# Cálculo de Programas

## Resolução - Ficha 04

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2026

**Exercício 1**

$$
\begin{array}{ll}
& [\underline{k}, \underline{k}] = \underline{k} \\
\equiv & \quad \{ \text{ universal-+ } \} \\
& \left\{ \begin{array}{l} \underline{k} = \underline{k} \cdot i_1 \\ \underline{k} = \underline{k} \cdot i_2 \end{array} \right. \\
\equiv & \quad \{ \text{ fusão-+ } \} \\
& \left\{ \begin{array}{l} \underline{k} = \underline{k} \\ \underline{k} = \underline{k} \end{array} \right. \\
\square &
\end{array}
$$

**Exercício 2**

$$
\begin{array}{ll}
& fac \cdot [\underline{0}, \text{succ}] = [\underline{1}, mul \cdot \langle \text{succ}, fac \rangle] \\
\equiv & \quad \{ \text{ universal-+ } \} \\
& \left\{ \begin{array}{l} fac \cdot [\underline{0}, \text{succ}] \cdot i_1 = \underline{1} \\ fac \cdot [\underline{0}, \text{succ}] \cdot i_2 = mul \cdot \langle \text{succ}, fac \rangle \end{array} \right. \\
\equiv & \quad \{ \text{ cancelamento-+ } \} \\
& \left\{ \begin{array}{l} fac \cdot \underline{0} = \underline{1} \\ fac \cdot \text{succ} = mul \cdot \langle \text{succ}, fac \rangle \end{array} \right. \\
\equiv & \quad \{ \text{ absorção-+, pointwise } \} \\
& \left\{ \begin{array}{l} \overline{fac\ 0}\ x = \underline{1}\ x \\ (\overline{fac} \cdot \text{succ})\ n = (mul \cdot \langle \text{succ}, fac \rangle)\ n \end{array} \right. \\
\equiv & \quad \{ \text{ def. const, def. split, def. comp } \} \\
& \left\{ \begin{array}{l} fac\ 0 = 1 \\ fac\ (\text{succ}\ n) = mul\ (\text{succ}\ n, fac\ n) \end{array} \right. \\
\equiv & \quad \{ \text{ def. succ, def. } mul \ \} \\
& \left\{ \begin{array}{l} fac\ 0 = 1 \\ fac\ (n+1) = (n+1) * fac\ n \end{array} \right.
\end{array}
$$

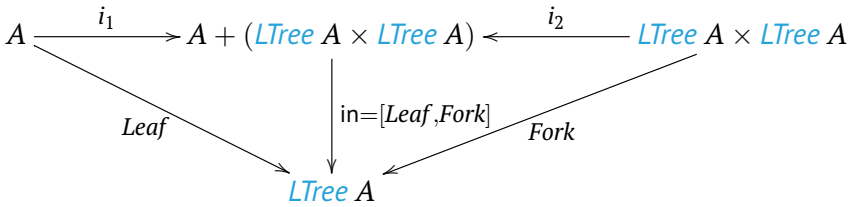
**Exercício 3**

$$out \cdot \text{in} = id$$

$$
\begin{array}{ll}
\equiv & \{\ \text{def. in, fusão-+, universal-+}\ \} \\
& \left\{ \begin{array}{l} out \cdot \underline{0} = id \cdot i_1 \\ out \cdot \mathsf{succ} = id \cdot i_2 \end{array} \right. \\
\equiv & \{\ \text{natural-}id\text{, absorção-const}\ \} \\
& \left\{ \begin{array}{l} \underline{out\ 0} = i_1 \\ out \cdot \mathsf{succ} = i_2 \end{array} \right. \\
\equiv & \{\ \text{pointwise, def. comp, def. succ, def. const}\ \} \\
& \left\{ \begin{array}{l} out\ 0 = i_1\ () \\ out\ (n+1) = i_2\ n \end{array} \right.
\end{array}
$$

## Exercício 5

$$
\begin{array}{ll}
& out \cdot \mathsf{in} = id \\
\equiv & \{\ \text{def. in, fusão-+, universal-+}\ \} \\
& \left\{ \begin{array}{l} out \cdot Leaf = id \cdot i_1 \\ out \cdot Fork = id \cdot i_2 \end{array} \right. \\
\equiv & \{\ \text{natural-}id\text{, natural-}id\text{, def. comp}\ \} \\
& \left\{ \begin{array}{l} out\ (Leaf\ a) = i_1\ a \\ out\ (Fork\ (x,y)) = i_2\ (x,y) \end{array} \right.
\end{array}
$$




## Exercício 6

$$
\begin{array}{ll}
& coassocl \cdot [id + i_1, i_2 \cdot i_2, \cdot] = id \\
\equiv & \{\ \text{fusão-+, universal-+}\ \} \\
& \left\{ \begin{array}{l} coassocl \cdot (id + i_1) = i_1 \\ coassocl \cdot i_2 \cdot i_2 = i_2 \end{array} \right. \\
\equiv & \{\ \text{def-+, fusão-+, universal-+}\ \} \\
& \left\{ \begin{array}{l} \left\{ \begin{array}{l} coassocl \cdot i_1 = i_1 \cdot i_1 \\ coassocl \cdot i_2 \cdot i_1 = i_1 \cdot i_2 \end{array} \right. \\ coassocl \cdot i_2 \cdot i_2 = i_2 \end{array} \right. \\
\equiv & \{\ \text{associação à direita}\ \} \\
& \left\{ \begin{array}{l} coassocl \cdot i_1 = i_1 \cdot i_1 \\ \left\{ \begin{array}{l} coassocl \cdot i_2 \cdot i_1 = i_1 \cdot i_2 \\ coassocl \cdot i_2 \cdot i_2 = i_2 \end{array} \right. \end{array} \right. \\
\equiv & \{\ \text{universal-+}\ \} \\
& \left\{ \begin{array}{l} coassocl \cdot i_1 = i_1 \cdot i_1 \\ coassocl \cdot i_2 = [i_1 \cdot i_2, i_2] \end{array} \right.
\end{array}
$$

$$
\begin{array}{ll}
\equiv & \{\ \text{universal-}+\ \} \\
& \textit{coassocl} = [i_1 \cdot i_1, [i_1 \cdot i_2, i_2]]
\end{array}
$$

## Exercício 8

$$
\begin{array}{ll}
& [\langle f, g\rangle, \langle h, k\rangle] = \langle [f, h], [g, k]\rangle \\
\equiv & \{\ \text{universal-}+\ \} \\
& \begin{cases} \langle f, g\rangle = \langle [f, h], [g, k]\rangle \cdot i_1 \\ \langle h, k\rangle = \langle [f, h], [g, k]\rangle \cdot i_2 \end{cases} \\
\equiv & \{\ \text{fusão-}+\ \} \\
& \begin{cases} \langle f, g\rangle = \langle [f, h] \cdot i_1, [g, k] \cdot i_1\rangle \\ \langle h, k\rangle = \langle [f, h] \cdot i_2, [g, k] \cdot i_2\rangle \end{cases} \\
\equiv & \{\ \text{cancelamento-}+\ \} \\
& \begin{cases} \langle f, g\rangle = \langle f, g\rangle \\ \langle h, k\rangle = \langle h, k\rangle \end{cases} \\
\square &
\end{array}
$$

## Exercício 9

$$
\begin{array}{l}
\textit{unglue} = \langle \mathsf{map}\ \ \textit{getLeft} \cdot \textit{filter}\ (\textit{isLeft} \cdot \pi_1), \mathsf{map}\ \ \textit{getRight} \cdot \textit{filter}\ (\textit{isRight} \cdot \pi_1)\rangle \\
\quad \textbf{where}\ \textit{getLeft}\ (i_1\ x, m) = (x, m) \\
\qquad \textit{getRight}\ (i_2\ x, m) = (x, m) \\
\textit{glue} = \widehat{(+\!+)} \cdot (\mathsf{map}\ \ (i_1 \times \textit{id}) \times \mathsf{map}\ \ (i_2 \times \textit{id}))
\end{array}
$$