Cálculo de Programas

Resolução - Ficha 05

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Exercício 1

Por $i_2 \cdot \pi_2$ inferimos $D = H$:

$$\begin{array}{c} id : A \to A \\ \pi_1 : B \times C \to B \\ i_2 : D \to E + D \\ \pi_2 : G \times H \to H \end{array}$$

$$\frac{\begin{array}{c} id : A \to A \\ \pi_1 : B \times C \to B \end{array}}{id + \pi_1 : A + B \times C \to A + B}$$

$$\frac{\begin{array}{c} id + \pi_1 : A + B \times C \to A + B \\ i_2 \cdot \pi_2 : G \times (B \times C) \to A + B \times C \end{array}}{\alpha : G \times (B \times C) \to A + B}$$

$$\frac{\dfrac{\begin{array}{c} i_2 : D \to E + D \\ \pi_2 : G \times H \to H \end{array}}{\begin{array}{c} i_2 : D \to E + D \\ \pi_2 : G \times D \to D \end{array}}}{i_2 \cdot \pi_2 : G \times D \to E + D}$$

Por $(id + \pi_1) \cdot i_2 \cdot \pi_2$ inferimos $A + B \times C = E + D$:

$$\Longrightarrow \begin{cases} A = E \\ B \times C = D \end{cases}$$

Exercício 2

$$\frac{\begin{array}{c} join : A + A \to A \\ dup : A \to A \times A \end{array}}{\alpha : A + A \to A \times A}$$

$$\begin{tikzcd} A \times A \arrow[d, "f \times g"'] & & A + A \arrow[ll, "dup \cdot join"'] \arrow[d, "f + g"] \\ B \times B & & B + B \arrow[ll, "dup \cdot join"] \end{tikzcd}$$

Propriedade grátis:

$$(f \times g) \cdot \alpha = \alpha \cdot (f + g)$$

Exercício 3

$$\begin{array}{ll} & \nabla \cdot (f + f) = f \cdot \nabla \\ \equiv & \quad \{ \text{ def-}+\text{, fusão-}+ \text{ } \} \\ & [\nabla \cdot i_1 \cdot f, \nabla \cdot i_2 \cdot f] = f \cdot \nabla \\ \equiv & \quad \{ \text{ universal-}+ \text{ } \} \\ & \begin{cases} \nabla \cdot i_1 \cdot f = f \cdot \nabla \cdot i_1 \\ \nabla \cdot i_2 \cdot f = f \cdot \nabla \cdot i_2 \end{cases} \end{array}$$

$$\equiv \quad \{ \ \nabla \cdot i_1 = id, \nabla \cdot i_2 = id \ \}$$

$$\begin{cases} id \cdot f = f \cdot id \\ id \cdot f = f \cdot id \end{cases}$$

$$\equiv \quad \{ \ \text{natural-}id \ \}$$

$$\begin{cases} f = f \\ f = f \end{cases}$$

$\square$

Exercício 4

$$f + g : A + B \to A' + B'$$
$$f + g \times h : A + C \times B \to A' + C' \times B'$$

$$\begin{tikzcd}
A + B \arrow[d, "f+g"'] & A + C \times B \arrow[l, "\alpha"'] \arrow[d, "f+g\times h"] \\
A' + B' & A' + C' \times B' \arrow[l, "\alpha"]
\end{tikzcd}$$

Podemos deduzir α como $\alpha = id + \pi_2$.

Exercício 5

$$\begin{tikzcd}
(A \times B) + (A \times C) \arrow[d, "(f\times g)+(f\times h)"'] & A \times (B + C) \arrow[l, "distr"'] \arrow[d, "f\times(g+h)"] \\
(A' \times B') + (A' \times C') & A' \times (B' + C') \arrow[l, "distr"]
\end{tikzcd}$$

Propriedade grátis:

$$((f \times g) + (f \times h)) \cdot distr = distr \cdot (f \times (g + h))$$

$$h \cdot distr \cdot (g \times (id + f)) = k$$

$$\equiv \quad \{ \ \text{propriedade grátis} \ \}$$

$$h \cdot ((g \times id) + (g \times f)) \cdot distr = k$$

$$\equiv \quad \{ \ \text{(F6)} \ \}$$

$$h \cdot ((g \times id) + (g \times f)) = k \cdot distr^{\circ}$$

$$\equiv \quad \{ \ distr^{\circ} = undistr \ \}$$

$$h \cdot ((g \times id) + (g \times f)) = k \cdot undistr$$

Exercício 6

$$(p \cdot h) \to (f \cdot h), (g \cdot h)$$

$$\equiv \quad \{ \ \text{def. condicional de McCarthy} \ \}$$

$$[f \cdot h, g \cdot h] \cdot (p \cdot h)\,?$$

$\equiv \quad \{$ absorção-$+$ $\}$

$[f,g] \cdot (h+h) \cdot (p \cdot h)\,?$

$\equiv \quad \{$ natural-guarda $\}$

$[f,g] \cdot p\,? \cdot h$

$\equiv \quad \{$ def. condicional de McCarthy $\}$

$(p \rightarrow f,g) \cdot h$

Exercício 7

$choose \cdot parallel\ p\ f\ g = p \rightarrow f,g$

$\equiv \quad \{$ def. *parallel*, def. *choose* $\}$

$(\pi_2 \rightarrow \pi_1 \cdot \pi_1, \pi_2 \cdot \pi_1) \cdot \langle\langle f,g\rangle,p\rangle = p \rightarrow f,g$

$\equiv \quad \{$ def. condicional de McCarthy $\}$

$[\pi_1 \cdot \pi_1, \pi_2 \cdot \pi_1] \cdot \pi_2\,? \cdot \langle\langle f,g\rangle,p\rangle = p \rightarrow f,g$

$\equiv \quad \{$ natural-guarda $\}$

$[\pi_1 \cdot \pi_1, \pi_2 \cdot \pi_1] \cdot (\langle\langle f,g\rangle,p\rangle + \langle\langle f,g\rangle,p\rangle) \cdot (\pi_2 \cdot \langle\langle f,g\rangle,p\rangle)? = p \rightarrow f,g$

$\equiv \quad \{$ cancelamento-$\times$, absorção-$+$ $\}$

$[\pi_1 \cdot \pi_1 \cdot \langle\langle f,g\rangle,p\rangle, \pi_2 \cdot \pi_1 \cdot \langle\langle f,g\rangle,p\rangle] \cdot p? = p \rightarrow f,g$

$\equiv \quad \{$ cancelamento-$\times$ $\}$

$[f,g] \cdot p? = p \rightarrow f,g$

$\equiv \quad \{$ def. condicional de McCarthy $\}$

$p \rightarrow f,g = p \rightarrow f,g$

$\square$

Exercício 8

Primeira propriedade:

$\langle(p \rightarrow f,h),(p \rightarrow g,i)\rangle$

$\equiv \quad \{$ def. condicional de McCarthy (twice) $\}$

$\langle[f,h] \cdot p?, [g,i] \cdot p?\rangle$

$\equiv \quad \{$ fusão-$\times$ $\}$

$\langle[f,h],g\ i\rangle \cdot p\,?$

$\equiv \quad \{$ lei da troca $\}$

$[\langle f,g\rangle,\langle h,i\rangle] \cdot p\,?$

$\equiv \quad \{$ def. condicional de McCarthy $\}$

$p \rightarrow \langle f,g\rangle,\langle h,i\rangle$

Segunda propriedade:

$p \rightarrow \langle f,g\rangle,\langle f,h\rangle$

$\equiv \quad \{$ (F11) $\}$

$$
\begin{array}{ll}
& \langle (p \to f, f), (p \to g, h) \rangle \\
\equiv & \quad \{ \text{ (F9) } \} \\
& \langle f, (p \to g, h) \rangle
\end{array}
$$

Terceira propriedade:

$$
\begin{array}{ll}
& p \to (p \to a, b), (p \to c, d) \\
\equiv & \quad \{ \text{ def. condicional de McCarthy } \} \\
& [[a, b] \cdot p?, [c, d] \cdot p?] \cdot p\,? \\
\equiv & \quad \{ \text{ absorção-}+ \text{ } \} \\
& [[a, b], [c, d]] \cdot (p\,? + p?) \cdot p\,? \\
\equiv & \quad \{ \text{ (F10) } \} \\
& [[a, b], [c, d]] \cdot (i_1 + i_2) \cdot p\,? \\
\equiv & \quad \{ \text{ absorção-}+\text{, cancelamento-}+ \text{ } \} \\
& [a, d] \cdot p\,? \\
\equiv & \quad \{ \text{ def. condicional de McCarthy } \} \\
& p \to a, d
\end{array}
$$

Exercício 9

$f = \perp$