

Cálculo de Programas

Resolução - Ficha 06

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Exercício 2

Exercício 1

$$\begin{array}{ll} ap \cdot (\bar{f} \times id) = f & \\ \equiv \{\text{pointwise}\} & \\ ap \cdot (\bar{f} \times id) (a, b) = f (a, b) & \\ \equiv \{\text{Def. comp, Def-}\times\} & \\ \bar{f} a b = f (a, b) & \\ \equiv \{\text{Def. comp, Def-}\times\} & \\ \bar{f} a b = f (a, b) & \\ \equiv \{\text{Curry}\} & \\ f (a, b) = f (a, b) & \\ \equiv \{\text{Curry}\} & \\ \hat{g} (a, b) = \hat{g} (a, b) & \\ \equiv \{\text{Uncurry}\} & \\ g a b = g a b & \end{array}$$

Exercício 3

$$\begin{array}{l} \overline{f \cdot (g \times h)} = \overline{ap \cdot (id \times h)} \cdot \bar{f} \cdot g \\ \equiv \{\text{Universal-exp}\} \\ f \cdot (g \times h) = ap \cdot ((\overline{ap \cdot (id \times h)} \cdot \bar{f} \cdot g) \times id) \\ \equiv \{\text{Natural-id, Functor-}\times\} \\ f \cdot (g \times h) = ap \cdot (\overline{ap \cdot (id \times h)} \times id) \cdot (\bar{f} \cdot g \times id) \\ \equiv \{\text{Cancelamento-exp}\} \\ f \cdot (g \times h) = ap \cdot (id \times h) \cdot ((\bar{f} \cdot g) \times id) \\ \equiv \{\text{Functor-}\times\} \\ f \cdot (g \times h) = ap \cdot ((\bar{f} \cdot g) \times h) \\ \equiv \{\text{Natural-id, Functor-}\times\} \\ f \cdot (g \times h) = ap \cdot (\bar{f} \times id) \cdot (g \times h) \\ \equiv \{\text{Cancelamento-exp}\} \\ f \cdot (g \times h) = f \cdot (g \times h) \end{array}$$

Exercício 4

$$\begin{aligned}
 & \text{flip} (\text{flip } f) = f \\
 & \equiv \{\text{Def. flip}\} \\
 & \overline{\text{flip } f \cdot \text{swap}} = f \\
 & \equiv \{\text{pointwise}\} \\
 & \overline{\text{flip } f \cdot \text{swap}} a b = f a b \\
 & \equiv \{\text{Curry}\} \\
 & (\widehat{\text{flip } f \cdot \text{swap}}) (a, b) = f a b \\
 & \equiv \{\text{Def. swap}\} \\
 & \widehat{\text{flip } f} (b, a) = f a b \\
 & \equiv \{\text{Uncurry}\} \\
 & \text{flip } f b a = f a b \\
 & \equiv \{\dots\} \\
 & \dots
 \end{aligned}
 \quad
 \begin{aligned}
 & \text{flip } f x y = f y x \\
 & \equiv \{\text{Def. flip}\} \\
 & \overline{\widehat{f} \cdot \text{swap}} f x y = f y x \\
 & \equiv \{\text{Curry}\} \\
 & (\widehat{f} \cdot \text{swap}) (x, y) = f y x \\
 & \equiv \{\text{Def. comp, Def. swap}\} \\
 & \widehat{f} (y, x) = f y x \\
 & \equiv \{\text{Uncurry}\} \\
 & f y x = f y x
 \end{aligned}$$

Exercício 5

$$\begin{aligned}
 & \text{junc} \cdot \text{unjunc} = id \\
 & \equiv \{\text{pointwise}\} \\
 & (\text{junc} \cdot \text{unjunc}) k = id k \\
 & \equiv \{\text{Natural-id, Def. comp, Def. unjunc}\} \\
 & \text{junc} (k \cdot i_1, k \cdot i_2) = k \\
 & \equiv \{\text{Def. junc}\} \\
 & [k \cdot i_1, k \cdot i_2] = k \\
 & \equiv \{\text{Fusão-+}\} \\
 & k \cdot [i_1, i_2] = k \\
 & \equiv \{\text{Reflexão-+, Natural-id}\} \\
 & k = k
 \end{aligned}
 \quad
 \begin{aligned}
 & \text{unjunc} \cdot \text{junc} = id \\
 & \equiv \{\text{pointwise}\} \\
 & (\text{unjunc} \cdot \text{junc}) (f, g) = id (f, g) \\
 & \equiv \{\text{Natural-id, Def. comp, Def. junc}\} \\
 & \text{unjunc} [f, g] = (f, g) \\
 & \equiv \{\text{Def. unjunc}\} \\
 & ([f, g] \cdot i_1, [f, g] \cdot i_2) = (f, g) \\
 & \equiv \{\text{Cancelamento-+}\} \\
 & (f, g) = (f, g)
 \end{aligned}$$

Exercício 6

$$\begin{aligned}
 & (for\ b\ i) \cdot in = [g_1, g_2] \cdot (id + for\ b\ i) \\
 & \equiv \{\text{Def. in, Fusão-+}, \text{Absorção-+}\} \\
 & [for\ b\ i \cdot zero, for\ b\ i \cdot succ] = [g_1 \cdot id, g_2 \cdot for\ b\ i] \\
 & \equiv \{\text{Natural-id, Eq-+}\} \\
 & \begin{cases} for\ b\ i \cdot zero = g_1 \\ for\ b\ i \cdot succ = g_2 \cdot for\ b\ i \end{cases} \\
 & \equiv \{\text{pointwise}\} \\
 & \begin{cases} (for\ b\ i \cdot zero)x = g_1 x \\ (for\ b\ i \cdot succ)n = (g_2 \cdot for\ b\ i)n \end{cases} \\
 & \equiv \{\text{Def. comp, Def. zero, Def. succ}\} \\
 & \begin{cases} for\ b\ i\ 0 = g_1 \\ for\ b\ i\ (n+1) = g_2\ (for\ b\ i\ n) \end{cases} \\
 & \equiv \{(F8)\} \\
 & \begin{cases} g_1 = i \\ g_2\ (for\ b\ i\ n) = b\ (for\ b\ i\ n) \end{cases} \\
 & \equiv \{f\ x = g\ x \implies f = g\} \\
 & \begin{cases} g_1 = i \\ g_2 = b \end{cases}
 \end{aligned}$$

Exercício 7

```

ghci> :{
ghci| for b i 0 = i
ghci| for b i n = b (for b i (n-1))
ghci| :}
ghci> :t +d for
for :: (t2 -> t2) -> t2 -> Integer -> t2
ghci> f = p2 . aux where aux = for (split (succ . p1) mul) (1,1)
ghci> :t f
f :: (Eq a, Num a) => a -> Integer
ghci> f 3
6
ghci> f 5
120
ghci> f 7
5040
ghci> -- f é a função que calcula o factorial

```

Nota: nas novas versões do Haskell, a syntax `for b i (n+1) = b (for b i n)` não é permitida.

Exercício 8

$$\begin{aligned}
 \left\{ \begin{array}{l} \text{for } b \ i \ 0 = i \\ \text{for } b \ i \ (n + 1) = b(\text{for } b \ i \ n) \end{array} \right. &\equiv \left\{ \begin{array}{l} (\text{for } b \ i) \cdot \underline{0} = \underline{i} \\ (\text{for } b \ i) \cdot \text{succ} = b \cdot \text{for } b \ i \end{array} \right. \\
 \left\{ \begin{array}{l} a + 0 = a \\ a + (n + 1) = 1 + (a + n) \end{array} \right. &\equiv \left\{ \begin{array}{l} (a+) \cdot \underline{0} = \underline{a} \\ (a+) \cdot \text{succ} = \text{succ} \cdot (a+) \end{array} \right. \\
 \text{for } b \ i = \emptyset [i, b] & \\
 (a+) = \emptyset [a, \text{succ}] & \\
 (a+) = \text{for succ } a &
 \end{aligned}$$

Exercício 9

```

int k(int n, int a) {
    int r = 0;
    int j;
    for (j = 1; j < n + 1; j++) {
        r = a + r;
    }
    return r;
};

```

Exercício 10

```

func :: Eq a => b -> [(a, b)] -> (a -> b)
func b = (maybe b id .) . flip lookup

a = [(140999000, "Manuel"), (200100300, "Mary"), (000111222, "Teresa")]

b = [(140999000, "PT"), (200100300, "UK")]

c = [(140999000, "Braga"), (200100300, "Porto"), (151999000, "Lisbon")]

```