

7.

Exercício 7.1

a)

$$\frac{31}{12}$$

b)

$$-1 + (-1 + e) e$$

c)

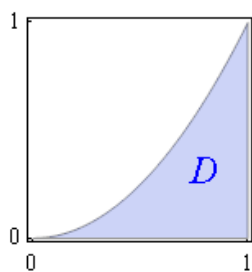
$$\frac{7 \sin[1]}{9}$$

d)

$$-2 + \operatorname{Log}[16]$$

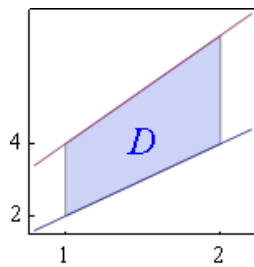
Exercício 7.2

a)



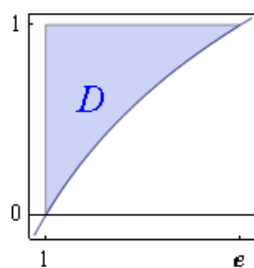
$$\int_0^1 \int_0^{x^2} dy dx = \frac{1}{3}$$

b)



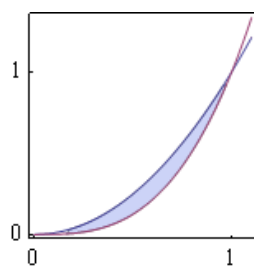
$$\int_1^2 \int_{2x}^{x+1} dy dx = \frac{5}{2}$$

c)



$$\int_0^1 \int_1^{e^y} (x+y) dx dy = \frac{1}{4} (-1 + e^2)$$

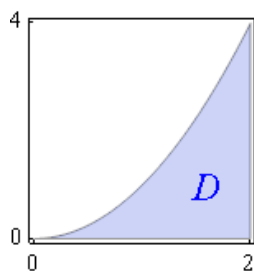
d)



$$\int_0^1 \int_{x^3}^{x^2} y dy dx = \frac{1}{35}$$

Exercício 7.3

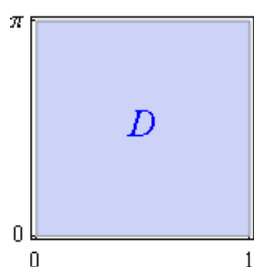
a)



$$\int_0^2 \int_0^{x^2} xy \, dy \, dx = \frac{16}{3}$$

$$\int_0^4 \int_{\sqrt{y}}^2 xy \, dx \, dy = \frac{16}{3}$$

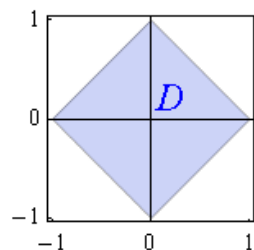
b)



$$\int_0^1 \int_0^\pi x \sin[x+y] \, dy \, dx = 2(-1 + \cos[1] + \sin[1])$$

$$\int_0^\pi \int_0^1 x \sin[x+y] \, dx \, dy = 2(-1 + \cos[1] + \sin[1])$$

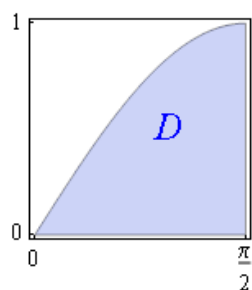
c)



$$\int_{-1}^0 \int_{-x-1}^{x+1} \text{Exp}[x+y] \, dy \, dx + \int_0^1 \int_{x-1}^{-x+1} \text{Exp}[x+y] \, dy \, dx = -\frac{1}{e} + e$$

$$\int_{-1}^1 \int_{y-1}^{1-y} \text{Exp}[x+y] \, dx \, dy + \int_{-1}^1 \int_{-y-1}^{y+1} \text{Exp}[x+y] \, dx \, dy = -\frac{1}{e} + e$$

d)

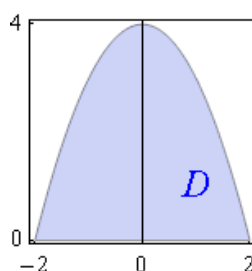


$$\int_0^{\frac{\pi}{2}} \int_0^{\sin[x]} x^2 + y^2 \, dy \, dx = -\frac{16}{9} + \pi$$

$$\int_0^1 \int_{\arcsin[y]}^{\frac{\pi}{2}} x^2 + y^2 \, dx \, dy = -\frac{16}{9} + \pi$$

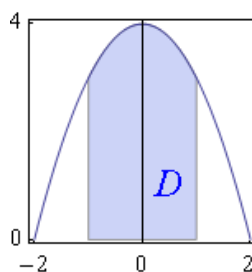
Exercício 7.4

a)



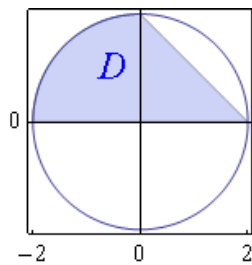
$$\int_{-2}^2 \int_0^{4-x^2} f[x, y] \, dy \, dx = \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} f[x, y] \, dx \, dy$$

b)



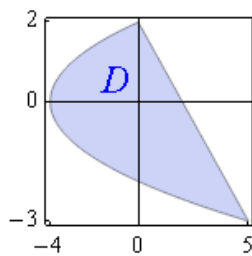
$$\int_{-1}^1 \int_0^{4-x^2} f[x, y] \, dy \, dx = \int_3^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} f[x, y] \, dx \, dy + \int_0^3 \int_{-1}^1 f[x, y] \, dx \, dy$$

c)



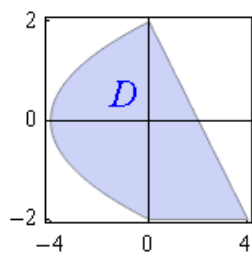
$$\int_0^2 \int_{-\sqrt{4-y}}^{2-y} f(x,y) dx dy = \int_{-2}^0 \int_0^{\sqrt{4-x^2}} f(x,y) dy dx + \int_0^2 \int_0^{2-x} f(x,y) dy dx$$

d)



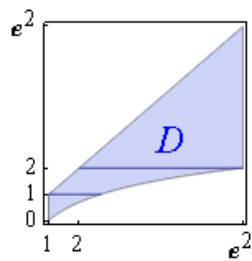
$$\int_{-3}^2 \int_{-4+y^2}^{2-y} f(x,y) dx dy = \int_{-4}^0 \int_{-\sqrt{x+4}}^{\sqrt{x+4}} f(x,y) dy dx + \int_0^5 \int_{-\sqrt{x+4}}^{2-x} f(x,y) dy dx$$

e)



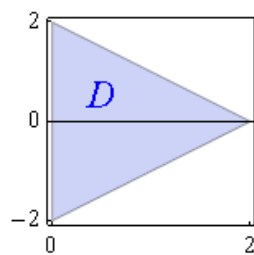
$$\int_{-2}^2 \int_{-4+y^2}^{2-y} f(x,y) dx dy = \int_{-4}^0 \int_{-\sqrt{x+4}}^{\sqrt{x+4}} f(x,y) dy dx + \int_0^4 \int_{-2}^{2-x} f(x,y) dy dx$$

f)



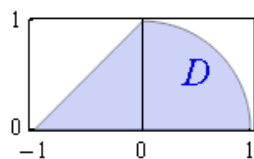
$$\int_1^{e^2} \int_{\log[x]}^x f[x, y] \, dy \, dx = \int_0^1 \int_0^{\exp[y]} f[x, y] \, dx \, dy + \int_1^2 \int_y^{\exp[y]} f[x, y] \, dx \, dy + \int_2^{e^2} \int_y^{e^2} f[x, y] \, dx \, dy$$

g)



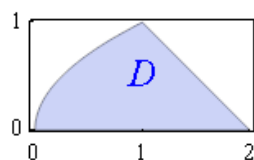
$$\int_{-2}^2 \int_0^{-|y|+2} f[x, y] \, dx \, dy = \int_0^2 \int_{x-2}^{2-x} f[x, y] \, dy \, dx$$

h)



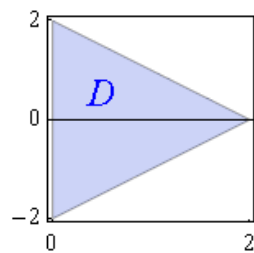
$$\int_0^1 \int_{y-1}^{\sqrt{1-y^2}} f[x, y] \, dx \, dy = \int_{-1}^0 \int_0^{x+1} f[x, y] \, dy \, dx + \int_0^1 \int_0^{\sqrt{1-x^2}} f[x, y] \, dy \, dx$$

i)



$$\int_0^1 \int_0^{\sqrt{x}} f[x, y] \, dy \, dx + \int_1^2 \int_0^{-x+2} f[x, y] \, dy \, dx = \int_0^1 \int_{y^2}^{2-y} f[x, y] \, dx \, dy$$

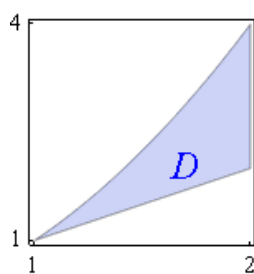
j)



$$\int_{-2}^0 \int_0^{y+2} f(x, y) dx dy + \int_0^2 \int_0^{-y+2} f(x, y) dx dy = \int_0^2 \int_{x-2}^{2-x} f(x, y) dy dx$$

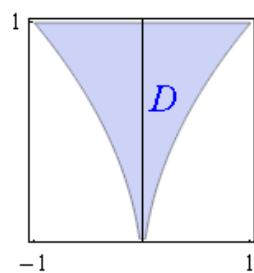
Exercício 7.5

a)



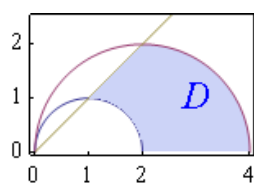
$$\text{Área} = \frac{5}{6}$$

b)



$$\text{Área} = \frac{2}{3}$$

Exercício 7.6



$$\text{Área} = \frac{3(2 + \pi)}{4}$$

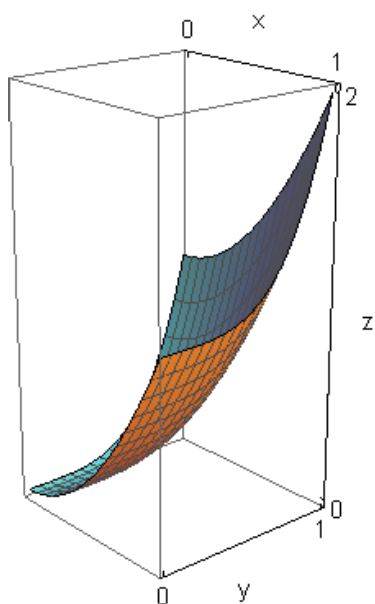
Exercício 7.7

$$\text{Área da circunferência} = 4 \int_0^r \int_0^{\sqrt{r^2 - x^2}} dy dx = \pi r^2$$

$$\text{Área da elipse} = 4 \int_0^a \int_0^{\frac{b}{a} \sqrt{a^2 - x^2}} dy dx = a b \pi$$

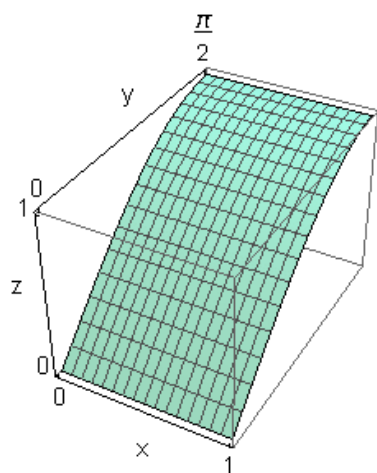
Exercício 7.8

a)



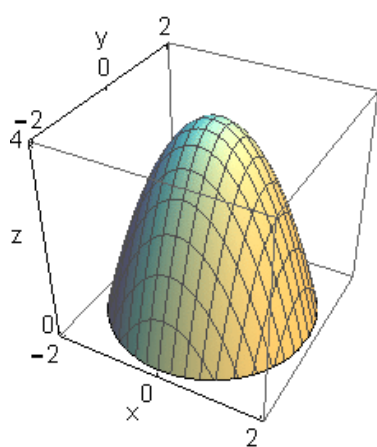
$$\text{volume} = \frac{8}{15}$$

b)



Volume = 1

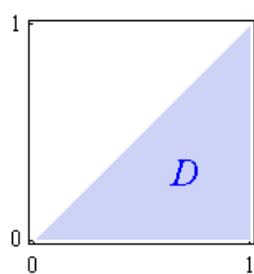
c)



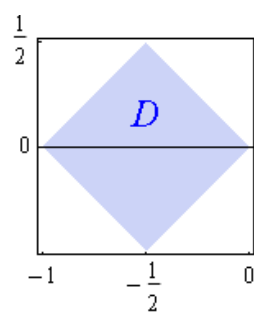
Volume = 8π

Exercício 7.9

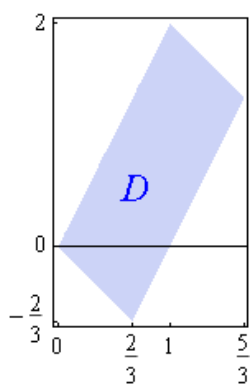
a)



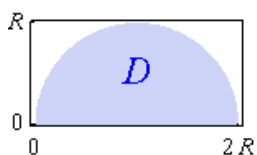
$\frac{1}{2}$

Exercício 7.10**a)**

$$\frac{1}{6}$$

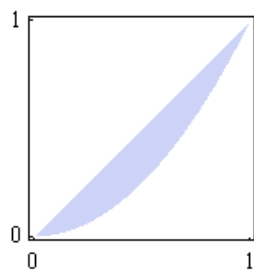
b)

$$\frac{1}{3}$$

Exercício 7.11**a)**

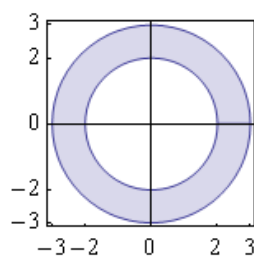
$$\frac{3 \pi R^4}{4}$$

b)



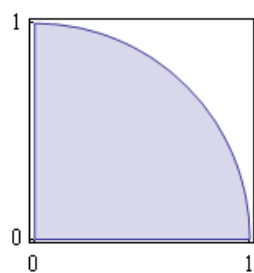
$$\frac{1}{6}$$

Exercício 7.12



$$\frac{38\pi}{3}$$

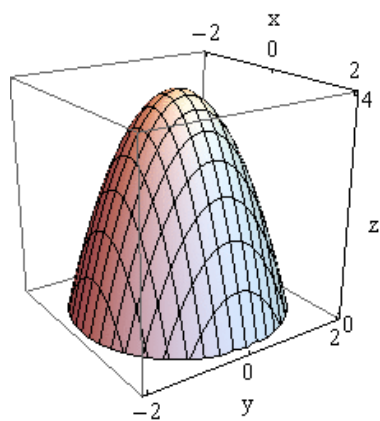
Exercício 7.13



$$\frac{1}{24}$$

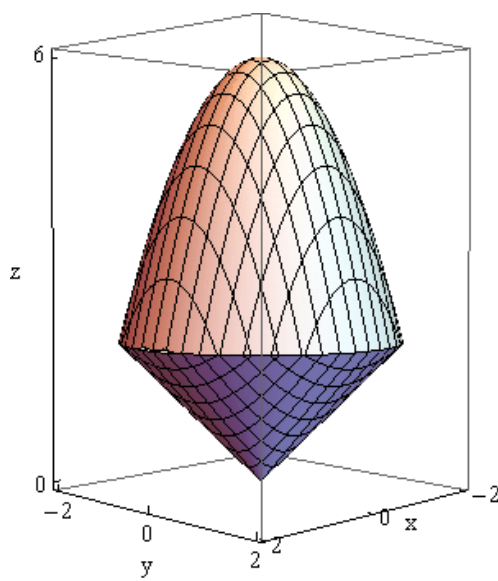
Exercício 7.14

a)



$$8\pi$$

b)



$$\frac{32\pi}{3}$$