Cálculo de Programas

Resolução - Ficha 06

Eduardo Freitas Fernandes

2026

Exercício 1

$$
\begin{aligned}
& ap \cdot (\overline{f} \times id) = f \\
\equiv \quad & \{ \text{ pointwise } \} \\
& ap \cdot (\overline{f} \times id)\ (a,b) = f\ (a,b) \\
\equiv \quad & \{ \text{ def. comp, def-}\times \} \\
& \overline{f}\ a\ b = f\ (a,b) \\
\equiv \quad & \{ \text{ def. curry } \} \\
& f\ (a,b) = f\ (a,b)
\end{aligned}
$$

$\square$

Exercício 2

$$
\begin{aligned}
& ap \cdot (\overline{f} \times id) = f \\
\equiv \quad & \{ \text{ pointwise } \} \\
& ap \cdot (\overline{f} \times id)\ (a,b) = f\ (a,b) \\
\equiv \quad & \{ \text{ def. comp, def-}\times \} \\
& \overline{f}\ a\ b = f\ (a,b) \\
\equiv \quad & \{\ f = \widehat{g}\ \} \\
& \overline{\widehat{g}}\ a\ b = \widehat{g}\ (a,b) \\
\equiv \quad & \{ \text{ def. curry } \} \\
& \widehat{g}\ (a,b) = \widehat{g}\ (a,b)
\end{aligned}
$$

$\square$

Exercício 3

$$
\begin{aligned}
& \overline{f \cdot (g \times h)} = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g \\
\equiv \quad & \{ \text{ universal-exp } \} \\
& f \cdot (g \times h) = ap \cdot ((\overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g) \times id) \\
\equiv \quad & \{ \text{ natural-}id\text{, functor-}\times \} \\
& f \cdot (g \times h) = ap \cdot (\overline{ap \cdot (id \times h)} \times id) \cdot (\overline{f} \cdot g \times id)
\end{aligned}
$$

$$
\begin{array}{ll}
\equiv & \quad \{\ \text{cancelamento-exp}\ \} \\
& f \cdot (g \times h) = ap \cdot (id \times h) \cdot ((\overline{f} \cdot g) \times id) \\
\equiv & \quad \{\ \text{functor-}\times\ \} \\
& f \cdot (g \times h) = ap \cdot ((\overline{f} \cdot g) \times h) \\
\equiv & \quad \{\ \text{natural-}id\text{, functor-}\times\ \} \\
& f \cdot (g \times h) = ap \cdot (\overline{f} \times id) \cdot (g \times h) \\
\equiv & \quad \{\ \text{cancelamento-exp}\ \} \\
& f \cdot (g \times h) = f \cdot (g \times h) \\
\Box &
\end{array}
$$

Exercício 4

$$
\begin{array}{ll}
& flip\ (flip\ f) = f \\
\equiv & \quad \{\ \text{def. } flip\ \} \\
& \overline{\widehat{(flip\ f)} \cdot swap} = f \\
\equiv & \quad \{\ \text{pointwise}\ \} \\
& \overline{\widehat{(flip\ f)} \cdot swap}\ a\ b = f\ a\ b \\
\equiv & \quad \{\ \text{def. curry}\ \} \\
& (\widehat{(flip\ f)} \cdot swap)\ (a, b) = f\ a\ b \\
\equiv & \quad \{\ \text{def. comp, def. } swap\ \} \\
& \widehat{(flip\ f)}\ (b, a) = f\ a\ b \\
\equiv & \quad \{\ \text{def. uncurry}\ \} \\
& flip\ f\ b\ a = f\ a\ b \\
\equiv & \quad \{\ \dots\ \} \\
& \dots
\end{array}
$$

$$
\begin{array}{ll}
& flip\ f\ x\ y = f\ y\ x \\
\equiv & \quad \{\ \text{def. } flip\ \} \\
& \overline{\widehat{f} \cdot swap}\ x\ y = f\ y\ x \\
\equiv & \quad \{\ \text{def. curry}\ \} \\
& (\widehat{f} \cdot swap)\ (x, y) = f\ y\ x \\
\equiv & \quad \{\ \text{def. comp, def. } swap\ \} \\
& \widehat{f}\ (y, x) = f\ y\ x \\
\equiv & \quad \{\ \text{def. uncurry}\ \} \\
& f\ y\ x = f\ y\ x \\
\Box &
\end{array}
$$

Exercício 5

$$
\begin{array}{ll}
 & junc \cdot unjunc = id \\
\equiv & \qquad \{\ \text{pointwise}\ \} \\
 & (junc \cdot unjunc)\ l = id\ k \\
\equiv & \qquad \{\ \text{def. } id\text{, def. comp, def. } unjunc\ \} \\
 & junc\ (k \cdot i_1, k \cdot i_2) = k \\
\equiv & \qquad \{\ \text{def. } junc\ \} \\
 & [k \cdot i_1, k \cdot i_2] = k \\
\equiv & \qquad \{\ \text{fusão-}{+}\ \} \\
 & k \cdot [i_1, i_2] = k \\
\equiv & \qquad \{\ \text{reflexão-}{+}\text{, natural-}id\ \} \\
 & k = k \\
\Box &
\end{array}
$$

$$
\begin{array}{ll}
 & unjunc \cdot junc = id \\
\equiv & \qquad \{\ \text{pointwise}\ \} \\
 & (unjunc \cdot junc)\ (f, g) = id\ (f, g) \\
\equiv & \qquad \{\ \text{def. } id\text{, def. comp, def. } junc\ \} \\
 & unjunc\ [\cdot, \cdot]\ f\ g = (f, g) \\
\equiv & \qquad \{\ \text{def. } unjunc\ \} \\
 & ([f, g] \cdot i_1, [f, g] \cdot i_2) = (f, g) \\
\equiv & \qquad \{\ \text{cancelamento-}{+}\ \} \\
 & (f, g) = (f, g) \\
\Box &
\end{array}
$$

Exercício 6

$$
\begin{array}{ll}
 & \mathsf{for}\ b\ i \cdot \mathsf{in} = [g_1, g_2] \cdot (id + \mathsf{for}\ b\ i) \\
\equiv & \qquad \{\ \text{def. in, fusão-}{+}\text{, absorção-}{+}\ \} \\
 & [\mathsf{for}\ b\ i \cdot \underline{0}, \mathsf{for}\ b\ i \cdot \mathsf{succ}] = [g_1 \cdot id, g_2 \cdot \mathsf{for}\ b\ i] \\
\equiv & \qquad \{\ \text{natural-}id\text{, eq-}{+}\ \} \\
 & \left\{ \begin{array}{l} \mathsf{for}\ b\ i \cdot \underline{0} = g_1 \\ \mathsf{for}\ b\ i \cdot \mathsf{succ} = g_2 \cdot \mathsf{for}\ b\ i \end{array} \right. \\
\equiv & \qquad \{\ \text{pointwise, def. comp, def. } \underline{0}\text{, def. succ}\ \} \\
 & \left\{ \begin{array}{l} \mathsf{for}\ b\ i\ 0 = g_1\ () \\ \mathsf{for}\ b\ i\ (n+1) = g_2\ \mathsf{for}\ b\ i\ n \end{array} \right. \\
\equiv & \qquad \{\ \text{(F8)}\ \} \\
 & \left\{ \begin{array}{l} g_1 = i \\ g_2\ \mathsf{for}\ b\ i\ n = b\ \mathsf{for}\ b\ i\ n \end{array} \right. \\
\equiv & \qquad \{\ \text{igualdade extensional (73)}\ \} \\
 & \left\{ \begin{array}{l} g_1 = i \\ g_2 = b \end{array} \right.
\end{array}
$$

Exercício 8

$$
\begin{array}{ll}
 & \left\{ \begin{array}{l} \mathsf{for}\ b\ i\ 0 = i \\ \mathsf{for}\ b\ i\ (n+1) = b\ \mathsf{for}\ b\ i\ n \end{array} \right. \\
\equiv & \qquad \{\ \text{pointfree}\ \} \\
 & \left\{ \begin{array}{l} \mathsf{for}\ b\ i \cdot \underline{0} = \underline{i} \\ \mathsf{for}\ b\ i \cdot \mathsf{succ} = b \cdot \mathsf{for}\ b\ i \end{array} \right.
\end{array}
$$

$$
\begin{array}{ll}
 & \left\{ \begin{array}{l} a + 0 = a \\ a + (n+1) = 1 + (a+n) \end{array} \right. \\
\equiv & \qquad \{\ \text{pointfree}\ \} \\
 & \left\{ \begin{array}{l} (a+) \cdot \underline{0} = \underline{a} \\ (a+) \cdot \mathsf{succ} = \mathsf{succ} \cdot (a+) \end{array} \right.
\end{array}
$$

Deduzimos então que $(a+) = \mathsf{for}\ \mathsf{succ}\ a$.

Exercício 9

```c
int k(int n, int a) {
      int r = 0;
      int j;
      for (j = 1; j < n + 1; j++) {
            r = a + r;
      }
      return r;
};
```

Exercício 10

$$func\ b = (maybe\ b\ id \cdot) \cdot flip\ lookup$$
$$a = [(140999000, \texttt{"Manuel"}), (200100300, \texttt{"Mary"}), (000111222, \texttt{"Teresa"})]$$
$$b = [(140999000, \texttt{"PT"}), (200100300, \texttt{"UK"})]$$
$$c = [(140999000, \texttt{"Braga"}), (200100300, \texttt{"Porto"}), (151999000, \texttt{"Lisbon"})]$$