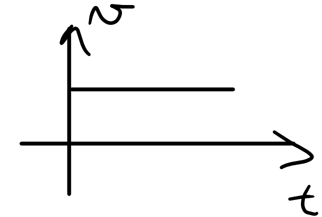
Fasores / Introdução à análise de circuitos em estado permanente senoidal.

$\omega = 2\pi f$ rad/s

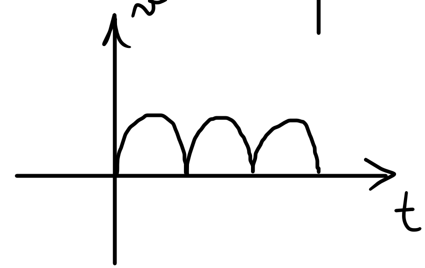
$\dot{V}$; $\dot{I}$

CC

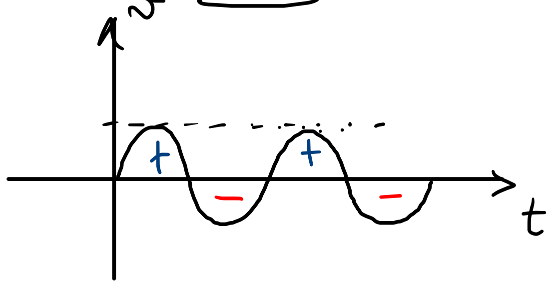


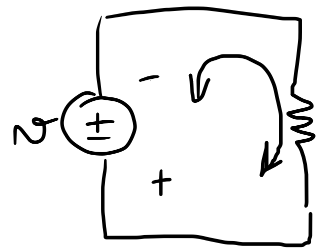
$v(t) = V_m \cos(\omega t + \theta)$

$$\boxed{V_m \cos(\omega t + \theta) \longleftrightarrow \underbrace{V_m \angle \theta}_{V \cdot e^{j\omega t}}}$$



CA





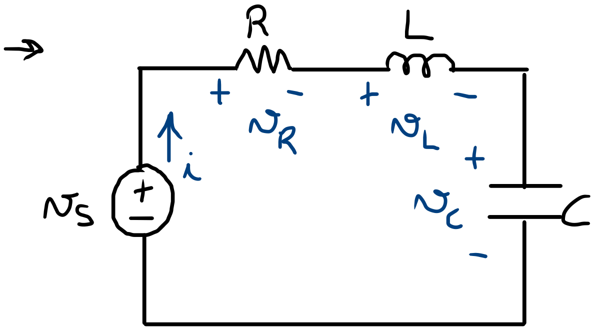
$z = a + jb$; $j = \sqrt{-1}$

$|z| = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Ex: $v(t) = 10\cos(10t + 30^\circ)$ (V) $\rightarrow \underbrace{10\angle 30^\circ}$ V

$$\begin{cases} I = (1+j)\ \text{A} \rightarrow I = \sqrt{2}\angle 45^\circ \rightarrow i(t) = \sqrt{2}\cos(\omega t + 45^\circ)\ \text{(A)} \\ i(t) = ? \end{cases}$$



$$v_s(t) = V_m \cos(\omega t) \ (V)$$

$$v_L + v_R + v_C = v_s$$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = v_s$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}\, i = \frac{1}{L}\frac{dv_s}{dt}$$

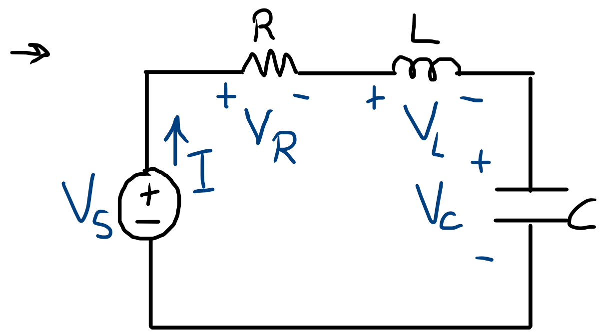
$$\begin{cases} \underline{V}_s = \underline{V}_s\, e^{j\omega t} \\ \underline{I} = \underline{I}\, e^{j\omega t} \end{cases}$$

$$j^2\omega^2 \underline{I}\cancel{e^{j\omega t}} + \frac{R}{L} j\omega \cancel{e^{j\omega t}\underline{I}} + \frac{1}{LC}\underline{I}\cancel{e^{j\omega t}} = \frac{1}{L} j\omega V_s \cancel{e^{j\omega t}}$$

$$-\omega^2 \underline{I} + j\frac{\omega R}{L}\underline{I} + \frac{1}{LC}\underline{I} = \frac{1}{L} j\omega V_s \quad \times \left(\frac{L}{j\omega}\right)$$

$$-\frac{\omega L}{j}\underline{I} + R.\underline{I} + \frac{1}{j\omega C}\underline{I} = V_s$$

$$\boxed{j\omega L.\underline{I} + R.\underline{I} + \frac{1}{j\omega C}.\underline{I} = V_s}$$



$$V_L + V_R + V_C = V_s$$

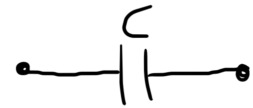
→ $v_S = V_m \cos(\omega t)$ (V)

$V_R = R \cdot I$

$Z_R = R$ [Ω]

$V_L = \underbrace{j\omega L} \cdot I$

$Z_L = j\omega L$ [Ω]



$V_C = \dfrac{1}{j\omega C} \cdot I$

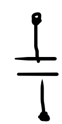
$Z_C = \dfrac{1}{j\omega C}$ [Ω]

$$Z_L \cdot I + Z_R \cdot I + Z_C \cdot I = V_S$$

$$\underbrace{Z_L + Z_R + Z_C} = \frac{V_S}{I} \rightarrow j\omega L + R + \frac{1}{j\omega C} \rightarrow \boxed{Z_{eq} = R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

→ De forma geral: $\boxed{Z = \underbrace{R}_{\text{Resistência}} + j\underbrace{X}_{\text{reatância}} \ [\Omega]}$

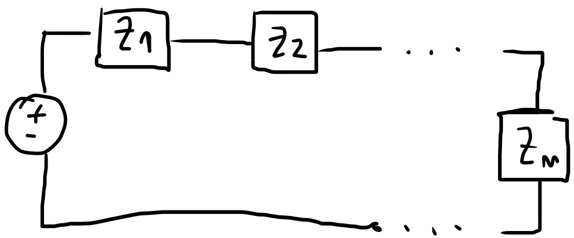
L → $X_L = \omega L$ [Ω]



C → $X_C = \dfrac{1}{\omega C}$ [Ω]

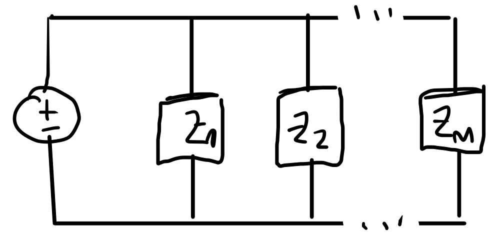
→ Associação de Impedâncias

Série:



$$Z_{eq} = Z_1 + Z_2 + \dots + Z_m$$

PARALELO:

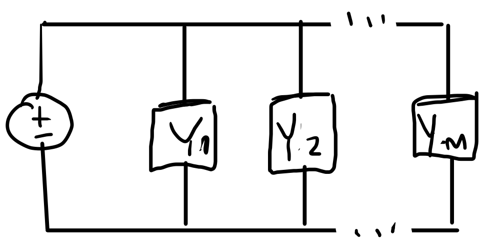


$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_M}$$

$$G = \frac{1}{R} \ [S]$$

→ Associação de Impedâncias

<u>PARALELO:</u>



Y: Admitância

$$Y = \frac{1}{Z} \ [\text{siemens}]$$

$$Y_{eq} = Y_1 + Y_2 + \dots + Y_m$$

Forma geral: $Y = G + jB \ [S]$ $\begin{cases} G: \text{Condutância} \\ B: \text{Susceptância} \end{cases}$

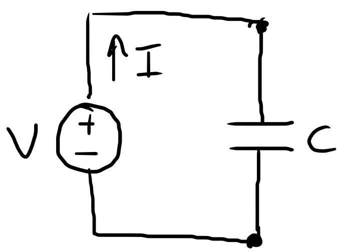
$$Y = \frac{1}{Z} = \frac{1}{R+jX} \cdot \frac{(R-jX)}{(R-jX)} = \frac{R-jX}{R^2+X^2} \Rightarrow Y = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$$

** Tensão e corrente

$-jX_c$

1) CAPACITOR

$$X_c = \frac{1}{\omega C} \;;\; Z_c = -jX_c$$



$$V = Z_c I \rightarrow Z_c = \frac{V}{I} \rightarrow -jX_c = \frac{V}{I} \Rightarrow$$

$$\Rightarrow X_c \angle -90^\circ = \frac{V}{I} \Rightarrow X_c \angle -90^\circ = \frac{|V| \angle \theta_v}{|I| \angle \theta_i} \rightarrow$$

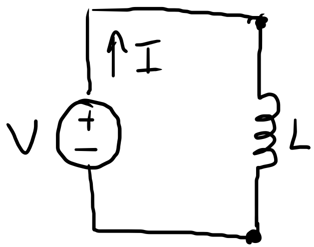
$$\begin{cases} V = |V| \angle \theta_v \\ I = |I| \angle \theta_i \end{cases}$$

$$\rightarrow \frac{|V|}{|I|} = X_c \;;\; \boxed{\theta_v - \theta_i = -90^\circ}$$

* A corrente está adiantada de 90° em relação a tensão!

** Tensão e Corrente

2) INDUTOR



$$X_L = \omega L \; ; \; Z_L = jX_L$$

$$V = Z_L \cdot I$$

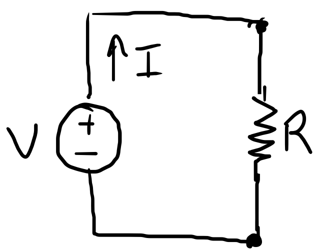
$$\frac{V}{I} = X_L \angle 90^\circ$$

$$\begin{cases} V = |V| \angle \theta_v \\ I = |I| \angle \theta_i \end{cases} \rightarrow \frac{|V|}{|I|} = X_L \; ; \; \boxed{\theta_v - \theta_i = 90^\circ}$$

* A corrente está atrasada de 90° em relação a tensão!

** Tensão e Corrente

3) RESISTOR



$$Z_R = R$$

$$V = R.I$$

$$\frac{V}{I} = R \rightarrow \frac{V}{I} = R\angle 0^\circ$$

$$\begin{cases} V = |V|\angle\theta_v \\ I = |I|\angle\theta_i \end{cases} \rightarrow \frac{|V|}{|I|} = R \; ; \; \boxed{\theta_v - \theta_i = 0^\circ}$$

* A corrente e a tensão estão em fase!