# Aula 3: Funções hiperbólicas

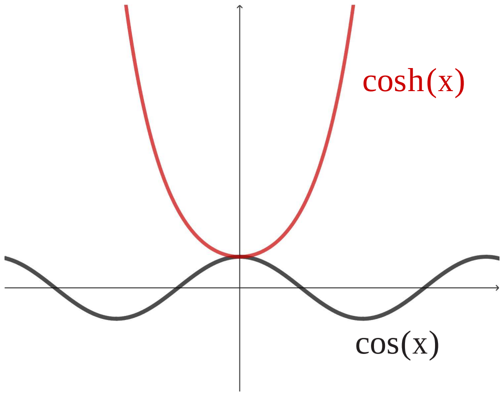
Fórmula de Euler: $\boxed{e^{ix} = \cos(x) + i\,\mathrm{sen}\,(x)}$ $\qquad (i^2 = -1)$

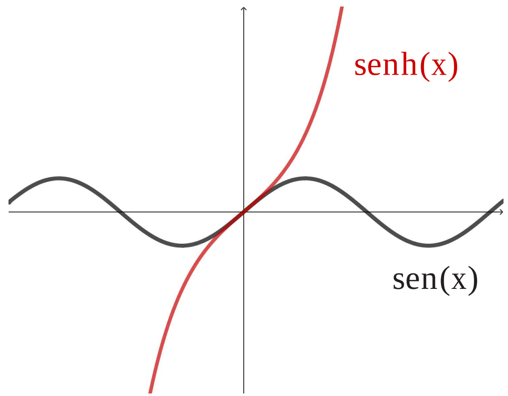
Daqui deduzimos:

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2} \quad , \quad \mathrm{sen}\,(x) = \frac{e^{ix} - e^{-ix}}{2i} \tag{1}$$

Eliminando a $i$ nas formulas anteriores obtemos o coseno e o seno hiperbólicos:

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \quad , \quad \mathrm{senh}\,(x) = \frac{e^{x} - e^{-x}}{2} \tag{2}$$



# Aula 3: Fórmulas hiperbólicas e trigonométricas

$\cosh(x) = \cos(ix)$

$\text{senh}\,(x) = -i\,\text{sen}\,(ix)$

$e^{ix} = \cos(x) + i\,\text{sen}\,(x)$

$\cosh^2(x) - \text{senh}^2(x) = 1$

$\text{tgh}\,(x) := \frac{\text{senh}\,(x)}{\cosh(x)} = \frac{e^{2x}-1}{e^{2x}+1}$

$\text{cotgh}\,(x) := \frac{\cosh(x)}{\text{senh}\,(x)} = \frac{e^{2x}+1}{e^{2x}-1}$

$\cosh(x+y) = \cosh(x)\cosh(y) + \text{senh}\,(x)\,\text{sen}\,(y)$

$\cosh(x-y) = \cosh(x)\cosh(y) - \text{senh}\,(x)\,\text{sen}\,(y)$

$\text{senh}\,(x+y) = \text{senh}\,(x)\cosh(y) + \cosh(x)\,\text{senh}\,(y)$

$\text{senh}\,(x-y) = \text{senh}\,(x)\cosh(y) - \cosh(x)\,\text{senh}\,(y)$

$\cosh(2x) = \cosh^2(x) + \text{senh}^2(x)$

$\text{senh}\,(2x) = 2\,\text{senh}\,(x)\cosh(x)$

$(\cosh(x))' = \text{senh}\,(x)$

$(\text{senh}\,(x))' = \cosh(x)$

$\cos(x) = \cosh(ix)$

$\text{sen}\,(x) = -i\,\text{senh}\,(ix)$

$e^{x} = e^{i(-ix)} = \cosh(x) + i\,\text{senh}\,(x)$

$\cos^2(x) + \text{sen}^2(x) = 1$

$\text{tg}\,x = \frac{\text{sen}\,(x)}{\cos(x)}$

$\text{cotg}\,x = \frac{\cos(x)}{\text{sen}\,(x)}$

$\cos(x+y) = \cos(x)\cos(y) - \text{sen}\,(x)\,\text{sen}\,(y)$

$\cos(x-y) = \cos(x)\cos(y) + \text{sen}\,(x)\,\text{sen}\,(y)$

$\text{sen}\,(x+y) = \text{sen}\,(x+y) = \text{sen}\,(x)\cos(y) + \cos(x)\,\text{sen}\,(y)$

$\text{sen}\,(x-y) = \text{sen}\,(x-y) = \text{sen}\,(x)\cos(y) - \cos(x)\,\text{sen}\,(y)$

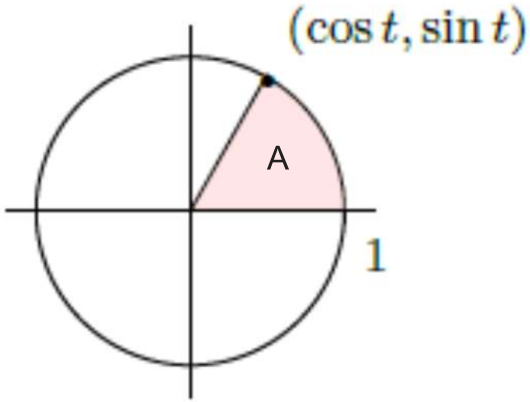
$\cos(2x) = \cos^2(x)\,\text{sen}^2(x)$

$\text{sen}\,(2x) = 2\,\text{sen}\,(x)\cos(x)$

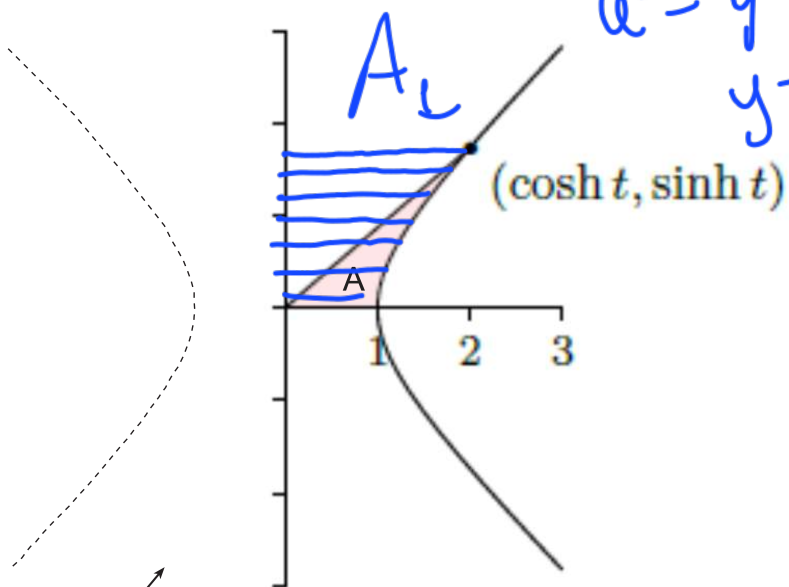
$(\cos(x))' = -\,\text{sen}\,(x)$

$(\text{sen}\,(x))' = \cos(x)$

# Aula 3: Semelhanças



circunferência unitária



hipérbole unitária

$$x^2 - y^2 = 1$$
$$y = \sqrt{1+x^2}$$

$$A = \pi \times 1^2 \times \frac{t}{2\pi}$$
$$= \frac{t}{2}$$

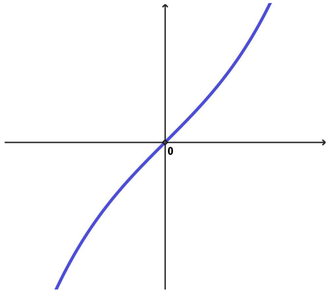
$$A = \frac{t}{2}$$

$$A_1 = \int_0^{\sinh t} \sqrt{1+x^2}\, dy$$

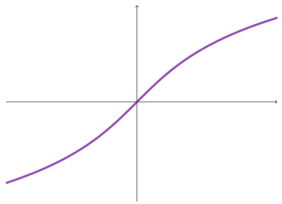
# Aula 3: Funções hiperbólicas inversas

pf: parte de f ao subdominio (f restrita ao subdominio)

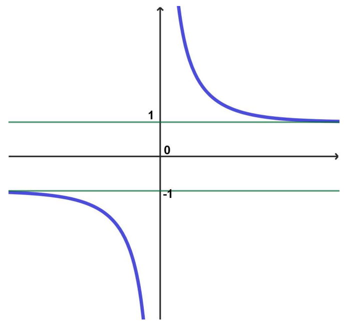
senh(x)



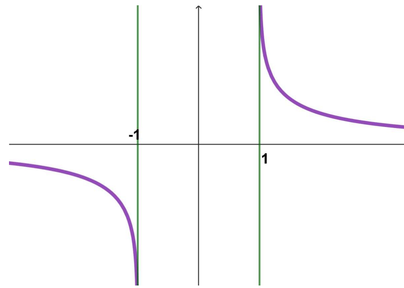

inversa $\longrightarrow$ $\text{senh}^{-1}(x)=\ln(x+\sqrt{x^2+1})$



cotgh(x)

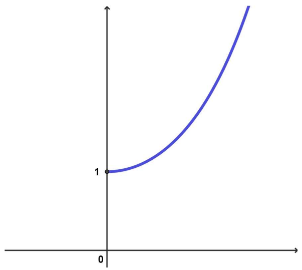



inversa $\longrightarrow$ $\text{cotgh}^{-1}(x)=\frac{1}{2}\ln(\frac{x+1}{x-1})$

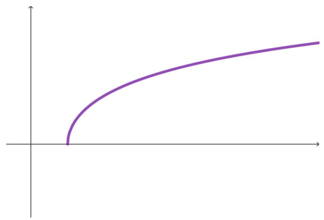



pcosh(x)

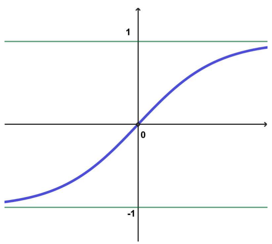
$x \geqslant 0$




inversa $\longrightarrow$ $\text{pcosh}^{-1}(x)=\ln(x+\sqrt{x^2-1})$



tgh(x)




inversa $\longrightarrow$ $\text{tgh}^{-1}(x)=\frac{1}{2}\ln(\frac{1+x}{1-x})$