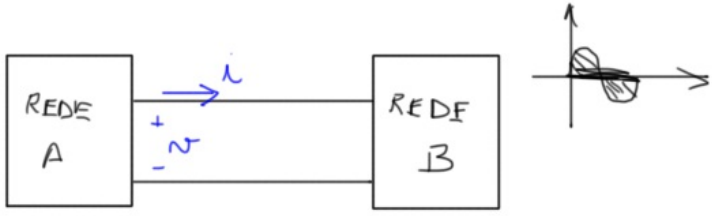
Potência Instantânea



$$\begin{cases} v = V_m \cos(\omega t + \theta_v)\ V \rightarrow V_m \angle \theta_v \\ i = I_m \cos(\omega t + \theta_i)\ A \rightarrow I_m \angle \theta_i \end{cases}$$

$$p = vi = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$* \cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

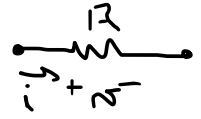
$$p = \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta_v + \theta_i) + \cos(\theta_v - \theta_i)\right]$$

Manipulando a expressão:

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\left[1 + \cos(2\omega t + 2\theta_i)\right] +$$

$$+ \frac{V_m I_m}{2}\left[\operatorname{sen}(\theta_v - \theta_i)\operatorname{sen}(2\omega t + 2\theta_i + 180^\circ)\right] \quad (i)$$

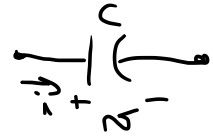
→ Resistor



$\theta_v = \theta_i$

$$p(t) = \frac{V_m I_m}{2}\left[1 + \cos(2\omega t + 2\theta_i)\right]$$

→ CAPACITOR



$$\theta_v - \theta_i = -90^\circ$$

$$p(t) = -\frac{V_m I_m}{2}\operatorname{sen}(2\omega t + 2\theta_i + 180^\circ)$$

→ Indutor

$$\theta_v - \theta_i = 90^\circ$$

$$p(t) = \frac{V_m I_m}{2}\operatorname{sen}(2\omega t + 2\theta_i + 180^\circ)$$

→ POTÊNCIA MÉDIA (ATIVA)

$$P = \frac{1}{T}\int_0^T p\,dt$$

De (i):

$$\boxed{P = \frac{V_m \cdot I_m}{2}\cos(\theta_v - \theta_i)\ [W]}$$

$$\begin{cases} P \text{ é máxima} \rightarrow \theta_v - \theta_i = 0^\circ \\ P \text{ é nula} \rightarrow \theta_v - \theta_i = \pm 90^\circ \end{cases}$$

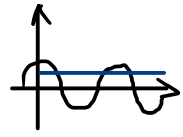
$\cos(\theta_v - \theta_i)$: FATOR DE POTÊNCIA.

Em termos de V_{ef} e I_{ef}:

$$\boxed{P = V_{ef} \cdot I_{ef} \cdot \cos(\theta_v - \theta_i)\ [W]}$$

* CONCEITO DE VALOR EFICAZ (RMS)

$$P = \frac{1}{T}\int_0^T \frac{v^2}{R}dt = \frac{1}{R}\left[\frac{1}{T}\int_0^T v^2\,dt\right]$$



$$\frac{V_{cc}^2}{R} = \frac{1}{R}\left[\frac{1}{T}\int_0^T v^2\,dt\right];\ v = V_m \cdot \cos(\omega t + \theta_v)\ (V)$$

$$V_{cc} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} \rightarrow \text{CÁLCULO DO VALOR EFICAZ}$$

$$V_{ef}^2 = \frac{1}{T}\int_0^T V_m^2 \cdot \cos^2(\omega t + \theta_v)\,dt$$

$$V_{ef}^2 = \frac{1}{T}\int_0^T V_m^2\left[\frac{1 + \cos(2\omega t + 2\theta_v)}{2}\right]dt$$

$$V_{ef}^2 = \frac{1}{T}\int_0^T \frac{V_m^2}{2}\,dt \rightarrow V_{ef}^2 = \frac{V_m^2}{2} \rightarrow \boxed{V_{ef} = \frac{V_m}{\sqrt{2}}}$$

→ Potência Reativa

Da segunda parte de (i):

$$P = \frac{V^2}{R} \; ; \; P = I^2 R$$

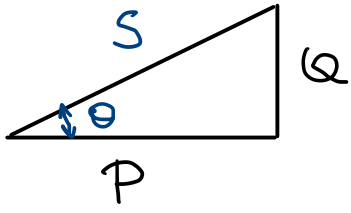
$$\begin{cases} Q = \dfrac{V_m I_m}{2} \cdot \operatorname{sen}(\theta_v - \theta_i) \; [VA_r] \\ Q = V_{ef} \cdot I_{ef} \cdot \operatorname{sen}(\theta_v - \theta_i) \; [VA_r] \end{cases}$$

CAPACITOR: $Q = \dfrac{V_{ef}^2}{X_C}$; $Q = I_{ef}^2 \cdot X_C$; $X_C = 1/\omega C$

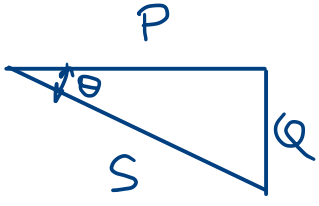
INDUTOR: $Q = \dfrac{V_{ef}^2}{X_L}$; $Q = I_{ef}^2 \cdot X_L$; $X_L = \omega L$

* Triângulo de Potências

$$\theta = \theta_V - \theta_i$$



$$\begin{cases} P = V_{ef} \cdot I_{ef} \cdot \cos\theta \ [W] \\ Q = V_{ef} \cdot I_{ef} \cdot \operatorname{sen}\theta \ [VA_r] \\ S = V_{ef} \cdot I_{ef} \ [VA] \end{cases}$$



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Potência Aparente

* Potência Complexa

$$\boxed{S = P \pm jQ}$$

$$S = V_{ef} \cdot I_{ef} \cdot \cos(\theta_v - \theta_i) \pm j\, V_{ef} \cdot I_{ef} \cdot \operatorname{sen}(\theta_v - \theta_i)$$

$$\boxed{S = V_{ef} \cdot I_{ef} \angle \theta_v - \theta_i}$$

$$\downarrow$$

$$\boxed{S = V \cdot I^*}$$

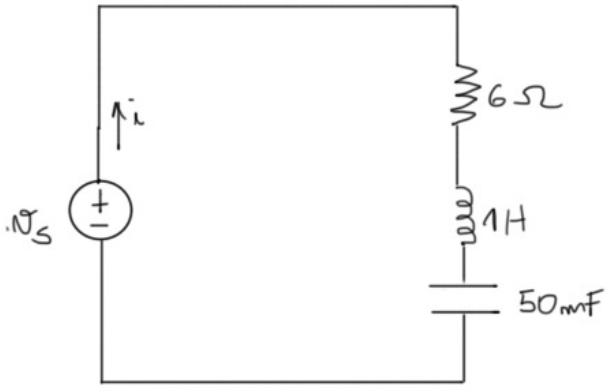
$$V = V_{ef} \angle \theta_v \quad ; \quad I = I_{ef} \angle \theta_i$$

$$V \cdot I = V_{ef} \cdot I_{ef} \angle \theta_v + \theta_i$$

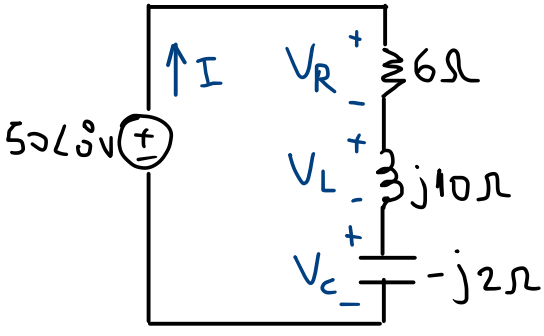
$$I^* = I_{ef} \angle -\theta_i$$

$$\begin{cases} I = a + jb \\ I^* = a - jb \end{cases}$$

EX: $v_s = 50\cos(\omega t)\ (V)$ $\omega = 10\ rad/s$

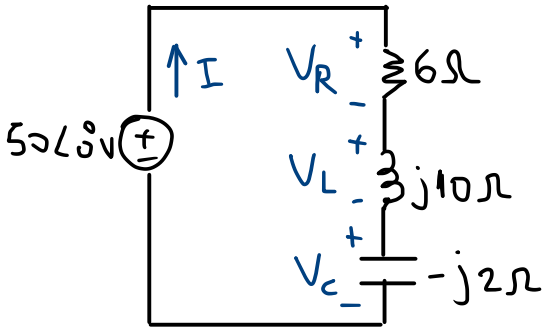


$$\begin{cases} I = 5\angle -53{,}13^\circ\ A \\ V_R = 30\angle -53{,}13^\circ\ V \\ V_L = 50\angle 36{,}87^\circ\ V \\ V_C = 10\angle -143{,}13^\circ\ V \end{cases}$$



CALCULAR P, Q e S

EX: $v_s = 50\cos(\omega t)\ (V)$ $\quad \omega = 10\ rad/s$



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→ Cálculos das potências:

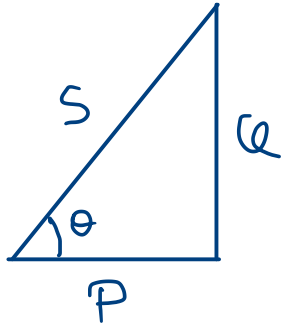
$P = ? \rightarrow P = V_{s_{ef}} \cdot I_{ef} \cdot \cos \measuredangle_I^{V_s} = \frac{50}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} \cdot \cos\left(0^\circ - (-53{,}13^\circ)\right) = \underline{75W}$

$Q = ? \rightarrow Q = V_{s_{ef}} \cdot I_{ef} \cdot \operatorname{sen} \measuredangle_I^{V_s} = \frac{50}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} \cdot \operatorname{sen}\left(0^\circ - (-53{,}13^\circ)\right) = \underline{100VAr}$

$S = ? \rightarrow S = V_{s_{ef}} \cdot I_{ef} = \frac{50}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} = \underline{125\,VA}$

* Triângulo de Potências

$$\theta = \theta_{V_S} - \theta_i$$



Cálculo do Fator de potência

$$\cos(0^\circ - (-53{,}13^\circ)) = \cos(53{,}13^\circ)$$

F.p = 0,6 indutivo ou atrasado

$$S = P + jQ$$