

1. (a)  $\Omega = \{(C_a, C_a), (C_a, C_o), (C_o, C_a), (C_o, C_o)\}$  ou  $\{(a_1, a_2) : a_1, a_2 \in \{C_a, C_o\}\}$  ou  $\{C_a, C_o\} \times \{C_a, C_o\}$ ;  
 (b)  $\Omega = \{(a_1, a_2) : a_1 \in \{C_a, C_o\}, a_2 \in \{1, 2, 3, 4, 5, 6\}\}$  ou  $\{C_a, C_o\} \times \{1, 2, 3, 4, 5, 6\}$ ;  
 (c)  $\Omega = \{(a_1, a_2, a_3) : a_1, a_2, a_3 \in \{1, 2, 3, 4, 5, 6\}\}$  ou  $\{1, 2, 3, 4, 5, 6\}^3$
2. a)  $\Omega = \{1, 2, 3, 4, 5, 6\}$  e  $P(\{i\}) = \frac{2^{i-1}}{63}$ ,  $i \in \{1, \dots, 6\}$ ;  
 b) Não;  
 c)  $A = \{2, 4, 6\}, P(A) = \frac{42}{63}$ ;  $B = \{3, 6\}, P(B) = \frac{36}{63}$ ;  $\overline{B} = \{1, 2, 4, 5\}, P(\overline{B}) = \frac{27}{63}$ ;  
 $C = \{1, 3, 5\}, P(C) = \frac{21}{63}$ ;  $A \cap B = \{6\}, P(A \cap B) = \frac{32}{63}$ ;  
 $A \cup B = \{2, 3, 4, 6\}, P(A \cup B) = \frac{46}{63}$ ;  $A \setminus B = \{2, 4\}, P(A \setminus B) = \frac{10}{63}$
3. a)  $\emptyset$ ;  $\emptyset$ ;  $\Omega$ ;  $\Omega$ ;  
 b)  $D = A \cap B$ ;  $E = B \cap C$ ;  $F = A \cup B \cup C$ ;  $G = \overline{F} = \overline{A} \cap \overline{B} \cap \overline{C}$ ;  
 $H = (A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap B \cap \overline{C}) \cup (\overline{A} \cap \overline{B} \cap C)$
4. (a) 0.32; (b) 0.68; (c) 0.12; (d) 0.24
5. (a) 0.485; (b) 0.515; (c) 0.025; (d) 0.035; (e) 0.295
6. (a) 0.2; (b) 0.575; (c) 0.044
7. (a) 0.72; (b)  $\frac{0.1}{0.28}$ ; (c) Não
8. Sim; Sim; Não; Não
9. (a) Não; (b) Falsa
10. (a) 0.1; 0.6; 0.3; 0.36; 0.42; (b)  $\frac{0.52}{0.64}$

1. (a)  $\frac{260}{630}$ ; (b)  $\frac{120}{630}$ ; (c)  $\frac{470}{630}$ ; (d)  $\frac{160}{630}$ ; (e)  $\frac{380}{630}$
2.  $P(A \Delta B) = P(A) + P(B) - 2P(A \cap B)$
3. (a) 0.32; (b) 0.68; (c) 0.03; (d) 0.03
4. (a)  $\frac{30}{36}$ ; (b)  $\frac{6}{30}$ ; (c)  $\frac{24}{30}$ ; (d)  $\frac{3}{6}$ ; (e) Não
5.  $\frac{2 \times \binom{13}{5}}{\binom{26}{5}} = \frac{9}{230}$
6. (a) 0.47; 0.55; 0.85; 0.83; (b) 0.691; (c) 0.309; (d) 0.333; (e) 0.717
7. (a)  $\Omega = \{C_a, C_o\}^{n-1}$ ;  
 (b) i.  $P(E_j) = \frac{1}{2}, j \in \{1, \dots, n\}$ ; ii. 0 e não são independentes
8.  $n > 6.644$  pelo que  $n = 7$ .
9. Se  $n = 2$  não são independentes; se  $n = 3$  são independentes.
10. (a) — (b) 0.719;  $\frac{0.665}{0.719}$ ;  $\frac{0.246}{0.281}$

1. (a) i.  $X : \left\{ \begin{array}{ccc} 0 & 1 & 2 \\ \frac{9}{36} & \frac{18}{36} & \frac{9}{36} \end{array} \right\}$ ,  $F_X(c) = \left\{ \begin{array}{lll} 0 & se & c < 0 \\ 9/36 & se & 0 \leq c < 1 \\ 27/36 & se & 1 \leq c < 2 \\ 1 & se & c \geq 2 \end{array} \right.$ ;

ii. igual à alínea anterior;

iii.  $Z : \left\{ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} \end{array} \right\}$ ,  $F_Z(c) = \left\{ \begin{array}{lll} 0 & se & c < 1 \\ 1/36 & se & 1 \leq c < 2 \\ 4/36 & se & 2 \leq c < 3 \\ 9/36 & se & 3 \leq c < 4 \\ 16/36 & se & 4 \leq c < 5 \\ 25/36 & se & 5 \leq c < 6 \\ 1 & se & c \geq 6 \end{array} \right.$ ;

vi.  $N : \left\{ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{11}{36} & \frac{9}{36} & \frac{7}{36} & \frac{5}{36} & \frac{3}{36} & \frac{1}{36} \end{array} \right\}$ ,  $F_N(c) = \left\{ \begin{array}{lll} 0 & se & c < 1 \\ 11/36 & se & 1 \leq c < 2 \\ 20/36 & se & 2 \leq c < 3 \\ 27/36 & se & 3 \leq c < 4 \\ 32/36 & se & 4 \leq c < 5 \\ 35/36 & se & 5 \leq c < 6 \\ 1 & se & c \geq 6 \end{array} \right.$ ;

v.  $W : \left\{ \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ \frac{6}{36} & \frac{10}{36} & \frac{8}{36} & \frac{6}{36} & \frac{4}{36} & \frac{2}{36} \end{array} \right\}$ ,  $F_W(c) = \left\{ \begin{array}{lll} 0 & se & c < 0 \\ 6/36 & se & 0 \leq c < 1 \\ 16/36 & se & 1 \leq c < 2 \\ 24/36 & se & 2 \leq c < 3 \\ 30/36 & se & 3 \leq c < 4 \\ 34/36 & se & 4 \leq c < 5 \\ 1 & se & c \geq 5 \end{array} \right.$ ;

vi.  $S : \left\{ \begin{array}{cccccccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \frac{5}{36} & \frac{4}{36} & \frac{3}{36} & \frac{2}{36} & \frac{1}{36} \end{array} \right\}$ ,  $F_S(c) = \left\{ \begin{array}{lll} 0 & se & c < 2 \\ 1/36 & se & 2 \leq c < 3 \\ 3/36 & se & 3 \leq c < 4 \\ 6/36 & se & 4 \leq c < 5 \\ 10/36 & se & 5 \leq c < 6 \\ 15/36 & se & 6 \leq c < 7 \\ 21/36 & se & 7 \leq c < 8 \\ 26/36 & se & 8 \leq c < 9 \\ 30/36 & se & 9 \leq c < 10 \\ 33/36 & se & 10 \leq c < 11 \\ 35/36 & se & 11 \leq c < 12 \\ 1 & se & c \geq 12 \end{array} \right.$ ;

vii.  $V : \left\{ \begin{array}{ccc} 0 & 1 & 2 \\ \frac{25}{36} & \frac{10}{36} & \frac{1}{36} \end{array} \right\}$ ,  $F_V(c) = \left\{ \begin{array}{lll} 0 & se & c < 0 \\ 25/36 & se & 0 \leq c < 1 \\ 35/36 & se & 1 \leq c < 2 \\ 1 & se & c \geq 2 \end{array} \right.$

(b) i.  $\frac{3}{4}$ ; ii.  $\frac{1}{4}$ ; iii.  $\frac{1}{4}$ ; iv.  $\frac{1}{4}$ ; v.  $\frac{1}{6}$ ; vi.  $\frac{5}{6}$ ; vii.  $\frac{1}{6}$ ; viii.  $\frac{1}{15}$

(c)  $\frac{4}{9}$

2. a)  $\Omega = \{(C_a, C_a), (C_a, C_o), (C_o, C_a), (C_o, C_o)\}$ ;

$\omega$	$X(\omega)$	$Y(\omega)$
$(C_a, C_a)$	2	0
b) i. $(C_a, C_o)$	1	1
$(C_o, C_a)$	1	1
$(C_o, C_o)$	0	2

- ii. Funções massa de probabilidade são iguais a:  $\begin{cases} 0 & \frac{1}{4} & \frac{1}{2} & \frac{2}{4} \end{cases}$ ,

$$\text{Funções de distribuição são iguais a: } F(c) = \begin{cases} 0 & \text{se } c < 0 \\ 1/4 & \text{se } 0 \leq c < 1 \\ 3/4 & \text{se } 1 \leq c < 2 \\ 1 & \text{se } c \geq 2 \end{cases}$$

Comentário:  $X$  e  $Y$  são funções distintas. Enquanto v.a.'s, têm em comum as funções que as caracterizam (a f.m.p. e a função de distribuição) e, por isso,  $X$  e  $Y$  são identicamente distribuídas (i.d.'s). Estas v.s.'s são ainda i.d.'s com as dos exercícios 1.(a)i. e 1.(a)ii..

$$3. (a) \text{ — } (b) F_X(c) = \begin{cases} 0 & \text{se } c < 0 \\ \frac{1}{2} + \frac{1}{4}(c-4) & \text{se } 0 \leq c < 4 \\ 1 & \text{se } 4 \leq c < 6 \\ 1 & \text{se } c \geq 6 \end{cases};$$

- (c) i.  $\frac{3}{16}$ ; ii.  $\frac{13}{16}$ ; iii. igual a ii.; iv. todas iguais a  $\frac{3}{8}$ ; (d)  $\frac{13}{16}$ ;  $\frac{11}{13}$

$$4. (a) F_T(c) = \begin{cases} 0 & \text{se } c < 0 \\ 1 - e^{-\lambda c} & \text{se } c \geq 0 \end{cases}; \quad (b) e^{-1}; \quad (c) \frac{1}{4}e^{-2} + \frac{3}{4}e^{-4}; \quad \frac{3e^{-4}}{e^{-2} + 3e^{-4}}$$

Exercícios Suplementares à Folha 2

1. (a) — (b) 0.65; 0.5; 0.5

$$(c) F_X(c) = \begin{cases} 0 & \text{se } c < 0 \\ 0.05 & \text{se } 0 \leq c < 1 \\ 0.15 & \text{se } 1 \leq c < 2 \\ 0.35 & \text{se } 2 \leq c < 3 \\ 0.5 & \text{se } 3 \leq c < 4 \\ 0.8 & \text{se } 4 \leq c < 5 \\ 1 & \text{se } c \geq 5 \end{cases}; \quad (d) \frac{0.15}{0.8}; \quad \frac{0.45}{0.8}; \quad \frac{0.3}{0.8}$$

$$2. (a) k = \frac{1}{4}; \quad F(c) = \begin{cases} 0 & \text{se } c < 1 \\ \frac{1}{8}(c^2 - 1) & \text{se } 1 \leq c < 3 \\ 1 & \text{se } c \geq 3 \end{cases}; \quad (b) \text{ i. } \frac{27}{32}; \quad \text{ii. igual a i.}; \quad \text{iii. } \frac{39}{128};$$

$$c) \frac{20}{27}$$

$$3. (a) F_X(c) = \begin{cases} 0 & \text{se } c < -1 \\ \frac{1}{2} + c + \frac{c^2}{2} & \text{se } -1 \leq c \leq 0 \\ \frac{1}{2} + c - \frac{c^2}{2} & \text{se } 0 < c \leq 1 \\ 1 & \text{se } c > 1 \end{cases}; \quad (b) 0, \quad \frac{7}{8}, \quad \frac{3}{8}, \quad \frac{1}{8}, \quad \frac{5}{9}$$

$$4. P(Y = 0) = 1 - e^{-\lambda a}, \quad F_Y(c) = \begin{cases} 0 & \text{se } c < 0 \\ 1 - e^{-\lambda(c+a)} & \text{se } c \geq 0 \end{cases}$$

[Sug.: Usar T.P.T. com a partição formada pelos acontecimentos  $(X \leq a)$  e  $(X > a)$ ]