Cálculo de Programas

Resolução - Ficha 01

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Exercício 1

$$
\begin{array}{ll}
& \pi_1 \cdot (f \times g)\ (x,y) \\
\equiv & \quad \{\ \text{Def. comp}\ \} \\
& \pi_1\ ((f \times g)\ (x,y)) \\
\equiv & \quad \{\ \text{(F1)}\ \} \\
& \pi_1\ (f\ x, g\ y) \\
\equiv & \quad \{\ \text{(F2)}\ \} \\
& f\ x \\
\equiv & \quad \{\ \text{(F2)}\ \} \\
& f\ (\pi_1\ (x,y)) \\
\equiv & \quad \{\ \text{Def. comp}\ \} \\
& f \cdot \pi_1
\end{array}
\qquad
\begin{array}{ll}
& \pi_2 \cdot (f \times g)\ (x,y) \\
\equiv & \quad \{\ \text{Def. comp}\ \} \\
& \pi_2\ ((f \times g)\ (x,y)) \\
\equiv & \quad \{\ \text{(F1)}\ \} \\
& \pi_2\ (f\ x, g\ y) \\
\equiv & \quad \{\ \text{(F2)}\ \} \\
& g\ y \\
\equiv & \quad \{\ \text{(F2)}\ \} \\
& g\ (\pi_2\ (x,y)) \\
\equiv & \quad \{\ \text{Def. comp}\ \} \\
& g \cdot \pi_2
\end{array}
\qquad
\begin{array}{ll}
& (f \times g)\ (x,y) \\
\equiv & \quad \{\ \text{(F1)}\ \} \\
& (f\ x, g\ y) \\
\equiv & \quad \{\ \text{(F2)}\ \} \\
& (f\ (\pi_1\ (x,y)), g\ (\pi_2\ (x,y))) \\
\equiv & \quad \{\ \text{Def. comp}\ \} \\
& (f \cdot \pi_1, g \cdot \pi_2) \\
\equiv & \quad \{\ \text{Def. split}\ \} \\
& \langle f \cdot \pi_1, g \cdot \pi_2 \rangle
\end{array}
$$

Exercício 2

$$
\begin{array}{ll}
& xor \cdot (and \times id)\ ((a,b),c) \\
\equiv & \quad \{\ \text{Def. comp}\ \} \\
& xor\ ((and \times id)\ ((a,b),c)) \\
\equiv & \quad \{\ \text{(F1)}\ \} \\
& xor\ (and\ (a,b), id\ c) \\
\equiv & \quad \{\ \text{Def. } and\text{, Def. } id\ \} \\
& xor\ (a \wedge b, c) \\
\equiv & \quad \{\ \text{Def. } xor\ \} \\
& (a \wedge b) \oplus c
\end{array}
$$

Exercício 4

$$
\begin{array}{ll}
& id = \langle f, g \rangle \\
\equiv & \quad \{\ \text{universal-}\times\ \} \\
& \begin{cases} \pi_1 \cdot id = f \\ \pi_2 \cdot id = g \end{cases}
\end{array}
$$

$$
\begin{array}{ll}
\equiv & \{ \text{ natural-}id \ \} \\
& \begin{cases} \pi_1 = f \\ \pi_2 = g \end{cases}
\end{array}
$$

Concluimos então que $id = \langle \pi_1, \pi_2 \rangle$. Seja $k = id$, ao aplicar a propriedade universal-$\times$ obtemos a propriedade reflexão-$\times$.

Exercício 5

$$
\begin{array}{ll}
& \langle h, k \rangle \cdot f = \langle h \cdot f, k \cdot f \rangle \\
\equiv & \{ \text{ (F7) } \} \\
& \begin{cases} \pi_1 \cdot \langle h, k \rangle \cdot f = h \cdot f \\ \pi_2 \cdot \langle h, k \rangle \cdot f = k \cdot f \end{cases} \\
\equiv & \{ \text{ cancelamento-}\times \ \} \\
& \begin{cases} h \cdot f = h \cdot f \\ k \cdot f = k \cdot f \end{cases}
\end{array}
$$

□

Exercício 6

$$
\begin{array}{ll}
& dup \cdot f = \langle f, f \rangle \\
\equiv & \{ \text{ pointwise, Def. comp } \} \\
& dup \ (f \ x) = \langle f, f \rangle \ x \\
\equiv & \{ \text{ Def. dup, Def. split } \} \\
& (f \ x, f \ x) = (f \ x, f \ x)
\end{array}
$$

□

Exercício 7

$$
\begin{array}{ll}
& \underline{b, a} = \langle \underline{b}, \underline{a} \rangle \\
\equiv & \{ \text{ universal-}\times \ \} \\
& \begin{cases} \pi_1 \cdot \underline{(b, a)} = \underline{b} \\ \pi_2 \cdot \underline{(b, a)} = \underline{a} \end{cases} \\
\equiv & \{ \text{ absorção-const } \} \\
& \begin{cases} \underline{\pi_1 \ (b, a)} = \underline{b} \\ \underline{\pi_2 \ (b, a)} = \underline{a} \end{cases} \\
\equiv & \{ \text{ cancelamento-}\times \ \} \\
& \begin{cases} \underline{b} = \underline{b} \\ \underline{a} = \underline{a} \end{cases}
\end{array}
$$

□

Exercício 8

$$
\begin{array}{ll}
& (g \times f) \cdot swap = swap \cdot (f \times g) \\
\equiv & \{ \text{ Def-}\times, \text{ Def. } swap \ \}
\end{array}
$$

$$
\begin{array}{ll}
 & \langle g \cdot \pi_1, f \cdot \pi_2 \rangle \cdot swap = \langle \pi_2, \pi_1 \rangle \cdot (f \times g) \\
\equiv & \quad \{ \text{ fusão-}\times \text{ (twice) } \} \\
 & \langle g \cdot \pi_1 \cdot swap, f \cdot \pi_2 \cdot swap \rangle = \langle \pi_2 \cdot (f \times g), \pi_1 \cdot (f \times g) \rangle \\
\equiv & \quad \{ \text{ Def. } swap, \text{ Def-}\times \text{ } \} \\
 & \langle g \cdot \pi_2 \cdot \langle \pi_2, \pi_1 \rangle, f \cdot \pi_2 \cdot \langle \pi_2, \pi_1 \rangle \rangle = \langle \pi_2 \cdot \langle f \cdot \pi_1, g \cdot \pi_2 \rangle, \pi_1 \cdot \langle f \cdot \pi_1, g \cdot \pi_2 \rangle \rangle \\
\equiv & \quad \{ \text{ cancelamento-}\times \text{ (twice) } \} \\
 & \langle g \cdot \pi_2, f \cdot \pi_1 \rangle = \langle g \cdot \pi_2, f \cdot \pi_1 \rangle \\
\Box &
\end{array}
$$

Exercício 9

$$acronym = \text{map } \ head \cdot words$$

$$String \xrightarrow{words} String^* \xrightarrow{\text{map } head} String$$

$$short = \widehat{(++)} \cdot (id \times (\text{'} \ \ \text{'} :)) \cdot \langle head, last \rangle \cdot words$$