Cálculo de Programas
Resolução - Ficha 11

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Exercício 1

$$
\begin{array}{l}
\quad mirror = (\!|\, in_2 \cdot \alpha \,|\!) \\
\equiv \{\text{Def. } in_2 \text{ , Def. } \alpha\} \\
\quad mirror = (\!|\, [Leaf, Fork] \cdot (id + swap) \,|\!) \\
\equiv \{\text{Absorção-+}\} \\
\quad mirror = (\!|\, [Leaf, Fork \cdot swap] \,|\!) \\
\equiv \{\text{Universal-cata}\} \\
\quad mirror \cdot in_{LTree} = [Leaf, Fork \cdot swap] \cdot (id + (mirror \times mirror)) \\
\equiv \{\text{Def. } in_{LTree}\text{, Fusão-+, Absorção-+, Eq-+}\} \\
\quad \begin{cases} mirror \cdot Leaf = Leaf \\ mirror \cdot Fork = Fork \cdot swap \cdot (mirror \times mirror) \end{cases} \\
\equiv \{\text{pointwise, Def. composição}\} \\
\quad \begin{cases} mirror\ (Leaf\ x) = Leaf\ x \\ mirror\ (Fork\ (l, r)) = Fork\ (swap\ (mirror\ l, mirror\ r)) \end{cases} \\
\equiv \{\text{Def. swap}\} \\
\quad \begin{cases} mirror\ (Leaf\ x) = Leaf\ x \\ mirror\ (Fork\ (l, r)) = Fork\ (mirror\ r, mirror\ l) \end{cases}
\end{array}
$$

Exercício 2

$$
\begin{array}{rl}
 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\Longleftarrow & \{\text{Fusão-cata}\} \\
 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\equiv & \{(\!|\, g \,|\!) \cdot in_2 = g \cdot G\ (\!|\, g \,|\!)\ \text{(F1)}\} \\
 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\Longleftarrow & \{\text{Leibniz}\} \\
 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\Longleftarrow & \{\text{Generalização de } (\!|\, g \,|\!) \text{ em f}\} \\
 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{array}
$$

Exercício 3

$$\begin{aligned} mirror &= (\!| g |\!) \\ mirror &= (\!| in_2 \cdot \alpha |\!) \end{aligned}$$

$$\begin{aligned} & (\!| g |\!) = (\!| in_2 \cdot \alpha |\!) \\ \equiv\ & \{\text{Def. } in_2\text{, Def. } \alpha\} \\ & (\!| g |\!) = (\!| [Leaf, Fork] \cdot (id + swap) |\!) \\ \equiv\ & \{\text{Absorção-+}\} \\ & (\!| g |\!) = (\!| [Leaf, Fork \cdot swap] |\!) \end{aligned}$$

Podemos então dizer que $g = [Leaf, Fork \cdot swap]$. Precisamos também de provar também que $id = (\!| g \cdot \alpha |\!)$.

$$\begin{aligned} & id = (\!| g \cdot \alpha |\!) \\ \equiv\ & \{\text{Def. g, Def. } \alpha\} \\ & id = (\!| [Leaf, Fork \cdot swap] \cdot (id + swap) |\!) \\ \equiv\ & \{\text{Absorção-+}\} \\ & id = (\!| [Leaf, Fork \cdot swap \cdot swap] |\!) \\ \equiv\ & \{swap \cdot swap = id\} \\ & id = (\!| [Leaf, Fork] |\!) \\ \equiv\ & \{[Leaf, Fork] = in_{LTree}\} \\ & id = (\!| in_{LTree} |\!) \\ \equiv\ & \{\text{Reflexão-cata}\} \\ & True \end{aligned}$$

Podemos então provar que $mirror$ é o seu próprio isomorfismo:

$$\begin{aligned} & (\!| g |\!) \cdot (\!| in_2 \cdot \alpha |\!) = (\!| g \cdot \alpha |\!) \\ \Longleftarrow\ & \{(\text{F3})\} \\ & G\ f \cdot \alpha = \alpha \cdot F\ f \\ \equiv\ & \{\text{G f = F f = id + f} \times \text{f}\} \\ & (id + f \times f) \cdot (id + swap) = (id + swap) \cdot (id + f \times f) \\ \equiv\ & \{\text{Functor-+}\} \\ & (id + (f \times f) \cdot swap) = (id + swap \cdot (f \times f)) \end{aligned}$$

Através da propriedade grátis da função $swap$ (i.e: $swap \cdot (f \times g) = (g \times f) \cdot swap$), podemos garantir a veracidade desta propriedade.

Exercício 4

$$
\begin{aligned}
& (\!|\, g \,|\!) \cdot T\ f = (\!|\, g \cdot B\ (f, id) \,|\!) \\
\equiv\ & \{\text{Def-map-cata}\} \\
& (\!|\, g \,|\!) \cdot (\!|\, in \cdot B\ (f, id) \,|\!) = (\!|\, g \cdot B\ (f, id) \,|\!) \\
\Longleftarrow\ & \{(\text{F3})\} \\
& G\ f \cdot B\ (f, id) = B\ (f, id) \cdot F\ f \\
\equiv\ & \{?????\} \\
& B\ (id, f) \cdot B\ (f, id) = B\ (f, id) \cdot B\ (id, f) \\
\equiv\ & \{\text{Functor-id-F (para Bi Functores)}\} \\
& B\ (f, f) = B\ (f, f)
\end{aligned}
$$

Exercício 5

$$
\begin{aligned}
& while\ p\ f\ g = tailr\ ((g + f) \cdot (\neg \cdot p)?) \\
\equiv\ & \{\text{Def. tailr}\} \\
& while\ p\ f\ g = [\![join, ((g + f) \cdot (\neg \cdot p)?)]\!] \\
\equiv\ & \{[\![f, g]\!] = f \cdot F [\![f, g]\!] \cdot g\} \\
& while\ p\ f\ g = join \cdot F\ (while\ p\ f\ g) \cdot ((g + f) \cdot (\neg \cdot p)?) \\
\equiv\ & \{\text{Def. join, Def. Functor}\} \\
& while\ p\ f\ g = [id, id] \cdot (id + (while\ p\ f\ g)) \cdot ((g + f) \cdot (\neg \cdot p)?) \\
\equiv\ & \{\text{Absorção-+ } (2\times)\} \\
& while\ p\ f\ g = [g, (while\ p\ f\ g) \cdot f] \cdot (\neg \cdot p)? \\
\equiv\ & \{\text{Def. Condicional de McCarthy}\} \\
& while\ p\ f\ g = (\neg \cdot p) \rightarrow g, (while\ p\ f\ g) \cdot f \\
\equiv\ & \{\text{pointwise}\} \\
& while\ p\ f\ g\ x = \textbf{if}\ not\ (p\ x)\ \textbf{then}\ g\ x\ \textbf{else}\ while\ p\ f\ g\ (f\ x)
\end{aligned}
$$

Exercício 6

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\begin{aligned}
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\equiv\ & \{\text{Def. tailr, Def. hylomorfismo}\} \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\Longleftarrow\ & \{\text{Leibniz}\} \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\Longleftarrow\ & \{\text{Fusão-ana, Def. Functor}\} \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

Exercício 7

$$
\begin{aligned}
& f \bullet [g, h] = [f \bullet g, f \bullet h] \\
\equiv\ & \{(\text{F9})\ (3\times)\} \\
& \mu \cdot T\ f \cdot [g, h] = [\mu \cdot T\ f \cdot g, \mu \cdot T\ f \cdot h] \\
\equiv\ & \{\text{Fusão-+}\} \\
& [\mu \cdot T\ f \cdot g, \mu \cdot T\ f \cdot h] = [\mu \cdot T\ f \cdot g, \mu \cdot T\ f \cdot h]
\end{aligned}
$$