



Universidade do Minho

Escola de Engenharia

Departamento de Informática

Programação em Lógica

LICENCIATURA EM ENGENHARIA INFORMÁTICA
MESTRADO integrado EM ENGENHARIA INFORMÁTICA

Inteligência Artificial
2025/26

From the last lesson

Limitations of problem solving problems

- Search systems are very efficient at solving problems that can be formalised by:
 - an initial state, actions and final state (or final states).
- but they are not able to solve problems that require **reasoning based on knowledge about the world**.
 - Because their *model of the world* is poor and their thinking is limited
 - e.g. medical diagnostics, expert systems in general,...
 - even in cases that are (apparently) solvable by search (using planning), it may be necessary to add explicit knowledge.

- Axioms: initial set of logical formulae
- Theorems: formulae derived from axioms and/or theorems (semantic consequences)
- Inference Rules: set of derivation rules
 - *Modus ponens* $\{ (A \text{ se } B), B \} \vdash A$ (sound - valid)
 - *Modus tollens* $\{ (A \text{ se } B), \neg A \} \vdash \neg B$ (sound - valid)
 - *Modus mistakens* $\{ (A \text{ se } B), A \} \vdash B$ (unsound – not valid)
- Inference system: union of axioms and derivation rules R
- Proof: sequence $\langle s_1, s_2, \dots \rangle$ of s_i that are axioms or are derivations using R and a subset of the members of the sequence that precede s_i ;
 - The sequence is a proof for s_n (derivation or deduction)
- Theory: union of the axioms and all the theorems derived using R
 - It is said to be consistent if there is no formula s such that, in theory T, there exists s and $\neg s$
- **None of these considerations take meaning into account!**
Only syntactic structure!!!

- Knowledge and Reasoning;
- Logic;
- Logic programming;
- **Knowledge-Based Systems.**



Synthetic Intelligence Lab

Logic Programming

- Logic Programming is a computer formalism that combines 2 basic principles:
 1. Uses logic to represent knowledge
(representation of assumptions and conclusions)
 1. Use Inference to manipulate knowledge
(establish logical relationships between assumptions and conclusions)
(mechanising proof procedures; reasoning)

Characterisation of Logic Programming **PROLOG**

- A programme in PROLOG is created by adding formulas called clauses
- The clauses can be of 3 types:
 - Facts: express something that is always true

p. son(xico,quim).
 - Rules: express something that is true, depending on the veracity of the conditions

p if q. father(josé,joão) if son(joão,josé).
 - Questions: express something that is true, depending on the veracity of the conditions

?q. ? father(josé,joão).
¬q. ¬father (Joseph, John).

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p if q.

- Horn's Clause (clausal notation of first-order logic)
 - It's a restricted version of Predicative Calculus
 - It's a well-formed formula
 - All formulas are universally quantified
 - All formulas are closed
 - Logical formulae only allow 1 term in the positive disjunction of literals

Horn clauses

- $\neg q \vee p_j$
- where $i \geq 0$
- $0 \leq j \leq 1$

- Reasoning:
 - A set of linked thoughts
 - Linking facts that lead to a conclusion
 - A mixture of the previous 2



- Reasoning:
 1. ...
 2. Logical train of thought
 3. ...
 4. [LOGIC] Discursive operation of thought by which we conclude that one or more propositions (premises) imply the truth (...) of another proposition (conclusion).

(in Dicionário da Língua Portuguesa, Porto Editora)



- Three men, António, Belmiro and Carlos, are Dulce, Eduarda and Filipa's spouses.
- We don't know who is married to whom.
- Their backgrounds are in engineering, law and medicine.
- We don't know who does what.
- Based on the data below, find out the name of each wife and the profession of each man:
 - The doctor is married to Filipa;
 - Carlos is a lawyer;
 - Eduarda is not married to Carlos;
 - Belmiro is not a doctor.

(adapted from: www.novaconcursos.com.br/portal/dicas/questao-raciocinio-logico-para-concursos)

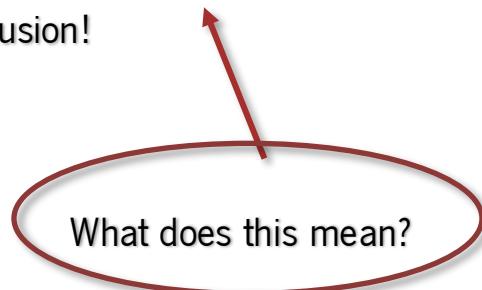
- In order to convince people that the answer is not "a lucky shot", you need to explain the reasons that lead you to your conclusion!

conclusion!



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conclusion!



- In order for the conclusion to be accepted, it is necessary to explain the reasoning mechanism applied, i.e. the **process of inference**.

- Inference:
 - It refers to the applied process that allows us to move from premises to conclusion.
 - Inference: deduction or conclusion.

(in Priberam Dictionary of the Portuguese Language)



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- Inference rules:
 - *Modus ponens*:
 - from: $P, P \rightarrow Q$
 - it follows: Q

▪ Inference:

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▪ Inference rules:

- *Modus ponens*: (*modus ponendo ponens* Latin meaning "the way that affirms by affirming")

- from: $P, P \rightarrow Q$
- it follows: Q

P	Q	$P \rightarrow Q$
V	V	V
V	F	F
F	V	V
F	F	V

- Inference:

- It refers to the applied process that allows us to move from premises to conclusion.
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- Inference rules:

- *Modus ponens*:

- from: $P, P \rightarrow Q$
 - it follows: Q

- *Modus tollens*:

- from: $P \rightarrow Q, \neg Q$
 - inferred: $\neg P$

- Inference:

- It refers to the applied process that allows us to move from premises to conclusion.
- Inference: deduction or conclusion.

(in Priberam Dictionary of the Portuguese Language)

- Inference rules:

- *Modus ponens*:

- from: $P, P \rightarrow Q$
- it follows: Q

- *Modus tollens* (Latin: way that negates by negation):

- from: $P \rightarrow Q, \neg Q$
- inferred: $\neg P$

P	Q	$\neg Q$	$P \rightarrow Q$
∨	∨	False	∨
∨	False	∨	False
False	∨	False	∨
False	False	∨	∨

- The *modus ponens* inference rule makes it possible to derive the conclusion of a clause as true by proving its conditions:

$$\{ (A \text{ if } B), B \} \vdash A$$

- The *modus ponens* inference rule makes it possible to derive the conclusion of a clause as true by proving its conditions:
$$\{ (A \text{ if } B), B \} \vdash A$$
- Applying the *modus tollens* rule of inference makes it possible to direct the search for proof to a specific point in the reasoning process:

$$\{ (A \text{ if } B), \neg A \} \vdash \neg B$$

- which makes it possible to develop a **mechanism of proof by contradiction**.

- Suppose we have a programme \square in which we want to determine the derivability of a question A:
 - i. Let's admit the negation of A:
 $\neg A$
 - ii. Insert the negation of A into the programme \square :
 $\mathcal{P} \cup \neg A$
- If a contradiction occurs
 $\{ A, \neg A \} \equiv \square$
it means that the initial question A is derivable from \mathcal{P} .

- Suppose we have a programme \square in which we want to determine the derivability of a question A:

i. Let's admit the negation of A:

$$\neg A$$

ii. Insert the negation of A into the programme \square :

$$\mathcal{P} \cup \neg A$$

- If a contradiction occurs

$$\{ A, \neg A \} \equiv \square$$

it means that the initial question A is derivable from \mathcal{P} .

- Modus tollens inference rule, with B = true

$$\{ (A \text{ se } B), \neg A \} \vdash \neg B$$

$$\{ (A \text{ se } \vee), \neg A \} \vdash \neg \vee$$

$$\{ A, \neg A \} \vdash \text{F}$$

$$\{ A, \neg A \} \vdash \square$$

Resolution Algorithm Application example

% son: Son,Father → {V, F}

```
son( joao,jose ).  
son( jose,manuel ).  
son( carlos,jose ).
```

% father: Father,Son → {V, F}

```
father( P,F ) :- son( F,P ).
```

Consider the programme \mathcal{P} shown opposite

Resolution Algorithm Application example (I)

- Is João José's son?

% son: Son,Father → {V, F}

son(joao,jose).
son(jose,manuel).
son(carlos,jose).

% father: Father,Son → {V, F}

father(P,F) :- son(F,P).

Let's take the question posed above

Resolution Algorithm Application example (I)

- Is João José's son?

$\neg \text{son}(\text{john}, \text{jose})$

% son: Son,Father $\rightarrow \{\forall, \exists\}$

son(joao,jose).
son(jose,manuel).
son(carlos,jose).

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father(P,F) :- son(F,P).

In logical terms, the question is described as follows

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father(P,F) :- son(F,P).

Of all the clauses in the programme \mathcal{P} , only those that offer conclusions about the predicate `father/2` will contribute to the development of the test

Resolution Algorithm Application example (I)

- Is João José's son?

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son(jose,manuel).
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% father: Father,Son $\rightarrow \{\forall, \exists\}$

father(P,F) :- son(F,P).

Of the three clauses listed, the first will be used in the first step of developing the (tree of) evidence

Resolution Algorithm Application example (I)

- Is João José's son?

$\neg \text{son}(\text{john}, \text{jose})$


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son(joao, jose).
son(jose, manuel).
son(carlos, jose).

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father(P, F) :- son(F, P).

The word joao unites with joao; the word jose unites with jose;

Resolution Algorithm Application example (I)

- Is João José's son?

```

¬son( john,jose )
  ↳ joao | joao, jose | jose
  ¬(true)
  
```

% son: Son,Father → { \forall , \exists }

```

son( joao,jose ).  

son( jose,manuel ).  

son( carlos,jose ).  

  
```

% father: Father,Son → { \forall , \exists }

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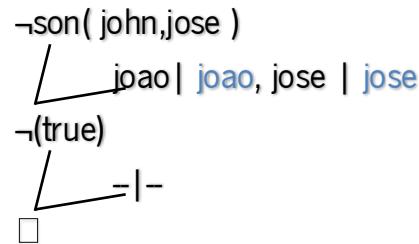
father( P,F ) :- son( F,P ).  

  
```

Under the conditions established by the unifications, which used the first clause of \mathcal{P} , the initial question is reduced to the current one, because it is a fact

Resolution Algorithm Application example (I)

- Is João José's son?



% son: Son,Father → {V, F}

son(joao,jose).
 son(jose,manuel).
 son(carlos,jose).

% father: Father,Son → {V, F}

father(P,F) :- son(F,P).

From which a contradiction can be drawn □ (regardless of any unifications)

Resolution Algorithm Application example (II)

- Is José João's father?

% son: Son,Father → {V, F}

```
son( joao,jose ).  
son( jose,manuel ).  
son( carlos,jose ).
```

% father: Father,Son → {V, F}

```
father( P,F ) :- son( F,P ).
```

You want to know if José is João's father

Resolution Algorithm Application example (II)

- Is José João's father?

$\neg\text{father}(\text{jose},\text{joao})$

% son: Son,Father $\rightarrow \{\forall, \exists\}$

son(joao,jose).
son(jose,manuel).
son(carlos,jose).

% father: Father,Son $\rightarrow \{\forall, \exists\}$

father(P,F) :- son(F,P).

In logical terms, the question is described as follows

Resolution Algorithm Application example (II)

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Of all the clauses in the \mathcal{P} programme, only those that offer conclusions about the `father/2` predicate will contribute to the development of the test

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son(carlos,jose).

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father(P,F) :- son(F,P).

The only clause of the `father/2` predicate that has been ticked will be used in the first step of developing the (proof) tree

Resolution Algorithm Application example (II)

- Is José João's father?

$\neg \text{father}(\text{jose}, \text{joao})$

└── $\text{jose} \mid \text{P}$, $\text{joao} \mid \text{F}$

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 son(jose,manuel).
 son(carlos,jose).

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father(P,F) :- son(F,P).

The term jose unifies with the variable P ; the term joao unifies with the variable E ;

Resolution Algorithm Application example (II)

- Is José João's father?

```

¬father( jose,joao )
  ↳ jose | P, joao | F
¬child( john,jose )

```

% son: Son,Father • {[],•}

son(joao,jose).
son(jose,manuel).
son(carlos,jose).

% father: Father,Son • {[],•}

father(P,F) :- son(F,P).

To continue the (tree of) proof, the initial question is reduced to proof of the conditions of the clause used to make the unifications

Resolution Algorithm

Application example (II)

- Is José João's father?

$\neg \text{father}(\text{jose}, \text{joao})$

joao | P, joao | F

$\neg \text{son}(\text{john}, \text{jose})$

% son: Son,Father → {V, F}

son(joao,jose).

son(jose,manuel).

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% father: Father,Son → {V, F}

`father(P,F) :- son(F,P).`

Of all the clauses in the programme \mathcal{P} , only those that offer conclusions about the predicate `son/2` will contribute to the development of the test.

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¬father( jose,joao )
  ↳ jose | P, joao | F
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Of the three clauses listed, the first will be used in the first step of developing the (tree of) evidence

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- Is José João's father?

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¬father( jose,joao )
  ↳ joao | P, joao | F
¬son( john,jose )
  ↳ joao | joao, jose | jose

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The word joao unites with joao; the word jose unites with jose;

Resolution Algorithm Application example (II)

- Is José João's father?

```

¬father( jose,joao )
  ↳ joao | P, joao | F
¬son( john,jose )
  ↳ joao | joao, jose | jose
¬(true)

```

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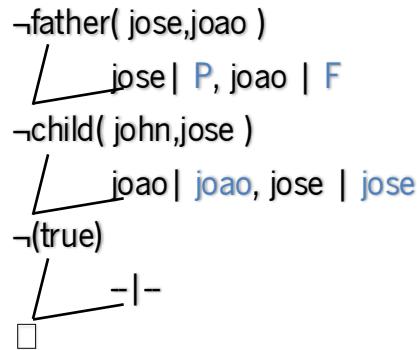
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From which a contradiction can be drawn □ (regardless of any unifications)

Resolution Algorithm Application example (III)

- Is José John's son?

% son: Son,Father → {V, F}

son(joao,jose).
son(jose,manuel).
son(carlos,jose).

% father: Father,Son → {V, F}

father(P,F) :- son(F,P).

Resolution Algorithm Application example (III)

- Is José John's son?

% son: Son,Father → {V, F}

son(joao,jose).
son(jose,manuel).
son(carlos,jose).

% father: Father,Son → {V, F}

father(P,F) :- son(F,P).

Let's take the question posed above

Resolution Algorithm Application example (III)

- Is José John's son?

$\neg \text{son}(\text{jose}, \text{joao})$

% son: Son,Father → {V, F}

son(joao,jose).
son(jose,manuel).
son(carlos,jose).

% father: Father,Son → {V, F}

father(P,F) :- son(F,P).

In logical terms, the question is described as follows; of the three clauses in the predicate `son/2`, the first will be used to develop the (tree of) evidence

Resolution Algorithm Application example (III)

- Is José John's son?

$\neg \text{son}(\text{jose}, \text{joao})$



~~jose~~ ~~joao~~, ...

% son: Son,Father $\rightarrow \{\forall, \exists\}$

son(joao,jose).
 son(jose,manuel).
 son(carlos,jose).

% father: Father,Son $\rightarrow \{\forall, \exists\}$

father(P,F) :- son(F,P).

The term jose does not unify with joao because they are two different constants, so the proof procedure cannot evolve through this branch of proof

Resolution Algorithm Application example (III)

- Is José John's son?

$\neg \text{son}(\text{jose}, \text{joao})$



~~jose~~ ~~joao~~, ...

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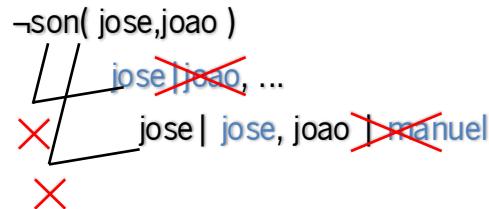
% father: Father,Son $\rightarrow \{\forall, \exists\}$

father(P,F) :- son(F,P).

Abandoning the first clause, the second is taken as an alternative to continue the development of the (tree of) proof;

Resolution Algorithm Application example (III)

- Is José John's son?



% son: Son,Father $\rightarrow \{\vee, \exists\}$

son(joao,jose).
son(jose,manuel).
son(carlos,jose).

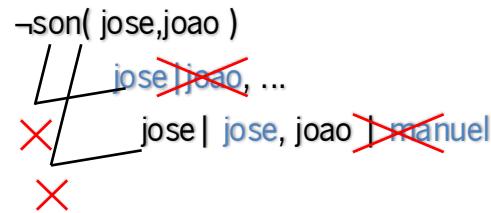
% father: Father,Son $\rightarrow \{\vee, \exists\}$

father(P,F) :- son(F,P).

The term **jose** unifies with **jose**; the term **joao** does not unify with **manuel** because they are two different constants, so the proof procedure cannot evolve through this branch of proof

Resolution Algorithm Application example (III)

- Is José John's son?



% son: Son,Father → {V, F}

son(joao,jose).
 son(jose,manuel).
son(carlos,jose).

% father: Father,Son → {V, F}

father(P,F) :- son(F,P).

Also abandoning the second clause, the third (and last) is taken as an alternative to give continuity to the development of the (tree of proof

Resolution Algorithm

Application example (III)

- Is José John's son?

$\neg \text{son}(\text{jose}, \text{joao})$

% son: Son,Father → {V, F}

```
son( joao,jose ).  
son( jose,manuel ).  
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```

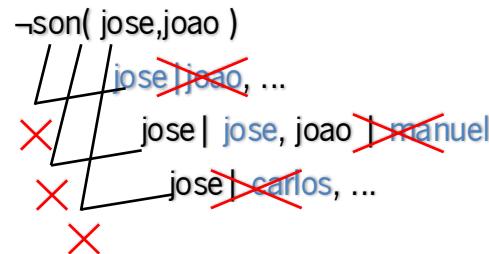
% father: Father,Son → {V, F}

`father(P,F) :- son(F,P).`

The term `jose` does not unify with `carlos` because they are two different constants, so the proof procedure cannot evolve through this branch of proof

Resolution Algorithm Application example (III)

- Is José John's son?



% son: Son,Father → {V, F}

son(joao,jose).
 son(jose,manuel).
 son(carlos,jose).

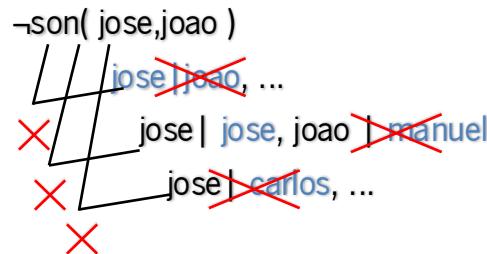
% father: Father,Son → {V, F}

father(P,F) :- son(F,P).

As there are no more clauses in the `son/2` predicate that can be considered as alternatives in the (proof) tree, the procedure of applying the resolution algorithm ends without reaching any contradiction

Resolution Algorithm Application example (III)

- Is José John's son?



~~Is José John's son?~~

% son: Son,Father $\rightarrow \{\vee, \exists\}$

son(joao,jose).
son(jose,manuel).
son(carlos,jose).

% father: Father,Son $\rightarrow \{\vee, \exists\}$

father(P,F) :- son(F,P).

This means that the initial question doesn't generate any contradiction at \mathcal{P} , so it's false.

Resolution Algorithm Application example (IV)

$\neg \text{son}(X, \text{jose})$

What is the meaning of this question?

Develop the evidence tree for this question

% son: Son,Father → {V, F}

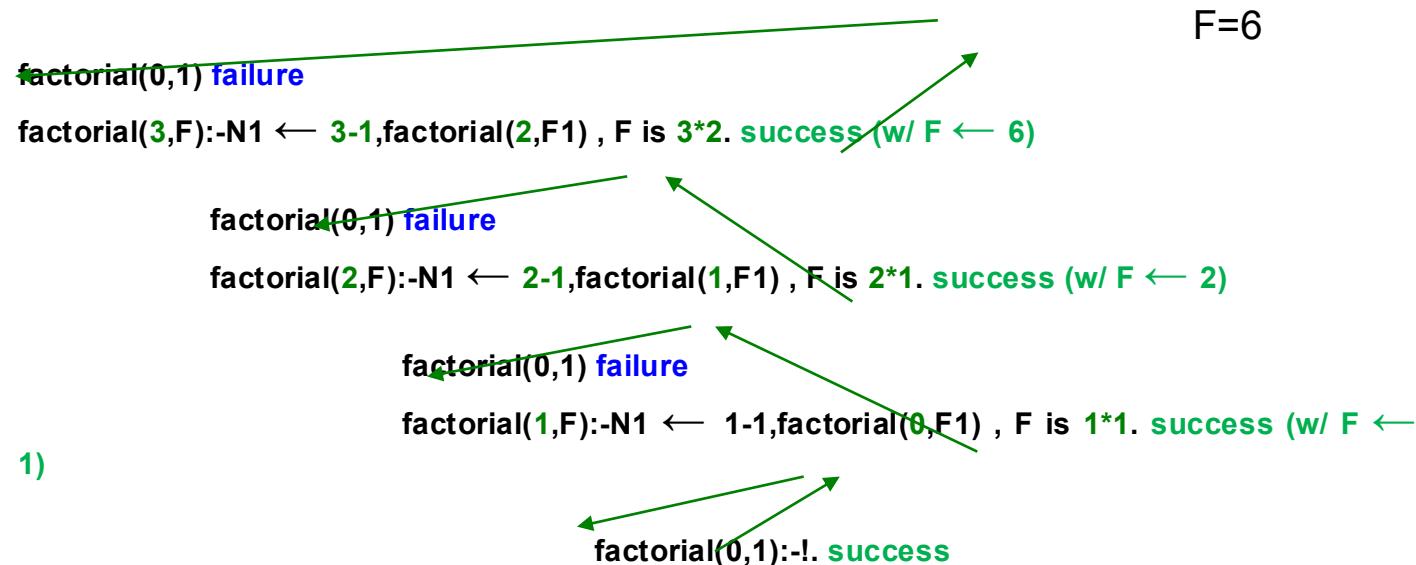
son(joao,jose).
son(jose,manuel).
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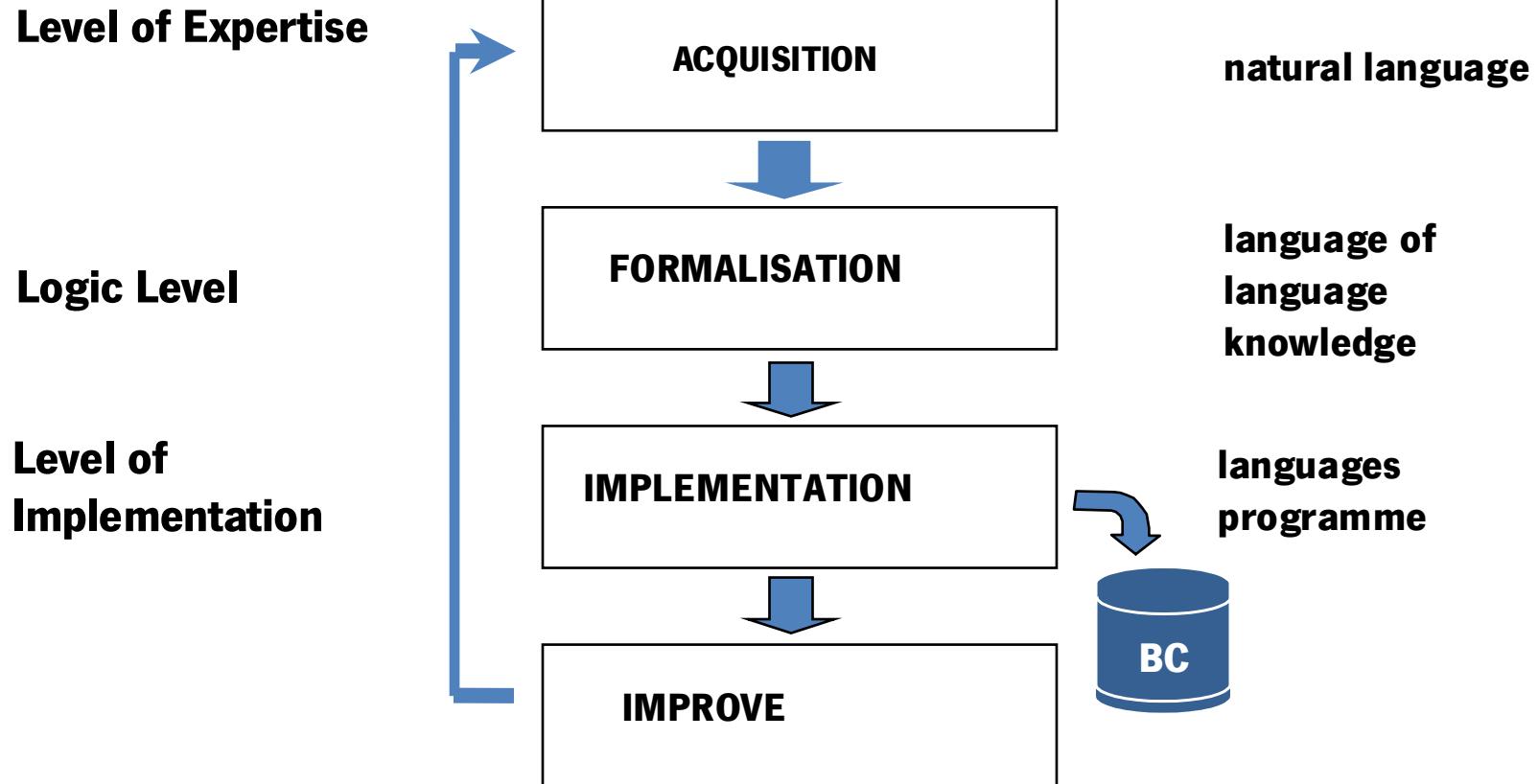
```
factorial(0,1):-!
factorial(N,F):-N1 is N-1,factorial(N1,F1),F is N*F1.
```

Let's see what happens when you call ?- factorial(3,F).



- These are systems that
 - They use explicitly represented knowledge to solve complex problems;
 - They manipulate knowledge and information;
 - They have "encrusted" the ability to reason:
 - The ability to define a set of steps for efficiently solving a problem;
 - The very mechanism of inference is knowledge.

Knowledge-Based Systems



▪ Deduction

- facts + inference rules => new facts
- cause → effect
 - If there is fire (cause), there is smoke (effect). There's fire here, so there's smoke here (new fact)
- It's the only type of inference that holds true
 - *truth-preserving*

▪ Abduction

- reverse deduction: from effect to cause
 - If there's smoke, there's fire. I saw smoke (effect), so there's fire here (cause)
- This type of inference maintains the falsehood
- We have generated a possible explanation that will have to be proven

- **Induction**

- part of the facts to generate (new) rules
 - fact1 + fact2 + fact3 → rule!
 - ex. Mr Joaquim, like Mrs Isabel, has the flu and a headache, so everyone who has the flu has a headache
- It transforms facts (knowledge in extension) into knowledge in the form of a hypothesis!

- **Analogue**

- facts + similarities + adaptation rules
- from facts, from the similarity between them, solves the problem without generating rules
 - e.g.: In the previous case of flu, I took an aspirin and it didn't work, so I'm not going to take aspirin in this similar case

- Deduction and Abduction
 - They are used in knowledge-based agents
- Induction and Analogy
 - They are used in machine learning
- Deduction: there are two large groups
 - Logic and the like
 - Treatment of uncertainty (e.g. probabilistic or *fuzzy*)

SWI-Prolog - A Free Software Prolog environment, licensed under the Lesser GNU public licence. This popular interpreter was developed by Jan Wielemaker. This is the interpreter we used while developing this book.

<http://www.swi-prolog.org/>

SICStus Prolog - Industrial strength Prolog environment from the Swedish Institute of Computer Science.

<http://www.sics.se/sicstus/>

GNU Prolog - Another more widely used free Prolog compiler developed by Daniel Diaz. <http://www.gprolog.org>

YAP Prolog - A Prolog compiler developed at the University of Porto and Federal University of Rio de Janeiro. Free for use in academic environments.

<http://www.ncc.up.pt/~vsc/Yap/>

Recommended Bibliography

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Universidade do Minho

Escola de Engenharia

Departamento de Informática

Logic Programming

BACHELOR'S DEGREE IN COMPUTER ENGINEERING

integrated master's degree in computer engineering

Artificial Intelligence

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