

Cálculo de Programas

Resolução - Ficha 02

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Exercício 1

$$\begin{aligned}\pi_1 \cdot (f \times g) (x, y) &= \{\text{def. composição}\} \\ &\quad \pi_1((f \times g)(x, y)) \\ &= \{(F1)\} \\ &\quad \pi_1(f \ x, g \ y) \\ &= \{(F2)\} \\ &\quad f \ x \\ &= \{(F2)\} \\ &\quad f(\pi_1(x, y)) \\ &= \{\text{def. composição}\} \\ &\quad f \cdot \pi_1\end{aligned}$$

$$\begin{aligned}\pi_2 \cdot (f \times g) (x, y) &= \{\text{def. composição}\} \\ &\quad \pi_2((f \times g)(x, y)) \\ &= \{(F1)\} \\ &\quad \pi_2(f \ x, g \ y) \\ &= \{(F2)\} \\ &\quad g \ y \\ &= \{(F2)\} \\ &\quad g(\pi_2(x, y)) \\ &= \{\text{def. composição}\} \\ &\quad g \cdot \pi_2\end{aligned}$$

$$\begin{aligned}(f \times g) (x, y) &= \{(F1)\} \\ &\quad (f \ x, g \ y) \\ &= \{(F2)\} \\ &\quad (f(\pi_1(x, y)), g(\pi_2(x, y))) \\ &= \{\text{def. composição}\} \\ &\quad (f \cdot \pi_1, g \cdot \pi_2) \\ &= \{\text{def. split}\} \\ &\quad \langle f \cdot \pi_1, g \cdot \pi_2 \rangle\end{aligned}$$

Exercício 2

$$\begin{aligned} xor \cdot (and \times id) ((a,b),c) &= \{\text{def. composição}\} \\ & xor((and \times id) ((a,b),c)) \\ &= \{(F1)\} \\ & xor(and(a,b), id\ c) \\ &= \{\text{def. and, def. id}\} \\ & xor(a \wedge b, c) \\ &= \{\text{def. xor}\} \\ & (a \wedge b) \oplus c \end{aligned}$$

Exercício 3

```
ghci> :l Cp.hs
ghci> xor (x,y) = x /= y
ghci> and (x,y) = x && y
ghci> f = xor . (and >< id)
ghci> f ((True, True), False)
True
ghci> f ((True, True), True)
False
ghci> f ((False, True), True)
True
ghci> f ((True, False), True)
True
```

Exercício 4

$$id = \langle f, g \rangle \iff \begin{cases} \pi_1 \cdot id = f \\ \pi_2 \cdot id = g \end{cases} \iff \begin{cases} \pi_1 = f \\ \pi_2 = g \end{cases} \iff id = \langle \pi_1, \pi_2 \rangle$$

Seja $k = id$, ao aplicar a propriedade *universal- \times* obtemos a propriedade *reflexão- \times* .

Exercício 5

$$\begin{aligned} & \underbrace{\langle h, k \rangle \cdot f}_k = \underbrace{\langle h \cdot f, k \cdot f \rangle}_{\langle h, f \rangle} \\ & \iff \{(F7)\} \\ & \begin{cases} \pi_1 \cdot \langle h, k \rangle \cdot f = h \cdot f \\ \pi_2 \cdot \langle h, k \rangle \cdot f = k \cdot f \end{cases} \\ & \iff \{\text{Cancelamento-}\times\} \\ & \begin{cases} h \cdot f = h \cdot f \\ k \cdot f = k \cdot f \end{cases} \end{aligned}$$

Exercício 6

$$\begin{aligned}
dup \cdot f \ x &= \{\text{natural-id}\} \\
&\quad dup \cdot f \cdot id \ x \\
&= \{\text{def. composição}\} \\
&\quad dup(f(id \ x)) \\
&= \{\text{def. dup}\} \\
&\quad (f(id \ x), f(id \ x)) \\
&= \{\text{def. composição}\} \\
&\quad \langle f \cdot id, f \cdot id \rangle \\
&= \{\text{fusão-x}\} \\
&\quad \langle f, f \rangle \cdot id \\
&= \{\text{natural-id}\} \\
&\quad \langle f, f \rangle
\end{aligned}$$

Exercício 7

$$\begin{aligned}
&\underbrace{(b, a)}_k = \underbrace{\langle \underline{b}, \underline{a} \rangle}_{\langle f, g \rangle} \\
&\iff \{\text{universal-}\times\} \\
&\quad \begin{cases} \pi_1 \cdot \underline{(b, a)} = \underline{b} \\ \pi_2 \cdot \underline{(b, a)} = \underline{a} \end{cases} \\
&\iff \{\text{absorção-const}\} \\
&\quad \begin{cases} \pi_1(b, a) = \underline{b} \\ \pi_2(b, a) = \underline{a} \end{cases} \\
&\iff \{\text{cancelamento-}\times\} \\
&\quad \begin{cases} \underline{b} = \underline{b} \\ \underline{a} = \underline{a} \end{cases}
\end{aligned}$$

Exercício 8

$$\begin{aligned}
&(g \times f) \cdot swap \\
&= \{\text{def-}\times\} \\
&\quad \langle g \cdot \pi_1, f \cdot \pi_2 \rangle \cdot swap \\
&= \{\text{fusão-}\times\} \\
&\quad \langle g \cdot \pi_1 \cdot swap, f \cdot \pi_2 \cdot swap \rangle \\
&= \{\text{def. swap}\} \\
&\quad \langle g \cdot \pi_1 \cdot \langle \pi_2, \pi_1 \rangle, f \cdot \pi_2 \cdot \langle \pi_2, \pi_1 \rangle \rangle \\
&= \{\text{cancelamento-}\times\} \\
&\quad \langle g \cdot \pi_2, f \cdot \pi_1 \rangle
\end{aligned}$$

$$\begin{aligned}
&swap \cdot (f \times g) \\
&= \{\text{def. swap}\} \\
&\quad \langle \pi_2, \pi_1 \rangle \cdot (f \times g) \\
&= \{\text{fusão-}\times\} \\
&\quad \langle \pi_2 \cdot (f \times g), \pi_1 \cdot (f \times g) \rangle \\
&= \{\text{def-}\times\} \\
&\quad \langle \pi_2 \cdot \langle f \cdot \pi_1, g \cdot \pi_2 \rangle, \pi_1 \cdot \langle f \cdot \pi_1, g \cdot \pi_2 \rangle \rangle \\
&= \{\text{cancelamento-}\times\} \\
&\quad \langle g \cdot \pi_2, f \cdot \pi_1 \rangle
\end{aligned}$$

Exercício 9

```
acronym :: String -> String
acronym = map head . words

short :: String -> String
short = uncurry (++) . (id >< (' ' :)) . split head last . words
```

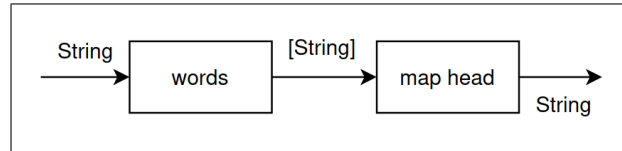


Figura 1: acronym

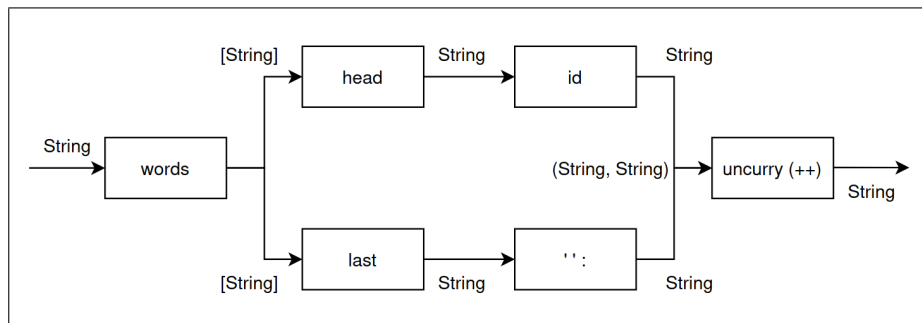


Figura 2: short