

# DEPARTAMENTO DE ELETRÓNICA, TELECOMUNICAÇÕES E INFORMÁTICA

## MESTRADO EM ENGENHARIA DE COMPUTADORES E TELEMÁTICA

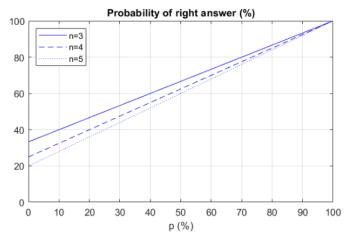
#### ANO 2025/2026

## MODELAÇÃO E DESEMPENHO DE REDES E SERVIÇOS

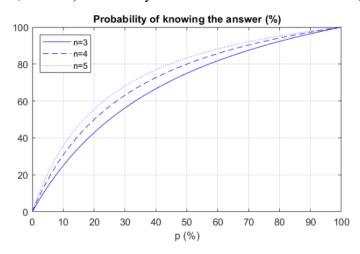
### PRACTICAL GUIDE

Consider a multiple-choice test such that each question has n multiple answers and only one is correct. Assume that the student has studied a percentage p (with  $0\% \le p \le 100\%$ ) of the test content. When a question addresses the content that the student has studied, he selects the right answer with 100% of probability. Otherwise, the student always selects randomly one of the n answers with a uniform distribution.

- **1.a.** When p = 60% and n = 4, determine the probability of the student to select the right answer. Answer: 70%
- **1.b.** When p = 70% and n = 5, determine the probability of the student to know the answer when he selects the right answer. Answer: 92.1%
- **1.c.** Draw a plot (with the same look as the plot below) with the probability of the student to select the right answer as a function of the probability p (consider the number of multiple answers n = 3, 4 and 5). What do you conclude from these results? Answer:

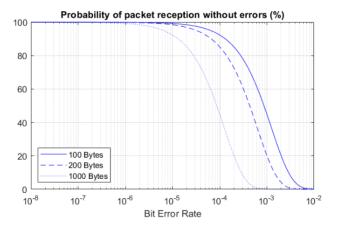


**1.d.** Draw a plot (with the same look as the plot below) with the probability of the student to know the answer when he selects the right answer as a function of the probability p (consider n = 3, 4 and 5). What do you conclude from these results? Answer:

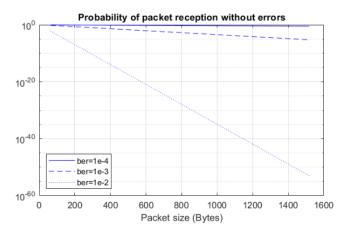


Consider a wireless link between multiple stations for data communications with a bit error rate (*ber*) of p. Assume that transmission errors in the different bits of a data frame are statistically independent (i.e., the number of errors of a data packet is a binomial random variable).

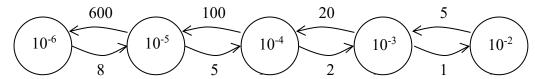
- **2.a.** Determine the probability of a data frame of 100 Bytes to be received without errors when  $p = 10^{-2}$ . Answer: 0.0322%
- **2.b.** Determine the probability of a data frame of 1000 Bytes to be received with exactly one error when  $p = 10^{-3}$ . Answer: 0.2676%
- **2.c.** Determine the probability of a data frame of 200 Bytes to be received with one or more errors when  $p = 10^{-4}$ . Answer: 14.7863%
- **2.d.** Draw a plot using a logarithmic scale for the X-axis (use the MATLAB function semilogx) with the same look as the plot below with the probability of a data frame (of size 100 Bytes, 200 Bytes or 1000 Bytes) being received without errors as a function of the *ber* (from  $p = 10^{-8}$  up to  $p = 10^{-2}$ ). What do you conclude from these results? Answer:



**2.e.** Draw a plot using a logarithmic scale for the Y-axis (use the MATLAB function semilogy) with the same look as the plot below with the probability of a data frame being received without errors (for  $p = 10^{-4}$ ,  $10^{-3}$  and  $10^{-2}$ ) as a function of the packet size (all integer values from 64 Bytes up to 1518 Bytes). What do you conclude from these results? Answer:

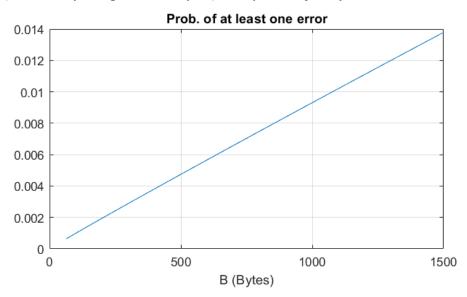


Consider a wireless link between multiple stations for data communications. The bit error rate (*ber*) introduced by the wireless link (due to the variation of the propagation and interference factors along with time) is approximately given by the following Markov chain:

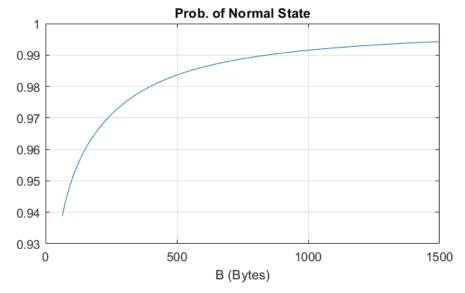


where the state transition rates are in number of transitions per hour. Consider that the link is in an interference state when its ber is at least  $10^{-3}$  and in a normal state, otherwise. Assume that all stations detect with a probability of 100% when the data frames sent by the other stations are received with errors. Determine:

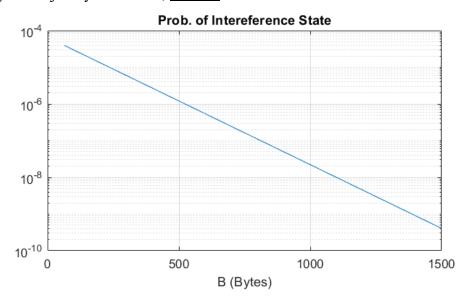
- **3.a.** the probability of the link being in each of the five states; <u>answer:</u>  $9.86 \times 10^{-1} (10^{-6}), 1.31 \times 10^{-2} (10^{-5}), 6.57 \times 10^{-4} (10^{-4}), 6.57 \times 10^{-5} (10^{-3}), 1.31 \times 10^{-5} (10^{-2})$
- **3.b.** the average percentage of time the link is in each of the five states; <u>answer:</u>  $9.86 \times 10^{-1} (10^{-6}), 1.31 \times 10^{-2} (10^{-5}), 6.57 \times 10^{-4} (10^{-4}), 6.57 \times 10^{-5} (10^{-3}), 1.31 \times 10^{-5} (10^{-2})$
- **3.c.** the average *ber* of the link; answer:  $1.38 \times 10^{-6}$
- **3.d.** the average holding time (in minutes) of the link in each of the five states; answer:  $7.5 \text{ min } (10^{-6})$ ,  $0.10 \text{ min } (10^{-5})$ ,  $0.59 \text{ min } (10^{-4})$ ,  $2.86 \text{ min } (10^{-3})$ ,  $12.0 \text{ min } (10^{-2})$
- **3.e.** the probability of the link being in the normal state and in interference state; <u>answer:</u> 0.999921 (normal),  $7.89 \times 10^{-5}$  (interference)
- **3.f.** the average *ber* of the link when it is in the normal state and when it is in the interference state; answer:  $1.18 \times 10^{-6}$  (normal),  $2.50 \times 10^{-3}$  (interference)
- **3.g.** considering a data frame of size *B* (in Bytes) sent by one source station to a destination station, draw a plot with the same look as the plot below of the probability of the packet being received by the destination station with at least one error as a function of the packet size (from 64 Bytes up to 1500 Bytes); analyze and justify the results; answer:



**3.h.** considering that a data frame of size *B* (in Bytes) sent by one source station is received with at least one error by the destination station, draw a plot with the same look as the plot below of the probability of the link being in the normal state as a function of the packet size (from 64 Bytes up to 1500 Bytes); analyze and justify the results; answer:



**3.i.** considering that a data frame of size *B* (in Bytes) sent by one source station is received without errors by the destination station, draw a plot with the same look as the plot below (use the MATLAB function semilogy) of the probability of the link being in the interference state as a function of the packet size (from 64 Bytes up to 1500 Bytes); analyze and justify the results; answer:

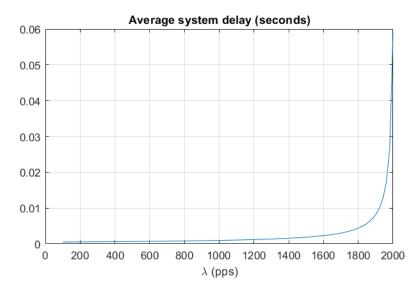


Consider an ideal link (i.e., with a ber = 0) from one router to another router with a capacity of C Mbps (1 Mbps =  $10^6$  bps) for IP communications. The link has a propagation delay of  $10 \mu s$  (1  $\mu s = 10^{-6}$  seconds). There is a very large queue at the output port of the link. The IP packet flow supported by the link is characterized by:

- (i) the packet arrivals are a Poisson process with rate  $\lambda$  pps (packets/second)
- (ii) the size of each IP packet is between 64 and 1518 bytes (the size includes the overhead of the Layer 2 protocol) with the probabilities: 19% for 64 bytes, 23% for 110 bytes, 17% for 1518 bytes and an equal probability for all other values (i.e., from 65 to 109 and from 111 to 1517).

Consider that  $\lambda = 1000$  pps and C = 10 Mbps. Determine:

- **4.a.** the average packet size (in Bytes) and the average packet transmission time of the IP flow; answer: 620.02 Bytes, 4.96×10<sup>-4</sup> seconds
- **4.b.** the average throughput (in Mbps) of the IP flow; answer: 4.96 Mbps
- **4.c.** the capacity of the link, in packets/second; <u>answer: 2016.06 pps</u>
- **4.d.** the average packet queuing delay and average packet system delay of the IP flow (the system delay is the queuing delay + transmission time + propagation delay) using the M/G/1 queuing model; answer: queuing  $-4.60 \times 10^{-4}$  seconds, system  $-9.66 \times 10^{-4}$  seconds
- **4.e.** draw a plot with the same look as the plot below with the average packet system delay as a function of the packet arrival rate  $\lambda$  (from  $\lambda = 100$  pps up to  $\lambda = 2000$  pps); analyze the results and take conclusions.



Consider the event driven simulator, implemented in the provided MATLAB function *Simulator1*, to estimate the performance of a point-to-point IP link between a company router and its ISP (Internet Service Provider). The simulator only considers the downstream direction, *i.e.*, from ISP to the company (usually, the direction with highest traffic load).

Simulator 1 considers a link of capacity C (in Mbps) and a queue of size f (in Bytes) with a FIFO (First-In-First-Out) scheduling discipline. The packet flow submitted to the link is characterized by: (i) an exponentially distributed time between packet arrivals with average  $1/\lambda$  and (ii) a random packet size between 64 and 1518 bytes with the probabilities: 19% for 64 bytes, 23% for 110 bytes, 17% for 1518 bytes and an equal probability for all other values (i.e., from 65 to 109 and from 111 to 1517).

Input parameters of *Simulator1*:

 $\lambda$  – packet rate, in packets per second (pps)

C — link capacity, in Mbps

*f* – queue size, in Bytes

P – total number of transmitted packets of a simulation run

Performance parameters estimated by Simulator 1:

PL – Packet Loss (%)

APD - Average Packet Delay (milliseconds)

MPD - Maximum Packet Delay (milliseconds)

TT - Transmitted Throughput (Mbps)

Stopping criterion of *Simulator1*:

> Time instant when the link finishes the transmission of the  $P^{th}$  packet (P is one of the input parameter); in Simulator I, the queued packets at the end of the simulation do not count for the performance estimation.

Simulator 1 makes use of the following variables:

Events: ARRIVAL (the arrival of a packet) and DEPARTURE (the transmission end of a packet).

<u>State variables</u>: STATE (binary variable indicating if the link is free or busy with the transmission of a packet), QUEUEOCCUPATION (occupation of the queue, in number of bytes, with the queued packets) and QUEUE (matrix with a variable number of rows and 2 columns where each column has the size and the arriving time instant of each packet in the queue).

<u>Statistical counters</u>: TOTALPACKETS (number of packets arrived to the system), LOSTPACKETS (number of packets dropped due to buffer overflow), TRANSPACKETS (number of transmitted packets), TRANSBYTES (sum of the bytes of the transmitted packets), DELAYS (sum of the delays of the transmitted packets), MAXDELAY (maximum delay among all transmitted packets).

Based on the statistical counters, the performance parameters are estimated at the end of the simulation as:

```
PL = 100 \times LostPackets / TotalPackets
```

 $APD = 1000 \times DELAYS / TRANSPACKETS$ 

 $MPD = 1000 \times MAXDELAY$ 

 $TT = 10^{-6} \times TRANSBYTES \times 8 / total simulated time$ 

**5.a.** Develop a MATLAB script to run *Simulator1* 10 times with a stopping criterion of P = 10000 at each run and to compute the estimated values and the 90% confidence intervals of all performance parameters when  $\lambda = 1800$  pps, C = 10 Mbps and f = 1.000.000 Bytes (~1 MByte). Results (recall that these are simulation results):

```
PacketLoss (%) = 0.00e+00 +- 0.00e+00

Av. Packet Delay (ms) = 4.67e+00 +- 3.07e-01

Max. Packet Delay (ms) = 2.54e+01 +- 2.90e+00

Throughput (Mbps) = 8.94e+00 +- 6.68e-02
```

**5.b.** Repeat the previous experiment now running *Simulator 1* 100 times. Compare these results with the previous ones and take conclusions. Results (recall that these are simulation results):

```
PacketLoss (%) = 0.00e+00 +- 0.00e+00

Av. Packet Delay (ms) = 4.32e+00 +- 1.01e-01

Max. Packet Delay (ms) = 2.27e+01 +- 7.71e-01

Throughput (Mbps) = 8.93e+00 +- 1.82e-02
```

**5.c.** Consider system modelled by an M/G/1 queueing model<sup>1</sup>. Determine the theoretical values of the packet loss, average packet delay and total throughput using the M/G/1 model for the parameters considered in experiments **5.a** and **5.b**. Compare these values with the simulation results of experiments **5.a** and **5.b** and take conclusions. Results:

```
Packet Loss (%) = 0.0000
Av. Packet Delay (ms) = 4.3883
Throughput (Mbps) = 8.9283
```

**5.d.** Repeat experiment **5.b** now considering f = 10.000 Bytes (~10 KBytes) Justify the differences between these results and the results of experiment **5.b**. Results (recall that these are simulation results):

```
PacketLoss (%) = 1.01e+00 +- 3.56e-02

Av. Packet Delay (ms) = 2.95e+00 +- 2.39e-02

Max. Packet Delay (ms) = 8.99e+00 +- 1.90e-02

Throughput (Mbps) = 8.75e+00 +- 1.60e-02
```

**5.e.** Repeat experiment **5.b** now considering f = 2.000 Bytes (~2 KBytes) Justify the differences between these results and the results of experiments **5.b** and **5.d**. Results (recall that these are simulation results):

```
PacketLoss (%) = 1.04e+01 +- 5.17e-02

Av. Packet Delay (ms) = 9.54e-01 +- 1.59e-03

Max. Packet Delay (ms) = 2.71e+00 +- 7.85e-03

Throughput (Mbps) = 7.12e+00 +- 1.02e-02
```

 $<sup>^{1}</sup>$  In Module 1 of the theoretical slides, check the M/G/1 queueing model (slide 53) and the resolution of Example 8 (slides 58-60).