Cálculo de Programas

Resolução - Ficha 03

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Exercício 1

$$
\begin{array}{ll}
 & assocl \cdot assocr = id \\
\equiv & \quad \{ \text{ Def. } assocl\text{, fusão-}\times\text{, reflexão-}\times\text{, eq-}\times \ \} \\
 & \begin{cases} (id \times \pi_1) \cdot assocr = \pi_1 \\ \pi_2 \cdot \pi_2 \cdot assocr = \pi_2 \end{cases} \\
\equiv & \quad \{ \text{ def-}\times\text{, universal-}\times \ \} \\
 & \begin{cases} \begin{cases} \pi_1 \cdot assocr = \pi_1 \cdot \pi_1 \\ \pi_1 \cdot \pi_2 \cdot assocr = \pi_2 \cdot \pi_1 \end{cases} \\ \pi_2 \cdot \pi_2 \cdot assocr = \pi_2 \end{cases} \\
\equiv & \quad \{ \text{ associação à direita } \} \\
 & \begin{cases} \pi_1 \cdot assocr = \pi_1 \cdot \pi_1 \\ \begin{cases} \pi_1 \cdot \pi_2 \cdot assocr = \pi_2 \cdot \pi_1 \\ \pi_2 \cdot \pi_2 \cdot assocr = \pi_2 \end{cases} \end{cases} \\
\equiv & \quad \{ \text{ universal-}\times \ \} \\
 & \begin{cases} \pi_1 \cdot assocr = \pi_1 \cdot \pi_1 \\ \pi_2 \cdot assocr = \langle \pi_2 \cdot \pi_1, \pi_2 \rangle \end{cases} \\
\equiv & \quad \{ \text{ natural-}id\text{, universal-}\times \} \\
 & assocr = \langle \pi_1 \cdot \pi_1, \pi_2 \times id \rangle
\end{array}
$$

Exercício 3

$$
\frac{f : A \to B \qquad g : C \to D}{f \times g : A \times C \to B \times D}
\qquad
\frac{f : A \to B \qquad g : A \to C}{\langle f, g \rangle : A \to B \times C}
\qquad
\frac{f : A \to B \qquad g : B \to C}{f \cdot g : A \to C}
$$

$$
\frac{\pi_2 : A \times B \to B \qquad \pi_1 : A \times B \to A}{\langle \pi_2, \pi_1 \rangle : A \times B \to B \times A}
\qquad
\frac{id : A \to A \qquad swap : B \times C \to C \times B}{id \times swap : A \times (B \times C) : A \times (C \times B)}
$$

$$
\frac{swap : D \times E \to E \times D \qquad id \times swap : A \times (B \times C) : A \times (C \times B)}{swap \cdot (id \times swap) : A \times (B \times C) \to (C \times B) \times A}
$$

$$(F0) : (g \times f) \cdot swap = swap \cdot (f \times g)$$

$$\begin{array}{ll}
& \beta \cdot (f \times (g \times h)) \\
\equiv & \quad \{ \text{ Def. } \beta \text{ } \} \\
& swap \cdot (id \times swap) \cdot (f \times (g \times h)) \\
\equiv & \quad \{ \text{ (F0) } \} \\
& (id \times swap) \cdot swap \cdot (f \times (g \times h)) \\
\equiv & \quad \{ \text{ (F0) } \} \\
& (swap \times id) \cdot ((g \times h) \times f) \cdot swap \\
\equiv & \quad \{ \text{ functor-}\times \text{ } \} \\
& ((swap \cdot (g \times h)) \times (id \cdot f)) \cdot swap \\
\equiv & \quad \{ \text{ (F0) } \} \\
& (((h \times g) \cdot swap) \times (f \cdot id)) \cdot swap \\
\equiv & \quad \{ \text{ functor-}\times \text{ } \} \\
& ((h \times g) \times f) \cdot (swap \times id) \cdot swap \\
\equiv & \quad \{ \text{ (F0) } \} \\
& ((h \times g) \times f) \cdot swap \ (id \times swap) \\
\equiv & \quad \{ \text{ Def. } \beta \text{ } \} \\
& ((h \times g) \times f) \cdot \beta
\end{array}$$

Exercício 4

$$\underline{k}\ x = \underline{k}\ (x) = \underline{k}\ (id\ x) = \underline{k} \cdot id = \underline{k} = k$$

Exercício 6

$$\frac{\begin{array}{c}\underline{False} : A \rightarrow Bool \\ id : A \rightarrow A\end{array}}{\langle \underline{False}, id \rangle : A \rightarrow Bool \times A} \qquad \frac{\begin{array}{c}f : A \rightarrow C \\ g : B \rightarrow C\end{array}}{[f, g] : A + B \rightarrow C}$$

$$\frac{\begin{array}{c}\langle \underline{False}, id \rangle : A \rightarrow Bool \times A \\ \langle \underline{True}, id \rangle : A \rightarrow Bool \times A\end{array}}{[\langle \underline{False}, id \rangle, \langle \underline{True}, id \rangle] : A + A \rightarrow Bool \times A}$$

Exercício 7

$$\begin{array}{ll}
& \alpha = [\langle \underline{False}, id \rangle, \langle \underline{True}, id \rangle] \\
\equiv & \quad \{ \text{ universal-}+ \text{ } \} \\
& \left\{ \begin{array}{l} \alpha \cdot i_1 = \langle \underline{False}, id \rangle \\ \alpha \cdot i_2 = \langle \underline{True}, id \rangle \end{array} \right. \\
\equiv & \quad \{ \text{ pointwise } \} \\
& \left\{ \begin{array}{l} (\alpha \cdot i_1) \cdot a = \langle \underline{False}, id \rangle\ a \\ (\alpha \cdot i_2) \cdot a = \langle \underline{True}, id \rangle\ a \end{array} \right.
\end{array}$$

$$\equiv \quad \{ \text{ Def. composição, Def. split } \}$$

$$\begin{cases} \alpha\ (i_1\ a) = (\underline{False}\ a, id\ a) \\ \alpha\ (i_2\ a) = (\underline{True}\ a, id\ a) \end{cases}$$

$$\equiv \quad \{ \text{ Def. const, Def. id } \}$$

$$\begin{cases} \alpha\ (i_1\ a) = (False, a) \\ \alpha\ (i_2\ a) = (True, a) \end{cases}$$

Exercício 8

$$\frac{\pi_1 : A \times B \to A \qquad id : C \to C}{\pi_1 \times id : (A \times B) \times C \to A \times C} \qquad \frac{\pi_2 : A \times B \to B \qquad \pi_1 : (A \times B) \times C \to A \times B}{\pi_2 \cdot \pi_1 : (A \times B) \times C \to B}$$

$$\frac{\pi_1 \times id : (A \times B) \times C \to A \times C \qquad \pi_2 \cdot \pi_1 : (A \times B) \times C \to B}{\langle \pi_1 \times id, \pi_2 \cdot \pi_1 \rangle : (A \times B) \times C \to (A \times C) \times B}$$

$$xr \cdot \langle \langle f, g \rangle, h \rangle = \langle \langle f, h \rangle, g \rangle$$

$$\equiv \quad \{ \text{ universal-}\times \}$$

$$\begin{cases} \pi_1 \cdot xr \cdot \langle \langle f, g \rangle, h \rangle = \langle f, h \rangle \\ \pi_2 \cdot xr \cdot \langle \langle f, g \rangle, h \rangle = g \end{cases}$$

$$\equiv \quad \{ \text{ Def. } xr\text{, cancelamento-}\times \}$$

$$\begin{cases} (\pi_1 \times id) \cdot \langle \langle f, g \rangle, h \rangle = \langle f, h \rangle \\ \pi_2 \cdot \pi_1 \cdot \langle \langle f, g \rangle, h \rangle = g \end{cases}$$

$$\equiv \quad \{ \text{ absorção-}\times\text{, cancelamento-}\times \}$$

$$\begin{cases} \langle \pi_1 \cdot \langle f, g \rangle, id \cdot h \rangle = \langle f, h \rangle \\ \pi_2 \cdot \langle f, g \rangle = g \end{cases}$$

$$\equiv \quad \{ \text{ cancelamento-}\times \}$$

$$\begin{cases} \langle f, h \rangle = \langle f, h \rangle \\ g = g \end{cases}$$

$\square$

Exercício 9

type $Key = String$
type $Aut = String$
type $Pag = Int$
type $Bib = [(Key, [Aut])]$
type $Aux = [(Pag, [Key])]$
type $Ind = [(Aut, [Pag])]$

$mkInd :: (Bib, Aux) \to Ind$
$mkInd = \bot$