

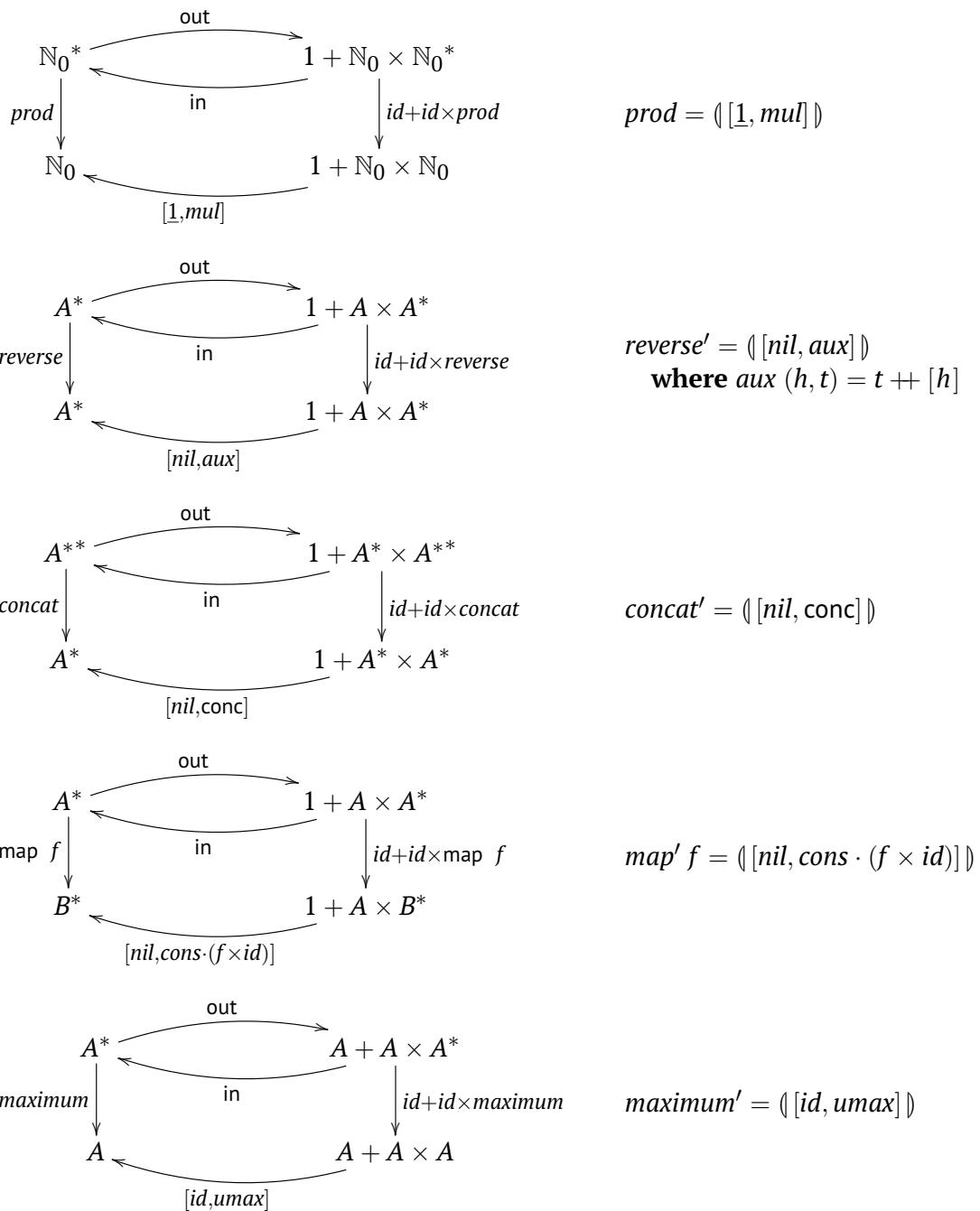
Cálculo de Programas

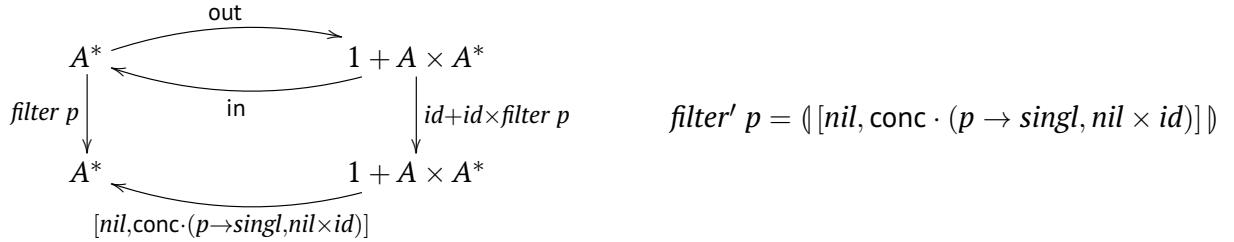
Resolução - Ficha 07

Eduardo Freitas Fernandes

2026

Exercício 1





Exercício 2

$$\begin{aligned}
 & sumprod a = (a*) \cdot sum \\
 \equiv & \{ \text{def. } sum, \text{def. } sumprod a \} \\
 & \langle [0, add \cdot ((a*) \times id)] \rangle = (a*) \cdot \langle [0, add] \rangle \\
 \Leftarrow & \{ \text{fusão-cata} \} \\
 & (a*) \cdot [0, add] = [0, add \cdot ((a*) \times id)] \cdot (id + id \times (a*)) \\
 \equiv & \{ \text{fusão-+}, \text{absorção-+} \} \\
 & [(a*) \cdot 0, (a*) \cdot add] = [0, add \cdot ((a*) \times id) \cdot (id \times (a*))] \\
 \equiv & \{ \text{eq-+} \} \\
 & \begin{cases} (a*) \cdot 0 = 0 \\ (a*) \cdot add = add \cdot ((a*) \times id) \cdot (id \times (a*)) \end{cases} \\
 \equiv & \{ \text{functor-}\times \} \\
 & \begin{cases} (a*) \cdot 0 = 0 \\ (a*) \cdot add = add \cdot ((a*) \times (a*)) \end{cases} \\
 \equiv & \{ \text{pointwise, def. } add, \text{def. } 0 \} \\
 & \begin{cases} a * 0 = 0 \\ a * (x + y) = (a * x) + (a * y) \end{cases}
 \end{aligned}$$

Exercício 3

$$\begin{aligned}
 & f \cdot length = \langle [0, 2+] \cdot \pi_2 \rangle \\
 \equiv & \{ \text{def. } length \} \\
 & f \cdot \langle [0, \text{succ} \cdot \pi_2] \rangle = \langle [0, 2+] \cdot \pi_2 \rangle \\
 \Leftarrow & \{ \text{fusão-cata} \} \\
 & f \cdot [0, \text{succ} \cdot \pi_2] = [0, (2+) \cdot \pi_2] \cdot (id + id \times f) \\
 \equiv & \{ \text{fusão-+}, \text{absorção-+}, \text{natural-id} \} \\
 & [f \cdot 0, f \cdot \text{succ} \cdot \pi_2] = [0, (2+) \cdot \pi_2 \cdot (id \times f)] \\
 \equiv & \{ \text{eq-+}, \text{natural-}\pi_2 \} \\
 & \begin{cases} f \cdot 0 = 0 \\ f \cdot \text{succ} \cdot \pi_2 = (2+) \cdot f \cdot \pi_2 \end{cases} \\
 \equiv & \{ \text{pointwise} \} \\
 & \begin{cases} (f \cdot 0) n = 0 n \\ (f \cdot \text{succ} \cdot \pi_2) (x, y) = ((2+) \cdot f \cdot \pi_2) (x, y) \end{cases}
 \end{aligned}$$

$$\equiv \{ \text{def. comp, def. } \pi_2 \}$$

$$\begin{cases} f 0 = 0 \\ f (y + 1) = 2 + f y \end{cases}$$

Dado que $2 + 0 = 2$ e $2 * 0 = 0$, concluimos que $f = (2*)$.

Exercício 4

$$\begin{aligned} & foldr \pi_2 i = f \\ \equiv & \{ \text{def. } foldr \} \\ & \langle [i, \widehat{\pi_2}] \rangle = f \\ \equiv & \{ \text{universal-cata, } \widehat{f} = f \} \\ & f \cdot \text{in} = [\underline{0}, \pi_2] \cdot (id + id \times f) \\ \equiv & \{ \text{def. in, fusão-+, absorção-+, eq-+} \} \\ & \begin{cases} f \cdot \text{nil} = i \\ f \cdot \text{cons} = \pi_2 \cdot (id \times f) \end{cases} \\ \equiv & \{ \text{pointwise} \} \\ & \begin{cases} f [] = i \\ f (h : t) = f t \end{cases} \end{aligned}$$

Podemos concluir que $foldr \pi_2 i$ é a função constante i .

Exercício 5

$$\begin{aligned} & f \cdot \text{for } f i = \text{for } f f i \\ \equiv & \{ \text{def. for } b i \} \\ & f \cdot \langle [i, f] \rangle = \langle [f i, f] \rangle \\ \Leftarrow & \{ \text{fusão-cata} \} \\ & f \cdot [i, f] = [f i, f] \cdot (id + f) \\ \equiv & \{ \text{fusão-+, absorção-+} \} \\ & [f \cdot i, f \cdot f] = [f i \cdot id, f \cdot f] \\ \equiv & \{ \text{eq-+, natural-id, absorção-const} \} \\ & \begin{cases} f i = f i \\ f \cdot f = f \cdot f \end{cases} \\ \square & \end{aligned}$$

Exercício 6

$$\begin{aligned} & \text{for } id i = \text{for } \underline{i} i \\ \equiv & \{ \text{def. for } \cdot \cdot \text{ (twice)} \} \\ & \langle [i, id] \rangle = \langle [i, i] \rangle \\ \equiv & \{ \text{universal-cata} \} \\ & \langle [i, id] \rangle \cdot \text{in} = [i, i] \cdot (id + \langle [i, id] \rangle) \end{aligned}$$

$$\begin{aligned}
&\equiv \{ \text{cancelamento-cata, absorção-+} \} \\
&[\underline{i}, id] \cdot (id + ([\underline{i}, id])) = [\underline{i}, \underline{i}] \\
&\equiv \{ \text{absorção-+} \} \\
&[\underline{i}, ([\underline{i}, id])] = [\underline{i}, \underline{i}] \\
&\equiv \{ \text{eq-+} \} \\
&\left\{ \begin{array}{l} \underline{i} = \underline{i} \\ ([\underline{i}, id]) = \underline{i} \end{array} \right. \\
&\equiv \{ \text{universal-cata} \} \\
&\left\{ \begin{array}{l} \text{true} \\ \underline{i} \cdot \text{in} = [\underline{i}, id] \cdot (id + \underline{i}) \end{array} \right. \\
&\equiv \{ \text{absorção-+} \} \\
&\underline{i} = [\underline{i}, \underline{i}] \\
&\square
\end{aligned}$$

A última expressão é verdadeira, pois o resultado de $[\underline{i}, \underline{i}]$ será sempre i .

Exercício 7

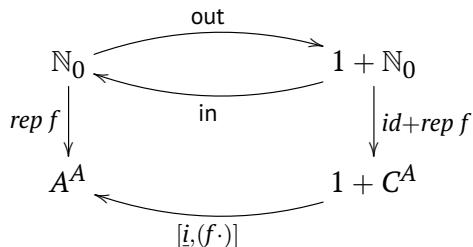
$$\begin{aligned}
&\underline{id} : B \rightarrow A^A \\
&f : C \rightarrow D \\
&(f \cdot) : C^E \rightarrow D^E
\end{aligned}$$

Temos de ter igualdade de tipos, ou seja:

$$A^A = D^E \implies \left\{ \begin{array}{l} A = E \\ A = D \end{array} \right.$$

Logo, temos:

$$\begin{aligned}
&\underline{id} : B \rightarrow A^A \\
&f : C \rightarrow A \\
&(f \cdot) : C^A \rightarrow A^A
\end{aligned}$$



Exercício 9

```

type Date = String
type Player = String
type Game = String
db1 = [
  ("2023-10-01", ["Game1", "Game2"]),
  ("2023-10-02", ["Game2", "Game3"])
]

```

```
]
db2 = [
    ("Game1", ["PlayerA", "PlayerB"]),
    ("Game2", ["PlayerA", "PlayerC"]),
    ("Game3", ["PlayerB", "PlayerC"])
]
f :: [(Date, [Game])] → [(Game, [Player])] → [(Player, [Date])]
f = ⊥
main :: IO ()
main = do
    let result = Main.f db1 db2
    print result
```