Cálculo de Programas
Resolução - Ficha 05

Eduardo Freitas Fernandes

2025

Exercício 1

$$
\begin{aligned}
& ap \cdot (\overline{f} \times id) = f \\
\equiv\ & \{\text{pointwise}\} \\
& ap \cdot (\overline{f} \times id)\ (a,b) = f\ (a,b) \\
\equiv\ & \{\text{Def. comp, Def-}\times\} \\
& \overline{f}\ a\ b = f\ (a,b) \\
\equiv\ & \{\text{Curry}\} \\
& f\ (a,b) = f\ (a,b)
\end{aligned}
$$

Exercício 2

$$
\begin{aligned}
& ap \cdot (\overline{f} \times id) = f \\
\equiv\ & \{\text{pointwise}\} \\
& ap \cdot (\overline{f} \times id)\ (a,b) = f\ (a,b) \\
\equiv\ & \{\text{Def. comp, Def-}\times\} \\
& \overline{f}\ a\ b = f\ (a,b) \\
\equiv\ & \{\text{f := uncurry g}\} \\
& \overline{\widehat{g}}\ a\ b = \widehat{g}\ (a,b) \\
\equiv\ & \{\text{Curry}\} \\
& \widehat{g}\ (a,b) = \widehat{g}\ (a,b) \\
\equiv\ & \{\text{Uncurry}\} \\
& g\ a\ b = g\ a\ b
\end{aligned}
$$

Exercício 3

$$
\begin{aligned}
& \overline{f \cdot (g \times h)} = \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g \\
\equiv\ & \{\text{Universal-exp}\} \\
& f \cdot (g \times h) = ap \cdot ((\overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g) \times id) \\
\equiv\ & \{\text{Natural-id, Functor-}\times\} \\
& f \cdot (g \times h) = ap \cdot (\overline{ap \cdot (id \times h)} \times id) \cdot (\overline{f} \cdot g \times id) \\
\equiv\ & \{\text{Cancelamento-exp}\} \\
& f \cdot (g \times h) = ap \cdot (id \times h) \cdot ((\overline{f} \cdot g) \times id) \\
\equiv\ & \{\text{Functor-}\times\} \\
& f \cdot (g \times h) = ap \cdot ((\overline{f} \cdot g) \times h) \\
\equiv\ & \{\text{Natural-id, Functor-}\times\} \\
& f \cdot (g \times h) = ap \cdot (\overline{f} \times id) \cdot (g \times h) \\
\equiv\ & \{\text{Cancelamento-exp}\} \\
& f \cdot (g \times h) = f \cdot (g \times h)
\end{aligned}
$$

Exercício 4

$$
\begin{aligned}
& flip\ (flip\ f) = f \\
\equiv\ & \{\text{Def. flip}\} \\
& \overline{\widehat{flip\ f} \cdot swap} = f \\
\equiv\ & \{\text{pointwise}\} \\
& \overline{\widehat{flip\ f} \cdot swap}\ a\ b = f\ a\ b \\
\equiv\ & \{\text{Curry}\} \\
& (\widehat{flip\ f} \cdot swap)\ (a, b) = f\ a\ b \\
\equiv\ & \{\text{Def. swap}\} \\
& \widehat{flip\ f}\ (b, a) = f\ a\ b \\
\equiv\ & \{\text{Uncurry}\} \\
& flip\ f\ b\ a = f\ a\ b \\
\equiv\ & \{...\} \\
& ...
\end{aligned}
$$

$$
\begin{aligned}
& flip\ f\ x\ y = f\ y\ x \\
\equiv\ & \{\text{Def. flip}\} \\
& \overline{\widehat{f} \cdot swap}\ f\ x\ y = f\ y\ x \\
\equiv\ & \{\text{Curry}\} \\
& (\widehat{f} \cdot swap)\ (x, y) = f\ y\ x \\
\equiv\ & \{\text{Def. comp, Def. swap}\} \\
& \widehat{f}\ (y, x) = f\ y\ x \\
\equiv\ & \{\text{Uncurry}\} \\
& f\ y\ x = f\ y\ x
\end{aligned}
$$

Exercício 5

$$
\begin{aligned}
& junc \cdot unjunc = id \\
\equiv\ & \{\text{pointwise}\} \\
& (junc \cdot unjunc)\ k = id\ k \\
\equiv\ & \{\text{Natural-id, Def. comp, Def. unjunc}\} \\
& junc\ (k \cdot i_1, k \cdot i_2) = k \\
\equiv\ & \{\text{Def. junc}\} \\
& [k \cdot i_1, k \cdot i_2] = k \\
\equiv\ & \{\text{Fusão-+}\} \\
& k \cdot [i_1, i_2] = k \\
\equiv\ & \{\text{Reflexão-+, Natural-id}\} \\
& k = k
\end{aligned}
$$

$$
\begin{aligned}
& unjunc \cdot junc = id \\
\equiv\ & \{\text{pointwise}\} \\
& (unjunc \cdot junc)\ (f, g) = id\ (f, g) \\
\equiv\ & \{\text{Natural-id, Def. comp, Def. junc}\} \\
& unjunc\ [f, g] = (f, g) \\
\equiv\ & \{\text{Def. unjunc}\} \\
& ([f, g] \cdot i_1, [f, g] \cdot i_2) = (f, g) \\
\equiv\ & \{\text{Cancelamento-+}\} \\
& (f, g) = (f, g)
\end{aligned}
$$

Exercício 6

$$
\begin{aligned}
& (for\ b\ i) \cdot in = [g_1, g_2] \cdot (id + for\ b\ i) \\
\equiv\ & \{\text{Def. in, Fusão-+, Absorção-+}\} \\
& [for\ b\ i \cdot zero, for\ b\ i \cdot succ] = [g_1 \cdot id, g_2 \cdot for\ b\ i] \\
\equiv\ & \{\text{Natural-id, Eq-+}\} \\
& \begin{cases} for\ b\ i \cdot zero = g_1 \\ for\ b\ i \cdot succ = g_2 \cdot for\ b\ i \end{cases} \\
\equiv\ & \{\text{pointwise}\} \\
& \begin{cases} (for\ b\ i \cdot zero)x = g_1\ x \\ (for\ b\ i \cdot succ)n = (g_2 \cdot for\ b\ i)n \end{cases} \\
\equiv\ & \{\text{Def. comp, Def. zero, Def. succ}\} \\
& \begin{cases} for\ b\ i\ 0 = g_1 \\ for\ b\ i\ (n+1) = g_2\ (for\ b\ i\ n) \end{cases} \\
\equiv\ & \{\text{(F8)}\} \\
& \begin{cases} g_1 = i \\ g_2\ (for\ b\ i\ n) = b\ (for\ b\ i\ n) \end{cases} \\
\equiv\ & \{\text{f x = g x} \implies \text{f = g}\} \\
& \begin{cases} g_1 = i \\ g_2 = b \end{cases}
\end{aligned}
$$

Exercício 7

```
ghci> :{
ghci| for b i 0 = i
ghci| for b i n = b (for b i (n-1))
ghci| :}
ghci> :t +d for
for :: (t2 -> t2) -> t2 -> Integer -> t2
ghci> f = p2 . aux where aux = for (split (succ . p1) mul) (1,1)
ghci> :t f
f :: (Eq a, Num a) => a -> Integer
ghci> f 3
6
ghci> f 5
120
ghci> f 7
5040
ghci> -- f é a função que calcula o factorial
```

Nota: nas novas versões do Haskell, a syntax `for b i (n+1) = b (for b i n)` não é permitida.

Exercício 8

Exercício 9

```c
int k(int n, int a) {
    int r = 0;
    int j;
    for (j = 1; j < n + 1; j++) {
        r = a + r;
    }
    return r;
};
```

Exercício 10

```haskell
func :: Eq a => b -> [(a, b)] -> (a -> b)
func b = (maybe b id .) . flip lookup

a = [(140999000, "Manuel"), (200100300, "Mary"), (000111222, "Teresa")]

b = [(140999000, "PT"), (200100300, "UK")]

c = [(140999000, "Braga"), (200100300, "Porto"), (151999000, "Lisbon")]
```