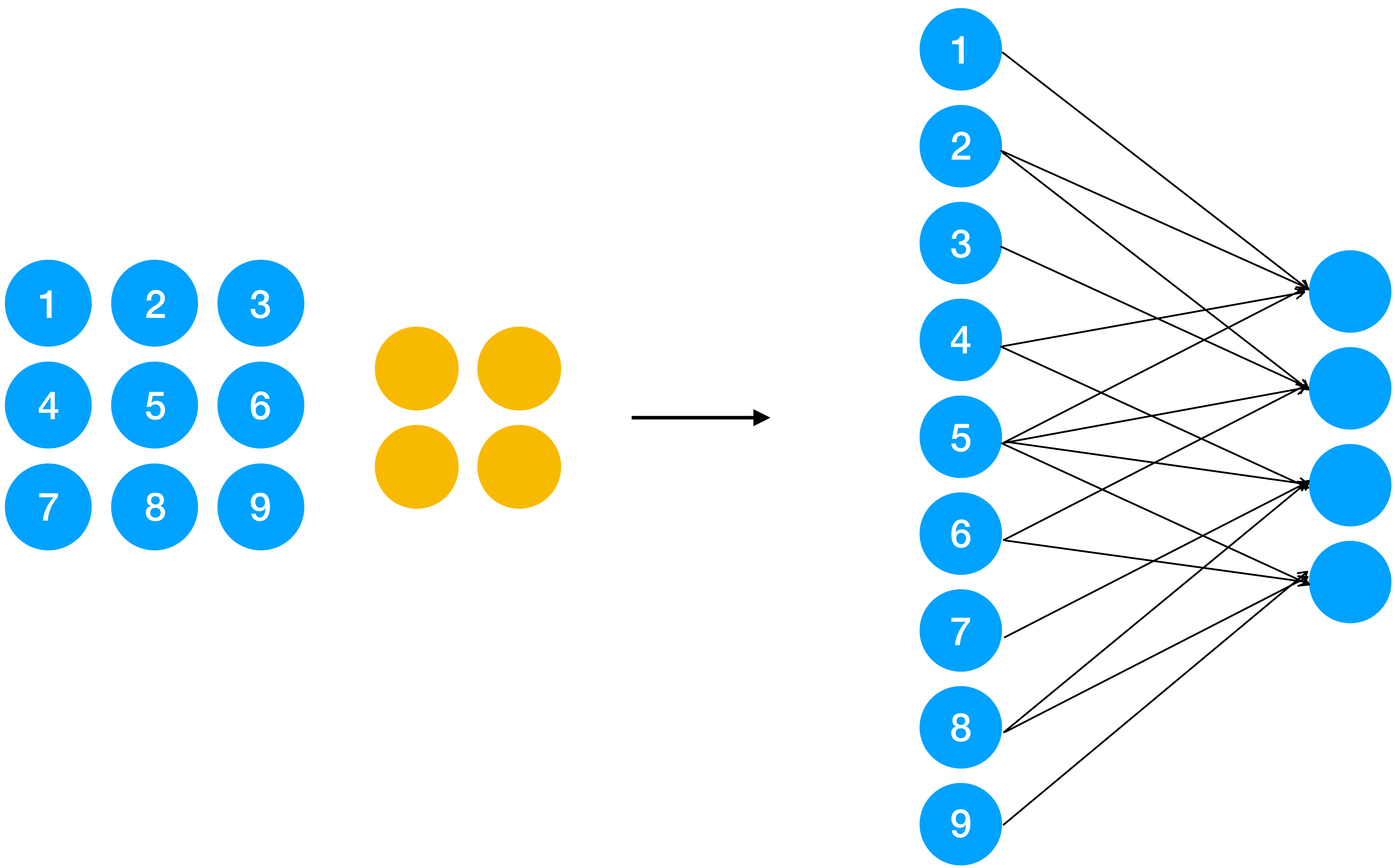

2*2卷积

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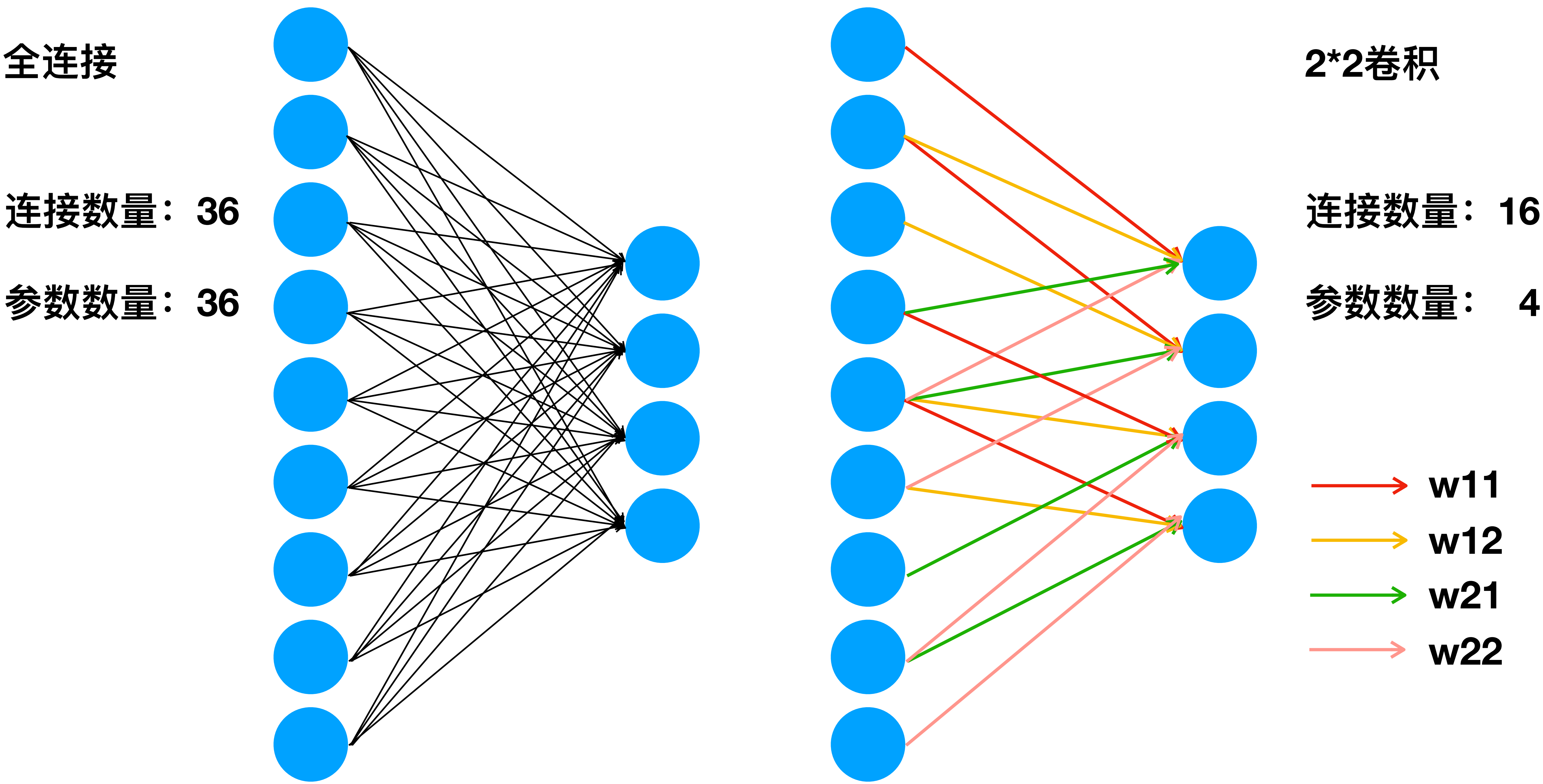
局部连接与权值共享



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CNN反向传播算法

与全连接神经网络相比

1. 池化层的前一层残差计算方式不同。

2. 卷积层的前一层残差计算方式不同。

3. 卷积核中的参数的偏导数计算方式不同。

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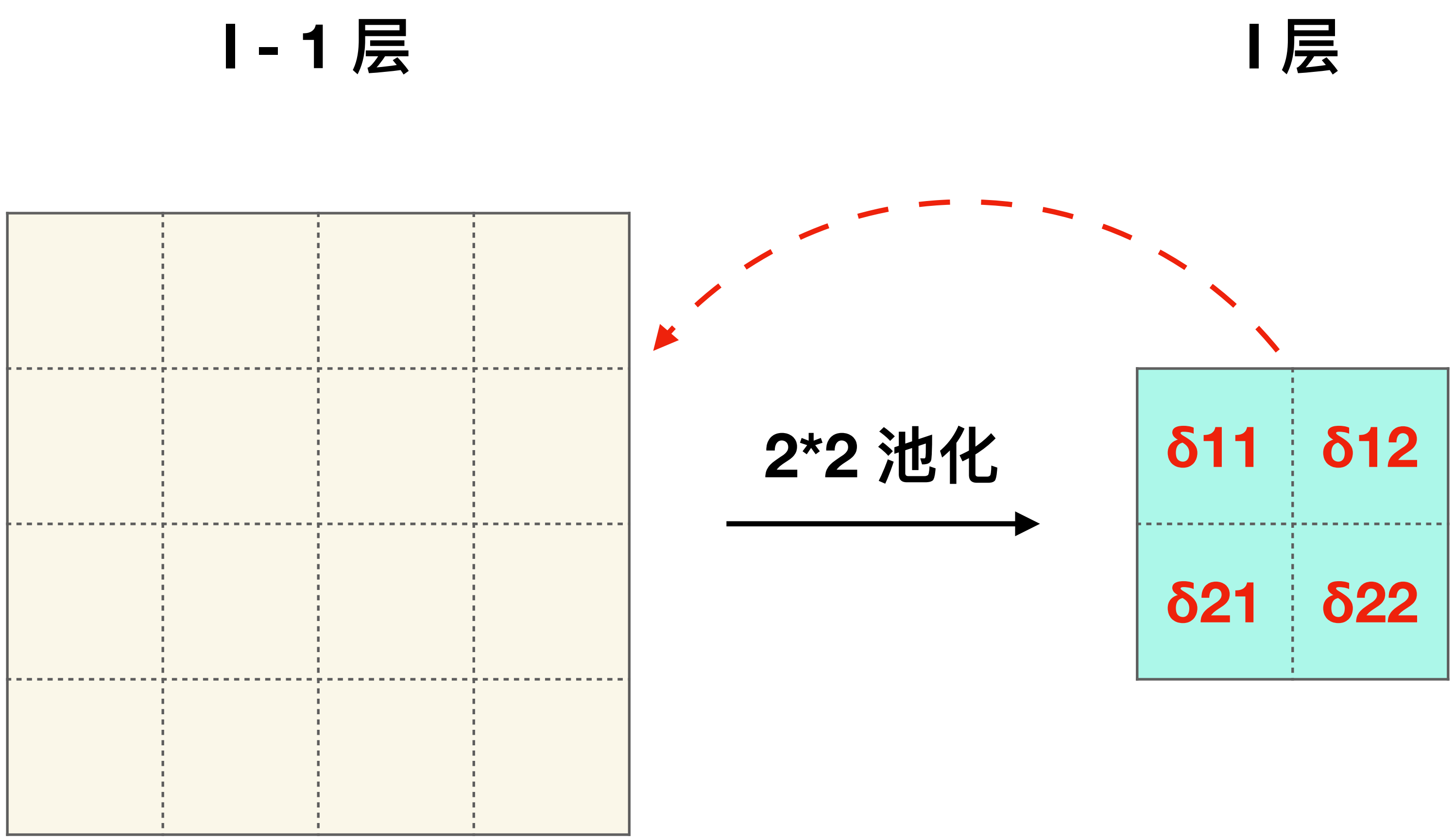
池化层的前一层的残差



一般的，池化层没有参数需要学习，仅需要将残差传递给前一层即可。

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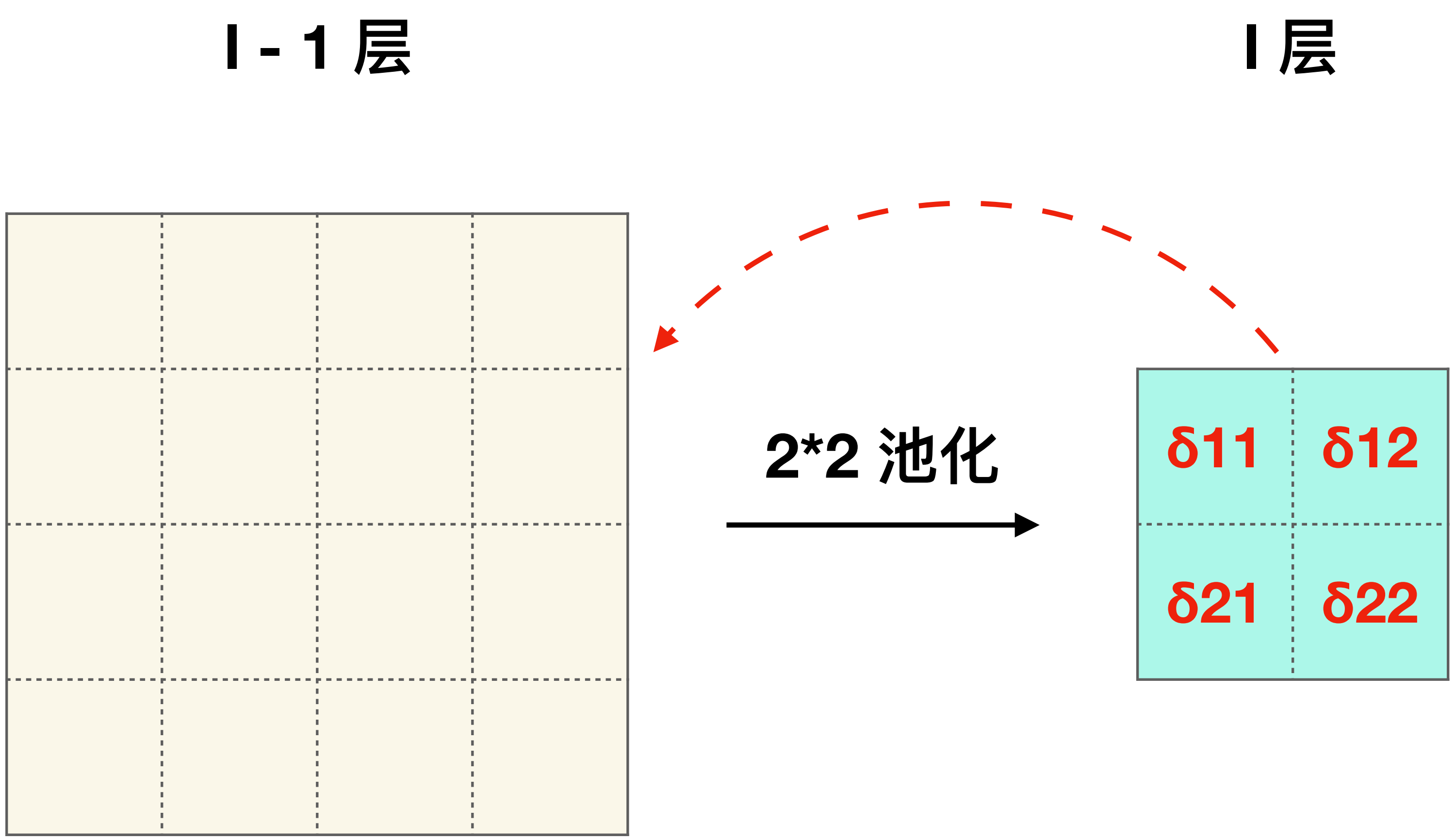
池化层前向传播



$a_{u,v}^{(l-1)}$ 表示l-1层的输出

$z_{p,q}^{(l)}$ 表示l层的输入

$a_{p,q}^{(l)}$ 表示l层的输出

$$a_{p,q}^{(l)} = z_{p,q}^{(l)} = pooling(a_{u,v}^{(l-1)})$$

为了方便，此处使用步长等于滑窗大小的池化方法。且不考虑池化层激活函数。

最大池化层的前一层残差

l - 1 层

1	8	4	3
7	2	1	9
6	1	2	4
8	7	6	0

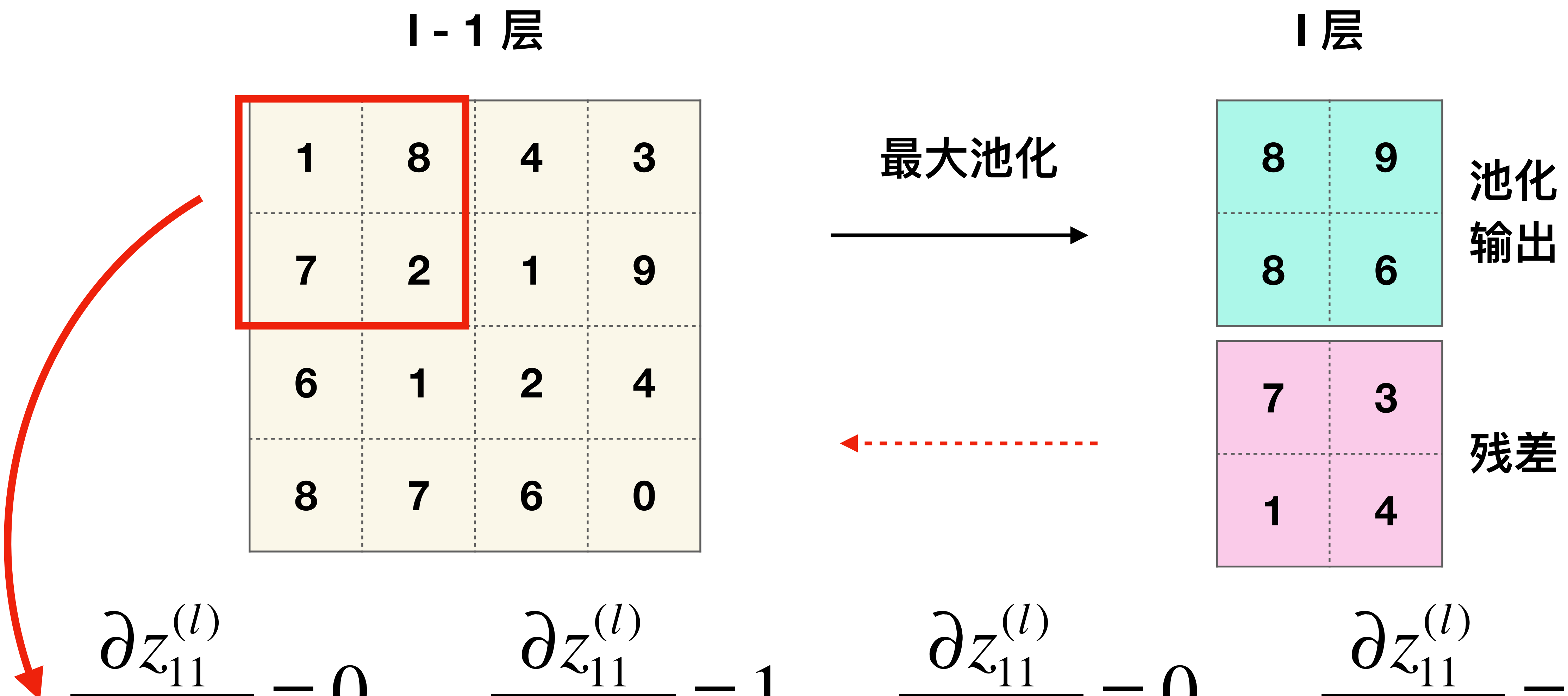
最大池化 →

l 层

8	9
8	6

$$a_{p,q}^{(l)} = z_{p,q}^{(l)} = \max(a_{(p-1)*m+1,(q-1)*n+1}^{(l-1)},\cdots,a_{p*m,q*n}^{(l-1)})$$ 此处的m、n指步长。

最大池化层的前一层残差



$$\frac{\partial z_{11}^{(l)}}{\partial a_{11}^{(l-1)}}=0 \qquad \frac{\partial z_{11}^{(l)}}{\partial a_{12}^{(l-1)}}=1 \qquad \frac{\partial z_{11}^{(l)}}{\partial a_{12}^{(l-1)}}=0 \qquad \frac{\partial z_{11}^{(l)}}{\partial a_{12}^{(l-1)}}=0$$

$$a_{12}^{(l-1)}=z_{11}^{(l)}$$

最大池化层的前一层残差

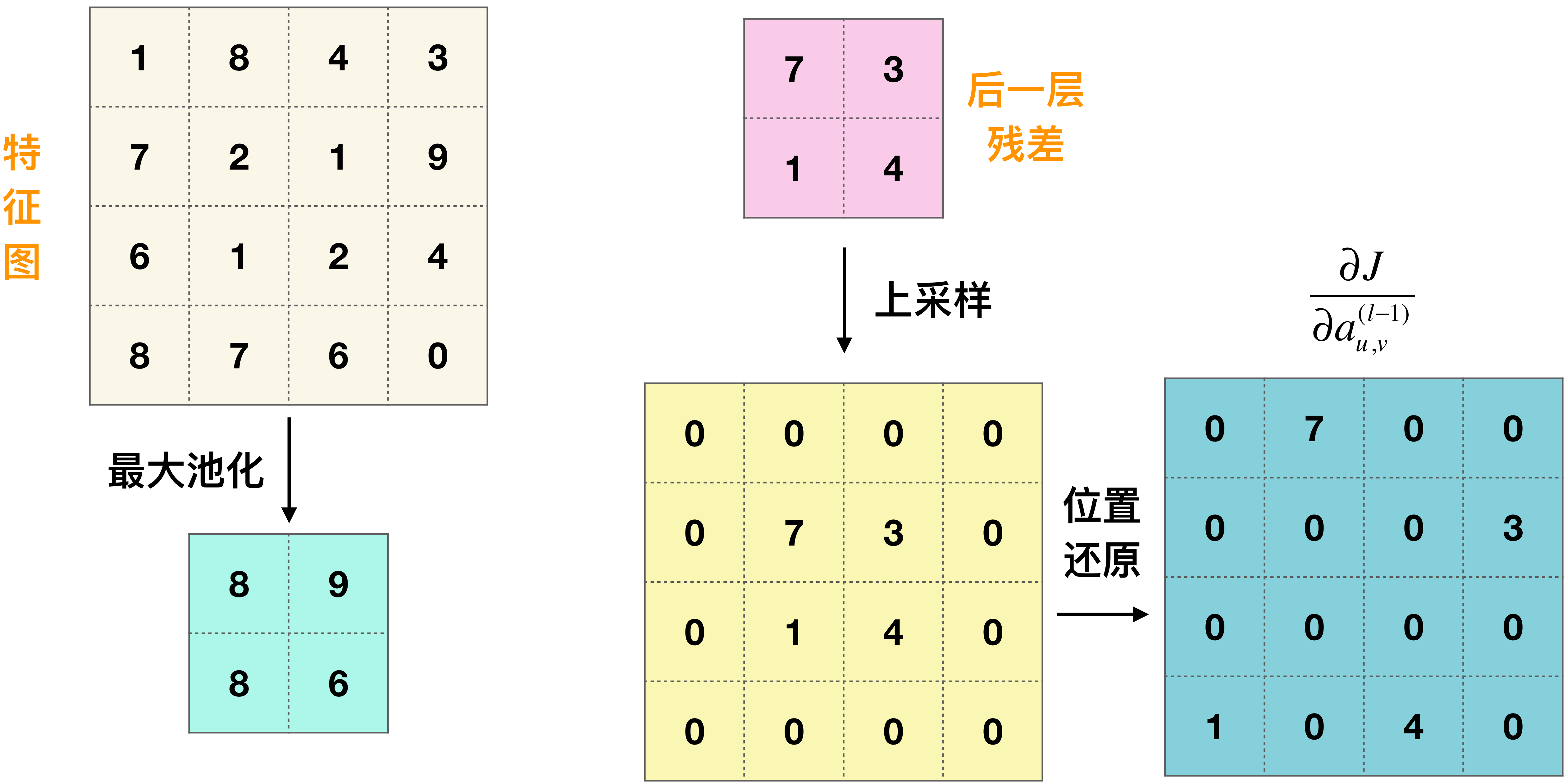
$$\delta_{u,v}^{(l-1)} = \frac{\partial J}{\partial a_{u,v}^{(l-1)}} \frac{\partial a_{u,v}^{(l-1)}}{\partial z_{u,v}^{(l-1)}} = \frac{\partial J}{\partial z_{p,q}^{(l)}} \frac{\partial z_{p,q}^{(l)}}{\partial a_{u,v}^{(l-1)}} \frac{\partial a_{u,v}^{(l-1)}}{\partial z_{u,v}^{(l-1)}} = \delta_{p,q}^{(l)} \frac{\partial z_{p,q}^{(l)}}{\partial a_{u,v}^{(l-1)}} \sigma'(z_{u,v}^{(l-1)})$$

当 $\frac{\partial z_{p,q}^{(l)}}{\partial a_{u,v}^{(l-1)}} = 0$ 则 $\delta_{u,v}^{(l-1)} = 0$

当 $\frac{\partial z_{p,q}^{(l)}}{\partial a_{u,v}^{(l-1)}} = 1$ 则 $\delta_{u,v}^{(l-1)} = \delta_{p,q}^{(l)} \sigma'(z_{u,v}^{(l-1)})$

$$\delta_{u,v}^{(l-1)} = upsample(\delta_{p,q}^{(l)}) \sigma'(z_{u,v}^{(l-1)})$$

最大池化层的前一层残差



平均池化层的前一层残差

l - 1 层

1	8	4	3
7	2	1	9
6	1	2	4
8	7	6	0

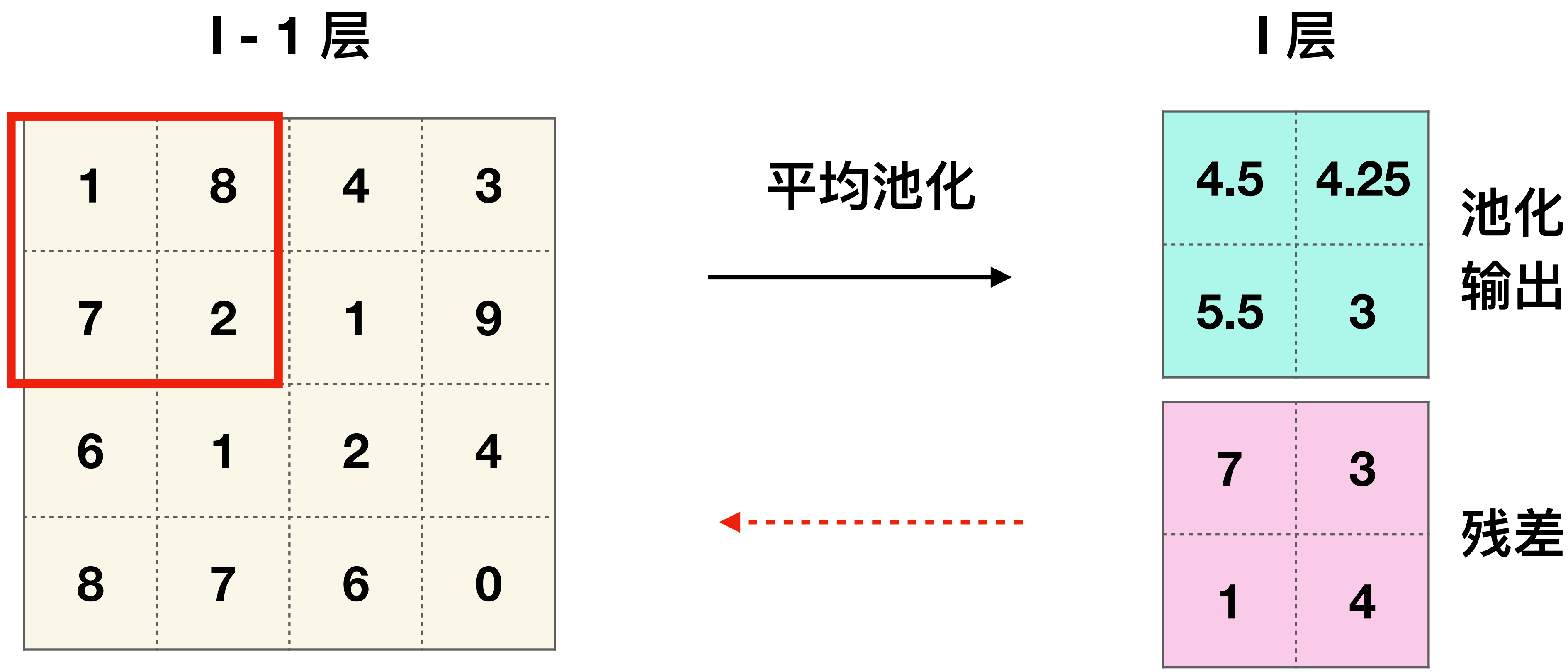
平均池化 →

l 层

4.5	4.25
5.5	3

$a_{p,q}^{(l)} = z_{p,q}^{(l)} = avg(sum(a_{(p-1)*m+1,(q-1)*n+1}^{(l-1)}, \cdots, a_{p*m,q*n}^{(l-1)}))$ 此处的m、n指步长。

平均池化层的前一层残差



$$\frac{\partial z_{11}^{(l)}}{\partial a_{11}^{(l-1)}} = \frac{1}{4} \qquad \frac{\partial z_{11}^{(l)}}{\partial a_{12}^{(l-1)}} = \frac{1}{4} \qquad \frac{\partial z_{11}^{(l)}}{\partial a_{12}^{(l-1)}} = \frac{1}{4} \qquad \frac{\partial z_{11}^{(l)}}{\partial a_{12}^{(l-1)}} = \frac{1}{4}$$

$$z_{11}^{(l)} = \frac{1}{4}(a_{11}^{(l-1)} + a_{12}^{(l-1)} + a_{21}^{(l-1)} + a_{22}^{(l-1)})$$

平均池化层的前一层残差

$$\delta_{u,v}^{(l-1)} = \frac{\partial J}{\partial a_{u,v}^{(l-1)}} \frac{\partial a_{u,v}^{(l-1)}}{\partial z_{u,v}^{(l-1)}} = \frac{\partial J}{\partial z_{p,q}^{(l)}} \frac{\partial z_{p,q}^{(l)}}{\partial a_{u,v}^{(l-1)}} \frac{\partial a_{u,v}^{(l-1)}}{\partial z_{u,v}^{(l-1)}} = \frac{1}{m*n} \delta_{p,q}^{(l)} \sigma'(z_{u,v}^{(l-1)})$$

此处m、n分别表示滑窗的高于宽。

平均池化层的前一层残差

特征图

1	8	4	3
7	2	1	9
6	1	2	4
8	7	6	0

平均池化

4.5	4.25
5.5	3

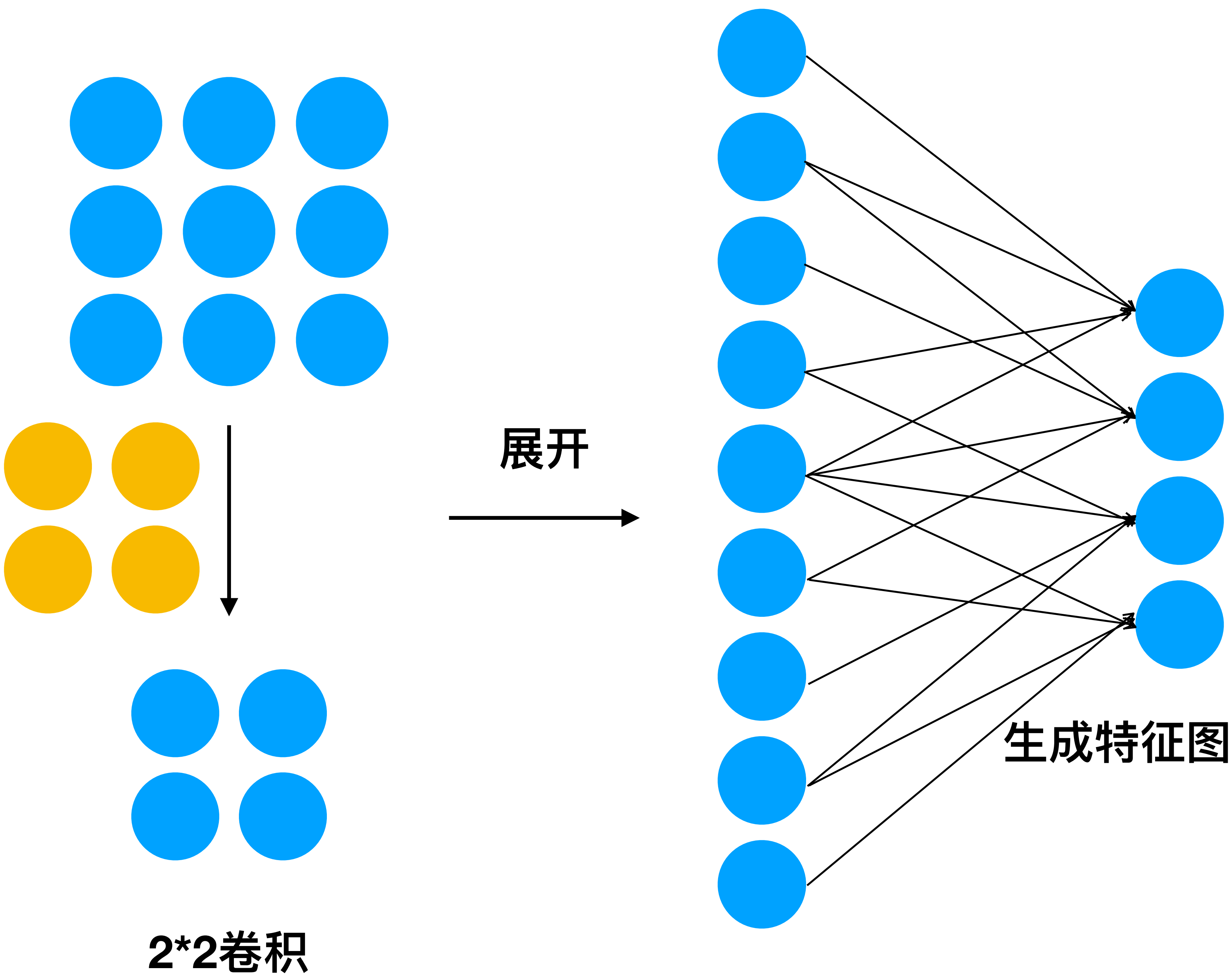
后一层残差

8	4
16	4

2	2	1	1
2	2	1	1
4	4	1	1
4	4	1	1

$$\frac{\partial J}{\partial a_{u,v}^{(l-1)}}$$

卷积的前一层的残差



卷积在正向传播时，卷积参数与输入运算，输出的是特征图。反向传播时，卷积参数与卷积层残差运算，输出的是前一层的残差。

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全连接ANN反向传播算法

全连接神经网络隐藏层的残差：

$$\delta_j^{(l)} = (w_j^{(l+1)})^{\mathrm{T}} \cdot \delta^{(l+1)} \cdot \sigma'(z_j^{(l)})$$

即 $$\delta_j^{(l)} \sim (w_j^{(l+1)})^{\mathrm{T}} \cdot \delta^{(l+1)}$$

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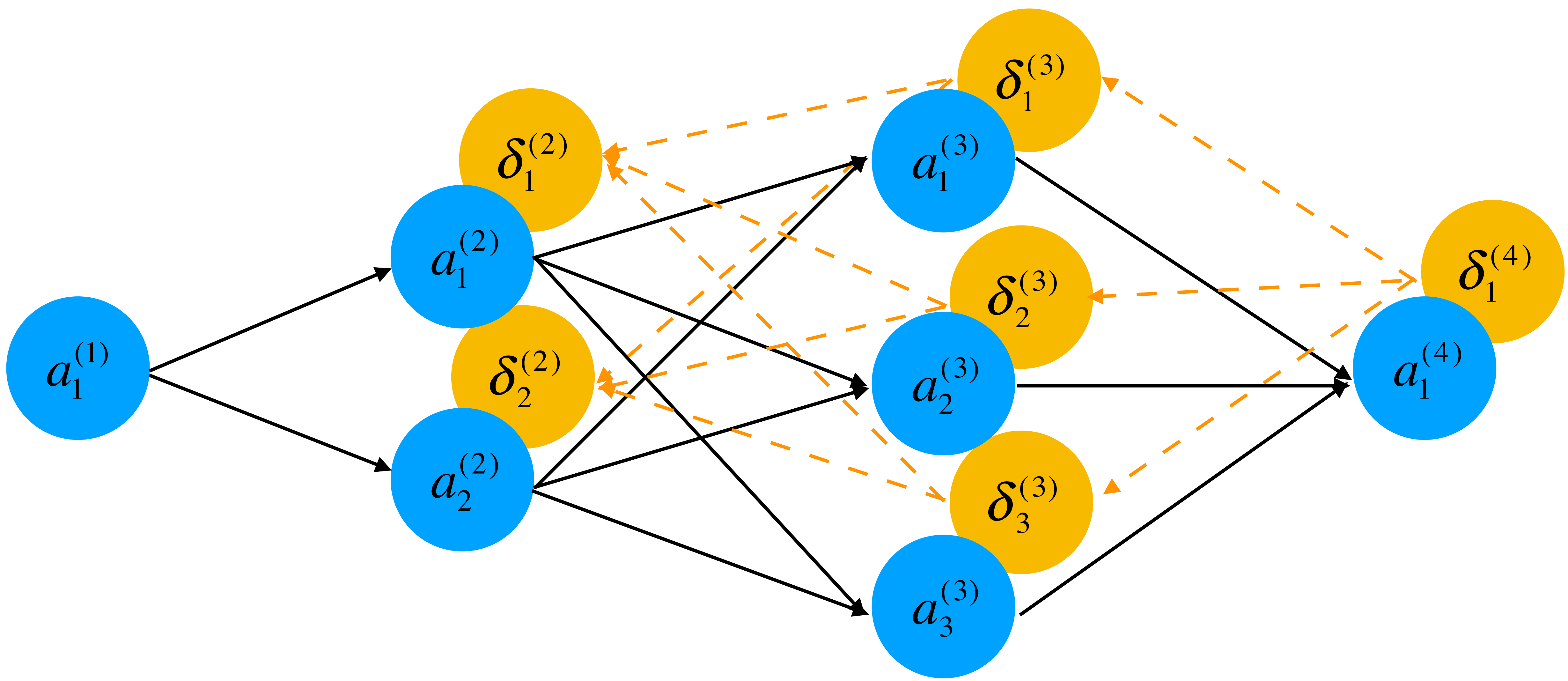
全连接ANN反向传播算法

全连接神经网络隐藏层的残差：



$$\delta_1^{(2)} \sim w_{11}^{(3)} \cdot \delta_1^{(3)} + w_{21}^{(3)} \cdot \delta_2^{(3)} + w_{31}^{(3)} \cdot \delta_3^{(3)}$$

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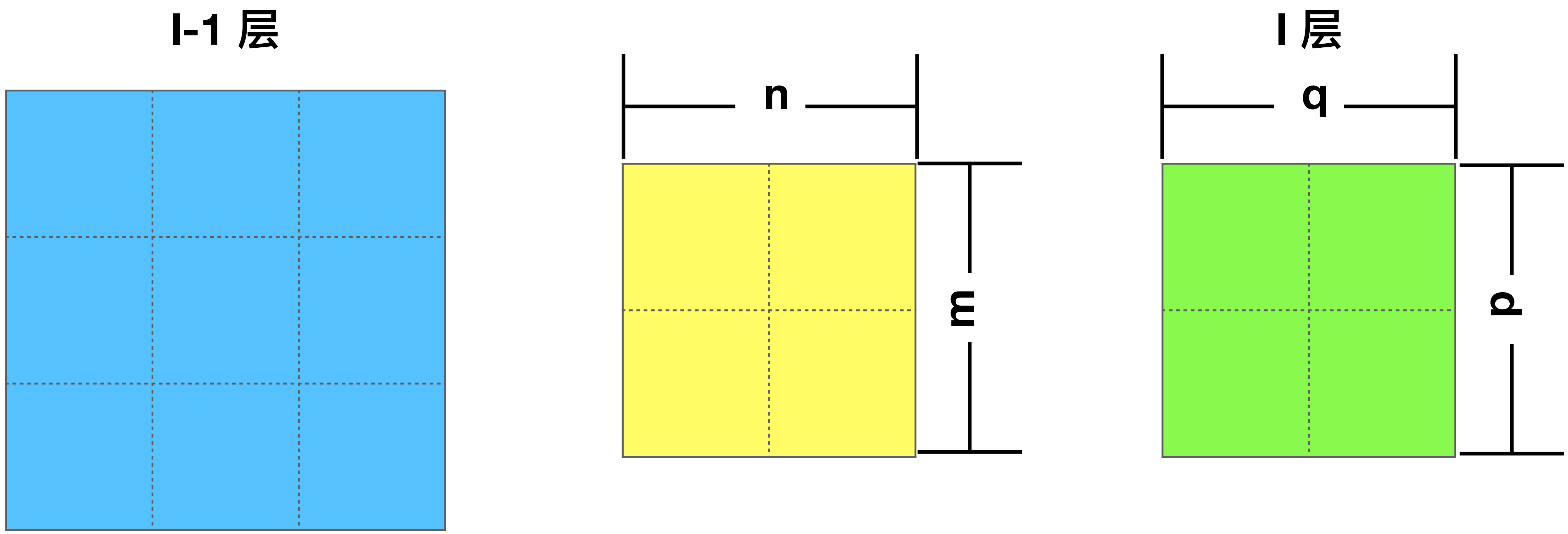
卷积的前向传播



输入　　卷积核　　特征图

$$z_{p,q}^{(l)} = \sum_{m} \sum_{n} w_{m,n}^{(l)} a_{p+m-1,q+n-1}^{(l-1)} + b^{(l)} \quad a_{p,q}^{(l)} = \sigma(z_{p,q}^{(l)})$$

为了计算方便，默认情况下步长为1，卷积核数量为1。

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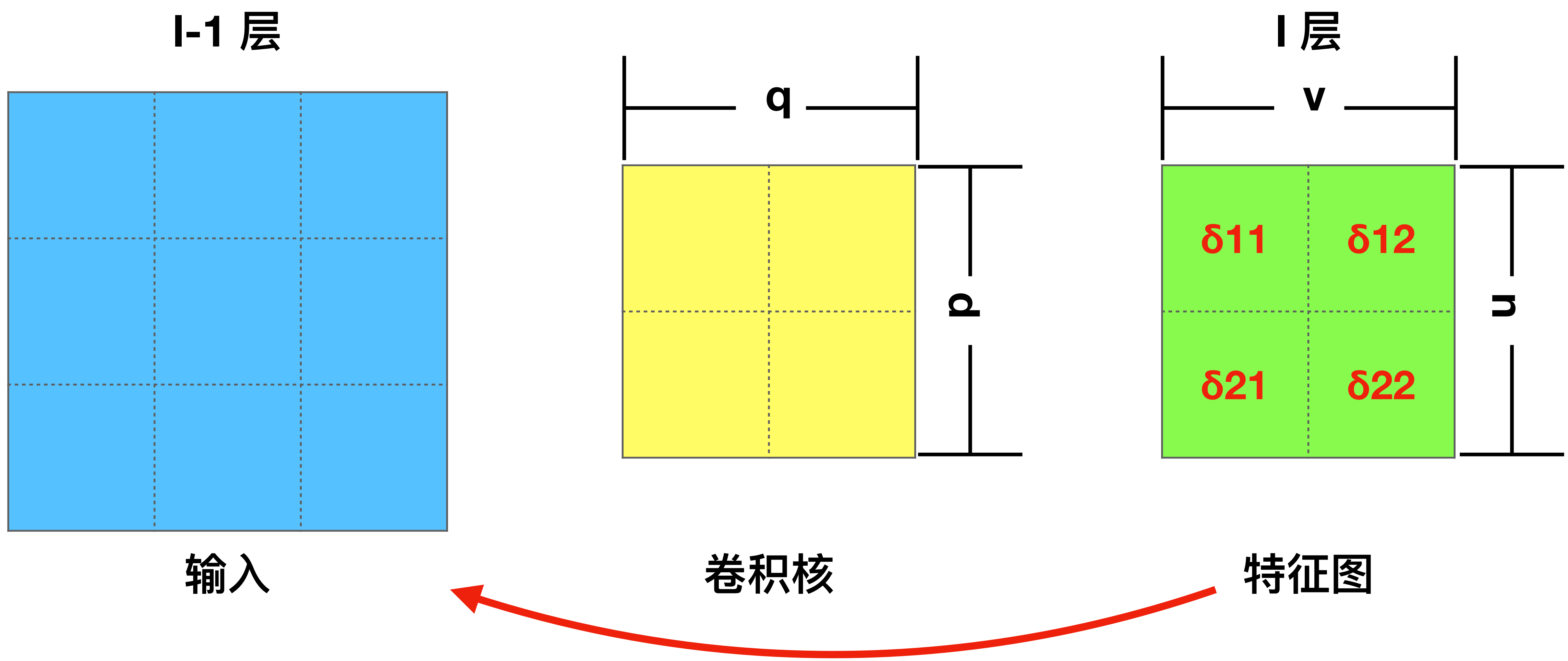
例子—前向传播



卷积层的输入

$$z_{11}^{(l)} = a_{11}^{(l-1)} w_{11}^{(l)} + a_{12}^{(l-1)} w_{12}^{(l)} + a_{21}^{(l-1)} w_{21}^{(l)} + a_{22}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{12}^{(l)} = a_{12}^{(l-1)} w_{11}^{(l)} + a_{13}^{(l-1)} w_{12}^{(l)} + a_{22}^{(l-1)} w_{21}^{(l)} + a_{23}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{21}^{(l)} = a_{21}^{(l-1)} w_{11}^{(l)} + a_{22}^{(l-1)} w_{12}^{(l)} + a_{31}^{(l-1)} w_{21}^{(l)} + a_{32}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{22}^{(l)} = a_{22}^{(l-1)} w_{11}^{(l)} + a_{23}^{(l-1)} w_{12}^{(l)} + a_{32}^{(l-1)} w_{21}^{(l)} + a_{33}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

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例子—残差

$$z_{11}^{(l)} = \boxed{a_{11}^{(l-1)} w_{11}^{(l)}} + a_{12}^{(l-1)} w_{12}^{(l)} + a_{21}^{(l-1)} w_{21}^{(l)} + a_{22}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{12}^{(l)} = a_{12}^{(l-1)} w_{11}^{(l)} + a_{13}^{(l-1)} w_{12}^{(l)} + a_{22}^{(l-1)} w_{21}^{(l)} + a_{23}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{21}^{(l)} = a_{21}^{(l-1)} w_{11}^{(l)} + a_{22}^{(l-1)} w_{12}^{(l)} + a_{31}^{(l-1)} w_{21}^{(l)} + a_{32}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{22}^{(l)} = a_{22}^{(l-1)} w_{11}^{(l)} + a_{23}^{(l-1)} w_{12}^{(l)} + a_{32}^{(l-1)} w_{21}^{(l)} + a_{33}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$\delta_{11}^{(l-1)}$ **残差仅仅与z11输入有关**

$$\frac{\partial z_{11}^{(l)}}{\partial a_{11}^{(l-1)}} = \frac{\partial (a_{11}^{(l-1)} w_{11}^{(l)})}{\partial a_{11}^{(l-1)}} = w_{11}^{(l)}$$

$$\delta_{11}^{(l-1)} = \frac{\partial J}{\partial z_{11}^{(l-1)}} = \frac{\partial J}{\partial z_{11}^{(l)}} \frac{\partial z_{11}^{(l)}}{\partial a_{11}^{(l-1)}} \frac{\partial a_{11}^{(l-1)}}{\partial z_{11}^{(l-1)}} = \delta_{11}^{(l)} w_{11}^{(l)} \sigma'(z_{11}^{(l-1)})$$

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例子—残差

$$z_{11}^{(l)} = a_{11}^{(l-1)} w_{11}^{(l)} + \boxed{a_{12}^{(l-1)} w_{12}^{(l)}} + a_{21}^{(l-1)} w_{21}^{(l)} + a_{22}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{12}^{(l)} = \boxed{a_{12}^{(l-1)} w_{11}^{(l)}} + a_{13}^{(l-1)} w_{12}^{(l)} + a_{22}^{(l-1)} w_{21}^{(l)} + a_{23}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{21}^{(l)} = a_{21}^{(l-1)} w_{11}^{(l)} + a_{22}^{(l-1)} w_{12}^{(l)} + a_{31}^{(l-1)} w_{21}^{(l)} + a_{32}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{22}^{(l)} = a_{22}^{(l-1)} w_{11}^{(l)} + a_{23}^{(l-1)} w_{12}^{(l)} + a_{32}^{(l-1)} w_{21}^{(l)} + a_{33}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$\delta_{11}^{(l-1)}$ **残差与z11、z12输入有关**

$$\frac{\partial z_{11}^{(l)}}{\partial a_{12}^{(l-1)}} = \frac{\partial (a_{12}^{(l-1)} w_{12}^{(l)})}{\partial a_{12}^{(l-1)}} = w_{12}^{(l)}$$

$$\frac{\partial z_{12}^{(l)}}{\partial a_{12}^{(l-1)}} = \frac{\partial (a_{12}^{(l-1)} w_{11}^{(l)})}{\partial a_{12}^{(l-1)}} = w_{11}^{(l)}$$

$$\delta_{12}^{(l-1)} = \frac{\partial J}{\partial z_{12}^{(l-1)}} = \frac{\partial J}{\partial z_{11}^{(l)}} \frac{\partial z_{11}^{(l)}}{\partial a_{12}^{(l-1)}} \frac{\partial a_{12}^{(l-1)}}{\partial z_{12}^{(l-1)}} + \frac{\partial J}{\partial z_{12}^{(l)}} \frac{\partial z_{12}^{(l)}}{\partial a_{12}^{(l-1)}} \frac{\partial a_{12}^{(l-1)}}{\partial z_{12}^{(l-1)}} = \delta_{11}^{(l)} w_{12}^{(l)} \sigma'(z_{12}^{(l-1)}) + \delta_{12}^{(l)} w_{11}^{(l)} \sigma'(z_{12}^{(l-1)})$$

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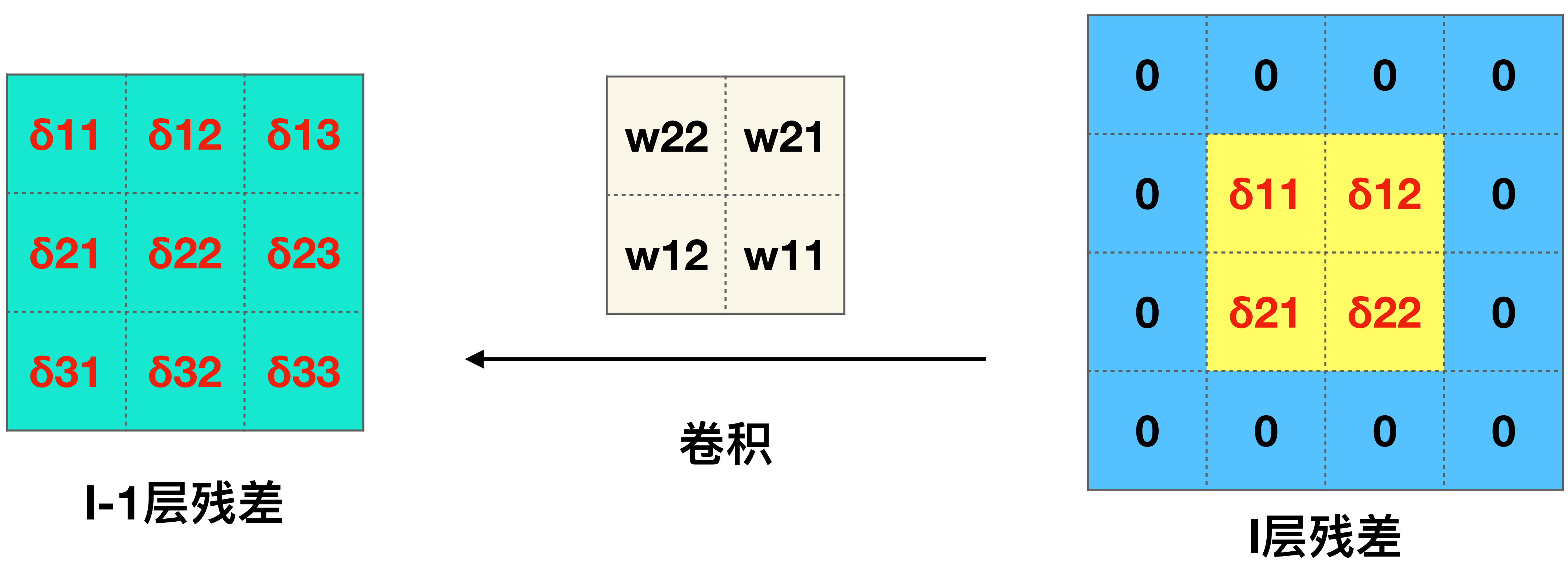
例子：残差规律

$$z_{11}^{(l)}=\boxed{a_{11}^{(l-1)}w_{11}^{(l)}}+\boxed{a_{12}^{(l-1)}w_{12}^{(l)}}+\boxed{a_{21}^{(l-1)}w_{21}^{(l)}}+\boxed{a_{22}^{(l-1)}w_{22}^{(l)}}+b^{(l)}$$
$$z_{12}^{(l)}=\boxed{a_{12}^{(l-1)}w_{11}^{(l)}}+\boxed{a_{13}^{(l-1)}w_{12}^{(l)}}+\boxed{a_{22}^{(l-1)}w_{21}^{(l)}}+\boxed{a_{23}^{(l-1)}w_{22}^{(l)}}+b^{(l)}$$
$$z_{21}^{(l)}=\boxed{a_{21}^{(l-1)}w_{11}^{(l)}}+\boxed{a_{22}^{(l-1)}w_{12}^{(l)}}+\boxed{a_{31}^{(l-1)}w_{21}^{(l)}}+\boxed{a_{32}^{(l-1)}w_{22}^{(l)}}+b^{(l)}$$
$$z_{22}^{(l)}=\boxed{a_{22}^{(l-1)}w_{11}^{(l)}}+\boxed{a_{23}^{(l-1)}w_{12}^{(l)}}+\boxed{a_{32}^{(l-1)}w_{21}^{(l)}}+\boxed{a_{33}^{(l-1)}w_{22}^{(l)}}+b^{(l)}$$



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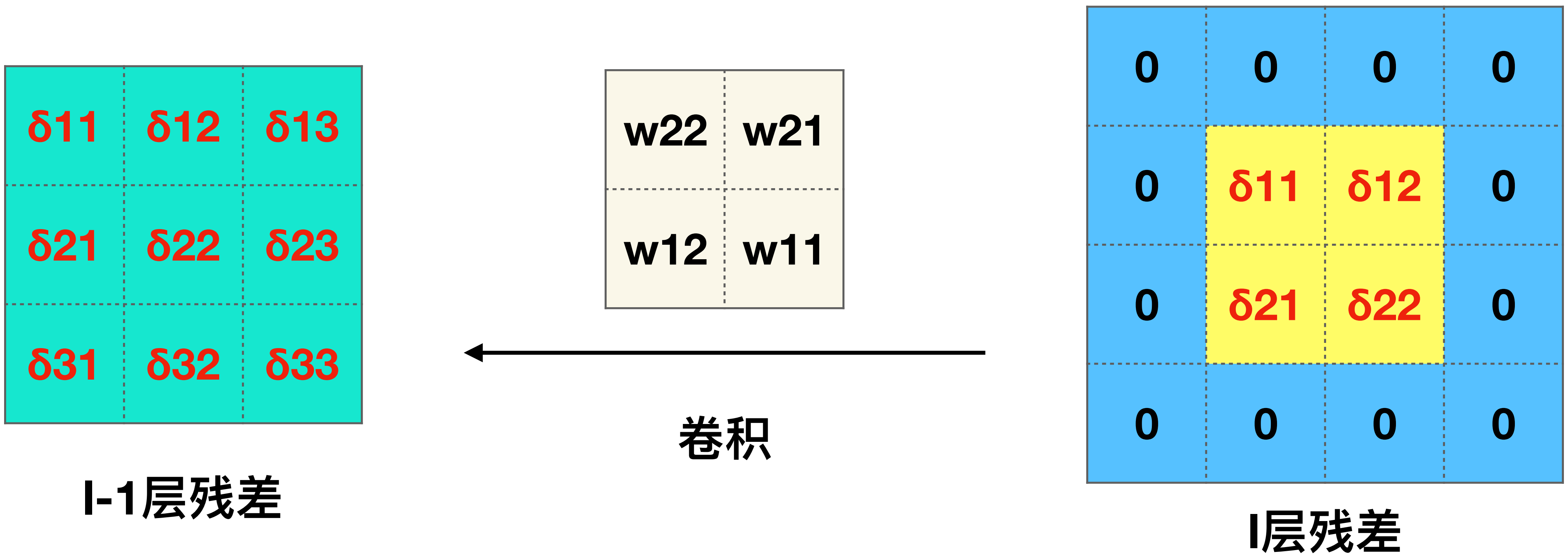
前一层残差计算方式



1. 将卷积权重翻转180°
2. 给当前层残差pad一圈0
3. 使用翻转后的卷积核对pad后的残差做卷积

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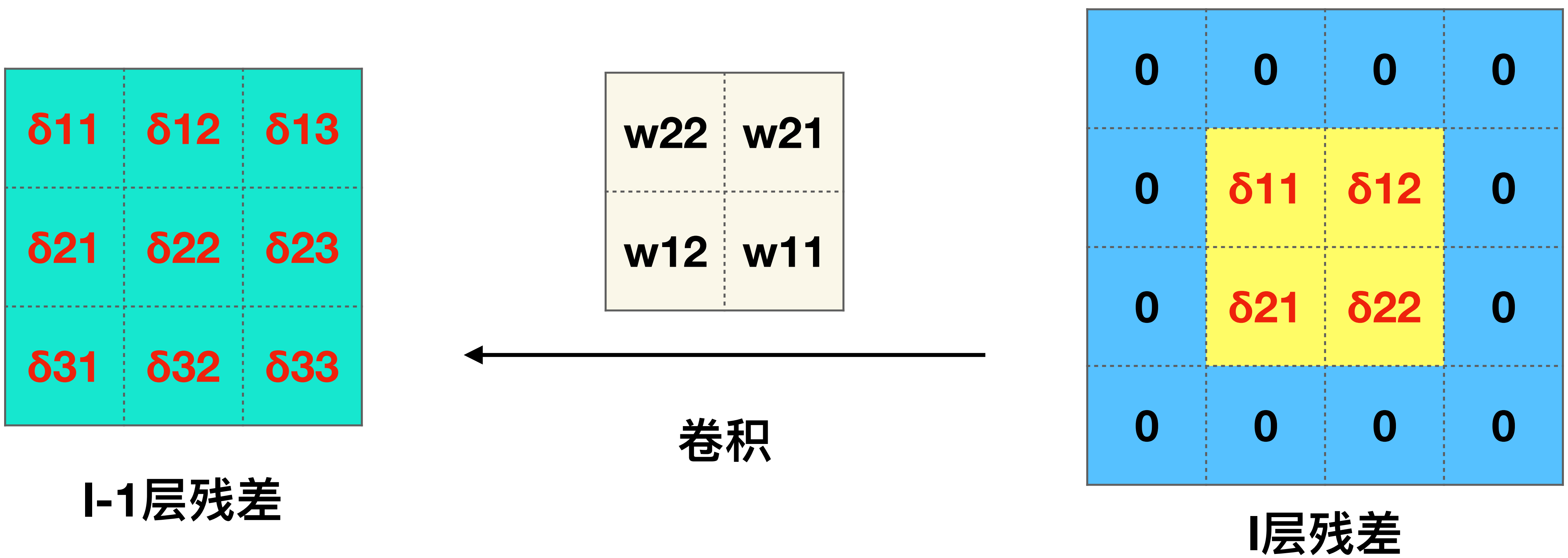
前一层残差计算公式



$$\delta_{u,v}^{(l-1)} = \frac{\partial J}{\partial z_{u,v}^{(l-1)}} = \frac{\partial J}{\partial a_{u,v}^{(l-1)}} \frac{\partial a_{u,v}^{(l-1)}}{\partial z_{u,v}^{(l-1)}} = \sum_m \sum_n w_{m,n}^{(l)} \delta_{m+u-1,n+v-1}^{l} \sigma'(z_{u,v}^{(l-1)})$$

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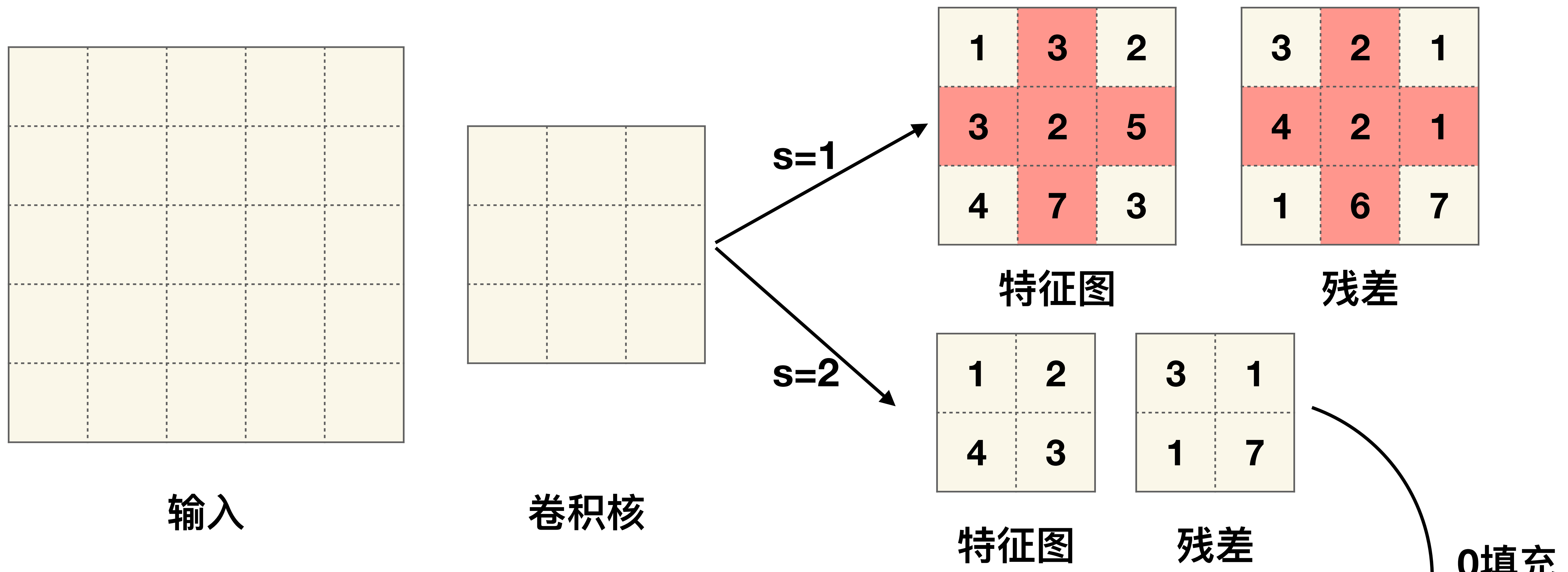
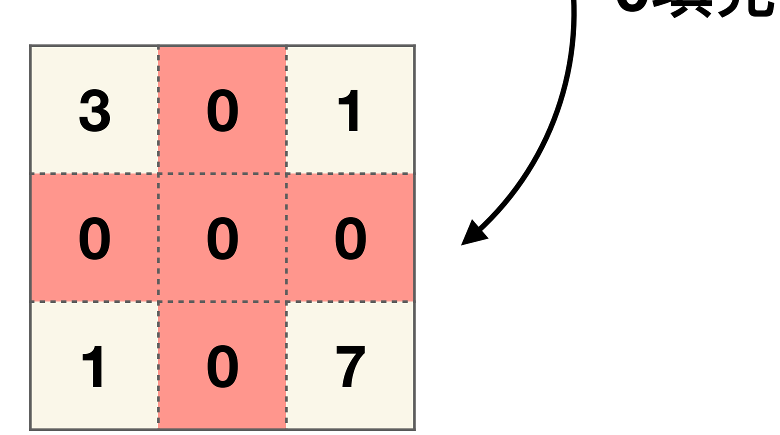
当步长不为1时





1. 将卷积权重翻转180°
2. 使用0填充残差对应位置
3. 在残差外层填充0
4. 使用填充后的残差与翻转后的权重卷积

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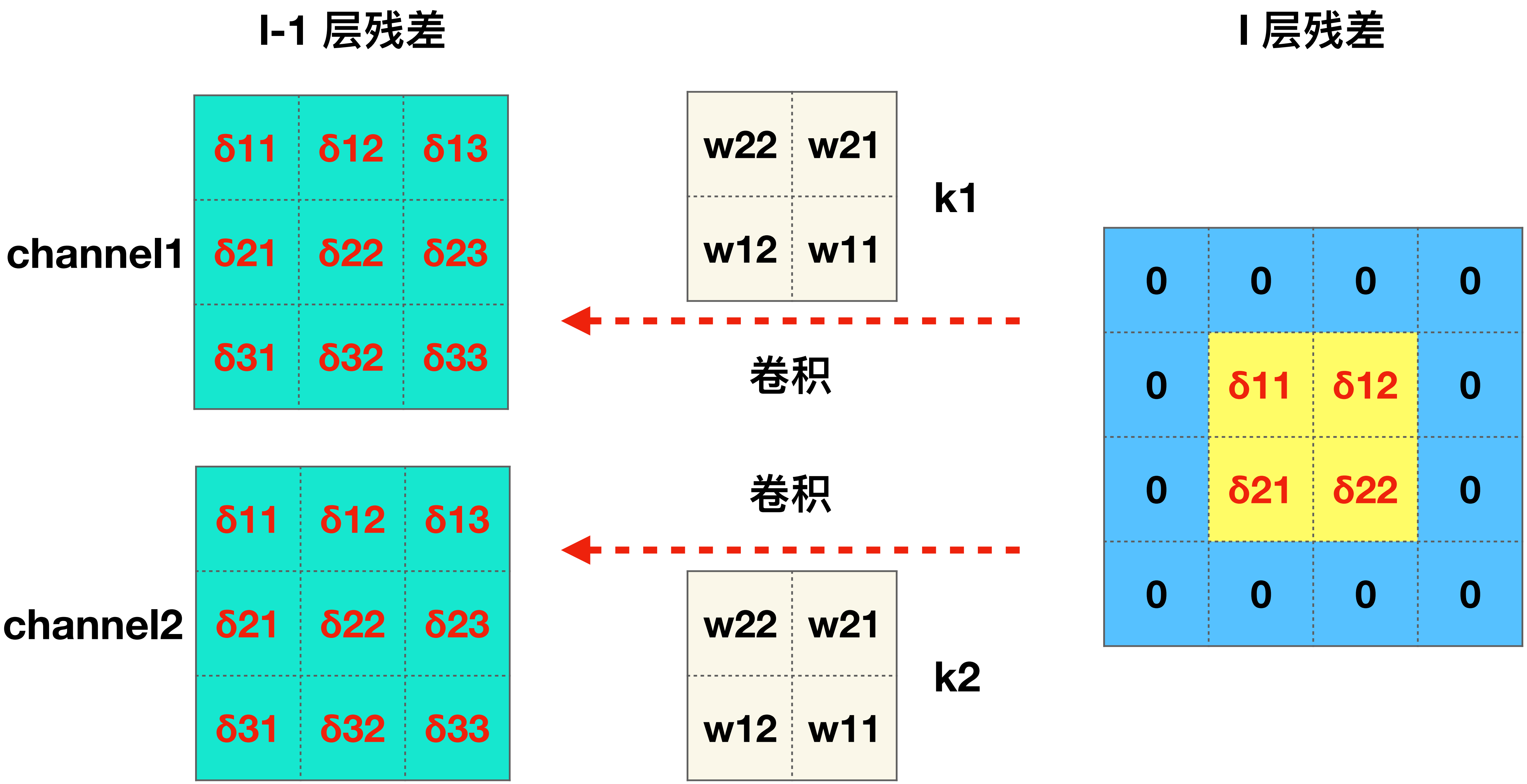
当卷积输入通道不为1时



k1、k2分布表示每个通道的卷积参数，k1、k2一起组成了一个3D卷积核。

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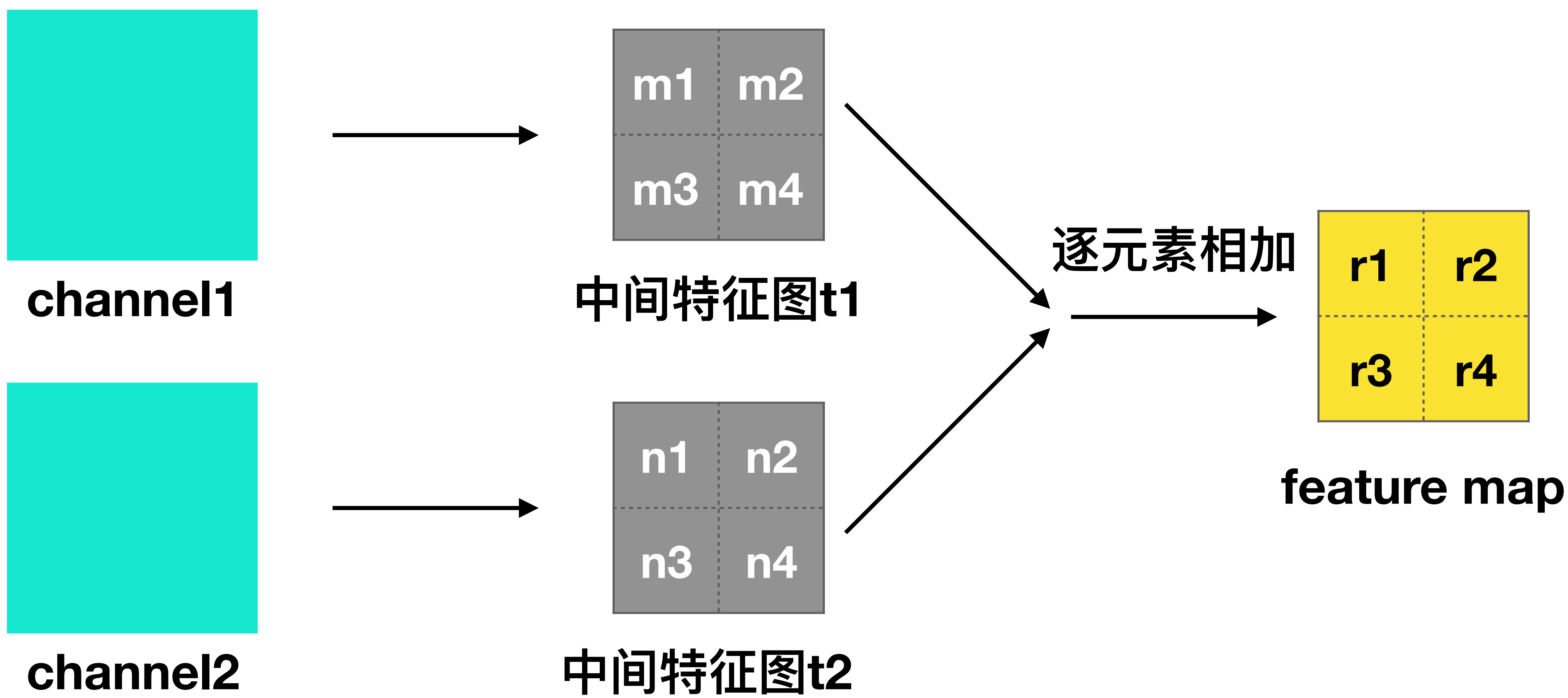
当卷积输入通道不为1时



$$\frac{\partial(x_1+x_2)}{\partial x_1}=\frac{\partial(x_1+x_2)}{\partial x_2}=1$$

中间特征图的残差等于特征图的残差。

$$\frac{\partial J}{\partial r}=\frac{\partial J}{\partial(t_1+t_2)}=\frac{\partial J}{\partial r}\frac{\partial r}{\partial x_1}=\frac{\partial J}{\partial r}\frac{\partial r}{\partial x_2}$$

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当卷积核数量不为1时

k1、k2分别表示两个卷积核。

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当输入的边界有0填充时

1. 卷积输入的边界处理不影响反向传播过程。

2. 计算填充0的边界的残差是冗余的，没有意义的。

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卷积核的梯度

卷积中参数：

1. 卷积中的参数都来自于卷积核。

2. 卷积核的参数与当前层的输入有关。

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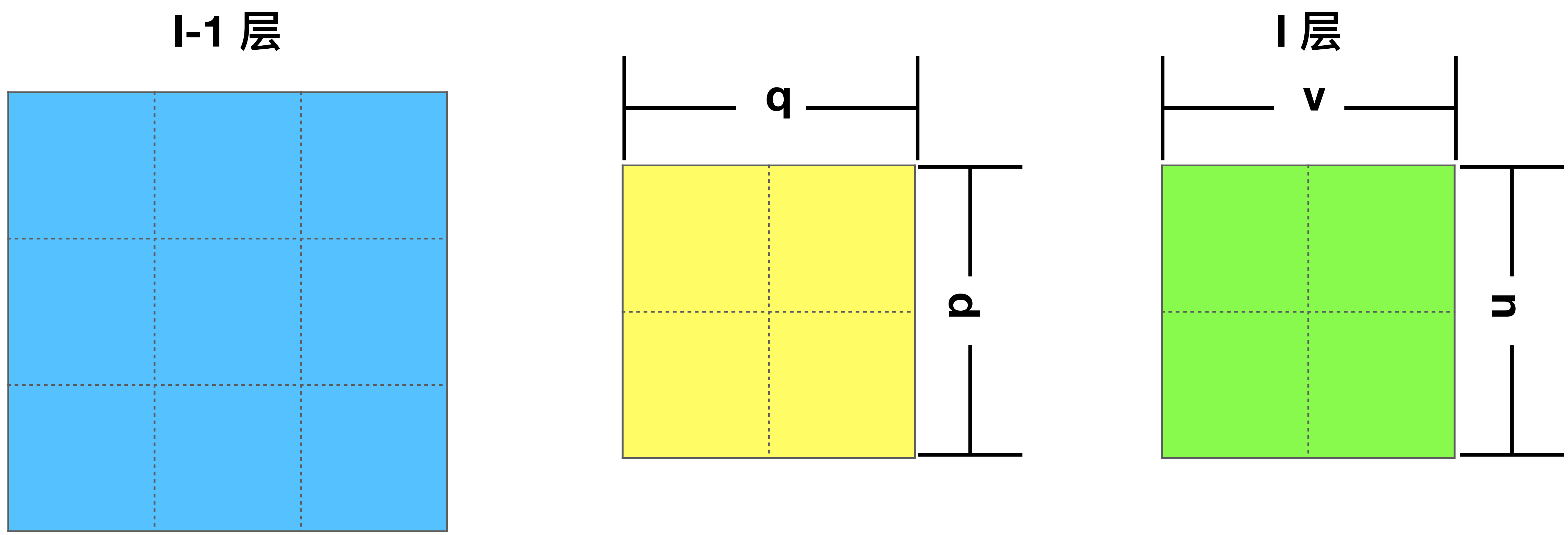
例子



输入　卷积核　特征图

$$z_{11}^{(l)} = a_{11}^{(l-1)} w_{11}^{(l)} + a_{12}^{(l-1)} w_{12}^{(l)} + a_{21}^{(l-1)} w_{21}^{(l)} + a_{22}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{12}^{(l)} = a_{12}^{(l-1)} w_{11}^{(l)} + a_{13}^{(l-1)} w_{12}^{(l)} + a_{22}^{(l-1)} w_{21}^{(l)} + a_{23}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{21}^{(l)} = a_{21}^{(l-1)} w_{11}^{(l)} + a_{22}^{(l-1)} w_{12}^{(l)} + a_{31}^{(l-1)} w_{21}^{(l)} + a_{32}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

$$z_{22}^{(l)} = a_{22}^{(l-1)} w_{11}^{(l)} + a_{23}^{(l-1)} w_{12}^{(l)} + a_{32}^{(l-1)} w_{21}^{(l)} + a_{33}^{(l-1)} w_{22}^{(l)} + b^{(l)}$$

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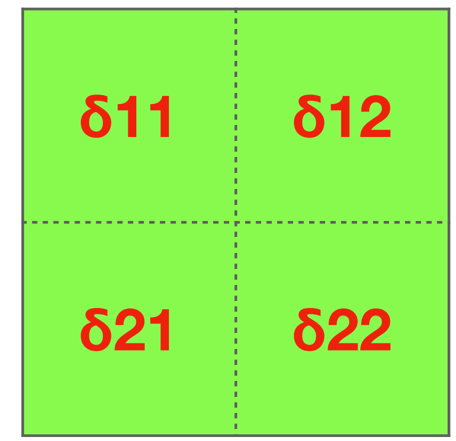
例子—求权重的梯度

$$z_{11}^{(l)} = a_{11}^{(l-1)}w_{11}^{(l)} + a_{12}^{(l-1)}w_{12}^{(l)} + a_{21}^{(l-1)}w_{21}^{(l)} + a_{22}^{(l-1)}w_{22}^{(l)} + b^{(l)}$$

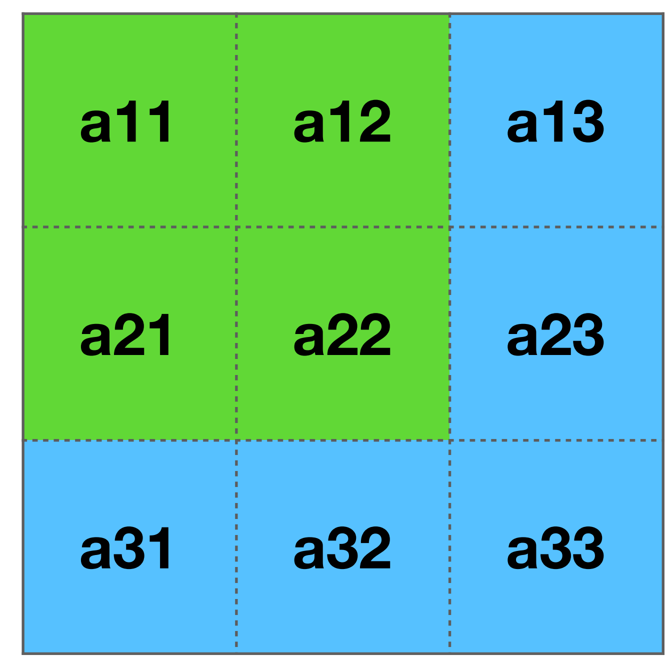
$$z_{12}^{(l)} = a_{12}^{(l-1)}w_{11}^{(l)} + a_{13}^{(l-1)}w_{12}^{(l)} + a_{22}^{(l-1)}w_{21}^{(l)} + a_{23}^{(l-1)}w_{22}^{(l)} + b^{(l)}$$

$$z_{21}^{(l)} = a_{21}^{(l-1)}w_{11}^{(l)} + a_{22}^{(l-1)}w_{12}^{(l)} + a_{31}^{(l-1)}w_{21}^{(l)} + a_{32}^{(l-1)}w_{22}^{(l)} + b^{(l)}$$

$$z_{22}^{(l)} = a_{22}^{(l-1)}w_{11}^{(l)} + a_{23}^{(l-1)}w_{12}^{(l)} + a_{32}^{(l-1)}w_{21}^{(l)} + a_{33}^{(l-1)}w_{22}^{(l)} + b^{(l)}$$



残差



输入

$$\frac{\partial J}{\partial w_{11}^{(l)}} = \frac{\partial J}{\partial z_{11}^{(l)}}\frac{\partial z_{11}^{(l)}}{\partial w_{11}^{(l)}} + \frac{\partial J}{\partial z_{12}^{(l)}}\frac{\partial z_{12}^{(l)}}{\partial w_{11}^{(l)}} + \frac{\partial J}{\partial z_{21}^{(l)}}\frac{\partial z_{21}^{(l)}}{\partial w_{11}^{(l)}} + \frac{\partial J}{\partial z_{22}^{(l)}}\frac{\partial z_{22}^{(l)}}{\partial w_{11}^{(l)}}$$

$$= \delta_{11}^{(l)}a_{11}^{(l-1)} + \delta_{12}^{(l)}a_{12}^{(l-1)} + \delta_{21}^{(l)}a_{21}^{(l-1)} + \delta_{22}^{(l)}a_{22}^{(l-1)}$$

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例子－求权重的梯度

$$z_{11}^{(l)}=a_{11}^{(l-1)}w_{11}^{(l)}+a_{12}^{(l-1)}w_{12}^{(l)}+a_{21}^{(l-1)}w_{21}^{(l)}+a_{22}^{(l-1)}w_{22}^{(l)}+b^{(l)}$$
$$z_{12}^{(l)}=a_{12}^{(l-1)}w_{11}^{(l)}+a_{13}^{(l-1)}w_{12}^{(l)}+a_{22}^{(l-1)}w_{21}^{(l)}+a_{23}^{(l-1)}w_{22}^{(l)}+b^{(l)}$$
$$z_{21}^{(l)}=a_{21}^{(l-1)}w_{11}^{(l)}+a_{22}^{(l-1)}w_{12}^{(l)}+a_{31}^{(l-1)}w_{21}^{(l)}+a_{32}^{(l-1)}w_{22}^{(l)}+b^{(l)}$$
$$z_{22}^{(l)}=a_{22}^{(l-1)}w_{11}^{(l)}+a_{23}^{(l-1)}w_{12}^{(l)}+a_{32}^{(l-1)}w_{21}^{(l)}+a_{33}^{(l-1)}w_{22}^{(l)}+b^{(l)}$$

δ11	δ12
δ21	δ22

残差

$$\frac{\partial J}{\partial w_{12}^{(l)}}=\frac{\partial J}{\partial z_{11}^{(l)}}\frac{\partial z_{11}^{(l)}}{\partial w_{12}^{(l)}}+\frac{\partial J}{\partial z_{12}^{(l)}}\frac{\partial z_{12}^{(l)}}{\partial w_{12}^{(l)}}+\frac{\partial J}{\partial z_{21}^{(l)}}\frac{\partial z_{21}^{(l)}}{\partial w_{12}^{(l)}}+\frac{\partial J}{\partial z_{22}^{(l)}}\frac{\partial z_{22}^{(l)}}{\partial w_{12}^{(l)}}$$
$$=\delta_{11}^{(l)}a_{12}^{(l-1)}+\delta_{12}^{(l)}a_{13}^{(l-1)}+\delta_{21}^{(l)}a_{22}^{(l-1)}+\delta_{22}^{(l)}a_{23}^{(l-1)}$$

a11	a12	a13
a21	a22	a23
a31	a32	a33

输入

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例子—求偏置值的梯度

$$z_{11}^{(l)} = a_{11}^{(l-1)}w_{11}^{(l)} + a_{12}^{(l-1)}w_{12}^{(l)} + a_{21}^{(l-1)}w_{21}^{(l)} + a_{22}^{(l-1)}w_{22}^{(l)} + b^{(l)}$$

$$z_{12}^{(l)} = a_{12}^{(l-1)}w_{11}^{(l)} + a_{13}^{(l-1)}w_{12}^{(l)} + a_{22}^{(l-1)}w_{21}^{(l)} + a_{23}^{(l-1)}w_{22}^{(l)} + b^{(l)}$$

$$z_{21}^{(l)} = a_{21}^{(l-1)}w_{11}^{(l)} + a_{22}^{(l-1)}w_{12}^{(l)} + a_{31}^{(l-1)}w_{21}^{(l)} + a_{32}^{(l-1)}w_{22}^{(l)} + b^{(l)}$$

$$z_{22}^{(l)} = a_{22}^{(l-1)}w_{11}^{(l)} + a_{23}^{(l-1)}w_{12}^{(l)} + a_{32}^{(l-1)}w_{21}^{(l)} + a_{33}^{(l-1)}w_{22}^{(l)} + b^{(l)}$$

δ11	δ12
δ21	δ22

残差

$$\frac{\partial J}{\partial b^{(l)}} = \frac{\partial J}{\partial z_{11}^{(l)}}\frac{\partial z_{11}^{(l)}}{\partial b^{(l)}} + \frac{\partial J}{\partial z_{12}^{(l)}}\frac{\partial z_{12}^{(l)}}{\partial b^{(l)}} + \frac{\partial J}{\partial z_{21}^{(l)}}\frac{\partial z_{21}^{(l)}}{\partial b^{(l)}} + \frac{\partial J}{\partial z_{22}^{(l)}}\frac{\partial z_{22}^{(l)}}{\partial b^{(l)}}$$

$$= \delta_{11}^{(l)} + \delta_{12}^{(l)} + \delta_{21}^{(l)} + \delta_{22}^{(l)}$$

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卷积核的梯度

权重的梯度

$$\frac{\partial J}{\partial w_{p,q}^{(l)}} = \sum_u \sum_v \delta_{u,v}^{(l)} a_{u+p-1,v+q-1}^{(l-1)}$$

将当前层的残差与前一层的输出做卷积。

偏置值的梯度

$$\frac{\partial J}{\partial b^{(l)}} = \sum_u \sum_v \delta_{u,v}^{(l)}$$

将当前层当前特征图的残差求和。

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CNN中的参数

- 卷积层的参数包括卷积核参数与激活函数的参数中的偏置值。偏置值数量等于生成的特征图的激活值数量（同一个特征图中的偏置值相同）。
- 池化层通常没有连接权重、偏置值和激活函数。
- 全连接层的参数。

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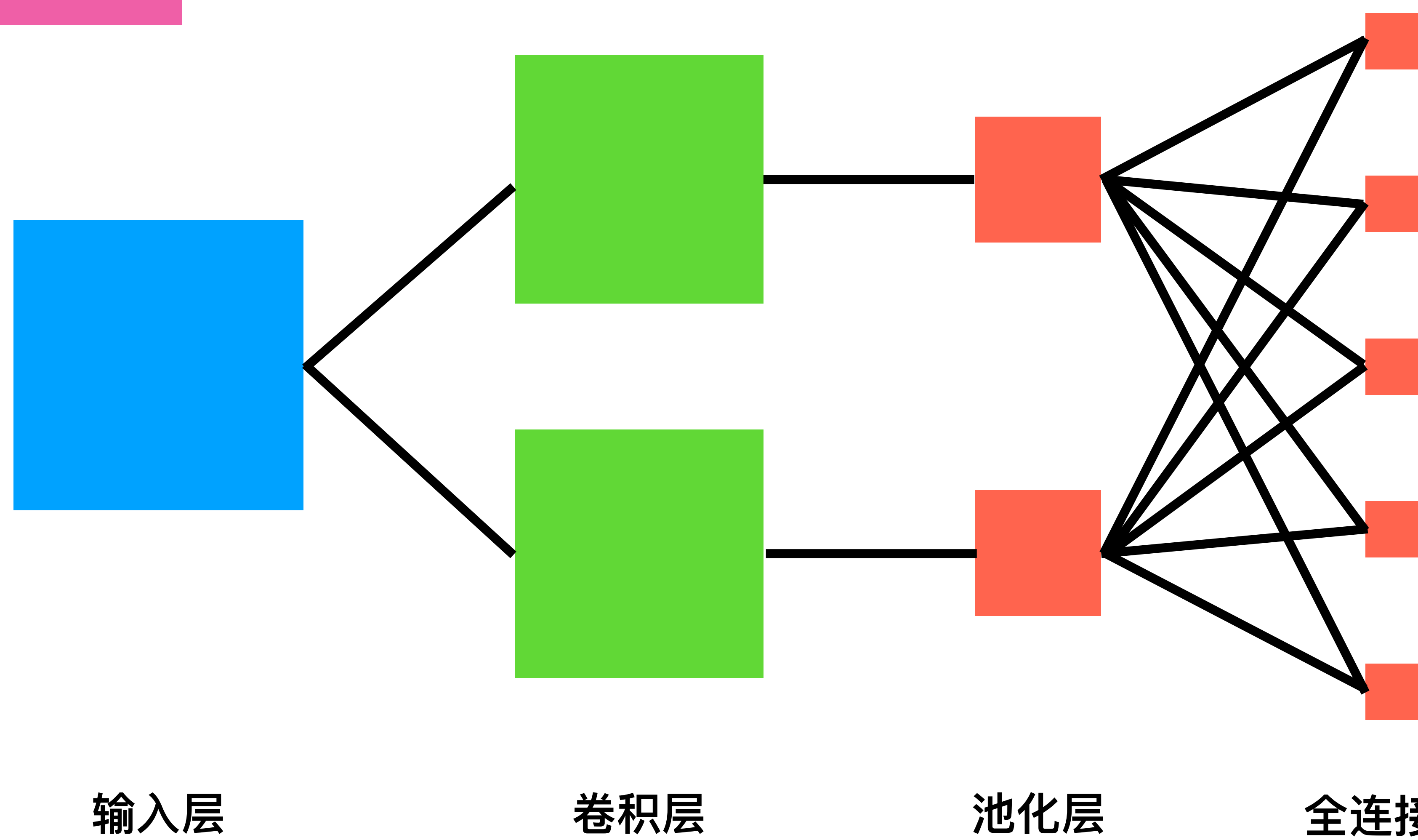
卷积层与池化层的灵活应用

应用方式一



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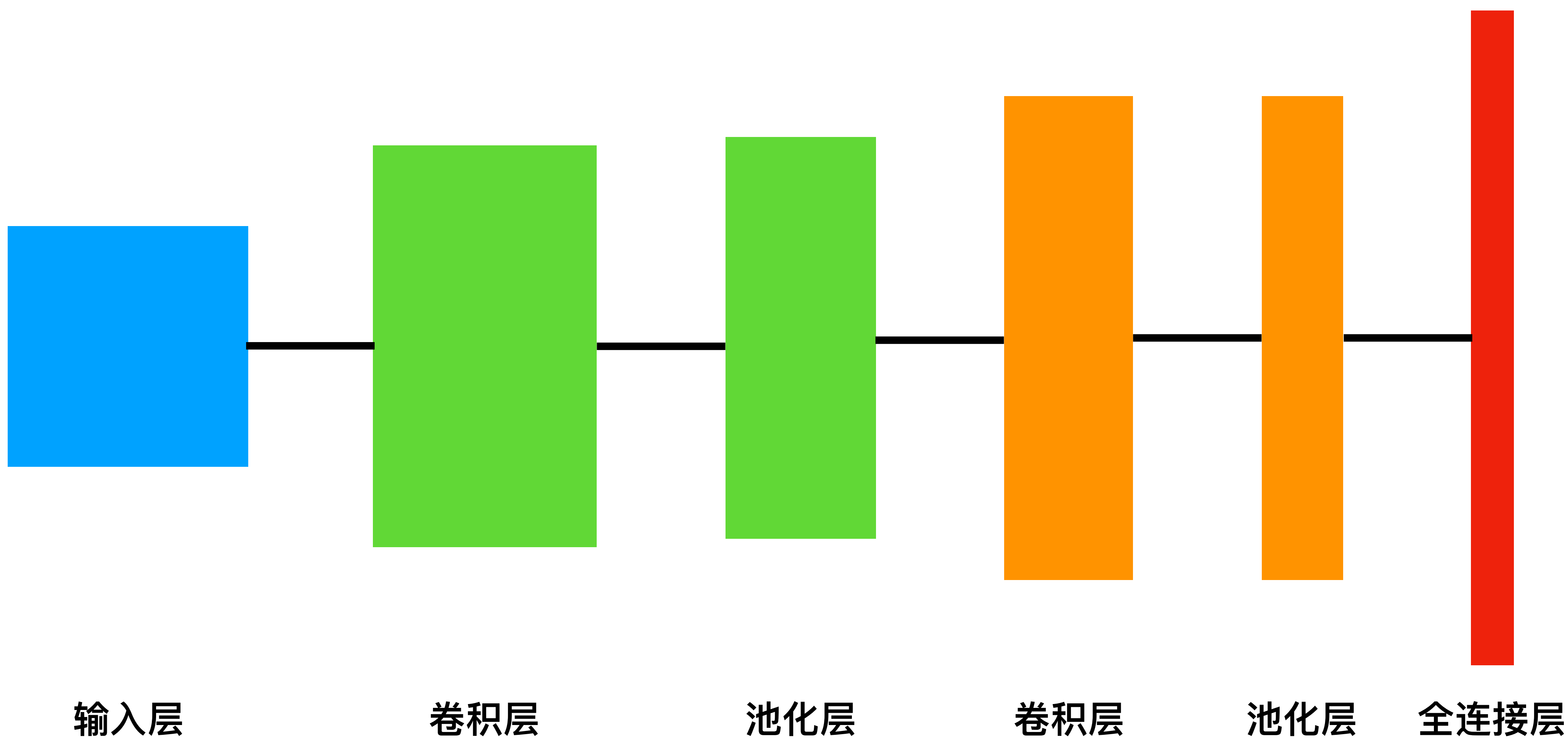
卷积层与池化层的灵活应用

应用方式二



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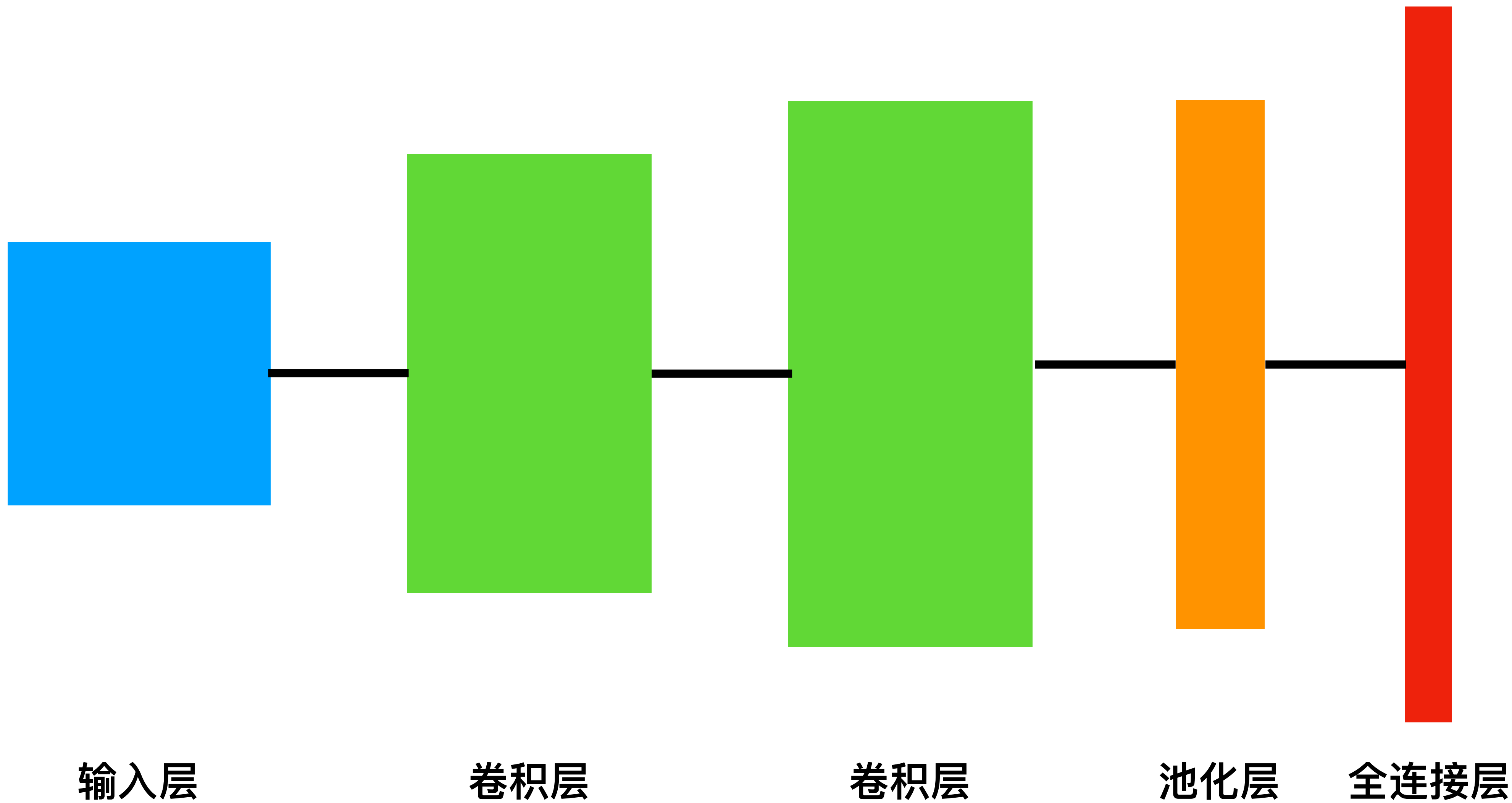
卷积层与池化层的灵活应用

应用方式三



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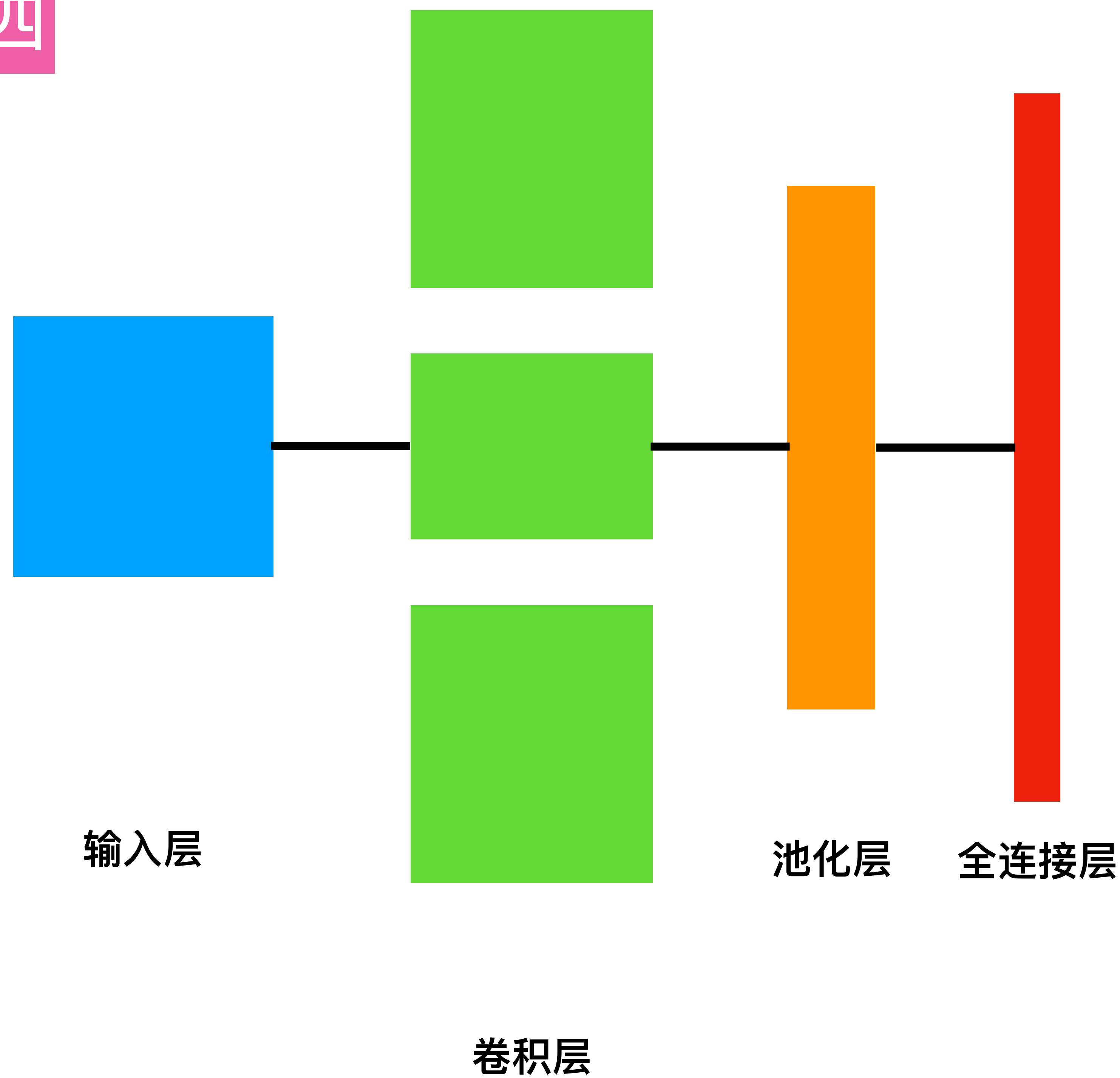
卷积层与池化层的灵活应用

应用方式四



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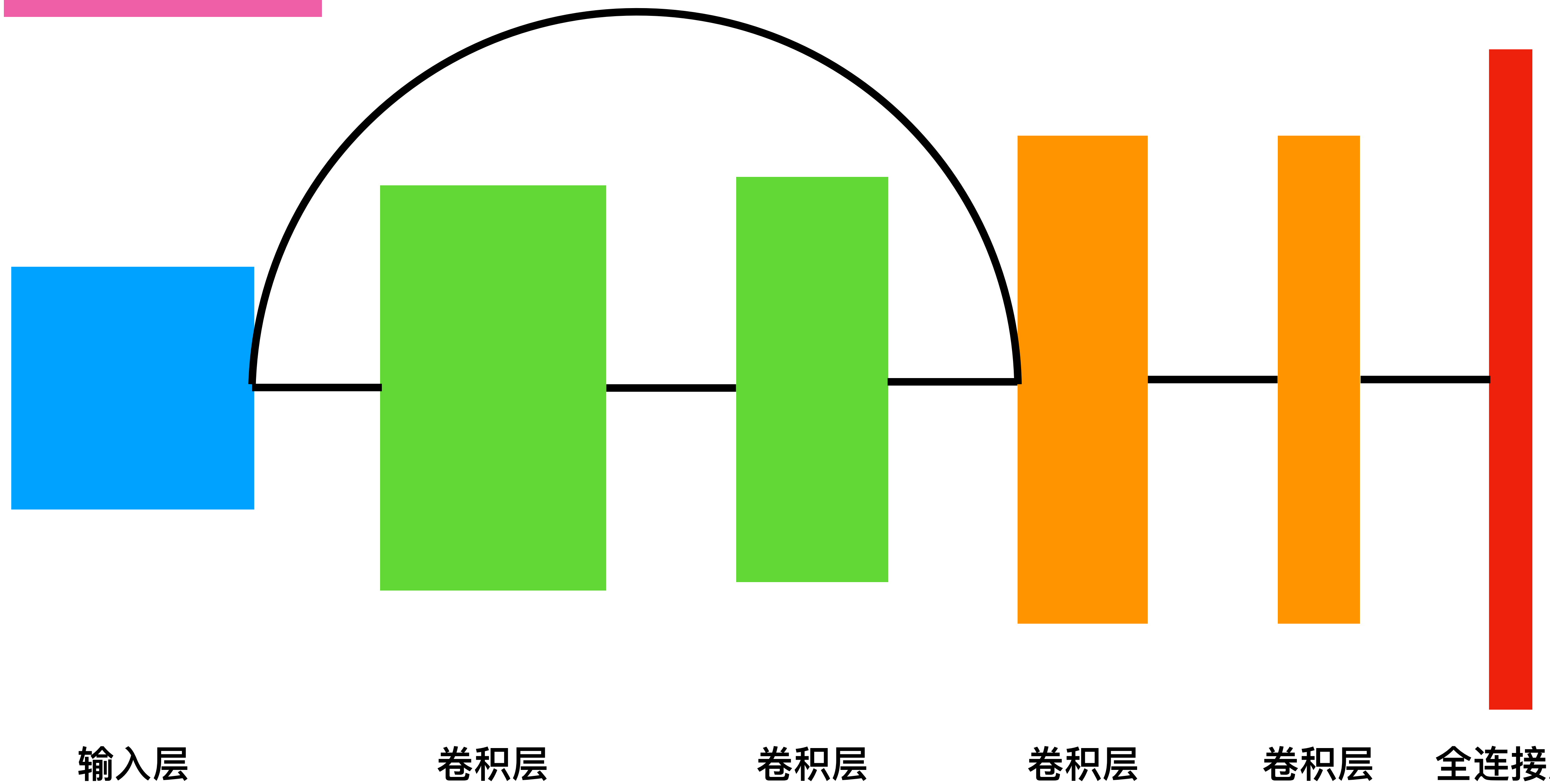
卷积层与池化层的灵活应用

应用方式五



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小结

- 局部连接与权值共享降低了连接数量与参数数量。
- 数学上的卷积需要翻转180°卷积核，CNN中的卷积核不需要翻转，即CNN中的卷积是信息处理中的互相关。
- 卷积层反向传播残差到前一层的计算方法是：翻转卷积核与补0的当前层残差做卷积。
- 卷积层连接权重的梯度计算方法是：使用当前层的残差与上一层的输出做卷积。
- 卷积层偏置值的梯度计算方法是：将当前某个特征图的残差累加作为此特征图偏置值的残差。（注意：一个特征图只有一个残差。）
- 卷积层与池化层可以灵活应用。

THANKS

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