

Cálculo de Programas

Resolução - Ficha 06

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Exercício 2

Exercício 1

$$\begin{aligned} & ap \cdot (\bar{f} \times id) = f \\ \equiv & \{\text{pointwise}\} \\ & ap \cdot (\bar{f} \times id) (a, b) = f (a, b) \\ \equiv & \{\text{Def. comp, Def-}\times\} \\ & \bar{f} a b = f (a, b) \\ \equiv & \{\text{Curry}\} \\ & f (a, b) = f (a, b) \end{aligned}$$

$$\begin{aligned} & ap \cdot (\bar{f} \times id) = f \\ \equiv & \{\text{pointwise}\} \\ & ap \cdot (\bar{f} \times id) (a, b) = f (a, b) \\ \equiv & \{\text{Def. comp, Def-}\times\} \\ & \bar{f} a b = f (a, b) \\ \equiv & \{f := \text{uncurry } g\} \\ & \widehat{g} a b = \widehat{g} (a, b) \\ \equiv & \{\text{Curry}\} \\ & \widehat{g} (a, b) = \widehat{g} (a, b) \\ \equiv & \{\text{Uncurry}\} \\ & g a b = g a b \end{aligned}$$

Exercício 3

$$\begin{aligned} & \overline{f \cdot (g \times h)} = \overline{ap \cdot (id \times h) \cdot \bar{f} \cdot g} \\ \equiv & \{\text{Universal-exp}\} \\ & f \cdot (g \times h) = ap \cdot (\overline{ap \cdot (id \times h) \cdot \bar{f} \cdot g} \times id) \\ \equiv & \{\text{Natural-id, Functor-}\times\} \\ & f \cdot (g \times h) = ap \cdot (\overline{ap \cdot (id \times h) \times id} \cdot (\bar{f} \cdot g \times id)) \\ \equiv & \{\text{Cancelamento-exp}\} \\ & f \cdot (g \times h) = ap \cdot (id \times h) \cdot ((\bar{f} \cdot g) \times id) \\ \equiv & \{\text{Functor-}\times\} \\ & f \cdot (g \times h) = ap \cdot ((\bar{f} \cdot g) \times h) \\ \equiv & \{\text{Natural-id, Functor-}\times\} \\ & f \cdot (g \times h) = ap \cdot (\bar{f} \times id) \cdot (g \times h) \\ \equiv & \{\text{Cancelamento-exp}\} \\ & f \cdot (g \times h) = f \cdot (g \times h) \end{aligned}$$

Exercício 4

$$\begin{aligned} & flip (flip f) = f \\ \equiv & \{\text{Def. flip}\} \\ & \overline{flip f \cdot swap} = f \\ \equiv & \{\text{pointwise}\} \\ & \overline{flip f \cdot swap} a b = f a b \\ \equiv & \{\text{Curry}\} \\ & (\widehat{flip f \cdot swap}) (a, b) = f a b \\ \equiv & \{\text{Def. swap}\} \\ & \widehat{flip f} (b, a) = f a b \\ \equiv & \{\text{Uncurry}\} \\ & flip f b a = f a b \\ \equiv & \{\dots\} \\ & \dots \end{aligned}$$

$$\begin{aligned} & flip f x y = f y x \\ \equiv & \{\text{Def. flip}\} \\ & \widehat{f \cdot swap} f x y = f y x \\ \equiv & \{\text{Curry}\} \\ & (\widehat{f \cdot swap}) (x, y) = f y x \\ \equiv & \{\text{Def. comp, Def. swap}\} \\ & \widehat{f} (y, x) = f y x \\ \equiv & \{\text{Uncurry}\} \\ & f y x = f y x \end{aligned}$$

Exercício 5

$$\begin{aligned} & junc \cdot unjunc = id \\ \equiv & \{\text{pointwise}\} \\ & (junc \cdot unjunc) k = id k \\ \equiv & \{\text{Natural-id, Def. comp, Def. unjunc}\} \\ & junc (k \cdot i_1, k \cdot i_2) = k \\ \equiv & \{\text{Def. junc}\} \\ & [k \cdot i_1, k \cdot i_2] = k \\ \equiv & \{\text{Fusão-+}\} \\ & k \cdot [i_1, i_2] = k \\ \equiv & \{\text{Reflexão-+, Natural-id}\} \\ & k = k \end{aligned}$$

$$\begin{aligned} & unjunc \cdot junc = id \\ \equiv & \{\text{pointwise}\} \\ & (unjunc \cdot junc) (f, g) = id (f, g) \\ \equiv & \{\text{Natural-id, Def. comp, Def. junc}\} \\ & unjunc [f, g] = (f, g) \\ \equiv & \{\text{Def. unjunc}\} \\ & ([f, g] \cdot i_1, [f, g] \cdot i_2) = (f, g) \\ \equiv & \{\text{Cancelamento-+}\} \\ & (f, g) = (f, g) \end{aligned}$$

Exercício 6

$$\begin{aligned} & (for\ b\ i) \cdot in = [g_1, g_2] \cdot (id + for\ b\ i) \\ \equiv & \{ \text{Def. in, Fusão-+}, \text{Absorção-+} \} \\ & [for\ b\ i \cdot zero, for\ b\ i \cdot succ] = [g_1 \cdot id, g_2 \cdot for\ b\ i] \\ \equiv & \{ \text{Natural-id, Eq-+} \} \\ & \begin{cases} for\ b\ i \cdot zero = g_1 \\ for\ b\ i \cdot succ = g_2 \cdot for\ b\ i \end{cases} \\ \equiv & \{ \text{pointwise} \} \\ & \begin{cases} (for\ b\ i \cdot zero)x = g_1\ x \\ (for\ b\ i \cdot succ)n = (g_2 \cdot for\ b\ i)n \end{cases} \\ \equiv & \{ \text{Def. comp, Def. zero, Def. succ} \} \\ & \begin{cases} for\ b\ i\ 0 = g_1 \\ for\ b\ i\ (n+1) = g_2\ (for\ b\ i\ n) \end{cases} \\ \equiv & \{ (F8) \} \\ & \begin{cases} g_1 = i \\ g_2\ (for\ b\ i\ n) = b\ (for\ b\ i\ n) \end{cases} \\ \equiv & \{ f\ x = g\ x \implies f = g \} \\ & \begin{cases} g_1 = i \\ g_2 = b \end{cases} \end{aligned}$$

Exercício 7

```
ghci> :{
ghci| for b i 0 = i
ghci| for b i n = b (for b i (n-1))
ghci| :}
ghci> :t +d for
for :: (t2 -> t2) -> t2 -> Integer -> t2
ghci> f = p2 . aux where aux = for (split (succ . p1) mul) (1,1)
ghci> :t f
f :: (Eq a, Num a) => a -> Integer
ghci> f 3
6
ghci> f 5
120
ghci> f 7
5040
ghci> -- f é a função que calcula o factorial
```

Nota: nas novas versões do Haskell, a syntax `for b i (n+1) = b (for b i n)` não é permitida.

Exercício 8

$$\begin{cases} \text{for } b \ i \ 0 = i \\ \text{for } b \ i \ (n + 1) = b(\text{for } b \ i \ n) \end{cases} \equiv \begin{cases} (\text{for } b \ i) \cdot \underline{0} = \underline{i} \\ (\text{for } b \ i) \cdot \text{succ} = b \cdot \text{for } b \ i \end{cases}$$

$$\begin{cases} a + 0 = a \\ a + (n + 1) = 1 + (a + n) \end{cases} \equiv \begin{cases} (a+) \cdot \underline{0} = \underline{a} \\ (a+) \cdot \text{succ} = \text{succ} \cdot (a+) \end{cases}$$

$$\text{for } b \ i = \llbracket [\underline{i}, b] \rrbracket$$

$$(a+) = \llbracket [\underline{a}, \text{succ}] \rrbracket$$

$$(a+) = \text{for } \text{succ } a$$

Exercício 9

```
int k(int n, int a) {  
    int r = 0;  
    int j;  
    for (j = 1; j < n + 1; j++) {  
        r = a + r;  
    }  
    return r;  
};
```

Exercício 10

```
func :: Eq a => b -> [(a, b)] -> (a -> b)  
func b = (maybe b id .) . flip lookup  
  
a = [(140999000, "Manuel"), (200100300, "Mary"), (000111222, "Teresa")]  
  
b = [(140999000, "PT"), (200100300, "UK")]  
  
c = [(140999000, "Braga"), (200100300, "Porto"), (151999000, "Lisbon")]
```