PROCEEDINGS OF SPIE

SPIEDigitalLibrary.org/conference-proceedings-of-spie

Flat hat glass diffractive optical beam shaper

Steffen Reichel, Uwe Petzold, Ralf Biertuempfel, Helge Vogt

Steffen Reichel, Uwe Petzold, Ralf Biertuempfel, Helge Vogt, "Flat hat glass diffractive optical beam shaper," Proc. SPIE 7194, Laser Resonators and Beam Control XI, 719403 (19 February 2009); doi: 10.1117/12.807932



Event: SPIE LASE: Lasers and Applications in Science and Engineering, 2009, San Jose, California, United States

Flat hat glass diffractive optical beam shaper

Steffen Reichel*^a, Uwe Petzold^{a, b}, Ralf Biertuempfel^a, Helge Vogt^a

^aSCHOTT AG, Advanced Optics, Hattenbergstr. 10, 55122 Mainz, Germany

^bTechnical University of Darmstadt, Dept. of Physics, Karolinenplatz 5, 64298 Darmstadt, Germany

ABSTRACT

Many laser applications need a homogeneous - so called flat hat - light distribution in the application area. However, many laser emit Gaussian shaped light. The technology of diffractive optical elements (DOE) can be used to shape the Gaussian beam into a flat hat beam at a compact length. SCHOTT presents a DOE design of a flat hat DOE beam shaper made out of optical glass. Here the material glass has the significant advantage of high laser durability, low scattering losses, high resistance to temperature, moisture, and chemicals compared to polymer DOEs. Simulations and measurements on different DOEs for different wavelength, laser beam width, and laser divergence are presented. Surprisingly the flat hat DOE beam shaper depends only weakly on wavelength and beam width but strongly on laser divergence. Based on the good agreement between simulation and measurement an improved flat hat DOE beam shaper is also presented.

Keywords: beam shaper, flat hat, Gaussian beam, glass, diffractive optics, diffractive optical element, DOE

1. INTRODUCTION

Applications, such as pico-projectors with laser sources or medical laser skin treatment, need a rectangular beam distribution at higher laser power. Unfortunately the fundamental mode (TEM00) of a laser is Gaussian shaped, i.e. the intensity distribution over the lateral dimension (i.e. radius) obeys a Gaussian curve. This means there is a strong intensity in the center (peak of the Gaussian curve) and a lower intensity by going off center. The above-mentioned applications need a homogeneous intensity distribution over a certain distance and zero intensity outside, thus a rectangular shaped light distribution.

Such a rectangular light distribution can be achieved for example by using a so-called rectangular light pipe [1]. Such a light pipe works as a mixing rod for the light and needs a certain numerical aperture (NA) of the input light and a certain length for the mixing. Both requirements are difficult in the laser field since laser light has low divergence and a lengthy device limits the compactness.

The technology of diffractive optical elements (DOE) can be used to overcome both topics. The DOE works for collimated light and enables beam shaping on a compact length. The DOE uses diffraction and thus the structure is on the length scale of the wavelength, i.e. around 1µm for green light. In this paper the basics of a DOE beam shaper (chapter 1) is described together with its realization in binary optics and its replication with precise pressing in glass. In chapter 2 the design of a glass diffractive optical beam shaper is shown together with simulations about wavelength dependency and laser beam quality of the source and its effects on the rectangular light distribution. Chapter 3 shows optical measurements on the designed and manufactured glass DOEs from SCHOTT. Measurement results as well as design results are compared in chapter 4 leading to an improved diffractive optical flat hat beam shaper as an outlook. A summary is presented in chapter 5.

1.1 Basic consideration of diffractive optics

The working principle of light pipes is total internal reflection leading to light mixing. In contrast DOEs use a wave optical behavior of light, namely diffraction, see Fig. 1.

*steffen.reichel@schott.com; phone +49 6131 66 3187; www.schott.com

Laser Resonators and Beam Control XI, edited by Alexis V. Kudryashov, Alan H. Paxton, Vladimir S. Ilchenko, Lutz Aschke, Proc. of SPIE Vol. 7194, 719403 · © 2009 SPIE · CCC code: 0277-786X/09/\$18 · doi: 10.1117/12.807932

Proc. of SPIE Vol. 7194 719403-1



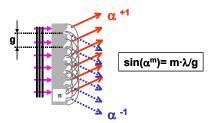


Fig. 1. Sketch of the working principle of diffraction assuming a diffraction structure realized in a material with refractive index n. Here the diffraction orders $m=\pm 1$ and -1 are shown together with the grating equation for normal incidence. The grating period g is also shown.

Diffraction is strongly visible if the structure is in the order of the wavelength. One example is diffraction on a slit where the slit width must be in the order of the wavelength. By diffraction on a slit the amplitude of the incident light is modulated, i.e. amplitude is 0 outside the slit and amplitude equals the incident power inside the slit. DOEs use phase modulation of the light instead of amplitude modulation. This means the phase of the incident light is changed in a determined manner by different optical path length (optical path length = product of geometrical length times refractive index). Fig. 1 shows also a phase modulation achieved by ramp function.

A DOE is typically designed to operate in the first diffraction order (m = +1) by choosing the correct height (for the used refractive index), and grating period. In order to maximize the power diffracted into the first order (+1) one typically uses the blaze technique [2]. Blazing has the effect that the +1st diffraction order becomes maximized and the zero diffraction order becomes nearly fully suppressed. The blaze angle coincides with the +1st diffraction order by choosing the correct height h of the DOE structure to [3], [5]

$$h = \frac{\lambda}{n(\lambda) - 1},\tag{1}$$

where λ is the vacuum wavelength and n is the refractive index of the material at wavelength λ . The diffraction angle α^{+1} for the first diffraction order (m = +1) is given by the distance DOE to image plane and the size of the flat hat beam at the image plane. With these 2 parameters the angle α^{+1} can be calculated and determines via the grating equation the grating period g.

1.2 Theory of diffractive optical elements

Now the basic parameters are determined namely the structure height h and the grating period g. In the next step the phase modulation must be calculated. In order to understand how this is done we repeat the result of scalar diffraction theory [3], [4]. In the case of Fraunhofer diffraction (= far field) the diffracted electric field on a screen (with coordinates x, y, z) can be calculated from:

$$E_{dif}(x, y, z) = K(x, y, z) \int_{-\infty - \infty}^{+\infty + \infty} A(x_0, y_0) \cdot e^{-ik \frac{xx_0 + yy_0}{z - z_0}} dx_0 dy_0,$$
 (2)

where K(x,y,z) is a factor that we ignore here for simplicity, $k=2\pi/\lambda$, and $A(x_0,y_0)$ is the pupil function at the position x_0 and y_0 (e.g. in the case of a slit $A(x_0,y_0)$ is a step function). Eq. 2 is the two dimensional Fourier transform of the pupil function. That is the diffracted electric field can be calculated by a Fourier transform of the pupil function in the case of Fraunhofer diffraction.

Now for designing a DOE the diffracted electric field is known (e.g. a flat hat distribution) and the pupil function can be calculated by inverse Fourier transform. The pupil function is the wanted phase function of the DOE. Eq. (2) is a simplification of the real circumstances but it demonstrates how difficult it is to calculate the DOE. In most practical applications the far-field approximation cannot be used and thus Fresnel diffraction approximation must be used. Moreover Eq. (2) assumes a homogeneous light distribution over the pupil, which is by laser illumination with Gaussian distribution not the case. Therefore the calculation of the DOE is a complex mathematical problem, which is iteratively solved [5]. Such iterative methods are used in commercial software to calculate the DOE ("pupil function") [6].

The design software needs as one parameter the grating period determined by the diffraction angle α^{+1} . Once the phase function with design software is known, e.g. by using [6], the height structure is determined by Eq. (1).

1.3 Binary optics for DOE manufacturing

The DOE phase function designed and calculated as described in Section 1.2 is typically a continuous distribution. For manufacturing reasons the continuous phase distribution is approximated by a stair case function. Such optic is called binary optics and can be realized by lithography [5]

a. 4 level binary optics b. 4 levels with 2 masks: mask 1 height h $h = \frac{\lambda}{n-1}$ approximation mask 2 1 2 3

Fig. 2. a. The approximation of a continuous phase by a 4 level staircase function. b. The N = 4 level phase function can be realized with 2 masks by lithography.

Fig. 2 shows the approximation of a continuous phase by a (in this case) 4 level stair case function as well as its realization with 2 masks using lithography. As it can be seen in Fig. 2 the stair case function is an approximation of the calculated (designed) phase and the more levels the better the approximation. Using scalar diffraction theory one can show that the diffraction efficiency η of the first diffraction order for binary optics obeys [5]:

$$\eta = \left[\frac{\sin(\pi/N)}{\pi/N}\right]^2,\tag{3}$$

where N is the number of levels used for the binary optics. This means the diffraction efficiency of N = 4 level binary optics is $\eta_4 = 81\%$, for 8 levels is $\eta_8 = 95\%$, and for 16 levels is $\eta_{16} = 99\%$ - the difference to 100% is stray light that is diffracted into other orders (zero, 2^{nd} , 3^{rd} , ... order). This demonstrates that for high diffraction efficiency (> 90%) one needs at least 8 or 16 levels resulting in 3 or 4 masks for the lithography. SCHOTT typically uses 8 or 16 levels binary optics for the manufacturing of a replication master from which the glass DOE is replicated by precise molding technique.

1.4 Precise molding technique for glass DOE manufacturing

The lithography process in combination with reactive ion etching in glass is a well-known technique for the production of multilevel DOEs. However, the manufacturing costs are fairly high and the processing times are quite long. This limits the technique to the production of small quantities. For mass production a different approach has to be used. Therefore, a fast replication technique is introduced which is capable to produce millions of diffractive optical elements within a short time. A replication master is manufactured by lithography.

SCHOTT has established a clean room facility where a variety range of glasses can be structured via a high precision glass molding process. This process is usually used for production of aspherical lenses. In such a molding process (see Fig. 3) a piece of glass is heated to a temperature, at which the glass has a certain viscosity that makes it deformable. At this point the glass is pressed into the final shape. Eventually, the glass is cooled down. The correct thermal management of the cooling step is essential for the accuracy of the optical element. The optical component may then be coated (with anti-reflective coating) and sold to the customer. In this process an additional grinding or polishing of the optical surfaces is not necessary.

However, not all glasses are suitable for precision glass molding. The range of glasses is limited by their tendency of sticking to the molds at elevated temperatures. A second limitation is the high molding temperature: This temperature must be low enough, to ensure a short process time and a long lifetime of the mold.

Among the SCHOTT low Tg glass portfolio the glasses P-LaSF47 ($n_d = 1.8016$), P-SK57 ($n_d = 1.5843$), N-LaF33 ($n_d = 1.7821$), and N-FK5 ($n_d = 1.4850$) have been designed to be especially suitable for manufacturing diffractive optical elements using the precision molding process. These glasses show a very high performance concerning repeatability and accuracy of the pressed structures.

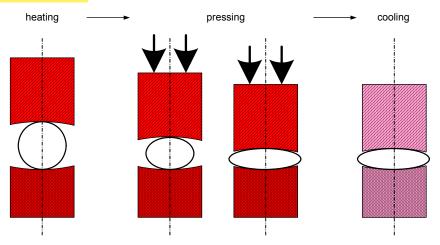


Fig. 3. Illustration of SCHOTT's pressing process. First step: heating of the glass preform. Second step: pressing the glass into the desired shape. Third step: cooling of the optical element.

2. FLAT HAT BEAM SHAPER DESIGN AND SIMULATION

The task of the flat hat beam shaper is to convert Gaussian distributed incident light (from a laser source) into a flat hat distribution, as depicted in Fig. 4.

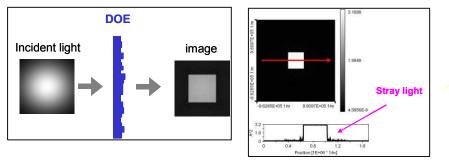
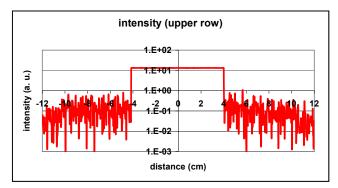


Fig. 4. Flat hat beam shaper converting Gaussian light into a rectangular light distribution (left). A cross section cut is also shown together with stray light (right).

The Gaussian distribution must be known, i.e. the 1/e² full width as well as the divergence angle (if any). Furthermore the distances source-DOE and DOE-image must be known. Afterwards - as described in Chapter 1 - a flat hat beam shaper can be designed by using an appropriate software tool [6]. The designer has to make sure that the homogeneity within the flat top is in the specified range. Since a DOE generates stray light (according to Eq. (3) where the diffraction efficiency is less than 100%) the designer has to make sure to distribute the stray light into an area where it does not disturb.

2.1 Simulation results for different wavelength

We designed a DOE that converts the Gaussian distribution into a flat hat. The Gaussian light was collimated, had a 1/e² full width of 2.0mm and the emission wavelength was 650nm. The flat hat was a square with dimensions 10cm x 10cm at a distance of 1.0m away from the DOE and the flat hat was inside a square with dimensions 50cm x 50cm wherein the stray light can be distributed. The DOE was sampled with 1024 pixels and had 16 levels. The result is shown in Fig. 5.



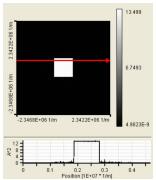
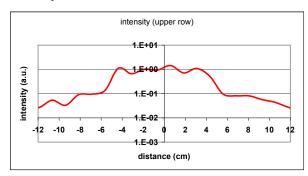


Fig. 5. Flat hat DOE simulation results for a 16 level DOE ($1/e^2$ full width of 2.0mm at 650nm). Left: line scan in logarithmic intensity scale of the flat hat design. Right: the image in 1m distance and the line scan (showing also the position of the line scan – called upper row in the following).

A perfect designed DOE generates a very good flat hat beam shaper as it can be seen from Fig. 5. This includes no manufacturing tolerances.

Now we intentionally simulated the same DOE only with 7 levels (obtained via the masks) accepting a poor diffraction efficiency in the +1 diffraction order in order to see some effects. This was done at 2 different wavelengths.



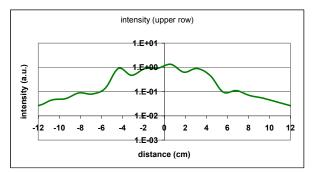


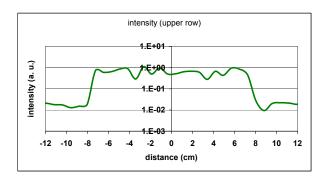
Fig. 6. Flat hat DOE simulation results in logarithmic intensity scale. Left: illuminated with a Gaussian beam at 650nm ($1/e^2$ full width of 2.0mm). Right: illuminated with a Gaussian beam at 532nm ($1/e^2$ full width of 2.0mm). The DOE had only 7 levels.

Fig. 6 shows the simulation results for the design wavelength of 650nm and for a wavelength of 532nm. Note the data have been average in order to obtain smooth curves for finding any trend (in contrast to Fig. 5 where the noisy data are displayed). The change due to the wavelength is small. The wavy behavior within a distance of about ± 5 cm is due to the low binary levels of just 7, as will be seen by comparing with Fig. 5.

2.2 Simulation of beam quality effects of the illuminating beam

Now another DOE designed at 532nm for collimated light was used to simulate the effect of beam quality. For that purpose we focused on the illumination with fully collimated light and diverging light with a diverging angle of $\pm 2.5^{\circ}$. The effect of beam width (1/e² full width) is not simulated since this effect is small (as will be seen in Section 3.2).

Fig. 7 shows the simulation result. The divergent illumination smoothes the borders and thus reduces the rectangular shape.



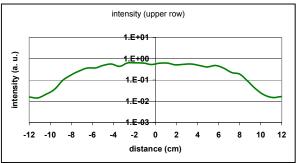


Fig. 7. Flat hat DOE simulation results in logarithmic intensity scale for a DOE illuminated at 532nm and with 2.6mm beam width (1/e² full width). Left: illuminated with fully collimated. Right: results for divergent illumination with a divergence angle of 2.5°.

3. MEASUREMENT RESULTS OF THE GLASS BEAM SHAPERS

According to the design presented in Chapter 2 SCHOTT manufactured binary optics master mold (as explained in Section 1.4) and pressed the DOEs into glass. The glass DOEs, made out of P-LaSF47, correspond to different designs. Glass DOEs have the advantage of high laser durability, low scattering losses, high resistance to temperature, moisture, and chemicals compared to polymer DOEs Therefore glass DOEs are preferred for laser applications. In our measurements the different DOEs were illuminated with laser light. In order to verify the performance of the manufactured DOE optically we simultaneously manufacture with the DOE test grating on the glass substrate.

The test grating was designed with a period $g = 8.0 \mu m$. Optically the grating was illuminated with a HeNe laser operating at 633nm and $1/e^2$ full width of 1.0mm. In a distance of 103cm the 1st diffraction order spot was measured at 8.15cm out of center (in transverse direction). The diffraction angle of the first order is thus given by $\tan(\alpha^{+1}) = 8.15/103$ which results in an angle of $\alpha^{+1} \approx 4.5^{\circ}$. According to the grating equation the period $g = \lambda/\sin(\alpha^{+1})$ can be calculated to $g = 8.02 \mu m$ which fits very well to the designed value.

Measurement uncertainties are ± 1 mm for the distance and $\pm 1\%$ for the intensity and are not shown in the Figures for simplicity.

3.1 Wavelength effects on the DOE

The DOE was designed for a wavelength of 650nm. In 100cm distance the flat hat should have a size of 10cm x 10cm and was measured by a line scan. In general the resolution for the measurement was limited by the detector size (about 0.75cm) and thus a steep border can only be resolved in a limited fashion.

Fig. 9 shows a photograph taken from the image for 2 different wavelengths. Here the DOE was illuminated with light from a semiconductor laser diode at 650nm and with a laser operating at 532nm.

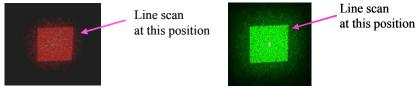
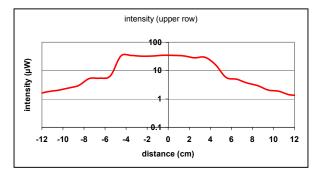


Fig. 8. Photograph of the image on the screen in 1.0m distance. Left: illuminated with a Gaussian beam at 650nm ($1/e^2$ full width of 2.0mm). Right: illuminated with a Gaussian beam at 532nm ($1/e^2$ full width of 2.0mm). The arrow indicates in both cases the position where the line scan was made (as displayed in Fig. 9). Here the 7 level DOE was used.

Although the wavelengths are quite different the image on the screen looks surprisingly good for both wavelength although this was only a 7 level DOE!

In Fig. 9 the described DOE was illuminated by the design wavelength of 650nm and by a wavelength of 532nm, now a line scan can be seen. By comparing both measurements one can see 2 effects: firstly the image illuminated by 532nm is

smaller (8.1cm instead of 10.0cm) due to the smaller diffraction angle for lower wavelengths (grating equation) and secondly the 532 nm image has smoother – unwanted – transition from the top (flat hat) to the border (e.g. compare positions at –4cm).



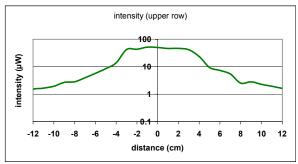
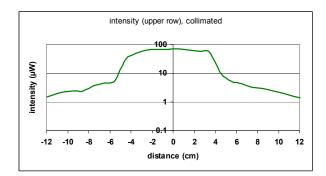


Fig. 9. Flat hat DOE in logarithmic intensity scale. Left: illuminated with a Gaussian beam at 650nm ($1/e^2$ full width of 2.0mm). Right: illuminated with a Gaussian beam at 532nm ($1/e^2$ full width of 2.0mm).

3.2 Beam quality effects of the illuminating beam

In the next measurement again a line scan was measured of the image generated by the DOE. Here we used a DOE designed for 532nm that generated a flat hat of $10 \text{cm} \times 10 \text{cm}$ at a distance of 55.5 cm (the same as was used in Section 2.2). The design assumed fully collimated light. Now we used a collimated laser beam at 532 nm and a divergent laser beam also at 532 nm (with $1/e^2$ full width of 2.6 mm). Illuminating a focusing lens with collimated light generated the divergent beam. The lens had a focal length of f = 3 cm and the DOE was in 6 cm distance from the lens. This generated a divergence beam of angle of about 2.5° (= arctan1.3mm/3cm).

The results are shown in Fig. 10 and show the strong effect of the divergence angle on the image quality. The image (line scan) of the DOE illuminated with a divergent beam has much smoother boarders, the image is "washed out".



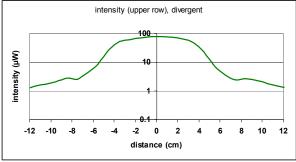
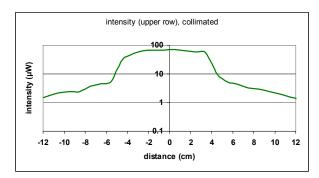


Fig. 10. Line scan measurements of the DOE illuminated with 532nm light and $1/e^2$ full width of 2.6mm. Left: results for collimated light illumination. Right: results for divergent illumination with a divergence angle of 2.5°.

In the next step the effect of the beam width $(1/e^2 \text{ full width})$ of the illuminating laser source was measured. Again a 532nm laser (collimated light) was used and the beam width was adjusted by an aperture (where the $1/e^2$ full width was measured).



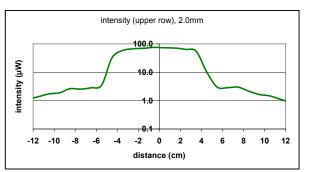


Fig. 11. Line scan measurements of the DOE illuminated with collimated light at 532nm and different $1/e^2$ full width. Left: results for a beam width $(1/e^2$ full width) of 2.6mm. Right: results for a beam width $(1/e^2$ full width) of 2.0mm.

Fig. 11 shows the effect for a beam width of 2.6mm and 2.0mm. The DOE was designed for 2.6mm and even a change by 24% (smaller width) has nearly no effect on the image.

4. ANALYISIS OF THE RESULTS AND IMPROVMENT

After designing, simulating and measuring different DOEs under different conditions the obtained results are now compared. First the correctness of the design software [6] must be tested since this software will be used for an improved design based on the experiences obtained and described in this publication.

4.1 Comparison of simulation and measurement

The test of the software is done for the 7 level DOE since this has (intentionally) some manufacturing errors namely only 7 levels. The design and measurement wavelength is 532nm the laser beam width 2.0mm (1/e² full width). Fig. 12 shows the result.

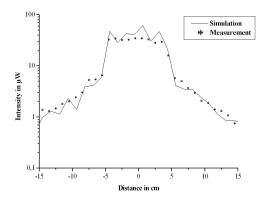


Fig. 12. Comparison between simulation and measurement for a 7 level DOE as described in Section 2.1. The simulation and measurement wavelength was 532nm, the beam width 2.0mm ($1/e^2$ full width). The solid line represents the simulation and the dots the measurement.

The comparison between simulation and measurement shows the very good agreement even in the case of only 7 levels. Therefore the simulation tool is well suited for the (improved) design of a flat hat diffractive optical beam shaper. In the simulation manufacturing tolerances - like sidewall slope lower than 90° or step tolerances - are not included. These manufacturing tolerances are the reasons for the small deviation between simulation and measurement in Fig. 12.

4.2 Conclusion from wavelength and beam quality effects

The wavelength dependency was investigated in the simulation as well as in the measurement, see Fig. 6 and Fig. 9. Both results show the surprising result that the shape (the rectangular light distribution) has only a weak dependency. Thus a

small change in the wavelength (for example due to scattering in the supplied laser charge) can nearly be ignored. The effect of the change in wavelength is reduced diffraction efficiency according to [7]:

$$\eta = \left[\frac{\sin\left(\pi \frac{\lambda_0}{\lambda} \cdot \frac{n(\lambda) - 1}{n(\lambda_0) - 1} - \pi\right)}{\pi \frac{\lambda_0}{\lambda} \cdot \frac{n(\lambda) - 1}{n(\lambda_0) - 1} - \pi} \right]^2, \tag{4}$$

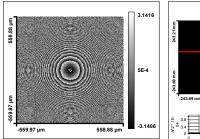
where λ_0 [λ] is the design [new] wavelength and $n(\lambda_0)$ or $n(\lambda)$ the refractive index at λ_0 or λ . If we ignore the wavelength dependency of the refractive index, since this is in our application a small effect, the efficiency changes from 100% to 89.6% assuming a continuous ideal phase (no binary optics approximation).

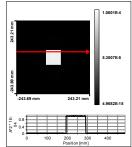
The effects of the beam quality were examined by a change in the beam width $(1/e^2)$ full width) and change from collimated light to diverging laser light. The change in beam width could be ignored, see Fig. 11, although the beam width was increased by 30% (from 2.0mm to 2.6mm - $1/e^2$ full width). Changing the condition from collimated light into diverging light showed a stronger effect, see Fig. 7 and Fig. 10. But we changed from 0° (collimated light) into diverging light with 2.5° divergence angle which is a rather strong divergence angle for laser light. Nevertheless the divergence angle of the laser beam should be known for the design with high accuracy!

4.3 Improved flat hat beam shaper design

Based on the analysis in Section 4.2 we started an optimized simulation. As explained the emission wavelength and the beam width is not as important as the divergence angle. We measured all parameters, the emission wavelength was 532nm and the beam width was measured to 2.6mm $(1/e^2 \text{ full width})$. The divergence angle is 0.5mrad.

The flat hat was again a square with dimensions 10cm x 10cm at a distance of 1.0m away from the DOE and the flat hat was inside a square with dimensions 50cm x 50cm wherein the stray light can be distributed. The DOE was sampled with 1024 pixels and had 16 levels. The grating period is thus 2.2 µm.





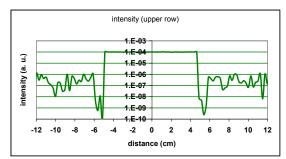


Fig. 13. New and improved design results based on the experiences described in this publication. Left: the DOE (phase function). Middle: the simulated image in 1m distance. Right: line scan of the image (corresponds to the red arrow indicated in the middle).

Fig. 13 shows the design result, which promises excellent homogeneity inside the flat hat (deviation from the flat hat of less than 0.8% RMS value) as well as a high diffraction efficiency of 98.6% in the +1 diffraction order. This DOE will now be manufactured and produced in glass.

5. SUMMARY

In this publication different designs of glass DOEs were explained. SCHOTT manufactures glass DOEs by using a precise pressing technology, which enables the mass production of DOEs. Hereby glass has the advantage of significant better thermal, mechanical, and chemical durability as well as a high light intensity as emitted by lasers.

We presented simulations and measurements on different DOEs. We demonstrated that the simulation fits excellent with the measurement. Moreover the effects of wavelength and beam quality were examined both by simulation and measurement.

Surprisingly we found that a change in wavelength and beam width has only a small effect on the image generated by the DOE. Care must be taken by the divergence of the laser source since this has a strong effect and must therefore be known for the design. Finally we used the obtained information for the design of an improved flat hat glass diffractive optical beam shaper that will be produced by SCHOTT in the next time.

6. ACKNOWLEDGEMENT

Part of this work was supported by the German Federal Ministry of Education and Research (BMBF), project MOLAS under FKZ 13N9374 and is herewith acknowledged.

REFERENCES

- W. Smith, [Modern Optical Engineering], McGraw Hill, New York, (1990).
- A. Pedrotti and L. Pedrotti, [Introduction to Optics], 2nd edition, Prentice Hall, Upper Saddle River (1993).
- A. Sommerfeld, [Optik], Verlag Harri Deutsch, Frankfurt (1989).

 M. Born and E. Wolf, [Principles op Optics], 7th edition, Cambridge university Press, Cambridge (1999).

 S. Sinzinger and J. Jahns, [Microoptics], 2nd edition, Wiley-VCH, Weinheim (2003).
- Software VirtualLab from LightTrans GmbH, D-07745 Jena, Germany
- B. Kleemann et al, "Design concepts for broadband high-efficiency DOEs", Journal of the European Optical Society, Rapid Publications 3 (April 2008).

Proc. of SPIE Vol. 7194 719403-10