Instituto Superior Técnico

Masters in
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Robotics

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Direct and Inverse Kinematics of Serial Manipulators

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1 Reference Frames

The first step to determine the direct and inverse kinematics is to determine the reference frames between the base frame, defined as the world frame, and the frame of the end-effector. 9 reference frames are used, which are represented in figure 1, through frames 0 to 8. It is important to notice that frames 7 and 8 overlap with each other even though all the axes' directions change, i.e, from the 7th frame to the 8th frame respectively, the X axis becomes the Z axis, the Z axis becomes the Y axis and the Y axis becomes the X axis. Each joint is represented by a degree of freedom and it is represented, in figure 1, by an arrow rotating along the Z axis.

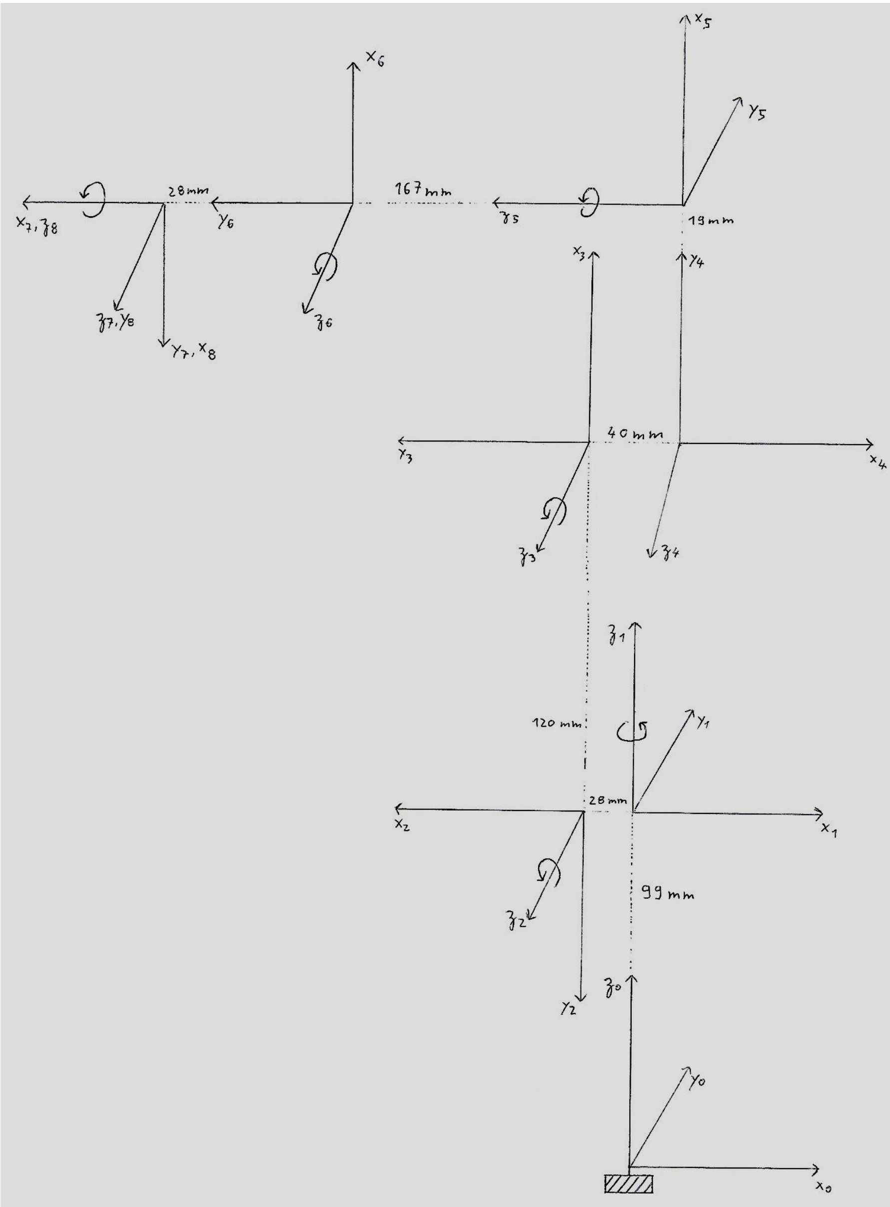

Figure 1: Chosen reference frames

After deciding how many and which reference frames to use, it is necessary to model the links and joints that will allow us to calculate the transformation between the different joints. To model these, we used the Denavit-Hartenberg (D-H) convention, consisting in four joint variables, two for linear joints and the other two for rotation joints, that are described by the set of reference frames. However, the notation used differs from the one explained in the theoretical classes. For each of the 8 links, we have 4 parameters: d, θ, a and α and we considered, for example, link 1 to be the link between frames 0 and 1, link 2 between the frames 1 and 2, and so on. That being said, the definition of the parameters used is the following:

- d : Depth along the Z axis of the previous joint.
- θ : Angle of rotation around the previous joint's Z axis to align the X axis of both frames — The angle about the previous z to align it's previous x with the new origin.
- a : Distance along the newly rotated X axis — The distance along the rotated x axis.
- α : Angle of rotation around the newly rotated X axis to place the Z axis in the desired orientation —Rotates about the new x axis to put z in it's desired orientation.

Finally, the parameter values for each link of figure 1 can be computed and these are presented in table 1.

Table 1: Table with the join variables for each link transformation

Link	d [mm]	θ [°]	a [mm]	α [°]
1	99	0	0	0
2	0	180	28	-90
3	0	-90	120	0
4	0	-90	40	0
5	0	90	19	-90
6	167	0	0	90
7	0	90	28	0
8	0	90	0	90

2 User Manual & Matlab Functions

This section contains instructions about how run the program created as well as how to use the MATLAB functions created. There are 4 function files and 1 script file. These are:

- *lab1.m* - script file that calls the direct kinematics and inverse kinematics functions with specific inputs;
- *direct_kinematics.m* - function file that does the direct kinematics based on the inputs and predefined frames;
- *inverse_kinematics.m* - function file that does the inverse kinematics based on the inputs and predefined frames;
- *create_trans_mat.m* - function file that computes the translation matrix based on the distance and desired axis;
- *create_rot_mat.m* - function file that computes the rotation matrix based on the angle and desired axis of rotation;

The functions are called by using the script *lab1*, with the desired inputs previous declared in *lab1.m*.

There are 4 functions, two of them being auxiliary functions used throughout the program:

- [position, orientation] = direct_kinematics(dofs);
- JointAngles = inverse_kinematics(position, orientation);
- [trans_mat] = create_trans_mat(distance, trans_axis);
- [rot_mat] = create_rot_mat(angle, rot_axis);

Explanations on what they compute and how to use them are presented below.

2.1 Direct Kinematics

The default position of the robot arm considered (every degree of freedom is 0°) is found in figure 2.

Each degree of freedom represents a joint and is identified in figure 1, represented by an arrow rotating along the z axis.
To calculate the position of the end-effector of the robot, the 8 transformations matrices were computed, along with a rotation matrix in the last frame, correspondent to the last degree of freedom. The product of this matrices result in the base-to-end transformation matrix.
The position of the end-effector (in world frame coordenates) is directly obtained from the first 3 entries of the last column of this matrix.
To calculate the orientation, it was used an X-Y-Z convention. The euler angles

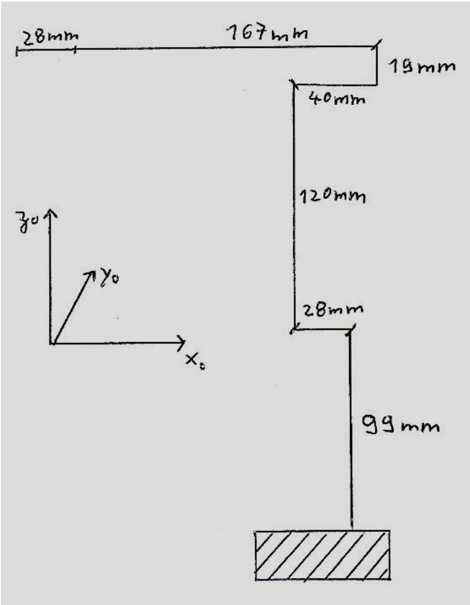

Figure 2: Default position

are obtained from the 3 first lines and columns of the base-to-end transformation matrix.

2.1.1 User Manual

This function receives an array with the 6 values corresponding to the 6 degrees of freedom and computes the position and orientation of the end effector. Example:

- Input: dofs($\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$) = [180, 45, 45, 0, 0, 0];
- Output: position(x, y, z), orientation(α, β, γ)
  - position = [131.853, 0, 28.8528];
  - orientation = [180, 0, 180];

2.2 Inverse Kinematics

To compute all the possible solutions for a given position and orientation, it's used the geometric method for $\theta_1, \theta_2, \theta_3$, and the algebraic method for $\theta_4, \theta_5, \theta_6$.
To compute θ_1, firstly it's obtained the position of frame 6. With this position, θ_1 it's directly obtained by $arctan(y_6, x_6)$.
For θ_r and θ_2, it was assumed a triangle connecting 3 different frames (frame 2, 3 and 6), with a distance 2-3 of 120 mm and a distance 3-6 of $\sqrt{(167-20)^2+19^2} \approx 128.41$ mm. With this, θ_3 and θ_2 are obtained, respectively, by

$$\theta_3 = \pm acos\left(\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}\right) \qquad \theta_2 = atan2(y,x) \pm acos\left(\frac{x^2+y^2+a_1^2-a_2^2}{2a_1\sqrt{x^2+y^2}}\right).$$

To calculate the 3 remaining angles, it is needed to compute $\left[{}^{0}_{4}T(\theta)\right]^{-1} \cdot {}^{base}_{tool}T$ and ${}^{5}_{8}T(\theta)$. Equaling both expressions, we can obtain the remaining angles by

$\theta_4 = atan2(\pm T(2,3); \pm T(1,3)), \theta_5 = \pm asin(T(3,3)), \theta_6 = atan2(\pm T(3,2); \pm T(3,1))$.

2.2.1 User Manual

This function receives the position, in millimetres, and the orientation angles, in degrees, of the end-effector in the world frame and computes all possible solutions in the joint angle space. Example:

- Input: position(x, y, z), orientation(α, β, γ)
  - position = [-183, 0, 238];
  - orientation = [0, 90, 180];
- Output: JointAngles(solution, degree of freedom)
  - JointAngles(1, :) = [180.00, 5.17, 81.49, 180.00, -3.34, 180.00];
  - JointAngles(2, :) = [180.00, 5.17, 81.49, -0.00, 3.34, -0.00];
  - JointAngles(3, :) = [180.00, 90.00, -81.49, 180.00, -81.49, 180.00];
  - JointAngles(4, :) = [180.00, 90.00, -81.49, -0.00, 81.49, -0.00];
  - JointAngles(5, :) = [0, 119.65, 44.66, 180.00, -105.68, 0.00];
  - JointAngles(6, :) = [0, 119.65, 44.66, -0.00, 105.68, -180.00];
  - JointAngles(7, :) = [0, 165.91, -44.66, 180.00, -148.75, 0.00];
  - JointAngles(8, :) = [0, 165.91, -44.66, -0.00, 148.75, -180.00];

2.3 Auxiliary Functions

2.3.1 Rotation Matrix

This function receives the desired angle and desired axis of rotation and computes the corresponding rotation matrix. Examples:

- Input: (90, 'X')
- Output: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- Input: (90, 'Y')
- Output: $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2.3.2 Translation Matrix

This function receives the desired translation distance along an axis and computes the corresponding translation matrix. Examples:

- Input: (50, 'Z');
- Output: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- Input: (75, 'Y');
- Output: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 75 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3 Test Results

This section contains the outputs of sets of tests, with different sets of inputs, for the direct and inverse kinematics.

3.1 Direct Kinematics

This subsection contains the outputs of sets of tests for the direct kinematics, using different sets of inputs. These outputs are presented in table 2, along with the inputs provided in each test.

Table 2: Table with the position and orientation of the end-effector for different inputs.

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	Position(x, y, z)	Orientation(α, β, γ)
0	0	0	0	0	0	$(-183, 0, 238)$	$(0, 90, 180)$
90	0	0	0	0	0	$(0, -183, 238)$	$(0, 90, 90)$
90	0	0	0	0	90	$(0, -183, 238)$	$(0, 0, 90)$
0	90	90	0	0	0	$(7, 0, 80)$	$(0, -90, -180)$
0	90	90	90	0	90	$(7, 0, 80)$	$(0, 90, 0)$
0	0	90	90	0	90	$(-47, 0, 64)$	$(180, 0, 180)$
0	0	0	0	90	0	$(-155, 0, 210)$	$(0, 0, 180)$
0	90	0	0	90	0	$(-139, 0, -28)$	$(0, -90, -180)$
0	0	45	0	45	0	$(-131.238, 0, 114.632)$	$(0, 0, 180)$
135	0	0	45	45	0	$(133.501, -113.702, 224)$	$(99.7356, 30, 125.264)$

3.2 Inverse Kinematics

This subsection contains the outputs of sets of tests for the inverse kinematics, using different sets of inputs. These outputs are presented in table 3, along with the inputs provided in each test.

Table 3: Table with the possible solutions for any given position and orientation.

Position	Orientation	Degrees of Freedom					
(x, y, z)[mm]	(α, β, γ)[mm]	$\theta_1[°]$	$\theta_2[°]$	$\theta_3[°]$	$\theta_4[°]$	$\theta_5[°]$	$\theta_6[°]$
(45, 0, 150)	(0, 0, 180)	0.00	1.09	142.22	180.00	−36.69	0.00
					0.00	36.69	−180.00
			154.62	-142.22	180.00	−167.60	0.00
					0.00	167.60	−180.00
		180.00	64.28	128.82	0.00	−13.10	0.00
					-180.00	13.10	−180.00
			201.20	-128.82	180.00	−107.63	180.00
					0.00	107.63	0.00
(150, 0, 150)	(0, 0, 180)	0.00	-24.00	108.47	180.00	−95.53	0.00
					0.00	95.53	−180.00
			89.95	-108.47	0.00	−161.38	180.00
					-180.00	161.38	0.00
		180.00	116.13	76.80	0.00	−12.93	0.00
					-180.00	12.93	−180.00
			196.01	-76.80	180.00	−60.80	180.00
					0.00	60.80	0.00
(250, 0, 150)	(0, 0, 180)	0.00	0.47	36.93	180.00	−142.59	0.00
					0.00	142.59	−180.00
			38.70	-36.93	180.00	−178.23	0.00
					0.00	178.23	−180.00

4 Singularities

If the position of frame 6 is (0,0,z), it is not possible to determine the angle θ_1, since

$$\theta_1 = atan2(y, x).$$

It also is not possible to determine θ_4 and θ_6 if $\theta_5 = \pm 90°$, since

$$\left.\begin{aligned} T(2,3) &= sin(\theta_4)cos(\theta_5) \\ T(1,3) &= cos(\theta_4)cos(\theta_5) \end{aligned}\right\} \frac{T(2,3)}{T(1,3)} = \frac{sin(\theta_4)}{cos(\theta_4)}\frac{cos(\theta_5)}{cos(\theta_5)}$$

and

$$\left.\begin{aligned} T(3,2) &= -sin(\theta_6)cos(\theta_5) \\ T(3,1) &= cos(\theta_6)cos(\theta_5) \end{aligned}\right\} \frac{T(3,2)}{T(3,1)} = -\frac{sin(\theta_6)}{cos(\theta_6)}\frac{cos(\theta_5)}{cos(\theta_5)}.$$