The Digital Age, Fourier Transforms and Spectra

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Abstract

It is crucial to sample at a rate such that one can resolve and characterize the incoming signal. It is shown that at the nyquist rate $v_{sig} = .5 * v_{samp}$ one can resolve and use the detected signal, without the alternatives of getting false data or wasting too many resources in terms of memory which becomes costly. Power spectra across frequency are used to help differentiate between to signals when mixed together and hekp diffrentiate between the mixed waves and 'noise' made by them. Important results include the calculation of tones from mixed signals and using fourier transforms to see the peaks of signals at frequecies that vary a little.

1 Introduction

In the larger scope of making important but slowly emerging dicoveries that could help mankind better know their place and role in the universe, we must amke use of our small-scale inventions. Utilization of small chips programmed to make incredibly fast computations that would not be practical for us to attempt by hand. Thanks to the wide variety of chips with storage and processing power today and in the future, we may be able to get to those grand discoveries faster. In the digital lab, code and knowledge of computer components lead to a proper understanding of digital sampling, Fourier Transforms (leading to power spectrums) and mixing, all with application to the radio astronomy field, especially when analyzing signals that could only be analyzed via radio astronomy. In the first section of the lab we sample at some fraction of a frequency to see what the minumum sample rate is to get good data, while taking the least computer resources to get that data. In the following section we mix two types of signals together and analyze their power spectrums, although this may be misleading because the power of the signal imput is not physical but shows a peak. Finally we work with the ROACH to create a filter that can be manipulated via coefficient inputs we can choose.

2 Week One: Sampling

2.1 Methods

For the first week of the lab, we needed to take control of the pulsar and the oscillator and from there take samples at some v_{sig} with some constant sample frequency of v_{samp} which was chosen to be 10 MHz. v_{sig} was chosen so that it would increase in increments of $.1v_{samp}$ for nine trials. From there we labeled our data points for which the signal was sampled at, at time intervals set up by v_{sig} this was done to test the Nyquist sampling rate.

2.2 Going to the Extremes

Before our data for v_{sig}, there should be a check on the extremes of Nyquist sampling, namely $v_{sig} = v_{samp}$ and $\frac{v_{sig}}{v_{samp}} = large$. In the latter case, when we come across this condition, our graph ends up looking like the figure below

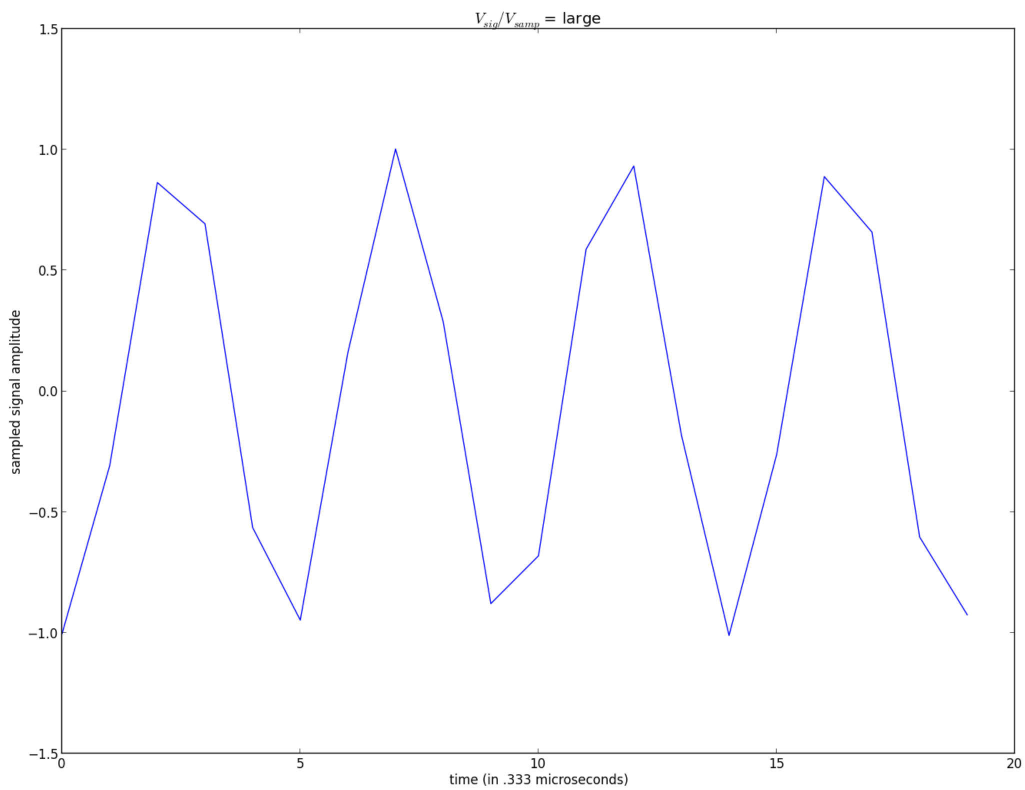

Figure 1: The plot of $\frac{v_{sig}}{v_{samp}} = large$ for $v_{sig} = 30MHz$ and $v_{samp} = 500Hz$.

at this sampling rate, we see that since the ratio between the signal and the sample frequency is so large, the data points taken are not sufficient to recreate the wave because our sampling frequency is much too slow to catch up with the maximum frequency and so we are left unable to charecterize to above wave.

For the case of $v_{sig} = v_{samp}$ my plot for the data did not come out to a straight line as it was predicted it would, it is ommitted here since it was an error on part of the equipment that it did not work. The reason this plot would be a straight line is because the sampling frequency would only sample at the period of the incoming signal, resulting in just a horizontal line when connecting the dots, there is no wave to analyze.

2.3 Nyquist Frequency

Now to demostrate an understanding of the Nyquist frequency, there is a graph below that gives the signal frequency in increments of .1 of the sample frequency, and of interest is the fifth graph.

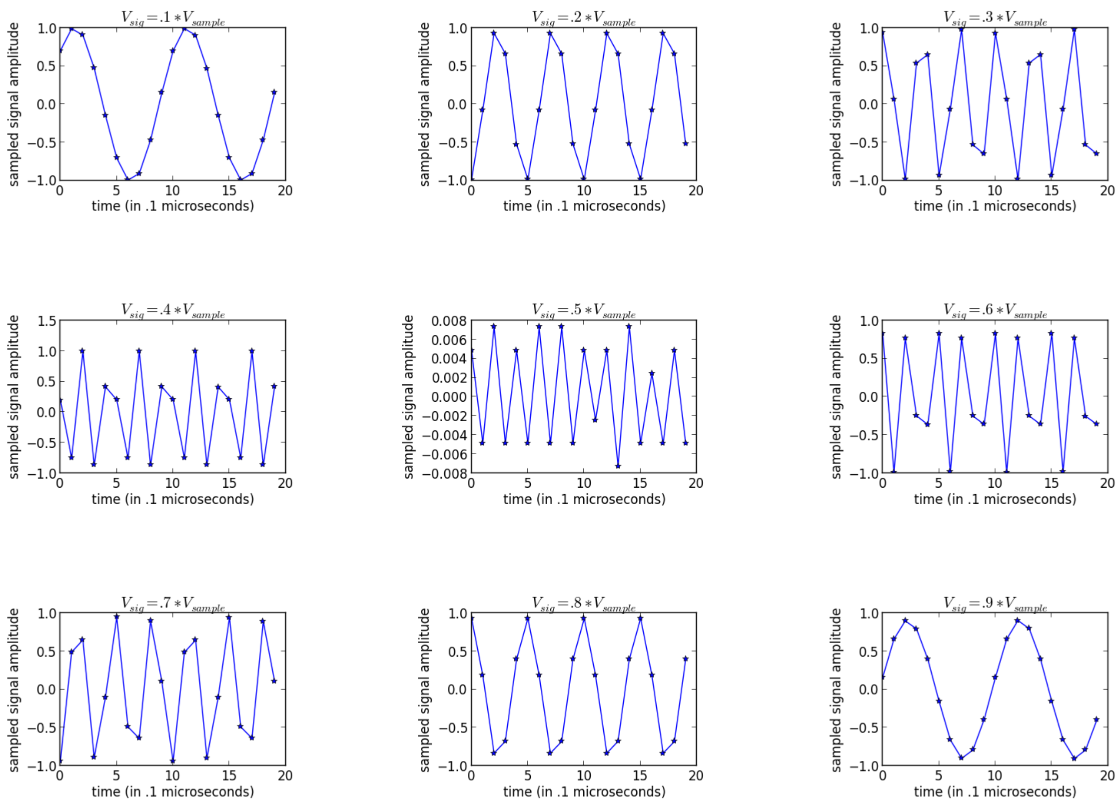

Figure 2: a series of plots demonstrating the declining effectives of sampling when you stray below the nyquist rate (at least twice the maximum frequency)

Here, we see the Nyquist criterion at work for a sampling frequency of 10 MHz, starting with the top left corner, we see that the wave is accurately represented since many points (represented by the starred data points) were able to be plotted along the line of the signal and thus we get an accurate representation of the wave. Looking at the top right plot we see now that the data points were plotted at larger intervals of time (we see less of a curve on th turning points of the signal), but overall, we can still characterize the signal. One important plot to note is the fifth plot taken right at the Nyquist sample rate. The reason the y-axis of this plot is different from the rest is that the pulsar sampler couldn't easily do the sampling at 10 Mhz, because of this, pulsar had to skip samples to simulate sampling at that frequency (it usually samples at 20 Mhz). Aside from these strange jump in the data, we see that at Nyquist sampling rate, the wave is barely resolved at that signal frequency. Now once we get past the fifth, we see that the next graphs look similar to the previous graphs we have looked at, although this may seem that the data points are still accurately representing the signal frequency, they aren't. The signal frquency is going through more cycles, and the data points can't accurately make a plot that represents that signal frquency. By the time we reach the ninth plot, the signal frequency undergoes many cycles and the data points were only able to record at a few points which definitely does not reproduce the graph. The reason Nyquist's criterion is preferred is because it is the best way to recreate an incoming signal using the least amount of computation power, you could make it more exact to the wavelength by making sure the sampling rate is higher than the signal rate but that takes up more time and resources on the computer than would be expcted since there are other devices that need computers to function, all simultaneously.

2.4 Power Spectrum

Here, the spectrum of the previous 9 plots was Fourier transformed and then squared to get rid of complex numbers that were a product of the imaginary number in the exponential, together with a frequency and sample number, the plots now look like

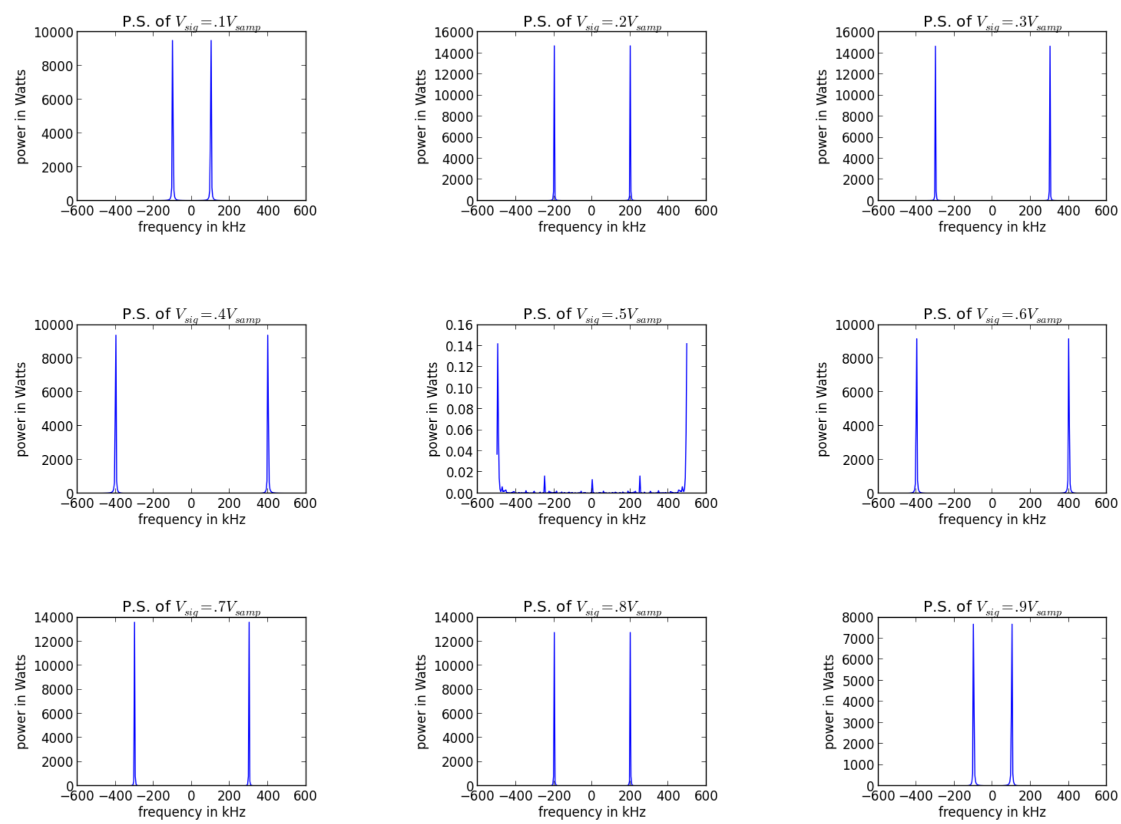

Figure 3: The DFT plots of our previous Nyquist plots

So these are the transfers to a power spectrum, and as we see, there are two spikes, the one to the right represents the real part of the sine wave while the second spike represents the imaginary part of the wave. The reason that they pop up, is from the afroementioned Fourier transform, the exponential term introduces a complex number which becomes negative when i is squared, and so hence, two spkies appear. There is also a pattern of the spikes getting further away from eachother until the Nyquist criterion and then come back together as ther start to violate that law. The place the spikes are at is prortional to v_{sig}, being a fraction of the sample frequency.

2.5 Spectral Leakage

Spectral leakage is represented by the following graphs which have all been modified to show the leakage(by taking the log of the previous set of plots), shown as the fringes along the curve of the spikes.

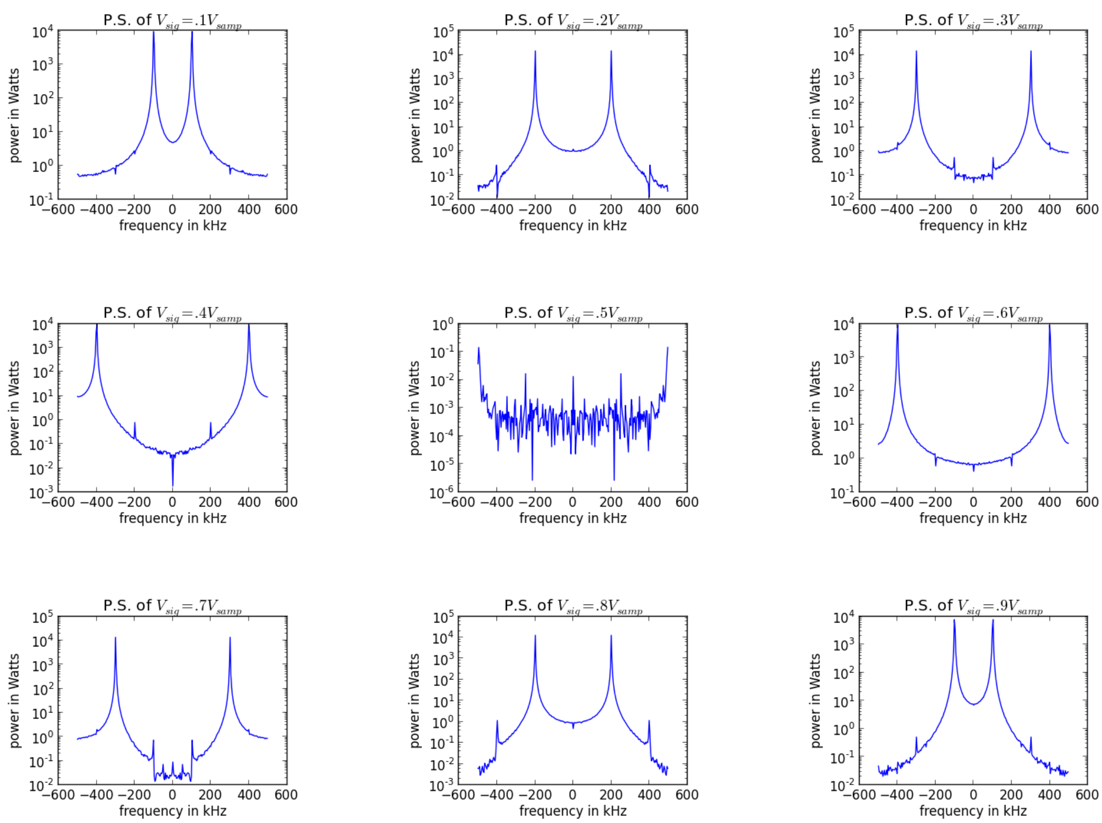

Figure 4: the leakage power shown as the 'fringe' peaks

This is the result of the signal in the time domain resulting in a convolution in the frequency domain. The small spikes that are seen are the leakage, showing up further away from the spectral peaks, those small spikes are not perfectly periodic in the Fast Fourier Transform. frequency bins are just the number of samples you decide to take over the sample frequency, here, there are two waves mixed together, these waves are periodic (being sine waves) and so when there's is a linear combination of them, within that bin one wave will fit in it while the other's wavelength will not fit within the bin and so a bit of it 'leaks' into the next bin, and so that is what is seen in the above graph.

2.6 Frequency Resolution

In regards to spectral resolution, it is defined as the siganl frequency over the number of data points you have. So, you could have the spikes be very close to gether, achieving very high resolution, but this takes alot of resources from the computer, slowing down your data collection time. We can't get infinite resolution because of limitations on our current hardware. To sample at rates good enough for our purposes, all we want is to stay near the Nyquist critereon, or have a signal frequency smaller than the sample to get adequate results.

3 Working with the ROACH

3.1 Methods

During the course of this week of the lab, Isaac Domalgalski developed the code to take the data the rest of the group needed. All other work henceforth all data that was worked with came from his code. This report used and mainpulated that data.

3.2 DSB Mixer Power Spectra

Having plotted the power spectrum of the sum and difference cases versus frequency we get this plot

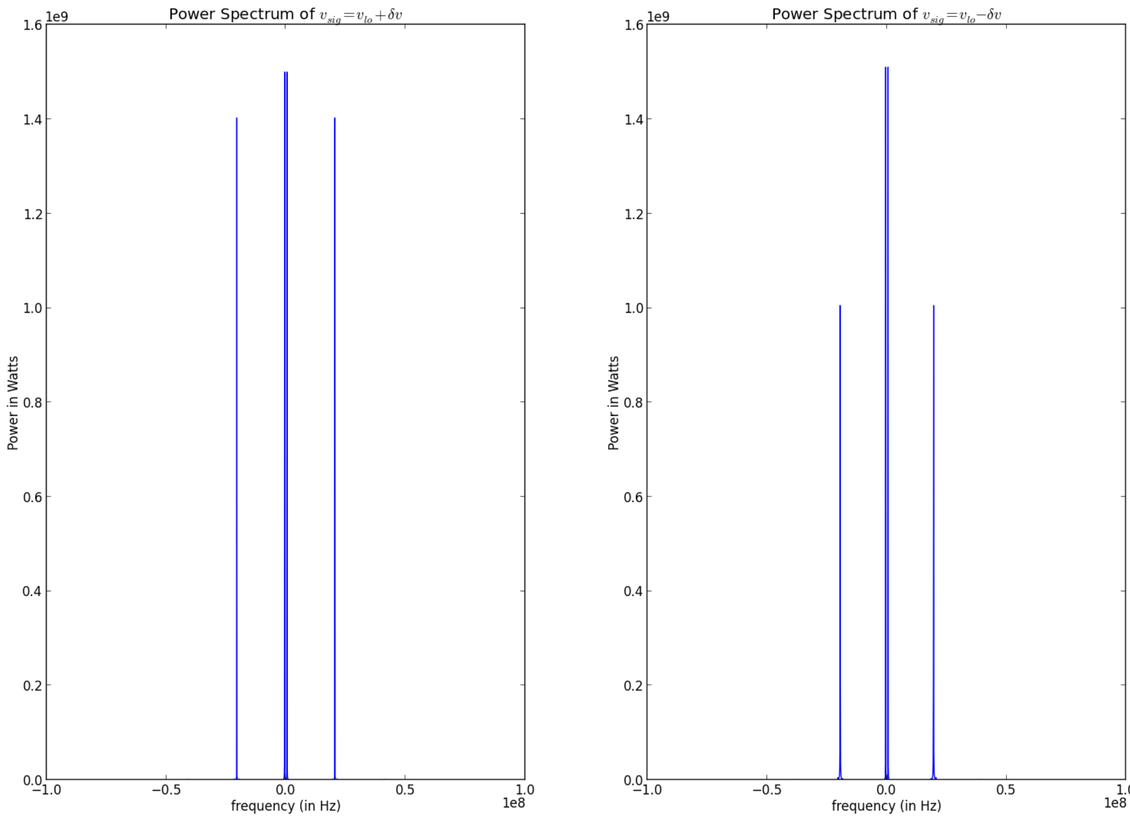

Figure 5: The power spectra for $\pm\delta v$.

An interesting thing that we see in these spectra is that the middle 'thick' line is actually two lines very close together which confirms the presence of two waves that been mixed together into one sine wave. Now, the upper sidebands refer to the bands (spikes) which are on the positive side of the x-axis(frequency); the real part of the sine wave. Whereas the lower sidebands refer to those on the negative x-axis; they correspond to an imaginary part of the sine wave.

We see the waveforms as pictured below, and we will focus on the $v_{sig} = v_{lo} + \delta v$ From observations made that day, it did look like what was seen on the oscilloscope's trace.

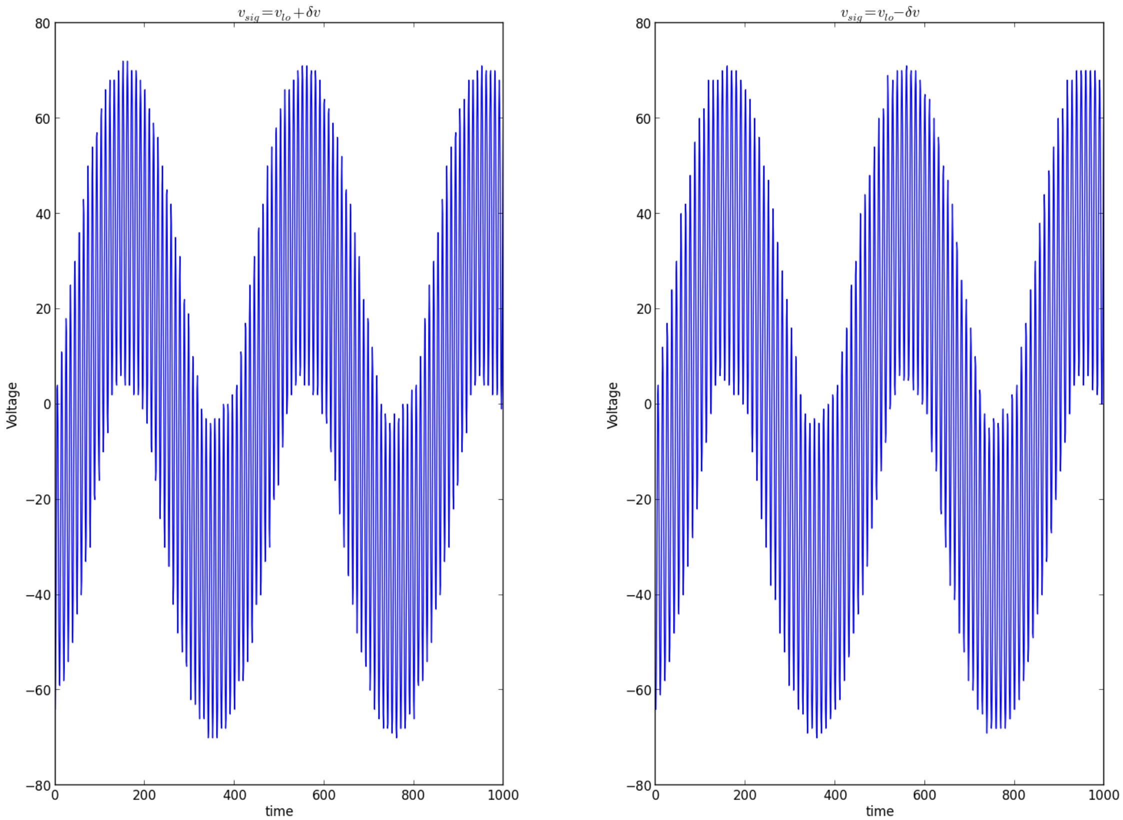

Figure 6: the Wave form of both mixed signals

Taking the Fourier Transform and zeroing out the sum frequency we then apply the inverse fourier transform on it and we retrieve a new signal

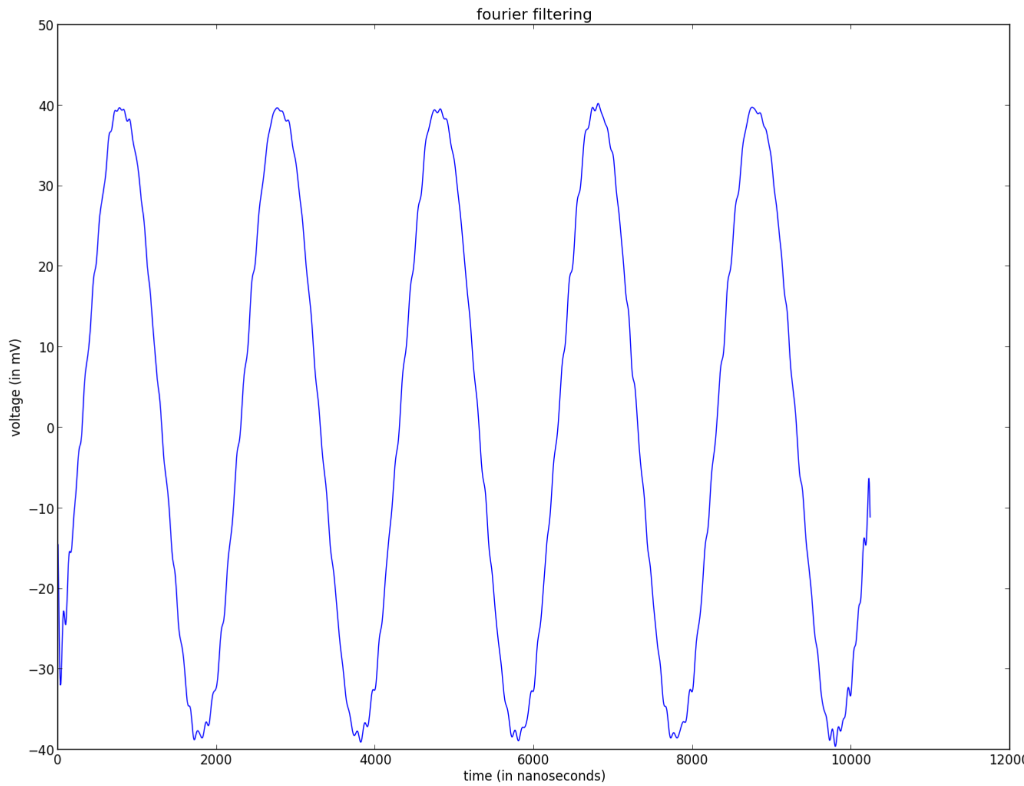

Figure 7: the original wave after zeroing and inverse fourier transforming

This new signal is δv the difference of v_{sig} and v_{lo} by zeroing out the sum after fourier transforming the waveform, we effectively destroyed the mixed wave. This new wave is the output of the mixer without the higher frequency part that was originally mixed in.

3.3 Digital Mixing and Power Spectra

So now, for the digital and analog power spectrum DSB mixing cases, we see,

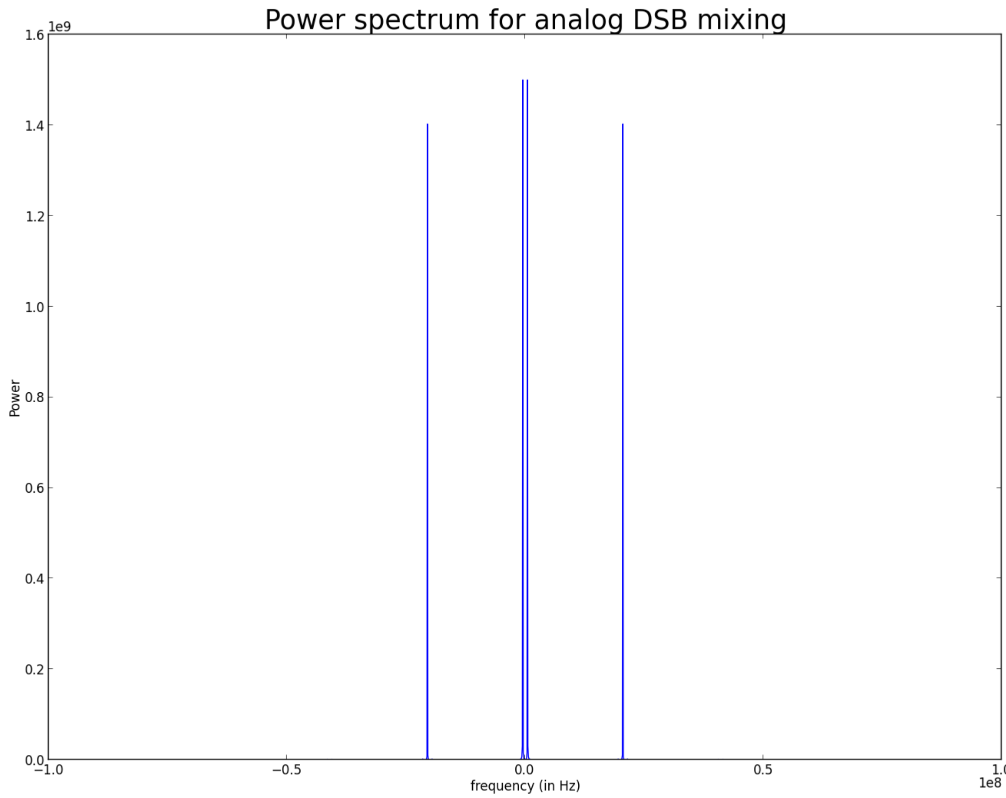

Figure 8: The power Spectrum of the analog DSB mixer

followed by the digital version

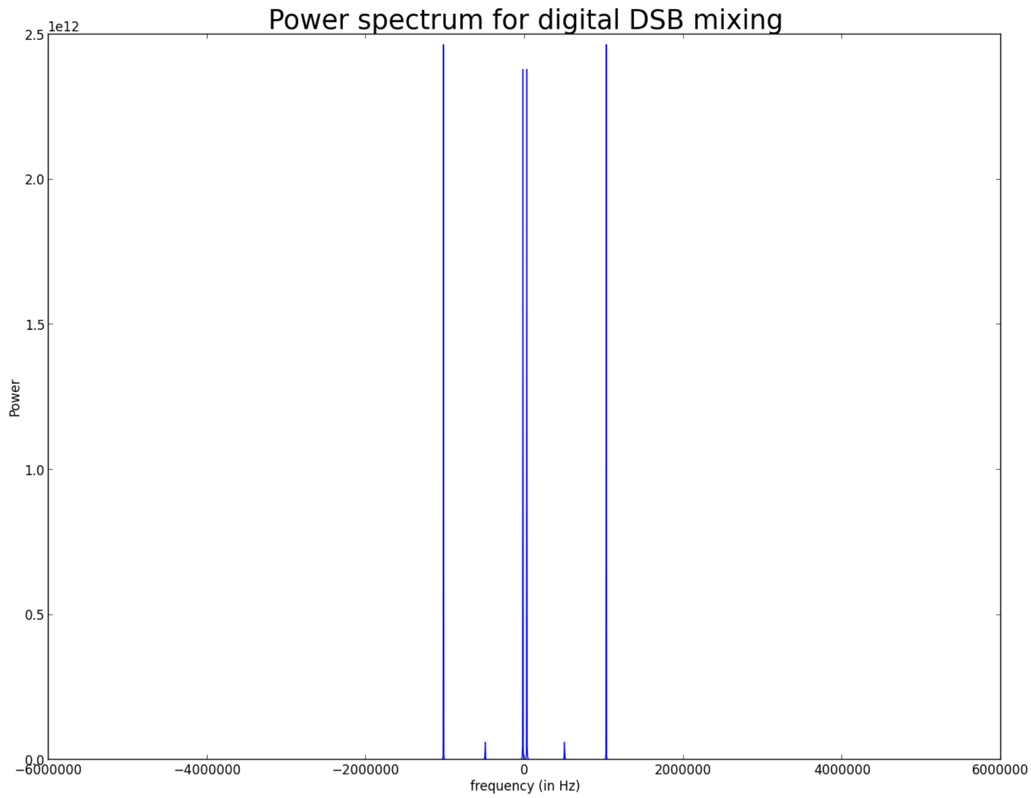

Figure 9: The power Spectrum of the digital DSB mixer

The main diffence between these two plots is the DC offput observed in figure 9 which is the result of the waves not being perfectly centered on 0 volts when being mixed digitally. Another difference in the graphs that is noticed is the scaling of the x-axis, the reason for this was the sampling frequency that was used, in the analog case, 200 MHz was used whereas the digital version used 20 MHz as the sampling frequency, thus why the x-axis is off by a factor of 10. The Waveform of the digital mixing case looks like

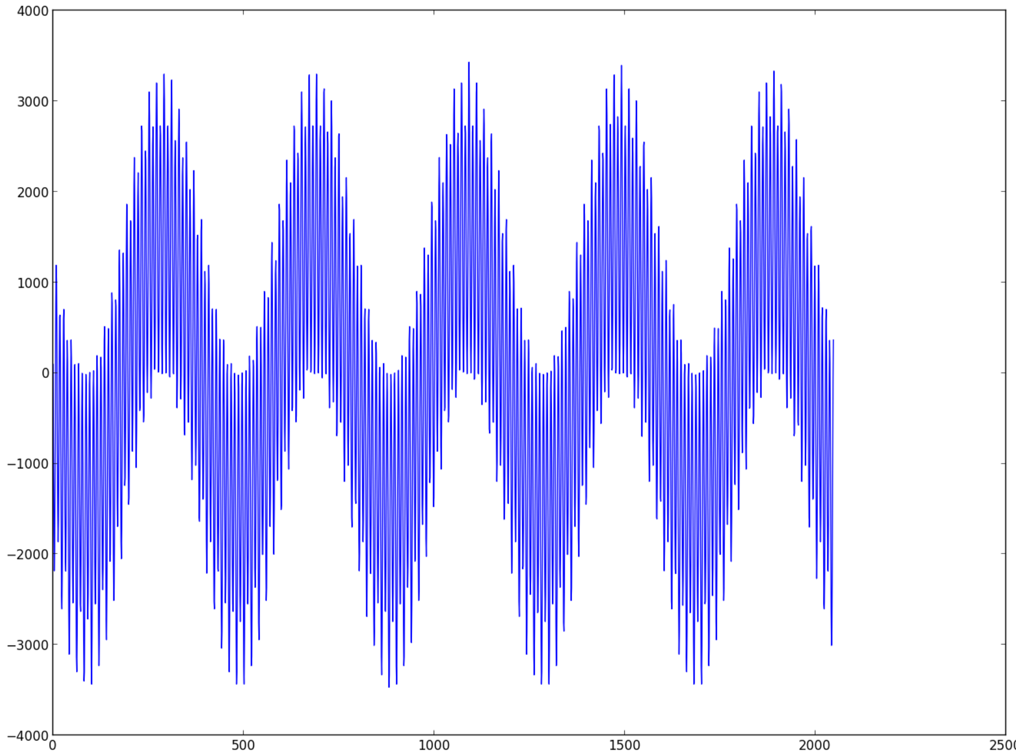

Figure 10: The Waveform of the digital DSB mixer

Here the waveform is plotted as a function of voltage versus time, this waveform includes the mixed signals we input.

Now we look at the power spectrum of the SSB mixer

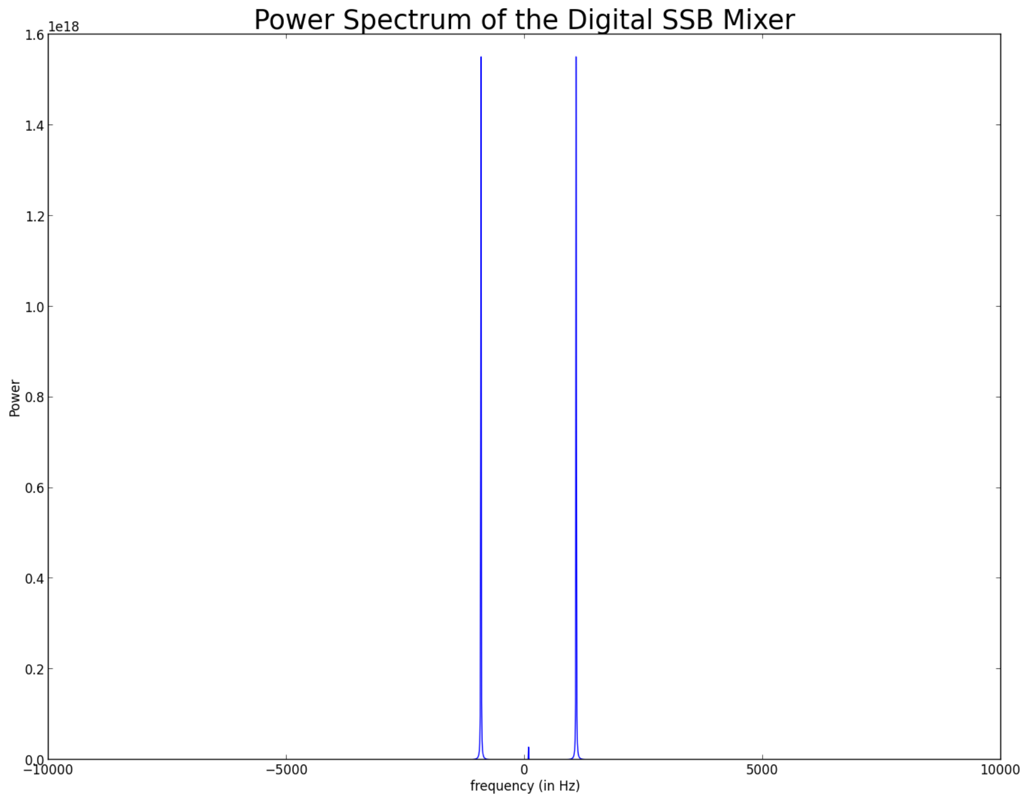

Figure 11: The power Spectrum of the digital SSB mixer

The reason the graph now looks like this is because the wave consisted of a real and imaginary part. When taking the power spectrum we needed to square the absolute value of the Linear combination of the cosine and sine components of the wave, with the sine component multiplied by the imaginary number, i because the power spectrum must be real. This multplication by i then forces the spikes to shift a liitle bit to the right, thus why the graph is shifted positively.

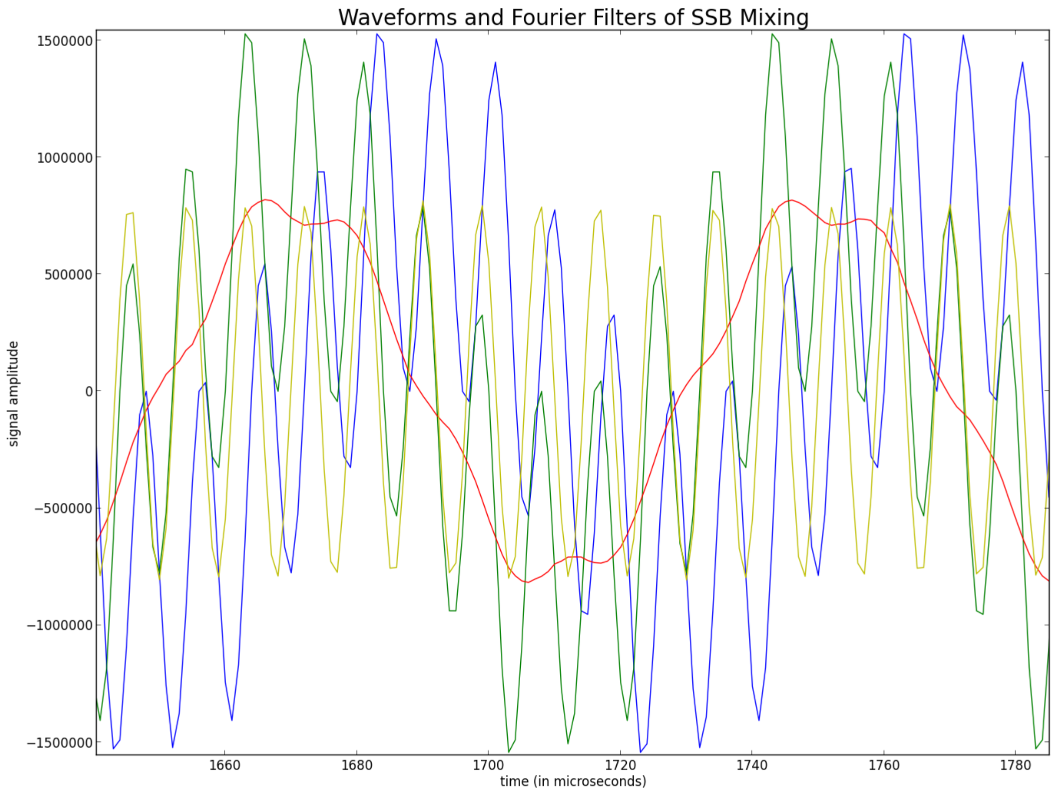

Figure 12: The Waveform and tones of the imaginary and real parts of the wave

Here, we present the waveform of the sine and cosine waves. The blue wave represents the cosine wave and the gren represents the sine waves which lags behind the cosine wave by a liitle bit. The red wave represents the tone we are looking for when the spike in frequency domain is zeroed out for the imaginary part of fourier transform and then inverse fourier transformed, so we see that this tone has a negative frequency since the previous power spectrum was offset positively.

In implementations of DSB and SSB mixing, we see that when we use DSB mixing, we get bands the lower and upper bands whereas in the SSB mixing we can only get either one or the other depending on the cosine and sine components of digital waves. If the cosine and sine waves were out out phase, we would expect to see errors in our power spectrum, mainly some spectral leakage.

4 Coefficients and Digital Down Conversion

In this section coefficients for the Finite Impulse Response filter will be calculated. FIR filters implement frequency domain filters with a tunable shape determined by those same coefficients that convoolve with an input wave form. In this experiement, an input wave of 20 MHz was chosen out out a series of other input waves. A diagram of a FIR filter is displayed below.

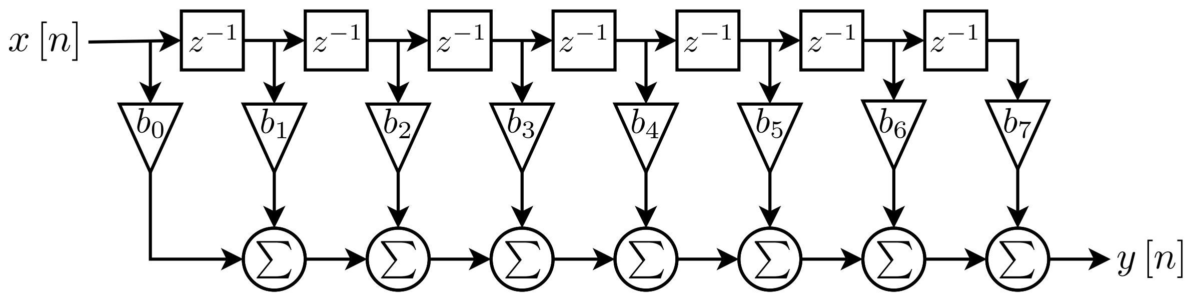

Figure 13: A digram of the FIR filter

Some signals x[n] propagate through the chain of delays, represented by the z^{-1} boxes. At each clock, N samples are multiplied by N coefficients $b_o ... b_{N-1}$ and summed.

4.1 Coefficients of the FIR Filter

Below we see the calclated coefficients of the FIR filter for an imput siganl of 20 MHz.

Coefficients for the FIR filter	
Floating Point	Fixed Binary
0.125	0.00100000000000000
−0.0517730712890625	1.11110010101111110
−0.125	1.11100000000000000
0.3017730712890625	0.01001101010000010
0.625	0.10100000000000000
0.317730712890625	0.01001101010000010
−0.125	0.11100000000000000
−0.0517730712890625	1.11110010101111110

Table 1: the coefficients used for digital down converting. We can see the sum of 32 bits on the binary fixed point column

when we pad out our expected coefficients with an array of 52 zeroes, we want to create a plot of desired bandpass filter, by concatenating an array of [1,1,1,0,0,0,1,1] with these zeroes then fourier transforming we get an ideal plot of

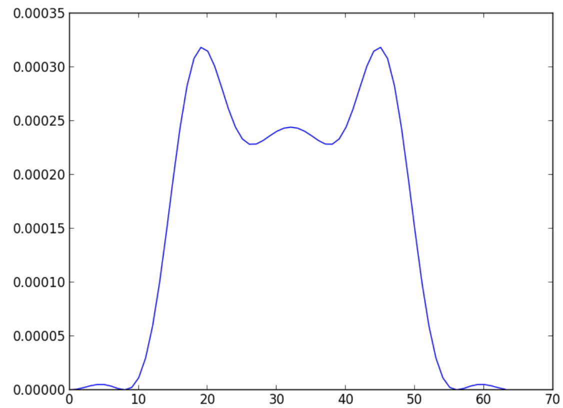

Figure 14: The desired bandpass we would like

this is the ideal shape we would want for this filter.

4.2 The Filter shape

There wasn't enough time to get the plots for this section of the lab. The true shape of the filter should not match up with the ideal one we calculated above, noise would distort the graph somewhat.

5 Conclusion/Discussion

Sampling with a signal is imporatnt in the field of astronomy because we want to have the minumum data to analyze various things in astronomy for practical reasons, computer space. Analyzation involves being able to resolve signals. There are still some things that are not understood at this end for instance, the whole of week 3, but mixing of signals and looking at their power spectra, waveforms is understood better