# Introduction to Computer Graphics

## 2023/2024

## Application Examples: 2D and 3D Transformations

### 2D Transformations

**1-** Given the square, defined by the vertices (2, 2), (3, 2), (3, 3) and (2, 3), it is to be rotated around its center by an angle of 90 degrees.

**a)** Determine the transformation matrix, in *Homogeneous Coordinates*, that accomplishes the desired rotation.

**b)** Compute the coordinates of the transformed vertices and draw the square resulting from the rotation.

**2-** Given the triangle, defined by the vertices (2, 0), (4, 2) and (-1, 5), determine the triangle resulting from applying a symmetry transformation relative to the $y = x$ straight-line.

**a)** Determine the transformation matrix, in *Homogeneous Coordinates*, that accomplishes the desired symmetry.

**b)** Compute the coordinates of the transformed vertices and draw the triangle resulting from the symmetry.

**3-** It is required to apply a **2D transformation** to the triangle defined by the vertices (−1, 1), (1, 1) and (0, −5).

The transformation to be applied is composed of a **90-degrees rotation**, relative to the **center of the triangle**, followed by a **symmetry** regarding the ***XX'* axis**.

**a)** Obtain the global transformation matrix representing the sequence of basic transformations that have to be applied.

**b)** Compute the coordinates of the triangle vertices after carrying out the required transformation.

**4-** The goal is to apply a **2D transformation** to the square defined by the following vertices (1,1), (3,1), (3,3) e (1,3).

The transformation to be applied is made up of a **scaling**, **relative to the square center** and with scaling factors $S_X = 2$ and $S_Y = 3$, followed by a **symmetry** relative to the *XX'* axis.

**a)** Obtain, using *Homogeneous Coordinates* and through the concatenation of elementary transformations, the global matrix representing the required transformation. Explain all the steps carried out.

**b)** Compute the coordinates of the square vertices after carrying out the required transformation.

### 3D Transformations

**5-** Consider the cube defined by the vertices:

| | | | |
|---|---|---|---|
| $V_1$ (0, 0, 0) | $V_2$ (0, 1, 0) | $V_3$ (1, 1, 0) | $V_4$ (1, 0, 0) |
| $V_5$ (0, 0, 1) | $V_6$ (1, 0, 1) | $V_7$ (1, 1, 1) | $V_8$ (0, 1, 1) |

Using *Homogeneous Coordinates*, determine the matrix that represents the transformation that is to be applied for the cube to rotate, by a 180 degrees angle, around the straight-line that passes through point (2, 0, 0) is parallel to the *YY'* axis.

**a)** Obtain, through the concatenation of elementary transformations, the matrix *M(180)* representing the desired transformation. Explain the steps carried out.

**b)** Compute the coordinates of the transformed cube vertices.

**c)** Draw the transformed cube and check if the desired transformation was effectively carried out.

**6-** Given a triangular prism defined by the following vertices:

| | | | | | |
|---|---|---|---|---|---|
| $V_1$ (1, 0, 0) | $V_2$ (3, 0, 0) | $V_3$ (2, 0, 3) | $V_4$ (1, 6, 0) | $V_5$ (3, 6, 0) | $V_6$ (2, 6, 3) |

We want to obtain, using *Homogeneous Coordinates*, the transformation matrix that is to be applied so that the prism is repositioned in the following way:

- edge $V_3V_6$ belongs to the *XX'* axis
- the center of edge $V_3V_6$ is at (0, 0, 0), vertex $V_3$ belongs to the negative semi-axis and vertex $V_6$ belongs to the positive semi-axis.

**a)** Obtain, through the concatenation of elementary transformations, the global matrix representing the required transformation. Explain all the steps carried out.

**b)** Compute the coordinates of the prism vertices after carrying out the required transformation.

**c)** Sketch the transformed prism and check if it is positioned as required.

**7-** Given a **cube** defined by the following vertices:

| | | | |
|---|---|---|---|
| $V_1$ (1, 0, 0) | $V_2$ (3, 0, 0) | $V_3$ (3, 0, 2) | $V_4$ (1, 0, 2) |
| $V_5$ (1, 2, 0) | $V_6$ (3, 2, 0) | $V_6$ (3, 2, 2) | $V_6$ (1, 2, 2) |

We want to obtain, using *Homogeneous Coordinates*, the transformation matrix that is to be applied so that the cube is transformed into a **quadrangular prism** with the following features:

- The **center** of the prism is at (0, 0, 0);
- The **base** of the prism is a square of side 2;
- The **height** of the prism is 10.

**a)** Obtain, through the concatenation of elementary transformations, the global matrix representing the required transformation. Explain all the steps carried out.

**b)** Compute the coordinates of the model vertices after carrying out the required transformation.

**c)** Sketch the transformed model and check if it is positioned as required.

**8-** Given a square pyramid defined by the following vertices:

| | | | | |
|---|---|---|---|---|
| $V_1$ (0, 0, 1) | $V_2$ (0, 0, 3) | $V_3$ (2, 0, 3) | $V_4$ (2, 0, 1) | $V_5$ (1, 2, 2) |

We want to obtain, using *Homogeneous Coordinates*, the transformation matrix that is to be applied so that the pyramid is repositioned in the following way:

- the centre of the square base is (0, 0, 0);
- vertices $V_1$ and $V_3$ belong to the *ZZ'* axis.

**a)** Obtain the rotation matrix $R_Y(\theta)$ which allows performing a $\theta$ degrees rotation around the *YY'* axis. Explain all the steps carried out.

**b)** Obtain, through the concatenation of elementary transformations, the global matrix representing the required transformation. Explain all the steps carried out.

**c)** Compute the coordinates of the pyramid vertices after carrying out the required transformation.

**d)** Sketch the transformed pyramid and check if it is positioned as required.