$$(u^2 \sin u)' = 2u \sin u + u^2 \cos u$$

$$(u(u)\,v(u))' = u'(u)\,v(u) + u(u)\,v'(u)$$

$$(uv)' = u'v + uv' \qquad (u \equiv u(u))$$

$$u'v = (uv)' - uv'$$

$$\boxed{\int u'v\,du = uv - \int u v'\,du}$$

Primitivacao por partes

$u = x$

③ $\int \ln u \, du = \int \underbrace{1}_{u'} \times \underbrace{\ln u}_{v} \, du$

$u \ln u - \int u \cdot \frac{1}{u} \, du$

$u \ln u - u + C, \; C \in \mathbb{R}$

$$\boxed{\int u'v \, du = uv - \int uv' \, du}$$

$\int \frac{u'}{u} \, du = \ln|u| + C, \; C \in \mathbb{R}$

① $\int \underbrace{u}_{v} \underbrace{\sec^2 u}_{u'} \, du =$

$= \underbrace{\operatorname{tg} u}_{u} \cdot \underbrace{u}_{v} - \int \underbrace{\operatorname{tg} u}_{u} \cdot \underbrace{1}_{v'} \, du$

$= u \operatorname{tg} u + \int \frac{-\sin u}{\cos u} \, du$

$= u \operatorname{tg} u + \ln|\cos u| + C, \; C \in \mathbb{R}$

$(\arctan u)' = \frac{1}{1+u^2}$

② $\int \underbrace{e^u}_{u'} \underbrace{\sin u}_{v} \, du = e^u \sin u - \int \underbrace{e^u}_{u_1'} \underbrace{\cos u}_{v_1} \, du$

$= e^u \sin u - \left[e^u \cos u - \int e^u \cdot (-\sin u) \, du \right]$

$= e^u (\sin u - \cos u) - \int e^u \sin u \, du$

$\int e^u \sin u \, du = e^u (\sin u - \cos u) - \int e^u \sin u \, du$

$2 \int e^u \sin u \, du = e^u (\sin u - \cos u) \quad \therefore \int e^u \sin u \, du = \frac{e^u}{2} (\sin u - \cos u) + C,$

$C \in \mathbb{R}$

Exercício 5.6 Calcule as seguintes famílias de primitivas:

5. $\int x \arctan x \, dx$;

$$(\arctan x)' = \frac{1}{1+x^2}$$

3. $\int \underbrace{\operatorname{sen}(2x)}_{u'} \underbrace{\operatorname{sen}(7x)}_{v} \, dx$;

1. $\int \arctan x \, dx$;

2. $\int \sec^3 x \, dx$;

$$5.\ \int \underbrace{x}_{u'} \underbrace{\arctan x}_{v}\, dx; = \frac{u^2}{2}\arctan u - \int \frac{u^2}{2}\cdot\frac{1}{1+u^2}\,du$$

$$= \frac{u^2}{2}\arctan u - \frac{1}{2}\int \frac{u^2}{u^2+1}\,du =$$

$$= \frac{u^2}{2}\arctan u - \frac{1}{2}\int \frac{u^2+1-1}{u^2+1}\,du =$$

$$= \frac{u^2}{2}\arctan u - \frac{1}{2}\int \left(1 - \frac{1}{u^2+1}\right)du$$

$$= \frac{u^2}{2}\arctan u - \frac{1}{2}\left(u - \operatorname{arctg} u\right) + C,\ C \in \mathbb{R}$$

$$3.\ \int \underbrace{\operatorname{sen}(2x)}_{u'}\, \underbrace{\operatorname{sen}(7x)}_{V}\, dx; =$$

$$= \underbrace{-\frac{\cos(2u)}{2}}_{u} \cdot \underbrace{\sin(7u)}_{V} - \int \underbrace{-\frac{\cos(2u)}{2}}_{u} \cdot \underbrace{7\cos(7u)}_{V'}\,du$$

$$= -\frac{1}{2}\cos(2u)\sin(7u) + \frac{7}{2}\int \underbrace{\cos(2u)}_{u_2'}\underbrace{\cos(7u)}_{V_2}\,du$$

$$= -\frac{1}{2}\cos(2u)\sin(7u) + \frac{7}{2}\left[\frac{\sin(2u)}{2}\cdot\cos(7u) - \int \frac{\sin(2u)}{2}\left(-7\sin(7u)\right)du\right]$$

$$= -\frac{1}{2}\cos(2u)\sin(7u) + \frac{7}{4}\sin(2u)\cos(7u) + \frac{49}{4}\boxed{\int \sin(2u)\sin(7u)\,du}$$

$$\underbrace{\left(1-\frac{49}{4}\right)}_{-\frac{45}{4}}\int \sin(2u)\sin(7u)\,du =$$

$$-\frac{1}{2}\cos(2u)\sin(7u) + \frac{7}{4}\sin(2u)\cos(7u)$$

$$\int \sin(2u)\sin(7u)\,du = -\frac{4}{45}\left(-\frac{1}{2}\cos(2u)\sin(7u) + \frac{7}{4}\sin(2u)\cos(7u)\right)$$

$$+C,\ C \in \mathbb{R}.$$

3. $\frac{2}{45}\cos(2x)\,\mathrm{sen}\,(7x) - \frac{7}{45}\,\mathrm{sen}\,(2x)\cos(7x) + C$;

Primitivação por Substituição: mudança de variável

Exemplo 5.4. Como calcular $\displaystyle\int \frac{x}{1+\sqrt{x}}dx$, com $x \in \mathbb{R}_0^+$?

$x = t^2$, $dx = 2t\,dt$ $\qquad t = \sqrt{x}$

$$\int \frac{x}{1+\sqrt{x}}dx = \int \frac{t^2}{1+t}\cdot 2t\,dt$$

$$= 2\int \frac{t^3}{t+1}dt = 2\int t^2 - t + 1 + \frac{(-1)}{t+1}\,dt$$

$$\begin{array}{l|l}
t^3 & t+1 \\ \hline
-t^3 - t^2 & t^2 - t + 1 \\ \hline
-t^2 & \\
t^2 + t & \\ \hline
t & \\
-t - 1 & \\ \hline
-1 &
\end{array}$$

$$= 2\left(\frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1|\right) + C$$

$$= \frac{2}{3}\left(\sqrt{x}\right)^3 - \left(\sqrt{x}\right)^2 + 2\sqrt{x} - 2\ln|\sqrt{x}+1| + C,\ C \in \mathbb{R}.$$

1. $\int \arctan x \, dx;$ $= \int \underbrace{1}_{u'} \cdot \underbrace{\arctan x}_{v} \, dx =$

$$= x \arctan x - \int x \cdot \frac{1}{1+x^2} \, dx =$$

$$= x \arctan x - \frac{1}{2} \int \frac{2x}{x^2+1} \, dx =$$

$$= x \arctan x - \frac{1}{2} \ln\left(x^2+1\right) + C, \; C \in \mathbb{R}.$$

2. $\int \sec^3 x\, dx$; $= \int \underbrace{\sec u}_{v} \cdot \underbrace{\sec^2 u}_{u'}\, du =$

$= tg u \cdot \sec u - \int tg u \cdot \frac{\sin u}{\cos^2 u}\, du =$

$= tg u \sec u - \int \frac{\sin^2 u}{\cos^3 u}\, du =$

$= tg u \sec u - \int \underbrace{\frac{\sin u}{\cos^3 u}}_{u'} \cdot \underbrace{\sin u}_{v}\, du =$

$= tg u \sec u - \left[\frac{1}{2\cos^2 u} \cdot \sin u - \int \underbrace{\frac{1}{2\cos^{\cancel{2}} u} \cdot \cancel{\cos} u}_{\sec u}\, du \right] =$

$= \underbrace{tg u \sec u - \frac{1}{2} tg u \sec u}_{1/2\, tg u \sec u} + \frac{1}{2} \ln|\sec u + tg u| + C,$

$C \in \mathbb{R}.$