Cálculo de Programas
Resolução - Ficha 05

Eduardo Freitas Fernandes

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Exercício 1

$$id : A \to B$$
$$\pi_1 : B \times C \to B$$
$$i_2 : D \to E + D$$
$$\pi_2 : G \times H \to H$$

Por $i_2 \cdot \pi_2$ inferimos $D = H$:

$$\cfrac{\cfrac{\begin{array}{c} i_2 : D \to E + D \\ \pi_2 : G \times H \to H \end{array}}{\begin{array}{c} i_2 : D \to E + D \\ \pi_2 : G \times D \to D \end{array}}}{i_2 \cdot \pi_2 : G \times D \to E + D}$$

$$\frac{\begin{array}{c} id : A \to B \\ \pi_1 : B \times C \to B \end{array}}{id + \pi_1 : A + B \times C \to A + B}$$

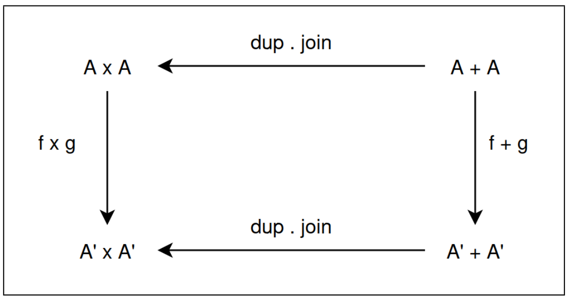
Por $(id + \pi_1) \cdot i_2 \cdot \pi_2$ inferimos $A + B \times C = E + D$:

$$\implies \begin{cases} A = E \\ B \times C = D \end{cases}$$

$$\frac{\begin{array}{c} id + \pi_1 : A + B \times C \to A + B \\ i_2 \cdot \pi_2 : G \times (B \times C) \to A + B \times C \end{array}}{\alpha : G \times (B \times C) \to A + B}$$

Exercício 2

$$\frac{\begin{array}{c} join : A + A \to A \\ dup : A \to A \times A \end{array}}{\alpha : A + A \to A \times A}$$



Propriedade grátis:

$$(f \times g) \cdot \alpha = \alpha \cdot (f + g)$$

Exercício 3

$$
\begin{aligned}
& \nabla \cdot (f + f) = f \cdot \nabla \\
\equiv\ & \{\text{Def-+, Fusão-+}\} \\
& [\nabla \cdot i_1 \cdot f, \nabla \cdot i_2 \cdot f] = f \cdot \nabla \\
\equiv\ & \{\text{Universal-+}\} \\
& \begin{cases} \nabla \cdot i_1 \cdot f = f \cdot \nabla \cdot i_1 \\ \nabla \cdot i_2 \cdot f = f \cdot \nabla \cdot i_2 \end{cases} \\
\equiv\ & \{\nabla \cdot i_1 = id, \nabla \cdot i_2 = id\} \\
& \begin{cases} id \cdot f = f \cdot id \\ id \cdot f = f \cdot id \end{cases} \\
\equiv\ & \{\text{Natural Id}\} \\
& \begin{cases} f = f \\ f = f \end{cases}
\end{aligned}
$$

Exercício 4

$$
\begin{aligned}
& f + h : A + B \to A' + B' \\
& f + g \times h : A + C \times B \to A' + C' \times B'
\end{aligned}
$$

$$
\begin{tikzcd}
\text{A + B} \arrow[d, "\text{f + h}"'] & & \text{A + C x B} \arrow[ll, "\text{alpha}"'] \arrow[d, "\text{f + g x h}"] \\
\text{A' + B'} & & \text{A' + C' x B'} \arrow[ll, "\text{alpha}"]
\end{tikzcd}
$$

$$\alpha = id + \pi_2$$

Exercício 5

$$
\begin{tikzcd}
\text{(A x B) + (A x C)} \arrow[d, "\text{(f x g) + (f x h)}"'] & & \text{A x (B + C)} \arrow[ll, "\text{distr}"'] \arrow[d, "\text{f x (g + h)}"] \\
\text{(A' x B') + (A' x C')} & & \text{A' x (B' + C')} \arrow[ll, "\text{distr}"]
\end{tikzcd}
$$

Propriedade grátis:

$$((f \times g) + (f \times h)) \cdot distr = distr \cdot (f \times (g + h))$$

$$
\begin{aligned}
& h \cdot distr \cdot (g \times (id + f)) = k \\
\equiv\ & \{\text{propriedade grátis}\} \\
& h \cdot ((g \times id) + (g \times f)) \cdot distr = k \\
\equiv\ & \{\text{(F6)}\} \\
& h \cdot ((g \times id) + (g \times f)) = k \cdot distr^{\circ} \\
\equiv\ & \{distr^{\circ} = undistr\} \\
& h \cdot ((g \times id) + (g \times f)) = k \cdot undistr
\end{aligned}
$$

Exercício 6

$$
\begin{aligned}
& (p \cdot h) \rightarrow (f \cdot h), (g \cdot h) \\
\equiv\ & \{\text{Def condicional de McCarthy}\} \\
& [f \cdot h, g \cdot h] \cdot (p \cdot h)? \\
\equiv\ & \{\text{Absorção-+}\} \\
& [f, g] \cdot (h + h) \cdot (p \cdot h)? \\
\equiv\ & \{\text{Natural-guarda}\} \\
& [f, g] \cdot p? \cdot h \\
\equiv\ & \{\text{Def condicional de McCarthy}\} \\
& (p \rightarrow f, g) \cdot h
\end{aligned}
$$

Exercício 7

$$
\begin{aligned}
& choose \cdot parallel p f g = p \rightarrow f, g \\
\equiv\ & \{\text{def. parallel, def. choose}\} \\
& (\pi_2 \rightarrow \pi_1 \cdot \pi_1, \pi_2 \cdot \pi_1) \cdot \langle \langle f, g \rangle, p \rangle = p \rightarrow f, g \\
\equiv\ & \{\text{Def condicional de McCarthy}\} \\
& [\pi_1 \cdot \pi_1, \pi_2 \cdot \pi_1] \cdot \pi_2? \cdot \langle \langle f, g \rangle, p \rangle = p \rightarrow f, g \\
\equiv\ & \{\text{Natural-guarda }\} \\
& [\pi_1 \cdot \pi_1, \pi_2 \cdot \pi_1] \cdot (\langle \langle f, g \rangle, p \rangle + \langle \langle f, g \rangle, p \rangle) \cdot (\pi_2 \cdot \langle \langle f, g \rangle, p \rangle)? = p \rightarrow f, g \\
\equiv\ & \{\text{Cancelamento-}\times\text{, Absorção-+}\} \\
& [\pi_1 \cdot \pi_1 \cdot \langle \langle f, g \rangle, p \rangle, \pi_2 \cdot \pi_1 \cdot \langle \langle f, g \rangle, p \rangle] \cdot p? = p \rightarrow f, g \\
\equiv\ & \{\text{Cancelamento-}\times\} \\
& [f, g] \cdot p? = p \rightarrow f, g \\
\equiv\ & \{\text{Def condicional de McCarthy}\} \\
& p \rightarrow f, g = p \rightarrow f, g
\end{aligned}
$$

Exercício 8

Primeira propriedade:

$$
\begin{aligned}
& \langle (p \to f, h), (p \to g, i) \rangle \\
\equiv\ & \{\text{Def condicional de McCarthy (2*)}\} \\
& \langle [f, h] \cdot p?, [g, i] \cdot p? \rangle \\
\equiv\ & \{\text{Fusão-}\times\} \\
& \langle [f, h], [g, i] \rangle \cdot p? \\
\equiv\ & \{\text{Lei da troca}\} \\
& [\langle f, g \rangle, \langle h, i \rangle] \cdot p? \\
\equiv\ & \{\text{Def condicional de McCarthy}\} \\
& p \to \langle f, g \rangle, \langle h, i \rangle
\end{aligned}
$$

Segunda propriedade:

$$
\begin{aligned}
& p \to \langle f, g \rangle, \langle f, h \rangle \\
\equiv\ & \{\text{(F11)}\} \\
& \langle (p \to f, f), (p \to g, h) \rangle \\
\equiv\ & \{\text{(F9)}\} \\
& \langle f, (p \to g, h) \rangle
\end{aligned}
$$

Terceira propriedade:

$$
\begin{aligned}
& p \to (p \to a, b), (p \to c, d) \\
\equiv\ & \{\text{Def condicional de McCarthy (3*)}\} \\
& [[a, b] \cdot p?, [c, d] \cdot p?] \cdot p? \\
\equiv\ & \{\text{Absorção-+}\} \\
& [[a, b], [c, d]] \cdot (p? + p?) \cdot p? \\
\equiv\ & \{\text{(F10)}\} \\
& [[a, b], [c, d]] \cdot (i1 + i2) \cdot p? \\
\equiv\ & \{\text{Absorção-+, Cancelamento-+}\} \\
& [a, d] \cdot p? \\
\equiv\ & \{\text{Def condicional de McCarthy}\} \\
& p \to a, d
\end{aligned}
$$

Exercício 9

```
f :: (Eq a) => [a] -> Either a (a, [a])
f = undefined
```