Estatística Aplicada - Ficha 9

1.

a)

- Maquina A

$$\bar{x}_A = \frac{3,2 + 4,1 + 3,5 + 3,0 + 3,1}{5} = 3,38$$

- Maquina B

$$\bar{x}_B = \frac{4,9 + 4,5 + 4,5 + 4,0 + 4,2}{5} = 4,42$$

- Maquina C

$$\bar{x}_C = \frac{3,0 + 2,9 + 3,7 + 3,5 + 4,2}{5} = 3,46$$

$$\bar{x}_{global} = \frac{3,38 + 4,42 + 3,46}{3} = 3,75$$

- $SPQ = \sum_{i=1}^{k} \sum_{j=1}^{m} (y_{ij} - \bar{y}_i)^2$

- $A = (3,2 - 3,75)^2 + (4,1 - 3,75)^2 + (3,5 - 3,75)^2 + (3,0 - 3,75)^2 + (3,1 - 3,75)^2$

$= (-0,55)^2 + (0,35)^2 + (-0,25)^2 + (0,75)^2 + (0,65)^2$

$= 1,4725$

- $B = (4,9 - 3,75)^2 + (4,5 - 3,75)^2 + (4,5 - 3,75)^2 + (4,0 - 3,75)^2 + (4,2 - 3,75)^2$

$= 2,7125$

- $C = (3,0 - 3,75)^2 + (2,9 - 3,75)^2 + (3,7 - 3,75)^2 + (3,5 - 3,75)^2 + (4,2 - 3,75)^2$

$= 1,5525$

- $SPQ = 1,4725 + 2,7125 + 1,5525 = 5,7375$

- $SQT = m \cdot \sum_{i=1}^{k} (\bar{x}_i - \bar{y})^2$

- $A = (3,38 - 3,75)^2 = 0,1369$
- $B = (4,42 - 3,75)^2 = 0,4489$
- $C = (3,46 - 3,75)^2 = (-0,29)^2 = 0,0841$

- $SQT = 5 \times (0,1369 + 0,4489 + 0,0841)$

$= 5 \times 0,6699$

$= 3,3496$

- Hipóteses:

$H_0: \mu_A = \mu_B = \mu_C$

$H_1: \exists i,j : \mu_i \neq \mu_j \quad i,j \in \{A,B,C\}$

$\bullet SQR = \sum_{i=1}^{k} \sum_{j=1}^{m} (y_{ij} - \bar{y}_i)^2$

$\bullet A = (3,3-3,38)^2 + (4,1-3,38)^2 + (3,6-3,38)^2 + (3,0-3,38)^2 + (3,7-3,38)^2$

$\quad = 0,788$

$\bullet B = (4,9-4,42)^2 + (4,6-4,42)^2 + (4,5-4,42)^2 + (4,0-4,42)^2 + (4,2-4,42)^2$

$\quad = 0,468$

$\bullet C = (3,0-3,46)^2 + (2,9-3,46)^2 + (3,7-3,46)^2 + (3,5-3,46)^2 + (4,2-3,46)^2$

$\quad = 1,132$

$\bullet SQR = 0,788 + 0,468 + 1,132 = 2,388$

$\bullet$ Graus de liberdade:

$\quad GL_{SQT} = K-1 = 3-1 = 2$

$\quad GL_{SQR} = k(m-1) = 3 \times (5-1) = 12$

$\circ F = \dfrac{SQT/GL_{SQT}}{SQR/GL_{SQR}} = \dfrac{3,3445/2}{3,388/12} \bigg\rfloor MQR = 3,42$

$\bullet F_{crítico} = 3,89$
  $\quad \hookrightarrow$ tabela 8: numerador 2
  $\qquad$ denominador 12
  $\qquad \alpha = 0,05$

$\bullet$ Como a ET pertence à região de rejeição então rejeita-se Ho, para um nível de confiança de 95%. Logo, existem diferenças significativas na tensão dos pesos devido às maquinas

$\therefore F > F_{crítico}$ então rejeitamos a hipotese nula
$\quad 3,42 > 3,89$

$\rightarrow$ Ñ existem diferenças significativas entre as máquinas

b) $1 - \alpha = 0,9 \Rightarrow \alpha = 0,1$

$\mu_B = 4,42 \qquad \mu_C = 3,46$

$\bullet t_{12;0,05} = 1,782$

$IC = (\mu_B - \mu_C) \pm t_{(1-\alpha;12)} \sqrt{MQR\left(\dfrac{1}{m_B} + \dfrac{1}{m_C}\right)}$

$= (4,42 - 3,46) \pm 1,782 \sqrt{0,199 \cdot \left(\dfrac{1}{5} + \dfrac{1}{5}\right)}$

$= 0,96 \pm 0,53$

$\bullet$ A diferença entre as médias das maquinas B e C está contida no intervalo $]0,457 ; 1,263[$ ao considerar o nível de confiança de 90%

2-
• Temperate?

$$\bar{x}_A = \frac{73 + 64 + 67 + 62 + 70}{5} = 67,2$$

$$\bar{x}_B = \frac{84 + 80 + 81 + 77}{4} = 80,5$$

$$\bar{x}_C = \frac{32 + 79 + 71 + 75}{4} = 76,75$$

• Total$_A$ = 336
• Total$_B$ = 322   T = 965
• Total$_C$ = 307

$$SQT = \left[\frac{(336)^2}{5} + \left(\frac{322^2}{4}\right) + \left(\frac{307^2}{4}\right)\right] - \frac{1}{13} \cdot 965^2 = 429,7677$$

• Total^2_A = 22658
• Total^2_B = 25946   Total^2_t = 72035
• Total^2_C = 23631

$$STQ = 72035 - \frac{965^2}{13} = 602,3077$$

$$SQR = 602,3077 - 429,7577 = 172,55$$

• GL
  • K-1 = 2
  • 13-3 = 10
  • 12

$$F = \frac{429,7577 / 2}{172,55 / 10} = 12,456$$

• H$_0$: Não existem diferenças entre o tempo de duração dos marcos
$$\mu_A = \mu_B = \mu_C \text{ ou } a_i = 0$$

• H$_1$: existem diferenças entre o tempo de duração dos marcos
$$\exists i \quad a_i \neq 0 \quad i = 1, 2, 3$$

• Região de rejeição:
$$F > C, \quad C = F_{K-1, N-K, \alpha}$$

$$F_{0,5; 2, 10} = 4,10$$

∴ Rejeita-se

4-

• Dato A : $\bar{x}_A = \dfrac{21 + 23 + 19 + 24 + 26 + 23}{6} = 22,5$

• Dato B : $\bar{x}_B = \dfrac{19 + 21 + 20 + 18 + 22 + 20}{6} = 20$

• Dato C : $\bar{x}_C = \dfrac{15 + 10 + 13 + 14 + 11 + 15}{6} = 13$

$\Big] \ 18,5$

• $A = (21 - 18,5)^2 + (23 - 18,5)^2 + (19 - 18,5)^2 + (24 - 18,5)^2 + (26 - 18,5)^2 + (23 - 18,5)^2$

$= 6,25 + 20,25 + 0,26 + 30,25 + 42,25 + 20,25 = 119,5$

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- $B = (19-18,5)^2 + (21-18,5)^2 + (20-18,5)^2 + (18-18,5)^2 + (22-18,5)^2 + (20-18,5)^2$

$= 0,25 + 6,25 + 2,25 + 0,25 + 12,25 + 2,25 = 23,5$

- $C = (15-18,5)^2 + (10-18,5)^2 + (13-18,5)^2 + (14-18,5)^2 + (11-18,5)^2 + (15-18,5)^2$

$= 12,25 + 72,25 + 30,25 + 20,25 + 56,25 + 12,25 = 203,5$

- $STQ = 119,5 + 23,5 + 203,5 = 346,5$

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- $A = (22,5-18,5)^2 = 16$
- $B = (20-18,5)^2 = 2,25$
- $C = (13-18,5)^2 = 30,25$
- $SQT = 6 \times (16 + 2,25 + 30,25) = 6 \times 48,5 = 291$

- $A = (21-22,5)^2 + (25-23,5)^2 + (19-22,5)^2 + (24-22,5)^2 + (20-22,5)^2 + (23-22,5)^2$

$= 2,25 + 0,25 + 12,25 + 2,25 + 6,25 + 0,25 = 23,5$

- $B = (19-20)^2 + (21-20)^2 + (20-20)^2 + (18-20)^2 + (22-20)^2 + (20-20)^2$

$= 1 + 1 + 0 + 4 + 4 + 0 = 10$

- $C = (15-13)^2 + (10-3)^2 + (13-13)^2 + (14-13)^2 + (11-13)^2 + (5-13)^2$

$= 4 + 9 + 0 + 1 + 4 + 4 = 22$

- $SQR = 23,5 + 10 + 22 = 55,5$

- Graus de liberdade

$GL_{SQT} = 3 - 1 = 2$

$GL_{SQR} = 16 - K = 18 - 3 = 15$

$F = \dfrac{SQT/2}{SQR/15} = 39,32$

- $F_{crítico} = 3,68$

  4×8

  numerador 2

  denominador 16

  $\propto = 0,05$

$\therefore F > F_{crítico}$ então rejeitamos a hipótese nula

5-

ⓐ

$\overline{x}_{m1} = \dfrac{12+2+8+1+7}{5} = 6$

$\overline{x}_{m2} = \dfrac{20+14+17+12+17}{5} = 16$ $\overline{x}_{global} = 10$

$\overline{x}_{m3} = \dfrac{13+7+13+8+14}{5} = 11$

$\overline{x}_{m4} = \dfrac{11+5+10+3+6}{5} = 7$

• $m_1 = (12-10)^2 + (2-10)^2 + (8-10)^2 + (1-10)^2 + (7-10)^2$
  $= 4 + 64 + 4 + 81 + 9 = 162$

• $m_2 = (20-10)^2 + (14-10)^2 + (17-10)^2 + (12-10)^2 + (17-10)^2$
  $= 100 + 16 + 49 + 4 + 49 = 173$

• $m_3 = (13-10)^2 + (7-10)^2 + (13-10)^2 + (8-10)^2 + (14-10)^2$
  $= 9 + 9 + 9 + 4 + 16 = 47$

• $m_4 = (11-10)^2 + (5-10)^2 + (10-10)^2 + (3-10)^2 + (6-10)^2$
  $= 10 + 25 + 0 + 49 + 16 = 100$

$STQ = 482$

$SQA = 201$

• $SQT = 5 \times (65) = 325$ • $m_1 = (12-6)^2 + (2-6)^2 + (8-6)^2 + (1-6)^2 + (7-6)^2$
$m_1 = (6-10)^2 = 16$ $= 36 + 16 + 4 + 25 + 1 = 82$
$m_2 = (16-10)^2 = 36$
$m_3 = (11-10)^2 = 1$ • $m_2 = (20-16)^2 + (14-16)^2 + (17-16)^2 + (12-16)^2 + (17-16)^2$
$m_4 = (7-10)^2 = 9$ $= 16 + 4 + 1 + 16 + 1 = 38$
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        $\parallel$ • $m_3 = (13-11)^2 + (7-11)^2 + (13-11)^2 + (8-11)^2 + (14-11)^2$
                                     $= 4 + 9 + 4 + 9 + 9 = 35$

• $m_4 = (11-7)^2 + (5-7)^2 + (10-7)^2 + (3-7)^2 + (6-7)^2$
  $= 16 + 4 + 9 + 16 + 1 = 46$