→ Algebra linear - exercícios Matrizes

1-

a) $M_{4\times 6}$

b) $a_{34} = \frac{1}{2}$

$L_2 = \begin{bmatrix} 2 & \sqrt{2} & 0 & 8 & 4 \end{bmatrix}$

$C_5 = \begin{bmatrix} -1 \\ 4 \\ 7 \\ 6 \end{bmatrix}$

c) Diagonal = $\begin{bmatrix} \underset{a_{11}}{0} & \underset{a_{22}}{\sqrt{2}} & \underset{a_{33}}{5} & \underset{a_{44}}{6} \end{bmatrix}$

2-

a) $M = [\text{mdc}(i,j)]$ $i = 1 \ldots 6$
$j = 1 \ldots 6$

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 1 & 3 & 1 & 1 & 3 \\ 1 & 2 & 1 & 4 & 1 & 2 \\ 1 & 1 & 1 & 1 & 5 & 1 \\ 1 & 2 & 3 & 2 & 1 & 6 \end{bmatrix}$$

• $i = 1$, $j = 1$ mdc = 1 $j = 4$ mdc = 1
$j = 2$ mdc = 1 $j = 5$ mdc = 1
$j = 3$ mdc = 1

• $i = 2$

b) $A = [2i \times (j-2)]$ $i = 1, 2, 3$
$j = 1, 2$

• $i = 1$, $j = 1 \Rightarrow 2 \times 1 \times (1-2) = -2$
$j = 2 \Rightarrow 2 \times 2 \times (2-2) = 0$

$$A = \begin{bmatrix} -2 & 0 \\ -4 & 0 \\ -6 & 0 \end{bmatrix}$$

• $i = 2$

c) $B = [b_{ij}]$ $i = 1, 2, 3$ $\quad b_{ij} = |1 + j - i|$

$j = 1, 2$

• $i = 1$, $j = 1 \Rightarrow |1+1-1| = 1$

• $i = 2$, $j = 1 \Rightarrow$

$$B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

d) $A + 2B$

$$A = \begin{bmatrix} -2 & 0 \\ -4 & 0 \\ -6 & 0 \end{bmatrix} + 2 \cdot B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 2 \\ -4 & 0 \end{bmatrix}$$

3-

a) $A = [a_{ij}]_{m \times m}$ $\quad m = 3$

$$a_{ij} = \begin{cases} 1 & \text{se } i + j \text{ é par} \\ 0 & \text{caso contrário} \end{cases}$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

b) $A = [a_{ij}]_{m \times m}$ $\quad m = 3$

$$a_{ij} = \begin{cases} 2i & \text{se } i > j \\ 0 & \text{se } i = j \\ 2j & \text{se } i < j \end{cases}$$

$$A = \begin{bmatrix} 0 & 4 & 6 \\ 4 & 0 & 6 \\ 6 & 6 & 0 \end{bmatrix}$$

4-

a) AC

$$A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & -2 & 3 \\ 1 & -1 & 2 \\ 4 & 1 & 0 \end{bmatrix}_{4\times3} \cdot C = \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} -6 \\ 10 \\ 1 \\ -14 \end{bmatrix}$$

- $2\times(-3) + 1\times(-2) + 2\times1 = -6$
- $-1\times(-3) + (-2)\times(-2) + 3\times1 = 10$
- $1\times(-3) + (-1)\times(-2) + 2\times1 = 1$
- $4\times(-3) + 1\times(-2) + 0\times1 = -14$

b) BC

$$B = \begin{bmatrix} 0 & 1 & 2 \\ 3 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{3\times3} \cdot C = \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} 0 \\ -5 \\ -2 \end{bmatrix}$$

c) LA

$$L = \begin{bmatrix} 3 & 0 & -1 & 4 \end{bmatrix}_{1\times4} \cdot A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & -2 & 3 \\ 1 & -1 & 2 \\ 4 & 1 & 0 \end{bmatrix}_{4\times3} = \begin{bmatrix} 21 & 8 & 4 \end{bmatrix}$$

- $3\times2 + 0\times(-1) + (-1)\times1 + 4\times4 = 21$
- $3\times1 + 0\times(-2) + (-1)\times(-1) + 4\times1 = 8$
- $3\times2 + 0\times3 + (-1)\times2 + 4\times0 = 4$

d) LD

$$L = \begin{bmatrix} 3 & 0 & -1 & 4 \end{bmatrix} \cdot D = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 10 & 4 \end{bmatrix}$$

- $3 \times 0 + 0 \times 3 + (-1) \times 0 + 4 \times 2 = 8$
- $3 \times 1 + 0 \times (-2) + (-1) \times 0 + 4 \times 1 = 7$
- $3 \times 2 + 0 \times 0 + (-1) \times 0 + 4 \times 1 = 10$
- $3 \times 0 + 0 \times 0 + (-1) \times 0 + 4 \times 1 = 4$

e) $[AB]_{32} = 5$

$$A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & -2 & 3 \\ 1 & -1 & 2 \\ 4 & 1 & 0 \end{bmatrix}_{4 \times 3} \cdot B = \begin{bmatrix} 0 & 1 & 2 \\ 3 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{3 \times 3} = \begin{bmatrix} 3 & 2 & 4 \\ -6 & 6 & -2 \\ -3 & 5 & 2 \\ 3 & 2 & 8 \end{bmatrix}$$

- $2 \times 0 + 1 \times 3 + 2 \times 0 = 3$

f) $[DA]_{32} = 0$

$$D = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 1 \end{bmatrix}_{4 \times 4} \cdot A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & -2 & 3 \\ 1 & -1 & 2 \\ 4 & 0 & 1 \end{bmatrix}_{4 \times 3} = \begin{bmatrix} 1 & -4 & 7 \\ 8 & 7 & 0 \\ 0 & 0 & 0 \\ 8 & 1 & 13 \end{bmatrix}$$

g) $B^2 = B \cdot B$

$$B = \begin{bmatrix} 0 & 1 & 2 \\ 3 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot B = \begin{bmatrix} 0 & 1 & 2 \\ 3 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ -6 & 7 & 6 \\ 3 & -2 & 0 \end{bmatrix}$$

h) CL

$$C = \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}_{3\times 1} \cdot L = \begin{bmatrix} 3 & 0 & -1 & 4 \end{bmatrix}_{1\times 4} = \begin{bmatrix} -9 & 0 & 3 & -12 \\ -6 & 0 & 2 & -8 \\ 3 & 0 & -1 & 4 \end{bmatrix}$$

6-

a)

- $2B - D$

$$2 \cdot \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 2 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 2 \\ 4 & 1 \\ 4 & -6 \end{bmatrix}$$

- AB

$$A = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}_{1\times 3} \cdot D = \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 2 & -2 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 19 & -4 \end{bmatrix}$$

- $A(2B)$

$$A = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}_{1\times 3} \cdot \left(2 \cdot B = \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 2 & -2 \end{bmatrix}_{3\times 2} \right) = \begin{bmatrix} 38 & -8 \end{bmatrix}$$

- $20 + 6 + 12$
- $4 + 0 + (-12) = -8$

• ABC

$$A \cdot B = \begin{bmatrix} 19 & -4 \end{bmatrix}_{1\times 2} \cdot C = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}_{2\times 2} = \begin{bmatrix} -8 & 23 \end{bmatrix}$$

• $C^2 = C \times C$

$$C = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$$

• $C^4 = C^2 \times C^2$

$$C^2 = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix} \cdot C^2 = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ -10 & 11 \end{bmatrix}$$

• $I_3\ B$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} \cdot B = \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 2 & -2 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 2 & -2 \end{bmatrix}$$

• $I_2\ B$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \cdot B = \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 2 & -2 \end{bmatrix}_{3\times 2}$$

$$2\times 2 \neq 3\times 2$$

logo ñ é possível realizar esta operação

• BI_2

$$B = \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 2 & -2 \end{bmatrix}_{3\times 2} \cdot I_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 2 & -2 \end{bmatrix}$$

b) $B + X = D$

$$B = \begin{bmatrix} 5 & 1 \\ 3 & 0 \\ 2 & -2 \end{bmatrix} + X = \begin{bmatrix} -5 & -1 \\ -1 & -1 \\ -2 & 4 \end{bmatrix} = D = \begin{bmatrix} 0 & 0 \\ 2 & -1 \\ 0 & 2 \end{bmatrix}$$

7 -

a) AB

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}_{2\times 4} \cdot B = \begin{bmatrix} 0 & -1 \\ -1 & \frac{1}{2} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{4\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

b) BA

$$B = \begin{bmatrix} 0 & -1 \\ -1 & 1/2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{4\times 2} \cdot A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}_{2\times 4} = \begin{bmatrix} -1 & 0 & 0 & -2 \\ -\frac{1}{2} & -2 & -3 & 1 \\ 1 & 2 & 3 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

b) A B

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot B = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix}$$

5 -

a) $A = \begin{bmatrix} 1 & 8 \\ 3 & -1 \\ 0 & 2 \\ 4 & -2 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 3 & 0 & 4 \\ 8 & -1 & 2 & -2 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & -4 & 1 \\ 0 & 0 & 0 \\ -3 & 1 & -1 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 0 & 3 \\ -4 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 3 & \frac{1}{3} \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 \\ 3 \\ \frac{1}{3} \end{bmatrix}$

8 - $X + A = 2(X - AB^T)$

$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$ $\quad B = [a_{ij}]_{3\times3}$ $\quad b_{ij} = \begin{cases} 1 \text{ se } i > j \\ 0 \text{ se } i = j \\ 2 \text{ se } i < j \end{cases}$

$B = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 1 \\ 2 & 2 & 0 \end{bmatrix}$ $\quad B^T = \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$

C. aux.

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix} \cdot B^T = \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 0 \\ 1 & 0 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

8- $X + A = 2(X - AB^T) \Leftrightarrow$

$\Leftrightarrow X + A = 2X - 2AB^T \Leftrightarrow X + 2X = -2AB^T + A \Leftrightarrow X = 2AB^T + A$

$\Leftrightarrow$

$X =$

$$+ \; A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix} = 2X \quad - 2 \cdot \quad A \cdot B = \begin{bmatrix} 2 & 5 & 0 \\ 1 & 0 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

$$\Leftrightarrow X = \quad A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix} + \quad 2AB = \begin{bmatrix} 4 & 10 & 0 \\ 2 & 0 & 4 \\ 4 & 4 & 12 \end{bmatrix}$$

$$\Leftrightarrow X = \begin{bmatrix} 5 & 9 & 3 \\ 2 & 1 & 4 \\ 5 & 6 & 12 \end{bmatrix}$$

9-

a) $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad B^T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad -B^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

11- $(A-B)(A+B) - (A+B)^2 + 2B^2 =$

$= \cancel{A^2} - \cancel{B^2} - \cancel{A^2} - 2AB - \cancel{B^2} + \cancel{2B^2} = -2AB$

12- $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix}$ $\quad A' = \begin{bmatrix} -1 & 1 & 1 \\ 3 & -2 & -2 \\ 3 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix}$ $\quad A' = \begin{bmatrix} -2 & 1 & 1 \\ 3 & 1 & -1 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 & 1 \\ 0 & -1 & 0 \\ -3 & 5 & 2 \end{bmatrix}$

c) $A = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 0 & 2 & -1 & 0 \\ 3 & 0 & -2 & 1 \end{bmatrix}_{3\times 4} \cdot A' = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 3 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}_{4\times 3} = \begin{bmatrix} 4 & 2 & 0 \\ -3 & 2 & -3 \\ 0 & 4 & 0 \end{bmatrix}$

13-

a) $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot A' \begin{bmatrix} & \\ & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$