

1. (a) $\Omega = \{(C_a, C_a), (C_a, C_o), (C_o, C_a), (C_o, C_o)\}$ ou $\{(a_1, a_2) : a_1, a_2 \in \{C_a, C_o\}\}$ ou $\{C_a, C_o\} \times \{C_a, C_o\}$;
(b) $\Omega = \{(a_1, a_2) : a_1 \in \{C_a, C_o\}, a_2 \in \{1, 2, 3, 4, 5, 6\}\}$ ou $\{C_a, C_o\} \times \{1, 2, 3, 4, 5, 6\}$;
(c) $\Omega = \{(a_1, a_2, a_3) : a_1, a_2, a_3 \in \{1, 2, 3, 4, 5, 6\}\}$ ou $\{1, 2, 3, 4, 5, 6\}^3$
2. a) $\Omega = \{1, 2, 3, 4, 5, 6\}$ e $P(\{i\}) = \frac{2^{i-1}}{63}$, $i \in \{1, \dots, 6\}$;
b) Não;
c) $A = \{2, 4, 6\}$, $P(A) = \frac{42}{63}$; $B = \{3, 6\}$, $P(B) = \frac{36}{63}$; $\bar{B} = \{1, 2, 4, 5\}$, $P(\bar{B}) = \frac{27}{63}$;
 $C = \{1, 3, 5\}$, $P(C) = \frac{21}{63}$; $A \cap B = \{6\}$, $P(A \cap B) = \frac{32}{63}$;
 $A \cup B = \{2, 3, 4, 6\}$, $P(A \cup B) = \frac{46}{63}$; $A \setminus B = \{2, 4\}$, $P(A \setminus B) = \frac{10}{63}$
3. a) \emptyset ; \emptyset ; Ω ; Ω ;
b) $D = A \cap B$; $E = B \cap C$; $F = A \cup B \cup C$; $G = \bar{F} = \bar{A} \cap \bar{B} \cap \bar{C}$;
 $H = (A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$
4. (a) 0.32; (b) 0.68; (c) 0.12; (d) 0.24
5. (a) 0.485; (b) 0.515; (c) 0.025; (d) 0.035; (e) 0.295
6. (a) 0.2; (b) 0.575; (c) 0.044
7. (a) 0.72; (b) $\frac{0.1}{0.28}$; (c) Não
8. Sim; Sim; Não; Não
9. (a) Não; (b) Falsa
10. (a) 0.1; 0.6; 0.3; 0.36; 0.42; (b) $\frac{0.52}{0.64}$

Exercícios Suplementares à Folha 1

1. (a) $\frac{260}{630}$; (b) $\frac{120}{630}$; (c) $\frac{470}{630}$; (d) $\frac{160}{630}$; (e) $\frac{380}{630}$
2. $P(A \Delta B) = P(A) + P(B) - 2P(A \cap B)$
3. (a) 0.32; (b) 0.68; (c) 0.03; (d) 0.03
4. (a) $\frac{30}{36}$; (b) $\frac{6}{30}$; (c) $\frac{24}{30}$; (d) $\frac{3}{6}$; (e) Não
5. $\frac{2 \times \binom{13}{5}}{\binom{26}{5}} = \frac{9}{230}$
6. (a) 0.47; 0.55; 0.85; 0.83; (b) 0.691; (c) 0.309; (d) 0.333; (e) 0.717
7. (a) $\Omega = \{C_a, C_o\}^{n-1}$;
(b) i. $P(E_j) = \frac{1}{2}$, $j \in \{1, \dots, n\}$; ii. 0 e não são independentes
8. $n > 6.644$ pelo que $n = 7$.
9. Se $n = 2$ não são independentes; se $n = 3$ são independentes.
10. (a) — (b) 0.719; $\frac{0.665}{0.719}$; $\frac{0.246}{0.281}$

1. (a) i. $X : \left\{ \begin{array}{ccc} 0 & \frac{1}{36} & \frac{2}{36} \\ \frac{9}{36} & \frac{18}{36} & \frac{9}{36} \end{array} \right. , F_X(c) = \left\{ \begin{array}{lll} 0 & se & c < 0 \\ \frac{9}{36} & se & 0 \leq c < 1 \\ \frac{27}{36} & se & 1 \leq c < 2 \\ 1 & se & c \geq 2 \end{array} ; \right.$

ii. igual à alínea anterior;

iii. $Z : \left\{ \begin{array}{ccccccc} \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \frac{11}{36} \end{array} \right. , F_Z(c) = \left\{ \begin{array}{lll} 0 & se & c < 1 \\ \frac{1}{36} & se & 1 \leq c < 2 \\ \frac{4}{36} & se & 2 \leq c < 3 \\ \frac{9}{36} & se & 3 \leq c < 4 \\ \frac{16}{36} & se & 4 \leq c < 5 \\ \frac{25}{36} & se & 5 \leq c < 6 \\ 1 & se & c \geq 6 \end{array} ; \right.$

vi. $N : \left\{ \begin{array}{ccccccc} \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \frac{1}{36} \end{array} \right. , F_N(c) = \left\{ \begin{array}{lll} 0 & se & c < 1 \\ \frac{11}{36} & se & 1 \leq c < 2 \\ \frac{20}{36} & se & 2 \leq c < 3 \\ \frac{27}{36} & se & 3 \leq c < 4 \\ \frac{32}{36} & se & 4 \leq c < 5 \\ \frac{35}{36} & se & 5 \leq c < 6 \\ 1 & se & c \geq 6 \end{array} ; \right.$

v. $W : \left\{ \begin{array}{ccccccc} 0 & \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{2}{36} \end{array} \right. , F_W(c) = \left\{ \begin{array}{lll} 0 & se & c < 0 \\ \frac{6}{36} & se & 0 \leq c < 1 \\ \frac{16}{36} & se & 1 \leq c < 2 \\ \frac{24}{36} & se & 2 \leq c < 3 \\ \frac{30}{36} & se & 3 \leq c < 4 \\ \frac{34}{36} & se & 4 \leq c < 5 \\ 1 & se & c \geq 5 \end{array} ; \right.$

vi. $S : \left\{ \begin{array}{ccccccccccccc} \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \frac{7}{36} & \frac{8}{36} & \frac{9}{36} & \frac{10}{36} & \frac{11}{36} & \frac{12}{36} \end{array} \right. , F_S(c) = \left\{ \begin{array}{lll} 0 & se & c < 2 \\ \frac{1}{36} & se & 2 \leq c < 3 \\ \frac{3}{36} & se & 3 \leq c < 4 \\ \frac{6}{36} & se & 4 \leq c < 5 \\ \frac{10}{36} & se & 5 \leq c < 6 \\ \frac{15}{36} & se & 6 \leq c < 7 \\ \frac{21}{36} & se & 7 \leq c < 8 \\ \frac{26}{36} & se & 8 \leq c < 9 \\ \frac{30}{36} & se & 9 \leq c < 10 \\ \frac{33}{36} & se & 10 \leq c < 11 \\ \frac{35}{36} & se & 11 \leq c < 12 \\ 1 & se & c \geq 12 \end{array} ; \right.$

vii. $V : \left\{ \begin{array}{ccc} 0 & \frac{1}{36} & \frac{2}{36} \end{array} \right. , F_V(c) = \left\{ \begin{array}{lll} 0 & se & c < 0 \\ \frac{25}{36} & se & 0 \leq c < 1 \\ \frac{35}{36} & se & 1 \leq c < 2 \\ 1 & se & c \geq 2 \end{array} \right.$

- (b) i. $\frac{3}{4}$; ii. $\frac{1}{4}$; iii. $\frac{1}{4}$; iv. $\frac{1}{4}$; v. $\frac{1}{6}$; vi. $\frac{5}{6}$; vii. $\frac{1}{6}$; viii. $\frac{1}{15}$
 (c) $\frac{4}{9}$

2. a) $\Omega = \{(C_a, C_a), (C_a, C_o), (C_o, C_a), (C_o, C_o)\}$;

ω	$X(\omega)$	$Y(\omega)$
(C_a, C_a)	2	0
(C_a, C_o)	1	1
(C_o, C_a)	1	1
(C_o, C_o)	0	2

ii. Funções massa de probabilidade são iguais a: $\begin{cases} 0 & se c < 0 \\ \frac{1}{4} & se 0 \leq c < 1 \\ \frac{1}{2} & se 1 \leq c < 2 \\ \frac{3}{4} & se c \geq 2 \end{cases}$,

$$\text{Funções de distribuição são iguais a: } F(c) = \begin{cases} 0 & se c < 0 \\ 1/4 & se 0 \leq c < 1 \\ 3/4 & se 1 \leq c < 2 \\ 1 & se c \geq 2 \end{cases}$$

Comentário: X e Y são funções distintas. Enquanto v.a.'s, têm em comum as funções que as caracterizam (a f.m.p. e a função de distribuição) e, por isso, X e Y são identicamente distribuídas (i.d.'s). Estas v.s.'s são ainda i.d.'s com as dos exercícios 1.(a)i. e 1.(a)ii..

3. (a) — (b) $F_X(c) = \begin{cases} 0 & se c < 0 \\ \frac{1}{8}c & se 0 \leq c < 4 \\ \frac{1}{2} + \frac{1}{4}(c-4) & se 4 \leq c < 6 \\ 1 & se c \geq 6 \end{cases}$;

(c) i. $\frac{3}{16}$; ii. $\frac{13}{16}$; iii. igual a ii.; iv. todas iguais a $\frac{3}{8}$; (d) $\frac{13}{16}$; $\frac{11}{13}$

4. (a) $F_T(c) = \begin{cases} 0 & se c < 0 \\ 1 - e^{-\lambda c} & se c \geq 0 \end{cases}$; (b) e^{-1} ; (c) $\frac{1}{4}e^{-2} + \frac{3}{4}e^{-4}$; $\frac{3e^{-4}}{e^{-2}+3e^{-4}}$

Exercícios Suplementares à Folha 2

1. (a) — (b) 0.65; 0.5; 0.5

(c) $F_X(c) = \begin{cases} 0 & se c < 0 \\ 0.05 & se 0 \leq c < 1 \\ 0.15 & se 1 \leq c < 2 \\ 0.35 & se 2 \leq c < 3 ; (d) \frac{0.15}{0.8}; \frac{0.45}{0.8}; \frac{0.3}{0.8} \\ 0.5 & se 3 \leq c < 4 \\ 0.8 & se 4 \leq c < 5 \\ 1 & se c \geq 5 \end{cases}$

2. (a) $k = \frac{1}{4}$; $F(c) = \begin{cases} 0 & se c < 1 \\ \frac{1}{8}(c^2 - 1) & se 1 \leq c < 3 ; (b) i. \frac{27}{32}; ii. \text{ igual a i.}; iii. \frac{39}{128}; \\ 1 & se c \geq 3 \end{cases}$

c) $\frac{20}{27}$

3. (a) $F_X(c) = \begin{cases} 0 & se c < -1 \\ \frac{1}{2} + c + \frac{c^2}{2} & se -1 \leq c \leq 0 ; (b) 0, \frac{7}{8}, \frac{3}{8}, \frac{1}{8}, \frac{5}{9} \\ \frac{1}{2} + c - \frac{c^2}{2} & se 0 < c \leq 1 \\ 1 & se c > 1 \end{cases}$

4. $P(Y = 0) = 1 - e^{-\lambda a}$, $F_Y(c) = \begin{cases} 0 & se c < 0 \\ 1 - e^{-\lambda(c+a)} & se c \geq 0 \end{cases}$

[Sug.: Usar T.P.T. com a partição formada pelos acontecimentos $(X \leq a)$ e $(X > a)$]