→ Álgebra linear - Determinantes

1-

a)

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\det_A = 1\times4 - 2\times3 = -2 \neq 0$ A é invertível

$B = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ $\det_B = 3\times4 - 6\times2 = 0$ logo B ñ é invertível

$C = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$ $\det_C = +1\times\begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix} - 0\begin{bmatrix} 4 & 6 \\ 0 & 3 \end{bmatrix} + 0\begin{bmatrix} 4 & 6 \\ 2 & 5 \end{bmatrix}$

$= 1\times(2\times3 - 0\times5) - 0\times(4\times3 - 6\times0) + 0\times(4\times5 - 6\times2)$

$= 6 - 0 + 0 = 6 \neq 0$ logo C é invertível

$D = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ $\det_D = +1\times\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} - 1\times\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} + 1\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$

$= 1\times(2\times3 - 2\times3) - 1(2\times3 - 2\times3) + 1(2\times3 - 2\times3)$

$= 0$ logo D ñ é invertível

$E = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\det_E = +(-1)\cdot\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} - 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$

$= (-1)(2\times1 - 0\times0) - 0 + 0$

$= -2 \neq 0$ logo E é invertível

b)

$\det 3A = 3^2\times(-2) = -18 \neq 0$ logo é invertível

$\det AB = \det A \cdot \det B = (-2)\times0 = 0$ ñ é invertível

$\det|-A^2| = (-1)^2 2^2 = 4$

$\det|A^{-1}| = -\frac{1}{2}$

$\det(-2C^T) = (-2)^3\, 6 = -48$

$\det|(CD)^T| = 0$

$\det|C^n E^{-1}| = -\frac{6^n}{2}$

2-

a)

$$A = \begin{bmatrix} 2 & 2 & 0 & -2 \\ 3 & 1 & 1 & 1 \\ -1 & -2 & 0 & 3 \\ -2 & 0 & 2 & -1 \end{bmatrix} \quad L_1 \leftarrow \tfrac{1}{2} L_1 \quad 2 \begin{bmatrix} 1 & 1 & 0 & -1 \\ 3 & 1 & 1 & 1 \\ -1 & -2 & 0 & 3 \\ -2 & 0 & 2 & -1 \end{bmatrix} \quad \begin{matrix} L_2 \leftarrow L_2 - 3L_1 \\ L_3 \leftarrow L_3 + L_1 \\ L_4 \leftarrow L_4 + 2L_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & -2 & 1 & 4 \\ 0 & -1 & 0 & 2 \\ 0 & 2 & 2 & -3 \end{bmatrix} \quad L_3 \leftrightarrow L_2 \quad -2 \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & -1 & 0 & 2 \\ 0 & -2 & 1 & 4 \\ 0 & 2 & 2 & -3 \end{bmatrix} \quad \begin{matrix} L_3 \leftarrow L_3 - 2L_2 \\ L_4 \leftarrow L_4 + 2L_2 \end{matrix} \quad \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 \end{bmatrix} \quad L_4 \leftarrow L_4 - 2L_3$$

$$-2 \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \begin{matrix} L_4 \leftarrow -L_4 \\ L_1 \leftarrow L_1 + L_4 \\ L_2 \leftarrow L_2 + 2L_4 \end{matrix} \quad -2 \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} L_1 \leftarrow L_1 - L_2 \\ L_2 \leftarrow -L_2 \end{matrix} \quad +2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= +2 \times (1 \times 1 \times 1 \times 1) = 2$$

b)

$$A = \begin{bmatrix} 2 & 2 & 0 & -2 \\ 3 & 1 & 1 & 1 \\ -1 & -2 & 0 & 3 \\ -2 & 0 & 2 & -1 \end{bmatrix} \quad \det A = 0 \times \begin{bmatrix} 3 & 1 & 1 \\ -1 & -2 & 3 \\ -2 & 0 & -1 \end{bmatrix} - 1 \begin{bmatrix} 3 & 2 & -2 \\ -1 & -2 & 3 \\ -2 & 0 & -1 \end{bmatrix} + 0 \times \begin{bmatrix} 2 & 2 & -2 \\ 3 & 1 & 1 \\ -2 & 0 & -1 \end{bmatrix} \quad (1)$$

$$-2 \begin{bmatrix} 2 & 2 & -2 \\ 3 & 1 & 1 \\ -1 & -2 & 3 \end{bmatrix} = 0 - 1 \begin{bmatrix} 3 & 2 & -2 \\ -1 & -2 & 3 \\ -2 & 0 & -1 \end{bmatrix} + 0 - 2 \begin{bmatrix} 2 & 2 & -2 \\ 3 & 1 & 1 \\ -1 & -2 & 3 \end{bmatrix}$$

$$= -1 \left(-2 \begin{bmatrix} -1 & 3 \\ -2 & -1 \end{bmatrix} + (-2) \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix} \right) - 2 \left(-3 \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix} + 1 \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix} - 1 \begin{bmatrix} 2 & 2 \\ -1 & -2 \end{bmatrix} \right) \quad (1)$$

$$= -1 \left(-2 \times (-1 \times (-1) - (-2) \times 3) - 2 (3 \times 1 - 2 \times (-2)) \right) - 2 \left(-3 (2 \times 3 - (-2 \times (-2))) + 1 (2 \times 3 - (-2) \times (-1)) - 1 (2 \times (-2) - 2 \times (-1)) \right) = 4$$

$$\begin{bmatrix} [illegible] & 2 & 0 & -2 \\ 3 & 1 & 1 & 1 \\ -1 & -2 & 0 & 3 \\ -2 & 0 & 2 & -1 \end{bmatrix} \xrightarrow[\text{de Gauss}]{\text{Pelo método}} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 0 & 2 \\ 0 & -2 & 1 & 4 \\ 0 & 2 & 2 & 3 \end{bmatrix}$$

método de Laplace

$$= 2 \times 1 \left(\begin{bmatrix} -1 & 0 & 2 \\ -2 & 1 & 4 \\ 2 & 2 & 3 \end{bmatrix} \right) = 2 \left(-1 \begin{vmatrix} -1 & 2 \end{vmatrix} \right.$$

a) $A_1 = \begin{bmatrix} 5 & -4 & 3 \\ 0 & -1 & 0 \\ 1 & -3 & 2 \end{bmatrix}$ $\det A_1 = -0 \times \begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix} + (-1) \times \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} - 0 \times \begin{bmatrix} 5 & -4 \\ 1 & -3 \end{bmatrix}$

$= -1 \times (5 \times 2 - 3 \times 1) = -7$

$A_2 = \begin{bmatrix} 1/2 & 4 & 1 & 3 \\ 0 & -1 & 0 & 3/2 \\ -1 & 0 & 2 & -2 \\ 3 & 0 & -1 & -1 \end{bmatrix}$ $\det A_2 = +(-1) \begin{bmatrix} 1/2 & 1 & 3 \\ -1 & 2 & -2 \\ 3 & -1 & -1 \end{bmatrix} + 3/2 \begin{bmatrix} 1/2 & 4 & 1 \\ -1 & 0 & 2 \\ 3 & 0 & -1 \end{bmatrix}$

$= (-1) \left(-(-1) \begin{bmatrix} 1 & 3 \\ -1 & -1 \end{bmatrix} + 2 \begin{bmatrix} 1/2 & 3 \\ 3 & -1 \end{bmatrix} - (-2) \begin{bmatrix} 1/2 & 1 \\ 3 & -1 \end{bmatrix} \right) + \frac{3}{2} \left(-4 \begin{bmatrix} 1/2 & 1 \\ 3 & -1 \end{bmatrix} \right)$

$-1 \quad -(-3)$

$= (-1) \left(1 \times (1 \times (-1) - (3 \times (-1)) + 2 \times \left(\left(\frac{1}{2} \times (-1)\right) - 3 \times 3 \right) + 2 \left(\frac{1}{2} \times (-1) - 1 \times 3 \right) \right)$

$+ \frac{3}{2} \left(-4 \times \left(\frac{1}{2} \times (-1) - 1 \times 3 \right) \right)$

$= (-1) \left(\overset{-1+3}{\underset{2}{-1-(-3)}} + 2\left(\overset{-\frac{19}{2}}{-\frac{1}{2} - 9} \right) + 2 \left(-\frac{1}{2} - 3 \right) \right) + \frac{3}{2} \left(-4 \left(-\frac{1}{2} - 3 \right) \right)$

$= (-1) \left(2 + 2\left(-\frac{19}{2} \right) + 2 \left(-\frac{7}{2} \right) \right) + \frac{3}{2} \left(-4 \left(-\frac{7}{2} \right) \right)$ $\frac{7}{2}$

$= (-1) (2 - 19 - 7) + \frac{3}{2} (14) = 24 \quad 21$

$A_3 = \begin{bmatrix} 1 & 0 & -3 & 4 \\ 0 & -2 & 4 & 0 \\ -1 & -2 & 3 & 1 \\ 0 & 4 & -4 & -5 \end{bmatrix}$ $\det_{A_3} = 1 \times \begin{bmatrix} -2 & 4 & 0 \\ -2 & 3 & 1 \\ 4 & -4 & -5 \end{bmatrix} - 1 \begin{bmatrix} 0 & -3 & 4 \\ -2 & 4 & 0 \\ 4 & -4 & -5 \end{bmatrix}$

$= +(-2)\begin{bmatrix} 3 & 1 \\ -4 & -5 \end{bmatrix} - 4 \begin{bmatrix} -2 & 1 \\ 4 & -5 \end{bmatrix} - 1 \left(-(-2) \begin{bmatrix} -3 & 4 \\ -4 & -5 \end{bmatrix} + 4 \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix} \right)$

$= -2(\overset{-15}{3 \times (-5)} - \overset{+4}{1 \times (-4)}) - 4(\overset{10}{-2 \times (-5)} - \overset{-4}{1 \times 4}) - 1\Big(2(\overset{+15}{3 \times (-5)} - \overset{+16}{4 \times (-4)})$

$+4(-3 \times 0 - 4 \times 4)$

$+4($

$= -2(-11) - 4(6) - 1(2(31) + 4(-16))$

$= 22 - 24 - 1(62 - 64) = -2 - 1(-2) = 0$

b)

6 -

$$A_t = \begin{bmatrix} 5 & t & 3 \\ t & -1 & 0 \\ 1 & -3 & -2 \end{bmatrix} \quad \det A_t = -t \begin{bmatrix} t & 3 \\ -3 & -2 \end{bmatrix} + (-1) \begin{bmatrix} 5 & 3 \\ 1 & -2 \end{bmatrix}$$

$$= -t\left(-2t - (-9)\right) - 1\overset{-13}{(-10 - 3)}$$

$$= 2t^2 - 9t + 13$$

- A equação $\det(A_t) = 0$ é equivalente a $2t^2 - 9t + 13 = 0$. Esta equação não tem soluções reais, pelo que A_t é invertível para todo o $t \in \mathbb{R}$