

1. (a) i. $X : \left\{ \begin{array}{ccc} 0 & 1 & 2 \\ \frac{9}{36} & \frac{18}{36} & \frac{9}{36} \end{array} \right\}$, $F_X(c) = \left\{ \begin{array}{lll} 0 & se & c < 0 \\ 9/36 & se & 0 \leq c < 1 \\ 27/36 & se & 1 \leq c < 2 \\ 1 & se & c \geq 2 \end{array} \right.$;

ii. igual à alínea anterior;

iii. $Z : \left\{ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} \end{array} \right\}$, $F_Z(c) = \left\{ \begin{array}{lll} 0 & se & c < 1 \\ 1/36 & se & 1 \leq c < 2 \\ 4/36 & se & 2 \leq c < 3 \\ 9/36 & se & 3 \leq c < 4 \\ 16/36 & se & 4 \leq c < 5 \\ 25/36 & se & 5 \leq c < 6 \\ 1 & se & c \geq 6 \end{array} \right.$;

vi. $N : \left\{ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{11}{36} & \frac{9}{36} & \frac{7}{36} & \frac{5}{36} & \frac{3}{36} & \frac{1}{36} \end{array} \right\}$, $F_N(c) = \left\{ \begin{array}{lll} 0 & se & c < 1 \\ 11/36 & se & 1 \leq c < 2 \\ 20/36 & se & 2 \leq c < 3 \\ 27/36 & se & 3 \leq c < 4 \\ 32/36 & se & 4 \leq c < 5 \\ 35/36 & se & 5 \leq c < 6 \\ 1 & se & c \geq 6 \end{array} \right.$;

v. $W : \left\{ \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ \frac{6}{36} & \frac{10}{36} & \frac{8}{36} & \frac{6}{36} & \frac{4}{36} & \frac{2}{36} \end{array} \right\}$, $F_W(c) = \left\{ \begin{array}{lll} 0 & se & c < 0 \\ 6/36 & se & 0 \leq c < 1 \\ 16/36 & se & 1 \leq c < 2 \\ 24/36 & se & 2 \leq c < 3 \\ 30/36 & se & 3 \leq c < 4 \\ 34/36 & se & 4 \leq c < 5 \\ 1 & se & c \geq 5 \end{array} \right.$;

vi. $S : \left\{ \begin{array}{cccccccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \frac{5}{36} & \frac{4}{36} & \frac{3}{36} & \frac{2}{36} & \frac{1}{36} \end{array} \right\}$, $F_S(c) = \left\{ \begin{array}{lll} 0 & se & c < 2 \\ 1/36 & se & 2 \leq c < 3 \\ 3/36 & se & 3 \leq c < 4 \\ 6/36 & se & 4 \leq c < 5 \\ 10/36 & se & 5 \leq c < 6 \\ 15/36 & se & 6 \leq c < 7 \\ 21/36 & se & 7 \leq c < 8 \\ 26/36 & se & 8 \leq c < 9 \\ 30/36 & se & 9 \leq c < 10 \\ 33/36 & se & 10 \leq c < 11 \\ 35/36 & se & 11 \leq c < 12 \\ 1 & se & c \geq 12 \end{array} \right.$;

vii. $V : \left\{ \begin{array}{ccc} 0 & 1 & 2 \\ \frac{25}{36} & \frac{10}{36} & \frac{1}{36} \end{array} \right\}$, $F_V(c) = \left\{ \begin{array}{lll} 0 & se & c < 0 \\ 25/36 & se & 0 \leq c < 1 \\ 35/36 & se & 1 \leq c < 2 \\ 1 & se & c \geq 2 \end{array} \right.$

(b) i. $\frac{3}{4}$; ii. $\frac{1}{4}$; iii. $\frac{1}{4}$; iv. $\frac{1}{4}$; v. $\frac{1}{6}$; vi. $\frac{5}{6}$; vii. $\frac{1}{6}$; viii. $\frac{1}{15}$

(c) $\frac{4}{9}$

2. a) $\Omega = \{(C_a, C_a), (C_a, C_o), (C_o, C_a), (C_o, C_o)\}$;

ω	$X(\omega)$	$Y(\omega)$
(C_a, C_a)	2	0
b) i. (C_a, C_o)	1	1
(C_o, C_a)	1	1
(C_o, C_o)	0	2

- ii. Funções massa de probabilidade são iguais a: $\begin{cases} 0 & \frac{1}{4} & \frac{2}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{cases}$,

$$\text{Funções de distribuição são iguais a: } F(c) = \begin{cases} 0 & \text{se } c < 0 \\ 1/4 & \text{se } 0 \leq c < 1 \\ 3/4 & \text{se } 1 \leq c < 2 \\ 1 & \text{se } c \geq 2 \end{cases}$$

Comentário: X e Y são funções distintas. Enquanto v.a.'s, têm em comum as funções que as caracterizam (a f.m.p. e a função de distribuição) e, por isso, X e Y são identicamente distribuídas (i.d.'s). Estas v.s.'s são ainda i.d.'s com as dos exercícios 1.(a)i. e 1.(a)ii..

$$3. (a) \text{ — } (b) F_X(c) = \begin{cases} 0 & \text{se } c < 0 \\ \frac{1}{2} + \frac{1}{4}(c-4) & \text{se } 0 \leq c < 4 \\ 1 & \text{se } 4 \leq c < 6 \\ 1 & \text{se } c \geq 6 \end{cases};$$

- (c) i. $\frac{3}{16}$; ii. $\frac{13}{16}$; iii. igual a ii.; iv. todas iguais a $\frac{3}{8}$; (d) $\frac{13}{16}$; $\frac{11}{13}$

$$4. (a) F_T(c) = \begin{cases} 0 & \text{se } c < 0 \\ 1 - e^{-\lambda c} & \text{se } c \geq 0 \end{cases}; \quad (b) e^{-1}; \quad (c) \frac{1}{4}e^{-2} + \frac{3}{4}e^{-4}; \quad \frac{3e^{-4}}{e^{-2}+3e^{-4}}$$

Exercícios Suplementares à Folha 2

1. (a) — (b) 0.65; 0.5; 0.5

$$(c) F_X(c) = \begin{cases} 0 & \text{se } c < 0 \\ 0.05 & \text{se } 0 \leq c < 1 \\ 0.15 & \text{se } 1 \leq c < 2 \\ 0.35 & \text{se } 2 \leq c < 3 \\ 0.5 & \text{se } 3 \leq c < 4 \\ 0.8 & \text{se } 4 \leq c < 5 \\ 1 & \text{se } c \geq 5 \end{cases}; \quad (d) \frac{0.15}{0.8}; \quad \frac{0.45}{0.8}; \quad \frac{0.3}{0.8}$$

$$2. (a) k = \frac{1}{4}; \quad F(c) = \begin{cases} 0 & \text{se } c < 1 \\ \frac{1}{8}(c^2 - 1) & \text{se } 1 \leq c < 3 \\ 1 & \text{se } c \geq 3 \end{cases}; \quad (b) \text{ i. } \frac{27}{32}; \quad \text{ii. igual a i.}; \quad \text{iii. } \frac{39}{128};$$

$$c) \frac{20}{27}$$

$$3. (a) F_X(c) = \begin{cases} 0 & \text{se } c < -1 \\ \frac{1}{2} + c + \frac{c^2}{2} & \text{se } -1 \leq c \leq 0 \\ \frac{1}{2} + c - \frac{c^2}{2} & \text{se } 0 < c \leq 1 \\ 1 & \text{se } c > 1 \end{cases}; \quad (b) 0, \quad \frac{7}{8}, \quad \frac{3}{8}, \quad \frac{1}{8}, \quad \frac{5}{9}$$

$$4. P(Y = 0) = 1 - e^{-\lambda a}, \quad F_Y(c) = \begin{cases} 0 & \text{se } c < 0 \\ 1 - e^{-\lambda(c+a)} & \text{se } c \geq 0 \end{cases}$$

[Sug.: Usar T.P.T. com a partição formada pelos acontecimentos $(X \leq a)$ e $(X > a)$]