Cálculo de Programas
Resolução - Ficha 01

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Exercício 1

$$
\begin{aligned}
\pi_1 \cdot (f \times g)\ (x, y) &= \{\text{def. composição}\} \\
&\quad \pi_1((f \times g)(x, y)) \\
&= \{(\text{F1})\} \\
&\quad \pi_1(f\ x, g\ y) \\
&= \{(\text{F2})\} \\
&\quad f\ x \\
&= \{(\text{F2})\} \\
&\quad f(\pi_1(x, y)) \\
&= \{\text{def. composição}\} \\
&\quad f \cdot \pi_1
\end{aligned}
$$

$$
\begin{aligned}
\pi_2 \cdot (f \times g)\ (x, y) &= \{\text{def. composição}\} \\
&\quad \pi_2((f \times g)(x, y)) \\
&= \{(\text{F1})\} \\
&\quad \pi_2(f\ x, g\ y) \\
&= \{(\text{F2})\} \\
&\quad g\ y \\
&= \{(\text{F2})\} \\
&\quad g(\pi_2(x, y)) \\
&= \{\text{def. composição}\} \\
&\quad g \cdot \pi_2
\end{aligned}
$$

$$
\begin{aligned}
(f \times g)\ (x, y) &= \{(\text{F1})\} \\
&\quad (f\ x, g\ y) \\
&= \{(\text{F2})\} \\
&\quad (f(\pi_1(x, y)), g(\pi_2(x, y))) \\
&= \{\text{def. composição}\} \\
&\quad (f \cdot \pi_1, g \cdot \pi_2) \\
&= \{\text{def. split}\} \\
&\quad \langle f \cdot \pi_1, g \cdot \pi_2 \rangle
\end{aligned}
$$

Exercício 2

$$
\begin{aligned}
xor \cdot (and \times id)((a,b),c) &= \{\text{def. composição}\}\\
&\quad xor((and \times id)((a,b),c))\\
&= \{\text{(F1)}\}\\
&\quad xor(and(a,b), idc)\\
&= \{\text{def. and e id}\}\\
&\quad xor(a \wedge b, c)\\
&= \{\text{def. xor}\}\\
&\quad (a \wedge b) \oplus c
\end{aligned}
$$

Exercício 3

```
ghci> :l Cp.hs
ghci> xor (x,y) = x /= y
ghci> and (x,y) = x && y
ghci> f = xor . (and >< id)
ghci> f ((True, True), False)
True
ghci> f ((True, True), True)
False
ghci> f ((False, True), True)
True
ghci> f ((True, False), True)
True
```

Exercício 4

$$
id = \langle f, g \rangle \iff \begin{cases} \pi_1 \cdot id = f \\ \pi_2 \cdot id = g \end{cases} \iff \begin{cases} \pi_1 = f \\ \pi_2 = g \end{cases} \iff id = \langle \pi_1, \pi_2 \rangle
$$

Seja $k = id$, ao aplicar a propriedade *universal*-$\times$ obtemos a propriedade *reflexão*-$\times$.

Exercício 5

$$
\begin{aligned}
&\underbrace{\langle h, k \rangle \cdot f}_{k} = \underbrace{\langle h \cdot f, k \cdot f \rangle}_{\langle h, f \rangle}\\
\iff &\{\text{(F7)}\}\\
&\begin{cases} \pi_1 \cdot \langle h, k \rangle \cdot f = h \cdot f \\ \pi_2 \cdot \langle h, k \rangle \cdot f = k \cdot f \end{cases}\\
\iff &\{\text{Cancelamento-}\times\}\\
&\begin{cases} h \cdot f = h \cdot f \\ k \cdot f = k \cdot f \end{cases}
\end{aligned}
$$

Exercício 6

$$
\begin{aligned}
dup \cdot f\ x = {} & \{\text{natural-id}\} \\
& dup \cdot f \cdot id\ x \\
= {} & \{\text{def. composição}\} \\
& dup(f(id\ x)) \\
= {} & \{\text{def. dup}\} \\
& (f(id\ x), f(id\ x)) \\
= {} & \{\text{def. composição}\} \\
& \langle f \cdot id, f \cdot id \rangle \\
= {} & \{\text{fusão-x}\} \\
& \langle f, f \rangle \cdot id \\
= {} & \{\text{natural-id}\} \\
& \langle f, f \rangle
\end{aligned}
$$

Exercício 7

$$
\begin{aligned}
& \underbrace{\underline{(b,a)}}_{k} = \underbrace{\langle \underline{b}, \underline{a} \rangle}_{\langle f,g \rangle} \\
\iff & \{\text{universal-}\times\} \\
& \begin{cases} \pi_1 \cdot \underline{(b,a)} = \underline{b} \\ \pi_2 \cdot \underline{(b,a)} = \underline{a} \end{cases} \\
\iff & \{\text{absorção-const}\} \\
& \begin{cases} \underline{\pi_1(b,a)} = \underline{b} \\ \underline{\pi_2(b,a)} = \underline{a} \end{cases} \\
\iff & \{\text{cancelamento-}\times\} \\
& \begin{cases} \underline{b} = \underline{b} \\ \underline{a} = \underline{a} \end{cases}
\end{aligned}
$$

Exercício 8

$$
\begin{aligned}
& (g \times f) \cdot swap \\
= {} & \{\text{def-}\times\} \\
& \langle g \cdot \pi_1, f \cdot \pi_2 \rangle \cdot swap \\
= {} & \{\text{fusão-}\times\} \\
& \langle g \cdot \pi_1 \cdot swap, f \cdot \pi_2 \cdot swap \rangle \\
= {} & \{\text{def. swap}\} \\
& \langle g \cdot \pi_1 \cdot \langle \pi_2, \pi_1 \rangle, f \cdot \pi_2 \cdot \langle \pi_2, \pi_1 \rangle \rangle \\
= {} & \{\text{cancelamento-}\times\} \\
& \langle g \cdot \pi_2, f \cdot \pi_1 \rangle
\end{aligned}
$$

$$
\begin{aligned}
& swap \cdot (f \times g) \\
= {} & \{\text{def. swap}\} \\
& \langle \pi_2, \pi_1 \rangle \cdot (f \times g) \\
= {} & \{\text{fusão-}\times\} \\
& \langle \pi_2 \cdot (f \times g), \pi_1 \cdot (f \times g) \rangle \\
= {} & \{\text{def-}\times\} \\
& \langle \pi_2 \cdot \langle f \cdot \pi_1, g \cdot \pi_2 \rangle, \pi_1 \cdot \langle f \cdot \pi_1, g \cdot \pi_2 \rangle \rangle \\
= {} & \{\text{cancelamento-}\times\} \\
& \langle g \cdot \pi_2, f \cdot \pi_1 \rangle
\end{aligned}
$$

Exercício 9

```haskell
acronym :: String -> String
acronym = map head . words

short :: String -> String
short = uncurry (++) . (id >< (' ':)) . split head last . words
```

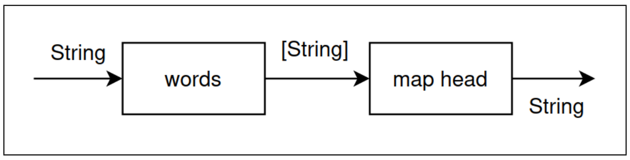

Figura 1: acronym

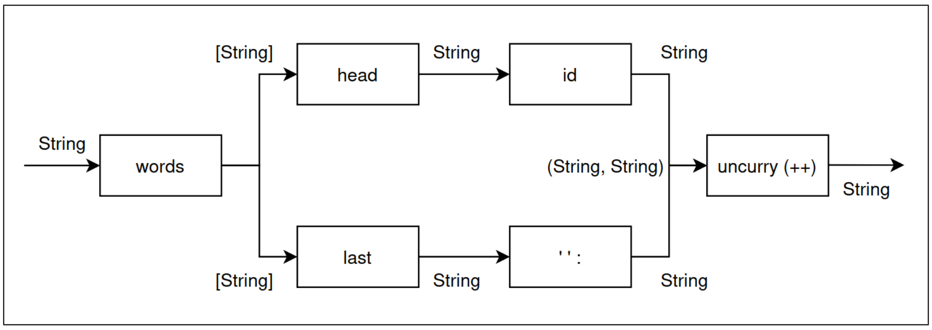

Figura 2: short