Poisson Image Editing

SIGGRAPH 2003

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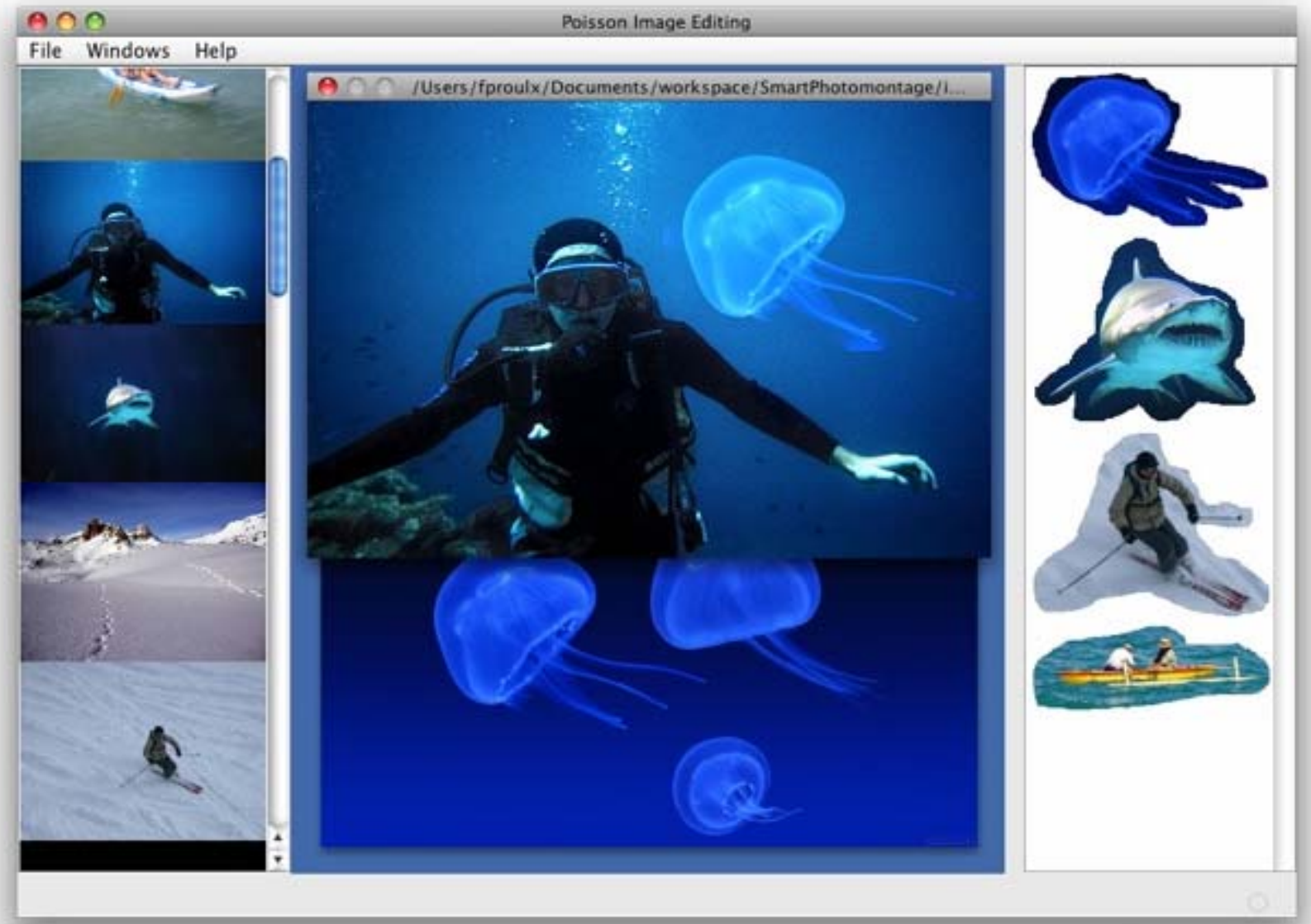
Poisson Image Editing
File
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Most Frequently Used PDE

- Wave Equation

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u + f(t, x, y, z)$$

- Heat Equation

$$\frac{\partial u}{\partial t} = c^2 \nabla^2 u + f(t, x, y, z)$$

Most Frequently Used PDE

- Poisson Equation, Steady State of Wave Equation and Heat Equation

$$\nabla^2 u = f(x, y, z)$$

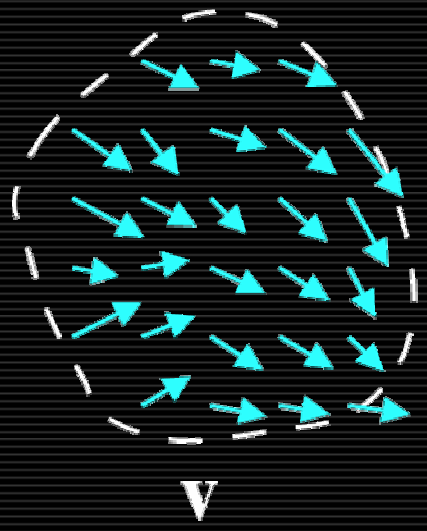
- Laplace's Equation

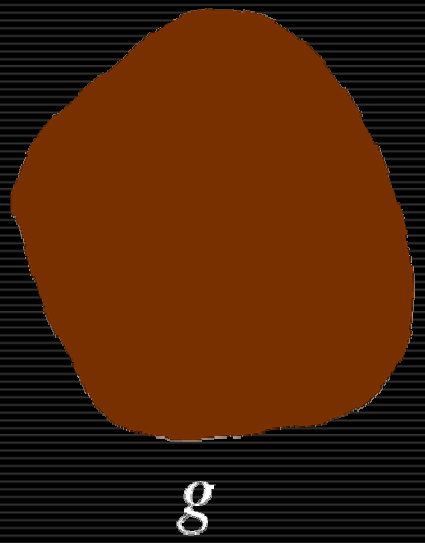
$$\nabla^2 u = 0$$

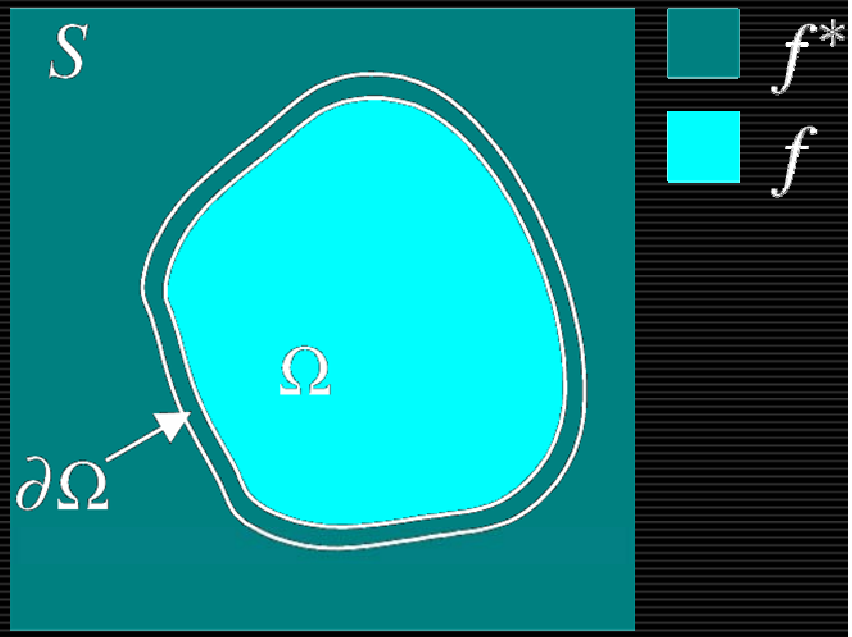
Boundary Conditions

- Dirichlet Boundary Conditions
  - Specify the value of the function on a surface
- Neumann Boundary Condition
  - Specify the normal derivative of the function on a surface

Guided Interpolation







- f: to be solved, f*: known region
- v: guided field, g: v is prob. gradient of g

Simple Interpolation

- Maximize the Smoothness

$$\begin{cases} \min_f \iint_\Omega \|\nabla f\|^2 \\ f|_{\partial\Omega} = f^*|_{\partial\Omega} \end{cases}$$

- Solution: Laplace Equation with Dirichlet Boundary Conditions

$$\begin{cases} \nabla^2 f = 0 \\ f|_{\partial\Omega} = f^*|_{\partial\Omega} \end{cases}$$

Example: Laplace's Equation. The functional corresponding to Laplace's equation is

$$\begin{aligned} I(\phi) &= \frac{1}{2}\int_V \|\nabla\phi\|^2\, dV \\ &= \frac{1}{2}\int_V \left(\phi_x^2 + \phi_y^2\right)\, dx\, dy \end{aligned} \tag{7.16}$$

in the 2D case. The factor of $1/2$ is included so that the functional can be identified as the energy stored by the field ϕ (for a particular physical problem there may also be an additional constant to make the units work out).

The Euler-Lagrange equation is

$$\frac{1}{2}\frac{\partial(\phi_x^2+\phi_y^2)}{\partial\phi} - \frac{1}{2}\frac{\partial}{\partial x}\frac{\partial(\phi_x^2+\phi_y^2)}{\partial\phi_x} - \frac{1}{2}\frac{\partial}{\partial y}\frac{\partial(\phi_x^2+\phi_y^2)}{\partial\phi_y} = 0 \tag{7.17}$$

Working the partial derivatives gives

$$0 - \frac{\partial}{\partial x}\phi_x - \frac{\partial}{\partial y}\phi_y = 0 \tag{7.18}$$

Notice that the partial derivatives are evaluated symbolically as if ϕ, ϕ_x, and ϕ_y were all independent variables. This can be understood intuitively by thinking of forming a parameterized variation of each of these functions with respect to a different small parameter and then using (7.10) separately for each function. Since the subscripts on ϕ_x and ϕ_y represent partial derivatives, Eq. (7.18) becomes

$$\frac{\partial\phi}{\partial x^2} + \frac{\partial\phi}{\partial y^2} = 0 \tag{7.19}$$

which is Laplace's equation. The functional can easily be modified to include a source term, so that Poisson's equation is obtained.

Guided Interpolation

- Interpolation-> minimization

$$\begin{cases} \min_f \iint_\Omega \|\nabla f - \mathbf{v}\|^2 \\ f|_{\partial\Omega} = f^*|_{\partial\Omega} \end{cases}$$

- Solution: Poisson Equation with Dirichlet Boundary Conditions

$$\begin{cases} \nabla^2 f = \nabla \cdot \mathbf{v} \\ f|_{\partial\Omega} = f^*|_{\partial\Omega} \end{cases}$$

- Relationship with Laplace case?

Or from vector field decomposition

- Helmholtz-Hodge decomp.

$$\mathbf{w} = \nabla\phi + \nabla \times v + \mathbf{h}$$

$$\min_{\phi} \int\int_{T} \|\nabla\phi - \mathbf{w}\|^2 dA,$$

$$\nabla^2\phi = \nabla \cdot \mathbf{w}$$

Discrete Poisson Solver

- Discretize the Minimization Directly

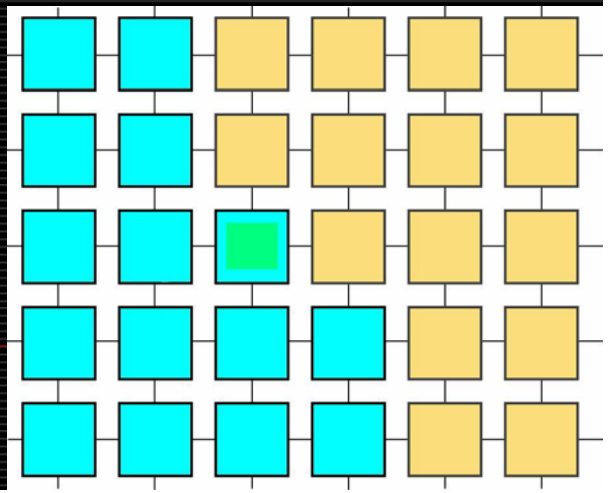
$$\min_{f|\Omega} \sum_{<p,q>\cap\Omega\neq\emptyset} (f_p - f_q - v_{pq})^2 \text{ with } f_p = f_p^* \text{ for } \forall p \in \partial\Omega$$

- Partial Derivative

$$\text{for } \forall p \in \Omega,\ |N_p| f_p - \sum_{q\in N_p\cap\Omega} f_q = \sum_{q\in N_p\cap\partial\Omega} f_q^* + \sum_{q\in N_p} v_{pq}$$

- Partial Derivative for Interior Points

$$|N_p| f_p - \sum_{q\in N_p} f_q = + \sum_{q\in N_p} v_{pq}$$



Discrete Poisson Solver

- Linear System of Equations
- Gauss-Seidel Method with Successive Overrelaxation
- V-cycle Multigrid
- Discretize Laplacian with Discrete Laplacian of Gaussian
- http://www.tau.ac.il/~stoledo/taucs/ Taucs

Seamless Cloning :Importing Gradients

- Importing Gradients from a Source Image

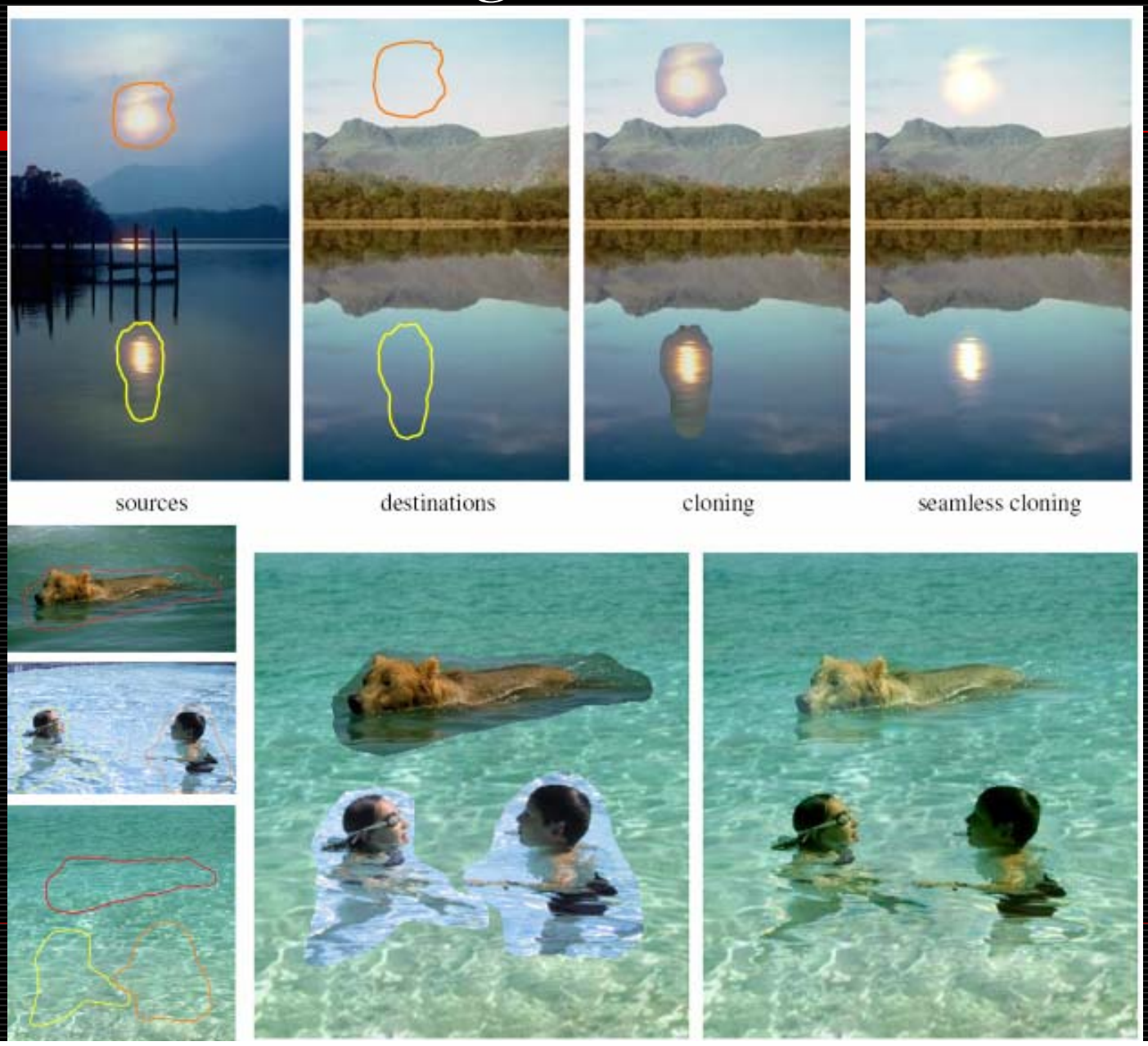
$$g$$

$$\mathbf{v} = \nabla g$$

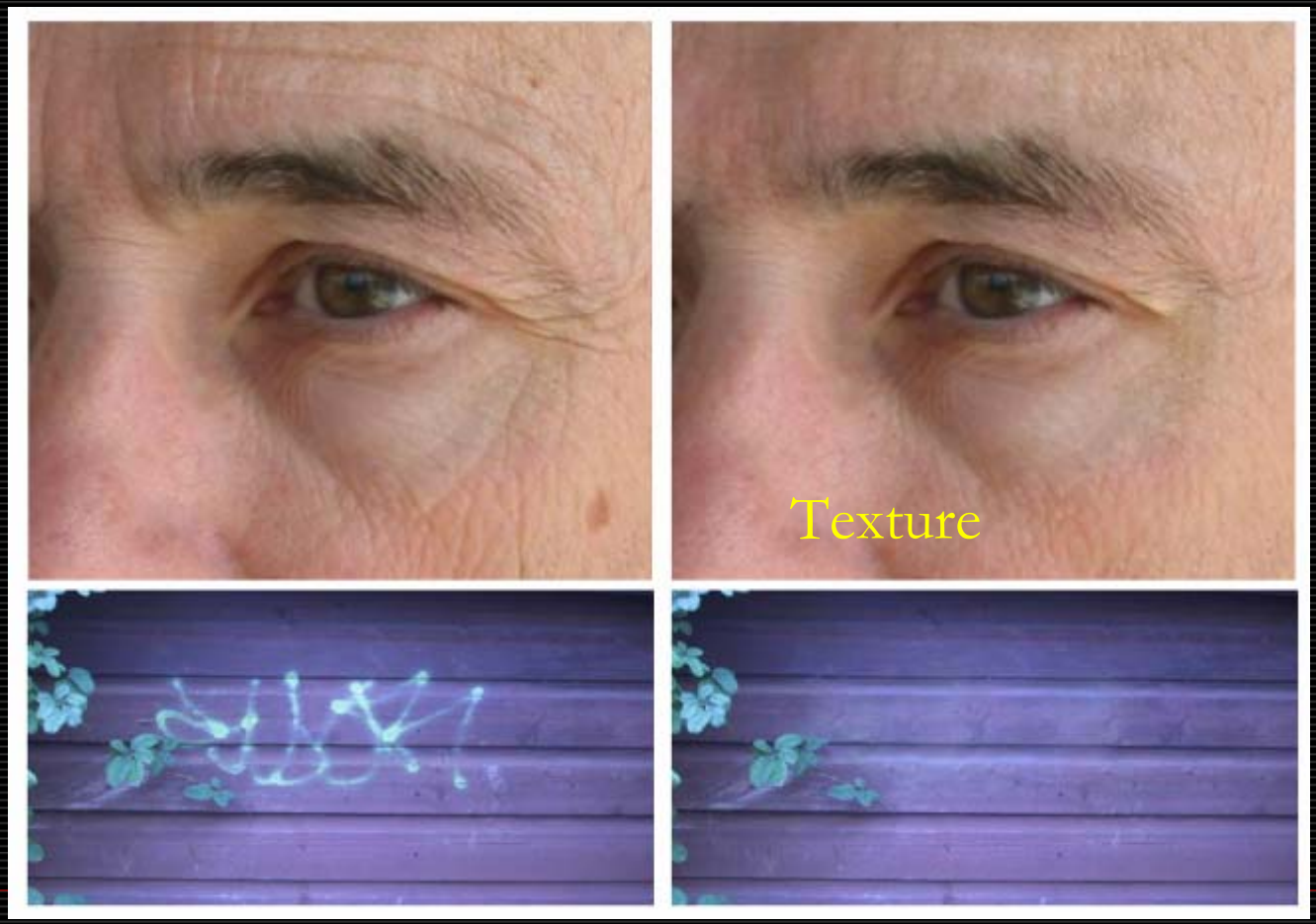
- Discretize

$$\text{for all } \langle p, q \rangle, v_{pq} = g_p - g_q$$

Seamless Cloning Results



Seamless Cloning Results

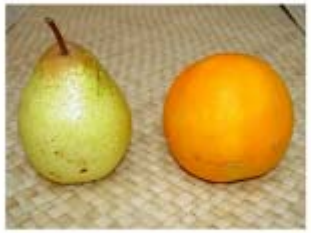


Transfer intensity only

source/destination

cloning

seamless cloning



swapped textures

Seamless Cloning: Mixing Gradients

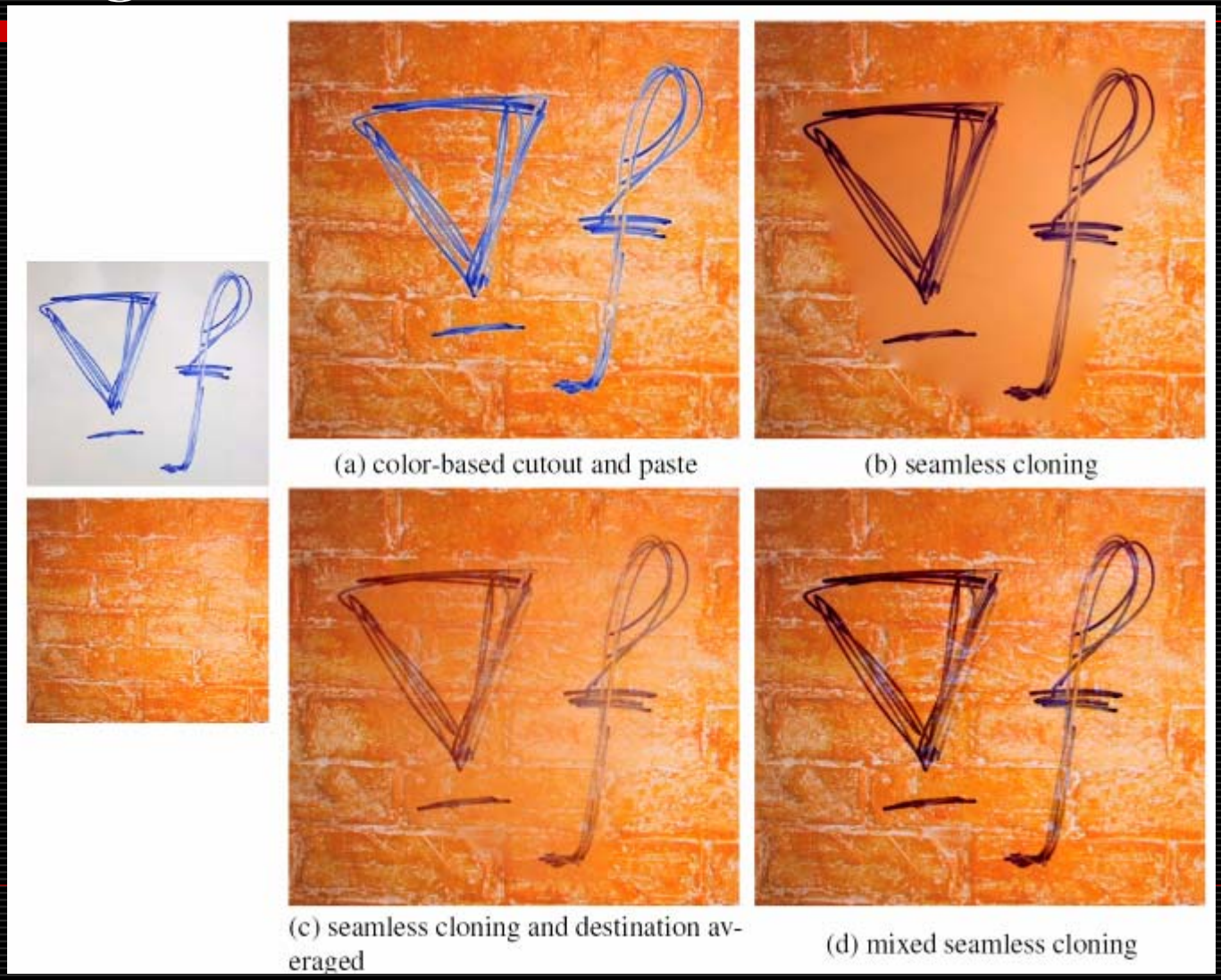
- Two Proposals
  - Define **v** as Linear Combination of Source and Destination Gradients
  - Select Stronger one from Source and Destination Gradients (not conservative!)

$$\text{for all } \mathbf{x} \in \Omega,\ \mathbf{v}(\mathbf{x}) = \begin{cases} \nabla f^*(\mathbf{x}) & \text{if } |\nabla f^*(\mathbf{x})| > |\nabla g(\mathbf{x})| \\ \nabla g(\mathbf{x}) & \text{otherwise.} \end{cases}$$

  - Discretization

$$v_{pq} = \begin{cases} f_p^* - f_q^* & \text{if } |f_p^* - f_q^*| > |g_p - g_q| \\ g_p - g_q & \text{otherwise,} \end{cases}$$

Mixing Gradients Results



(a) color-based cutout and paste

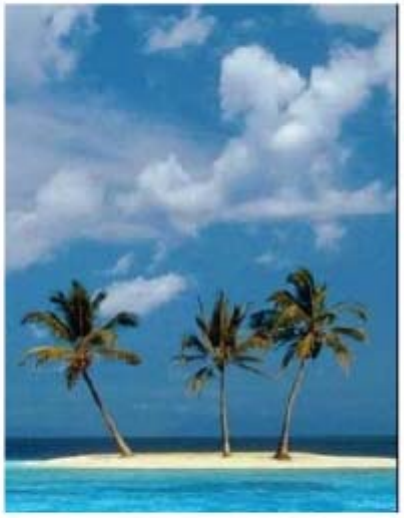
(b) seamless cloning

(c) seamless cloning and destination averaged

(d) mixed seamless cloning

Mixing Gradients Results

source



destination

Mixing Gradients Results

Just reduce terrible things ! Another solution

Texture Flattening

- Remain Only Salient Gradients

$$\text{for all } \mathbf{x} \in \Omega,\ \mathbf{v}(\mathbf{x}) = M(\mathbf{x}) \nabla f^*(\mathbf{x})$$

- Discretization

$$v_{pq} = \begin{cases} f_p - f_q & \text{if an edge lies between } p \text{ and } q \\ 0 & \text{otherwise,} \end{cases}$$

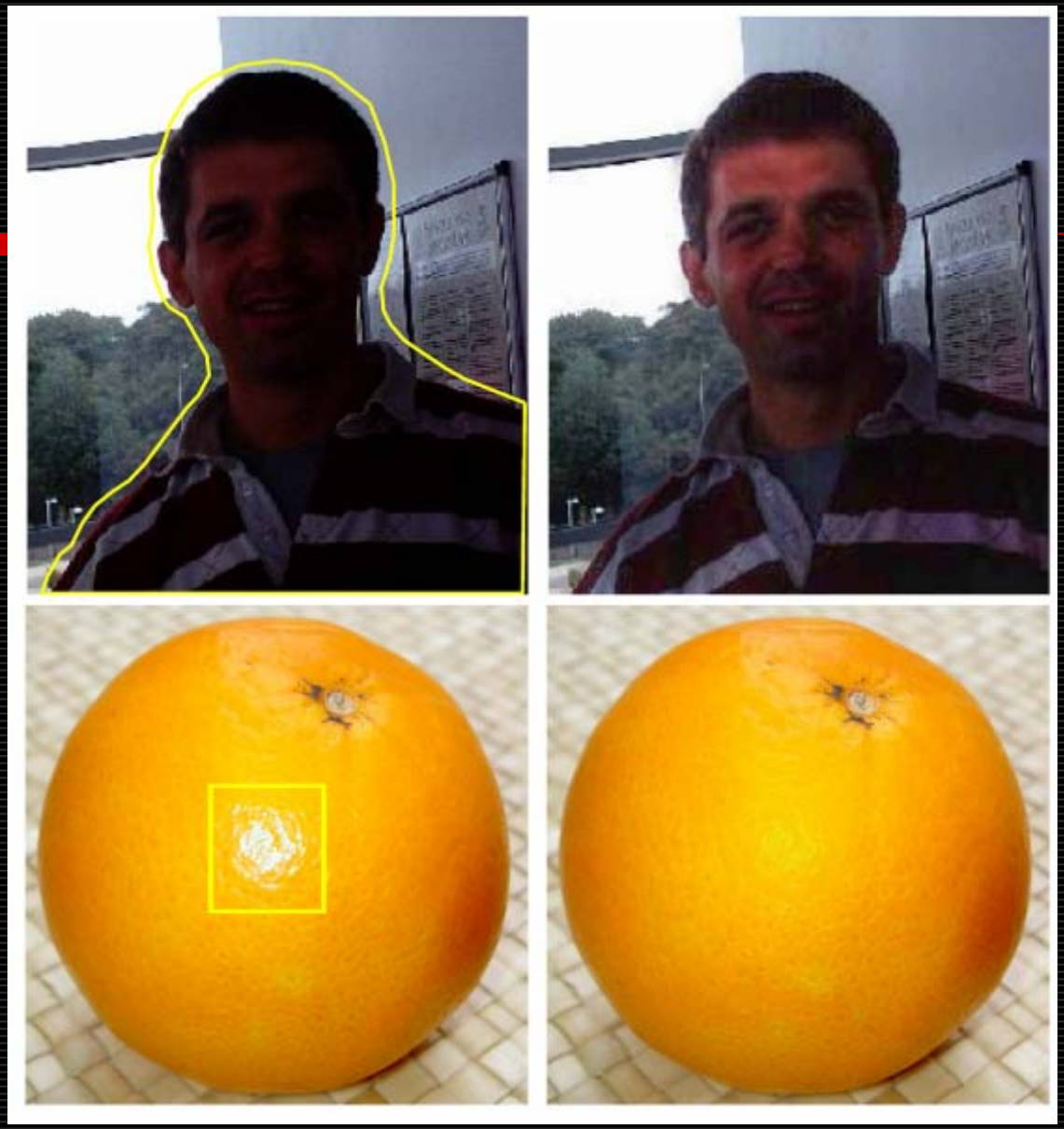
Texture Flattening

Edge mask

Local Illumination Changes

- Fattal Transformation

$$\mathbf{v} = \alpha^{\beta} |\nabla f^*|^{-\beta} \nabla f^*$$



Local Color Changes

- Mix two different colored version of original image
  - One provide f^* outside
  - One provide g inside

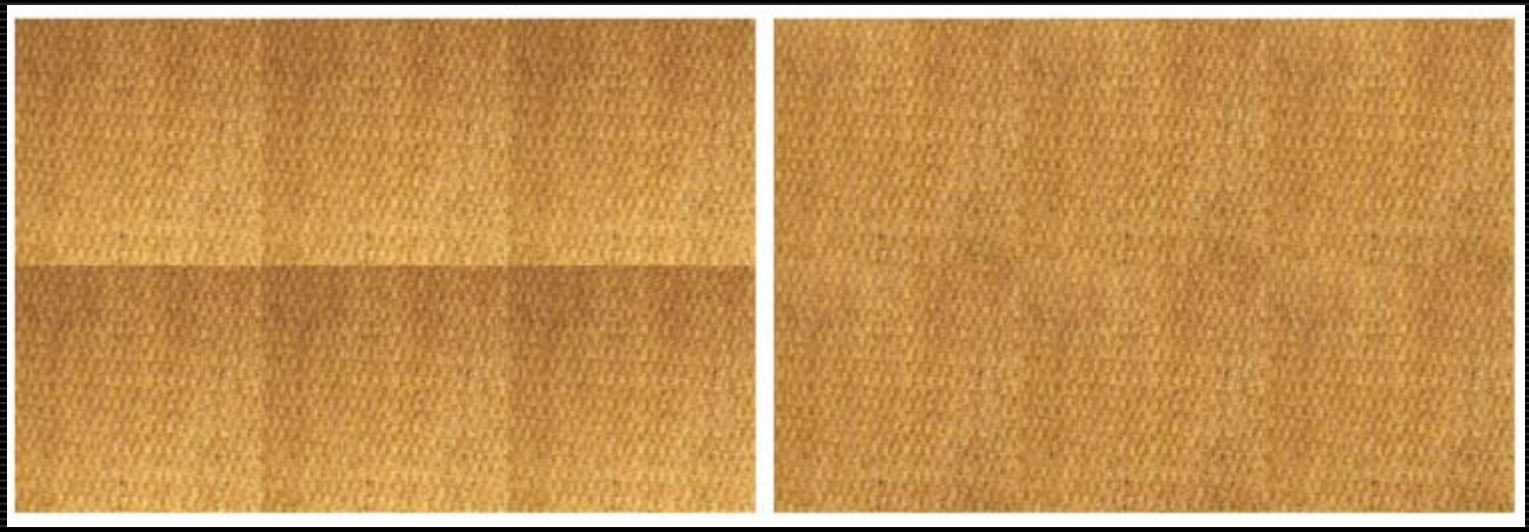
Local Color Changes

Monochrome of background
White in flower

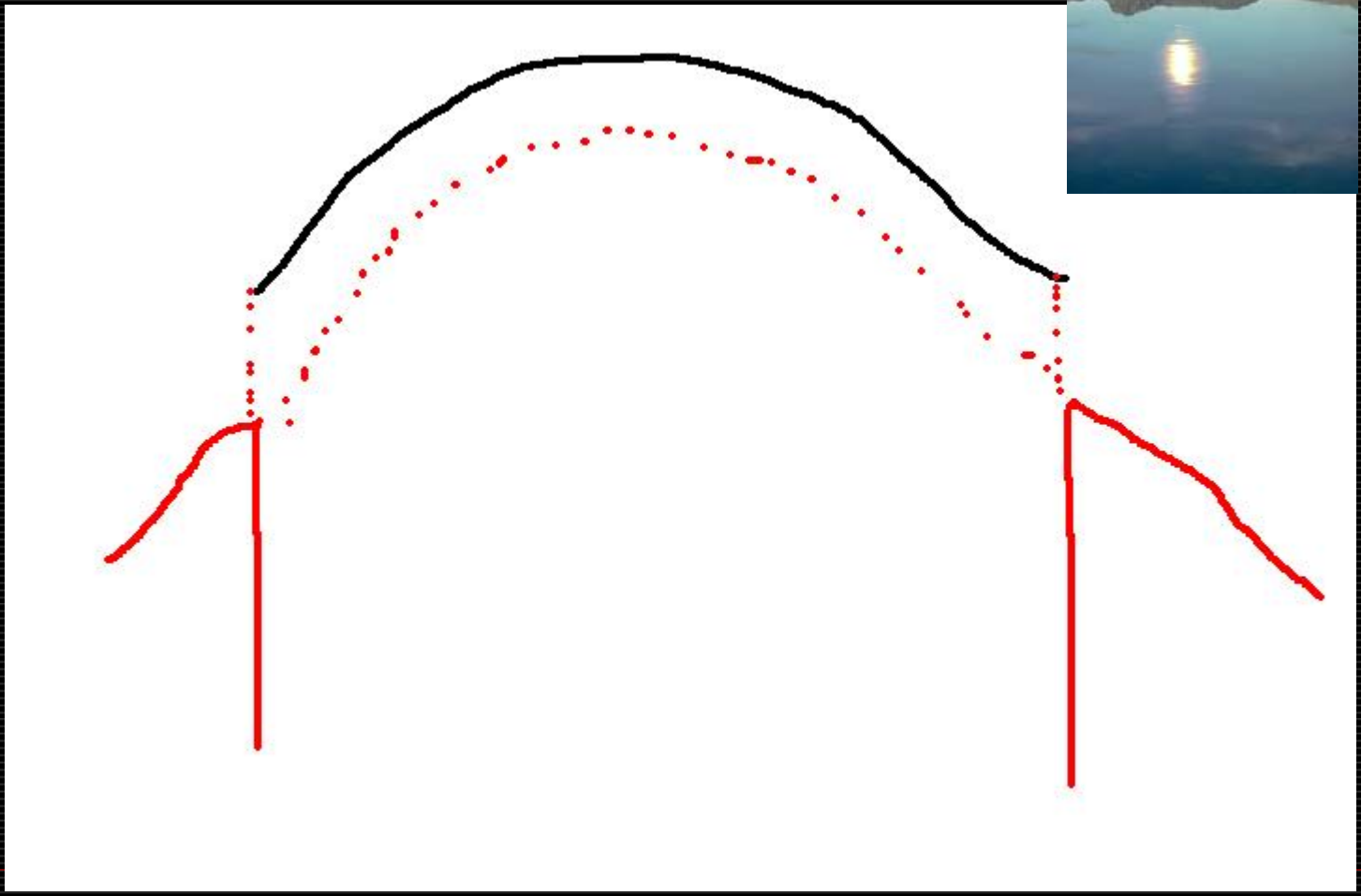
Seamless Tiling

- Select original image as g
- Boundary condition:
  - $f^*_{north} = f^*_{south} = 0.5(g_{north} + g_{south})$
  - Similarly for the east and west

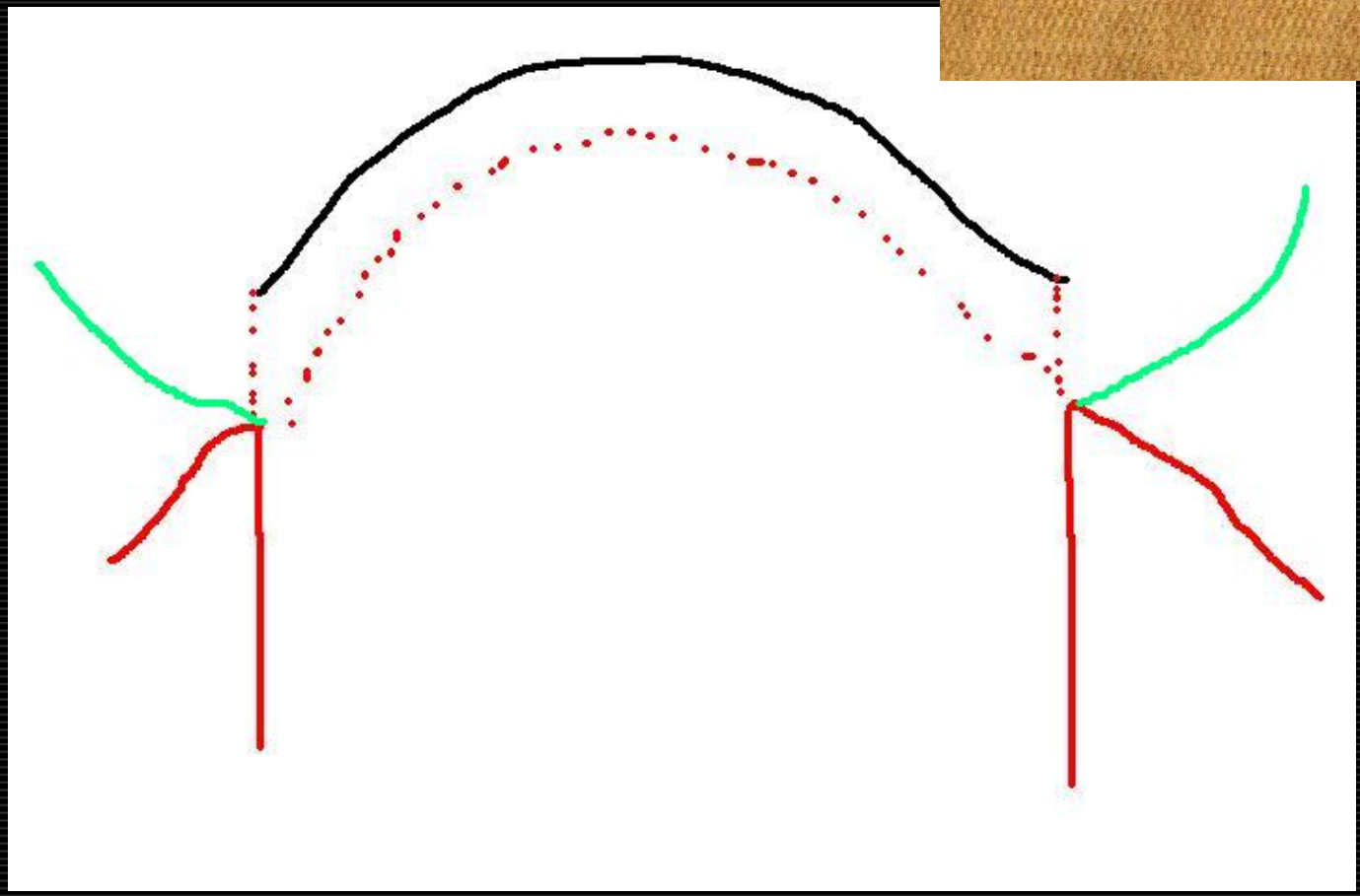
Seamless Tiling



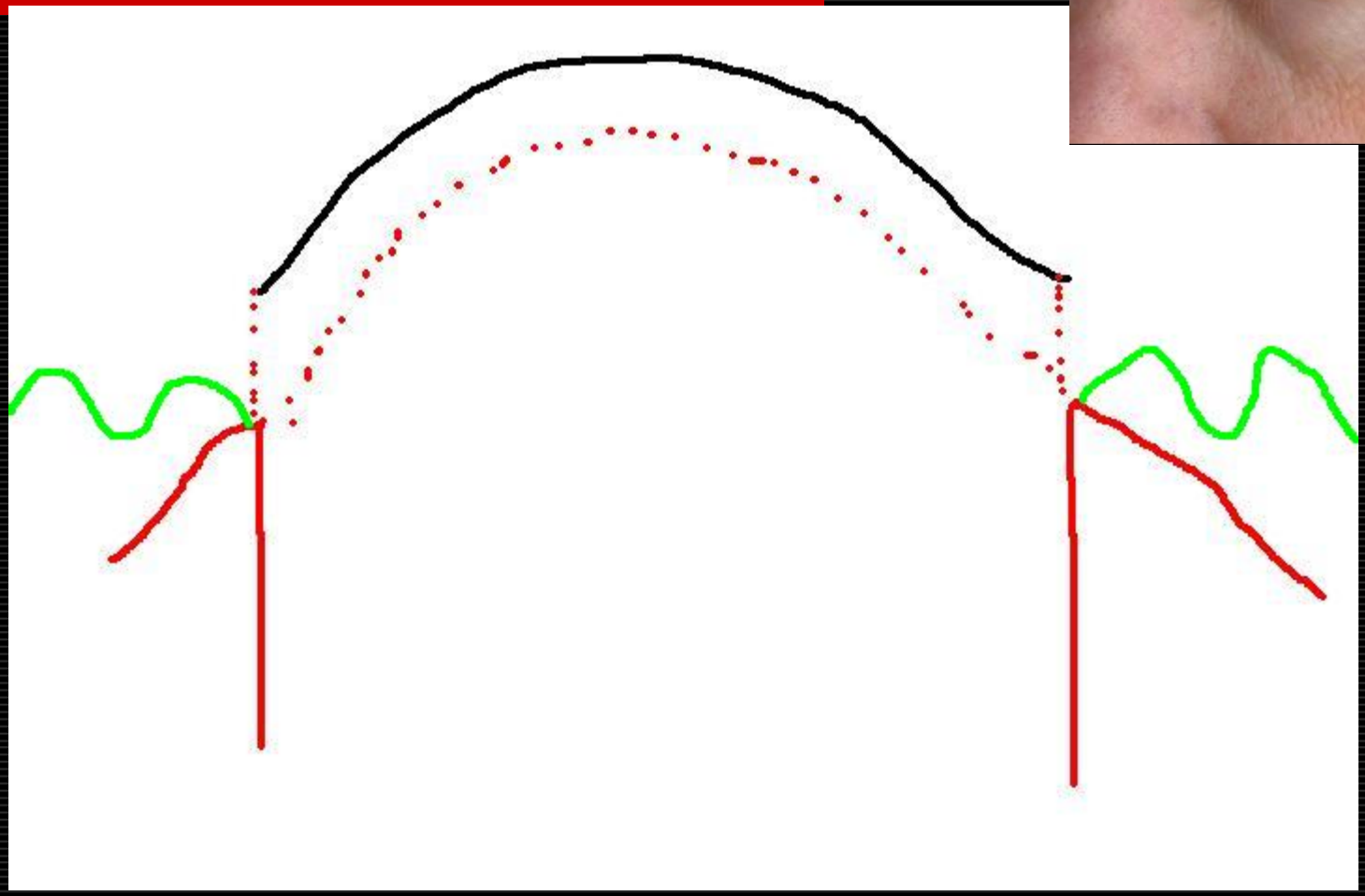
Discussion



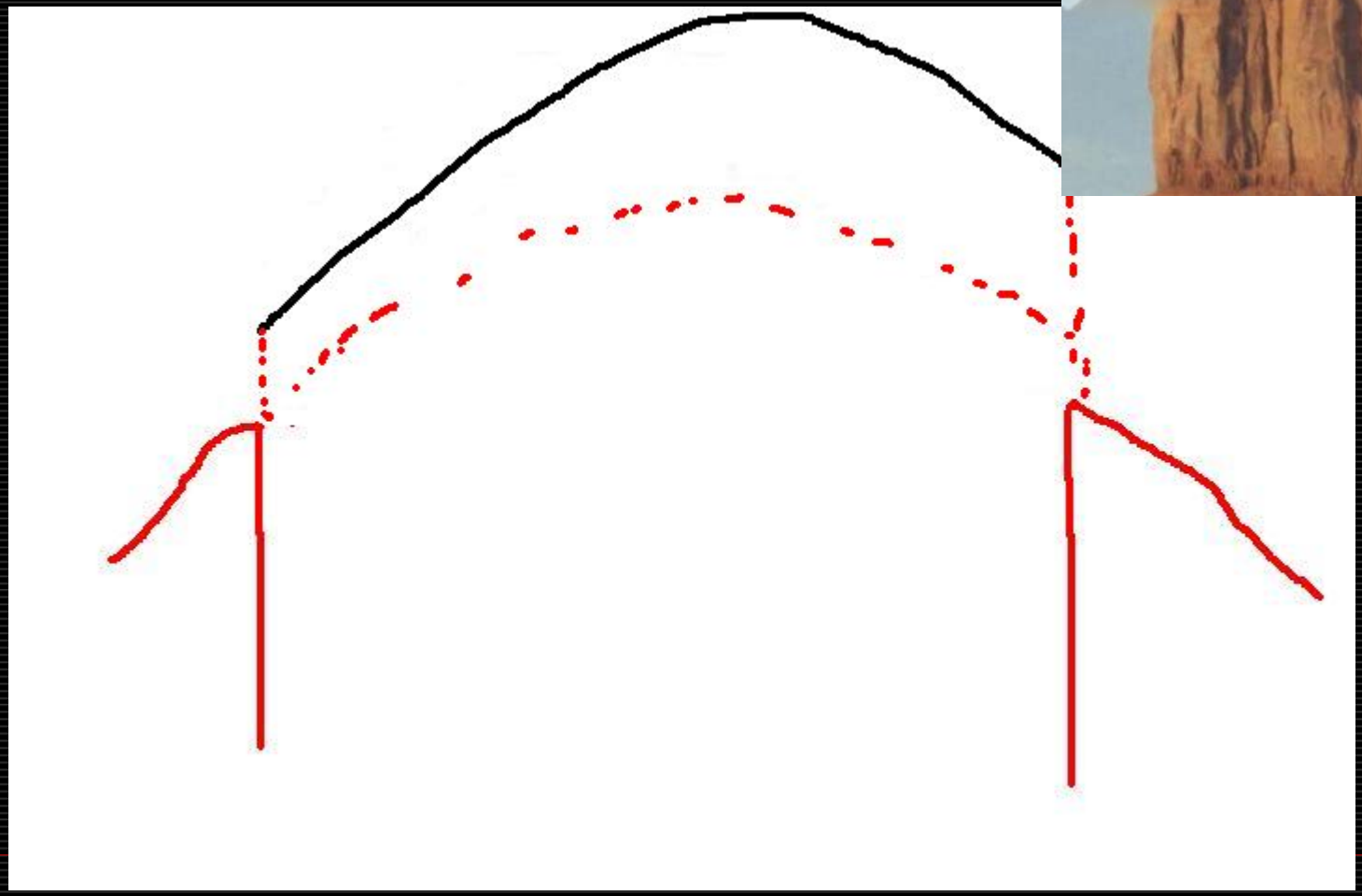
Discussion



Discussion



Discussion



Drag-and-Drop Pasting

SIGGRAPH 2006

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Jian Sun — Microsoft Research Asia
Chi-Keung Tang — The H.K. University of Sci. & Tech.
Heung-Yeung Shum — Microsoft Research Asia

Slides by the authors

Introduction to our method

- Our method improves the Poisson image editing with
  - a new boundary optimization algorithm,
  - an easier user interface,
  - and an integration of alpha values.

Poisson equations in images

- A case study

f_s

+

f_t

→

Poisson equations in images

- A case study

f_s

+

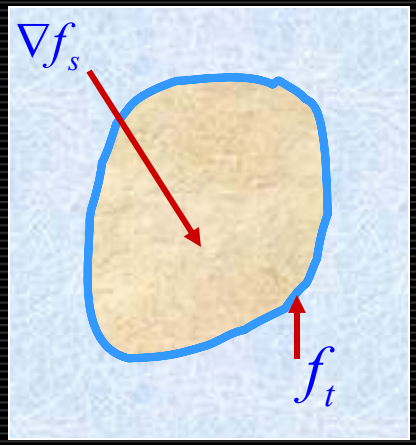
f_t

→

Poisson equations in images

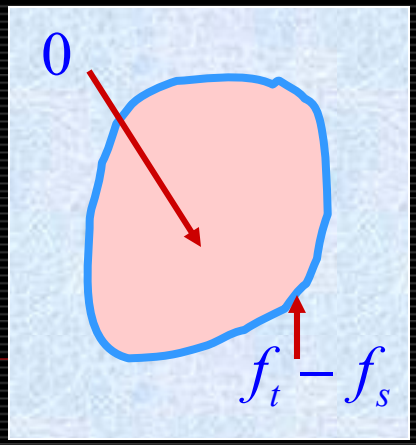
- The optimization problem in image blending [Perez et al. 2003]

$$\min_f \int_{p \in \Omega_0} |\nabla f - \nabla f_s|^2 \, dp \quad \text{with} \quad f|_{\partial\Omega_0} = f_t|_{\partial\Omega_0}$$



- Taking $f' = f - f_s$, we have

$$\min_{f'} \int_{p \in \Omega_0} |\nabla f'|^2 \, dp \quad \text{with} \quad f'|_{\partial\Omega_0} = f_t - f_s|_{\partial\Omega_0}$$



- What does it imply?

Poisson equations in images

- The minimization problem equals to solving the Laplace equation:

$$\Delta f' = 0 \text{ with } f'|_{\partial\Omega_0} = f_t - f_s|_{\partial\Omega_0}$$

- Image blending should take both the source and the target images into consideration.
- Property of solving the Laplace equation:

The variational energy $\int_{\Omega_0} |\nabla f'|^2$ *will approach zero if and only if all boundary pixels satisfy* $(f_t - f_s)|_{\partial\Omega_0} = k$, *where k is a constant value.*

Poisson equations in images

- Where is the optimal boundary $\partial\Omega$?
  - Inside the user drawn region
  - Outside the object of interest
- How is the object extracted?
  - Lazy snapping or Grabcut [Rother et al. 2004] (September 23)

$$\min \sum_{p \in \partial\Omega} ((f_t(p) - f_s(p)) - k)^2, \text{ s.t. } \partial\Omega \in \text{blue}$$

- How to optimize it?
  - Minimum color variance

Boundary optimization

$$E(\partial\Omega, k) = \sum_{p \in \partial\Omega} ((f_t(p) - f_s(p)) - k)^2, \text{ s.t. } \partial\Omega \in \text{blue}$$

- $\partial\Omega$ and k are all unknowns
- An iterative optimization
  - Initialize $\partial\Omega$ as the user drawn boundary.
  - Given new $\partial\Omega$, the optimal k is computed:

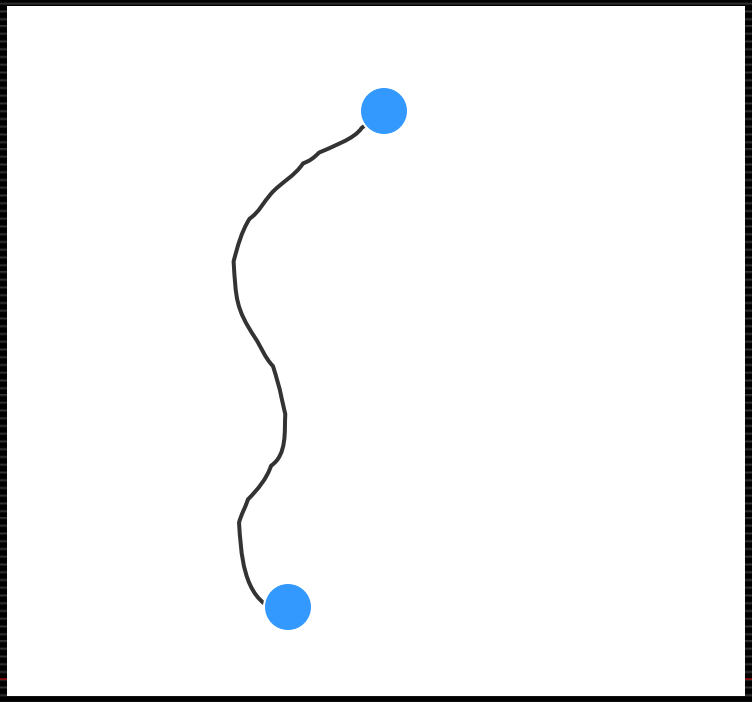
$$\frac{\partial E(\partial\Omega, k)}{\partial k} = 0$$

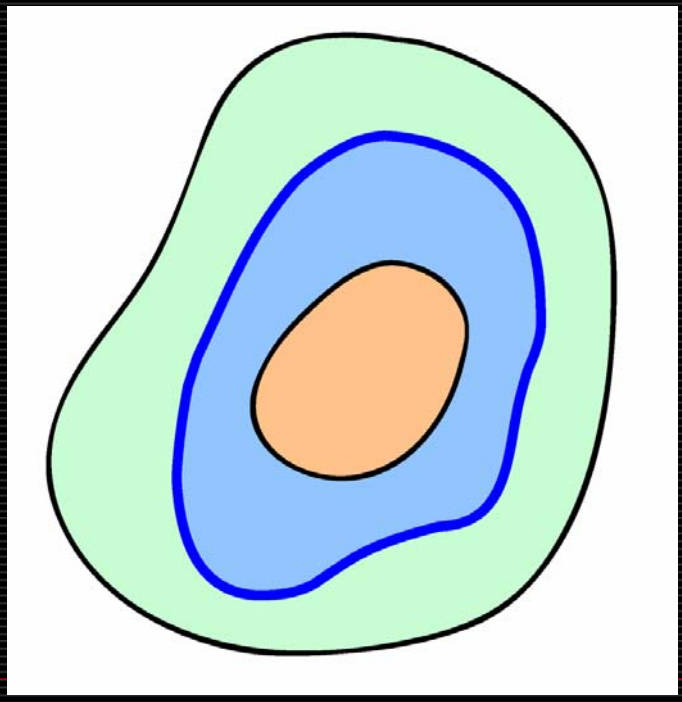
Shortest path problem

  - Given new k, optimize the boundary $\partial\Omega$.
  - Repeat the previous two steps until convergence.

Boundary optimization

- In 2D graph, computing the shortest path between any two points: Dynamic Programming
- Our problem is to compute a closed path

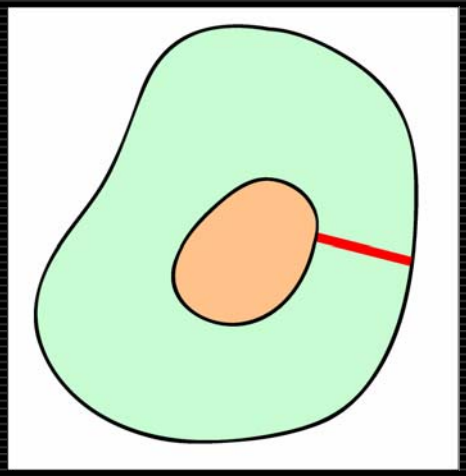


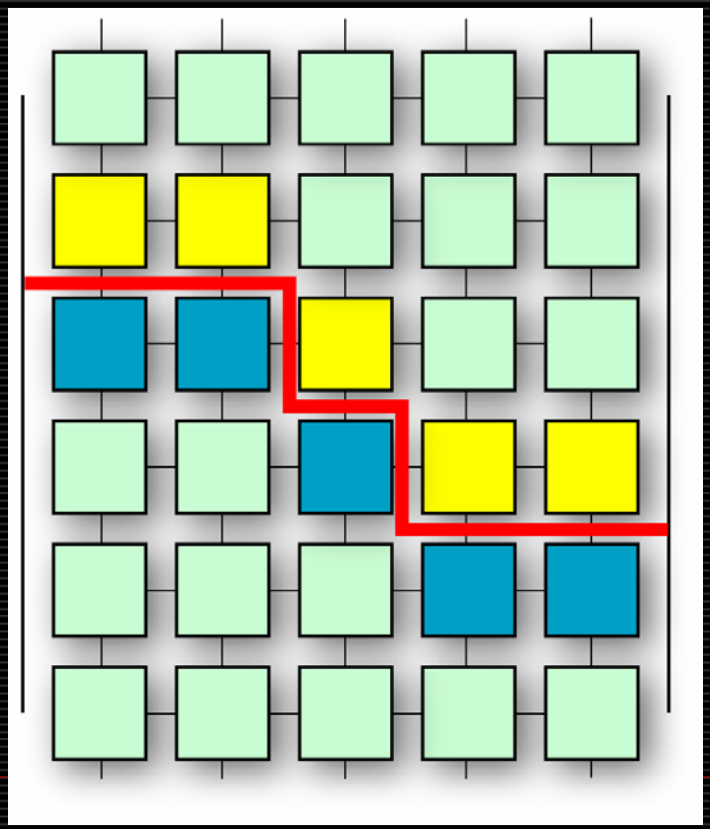


Boundary optimization

A shortest closed-path algorithm

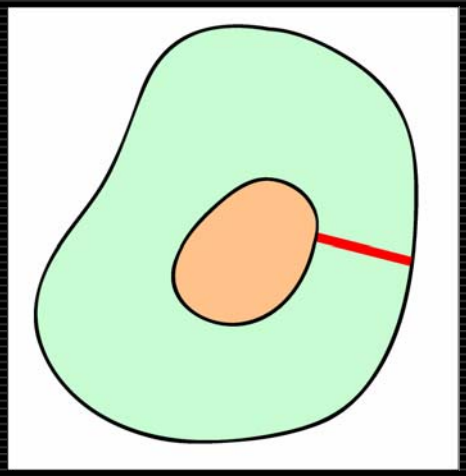
- Breaking closed boundary

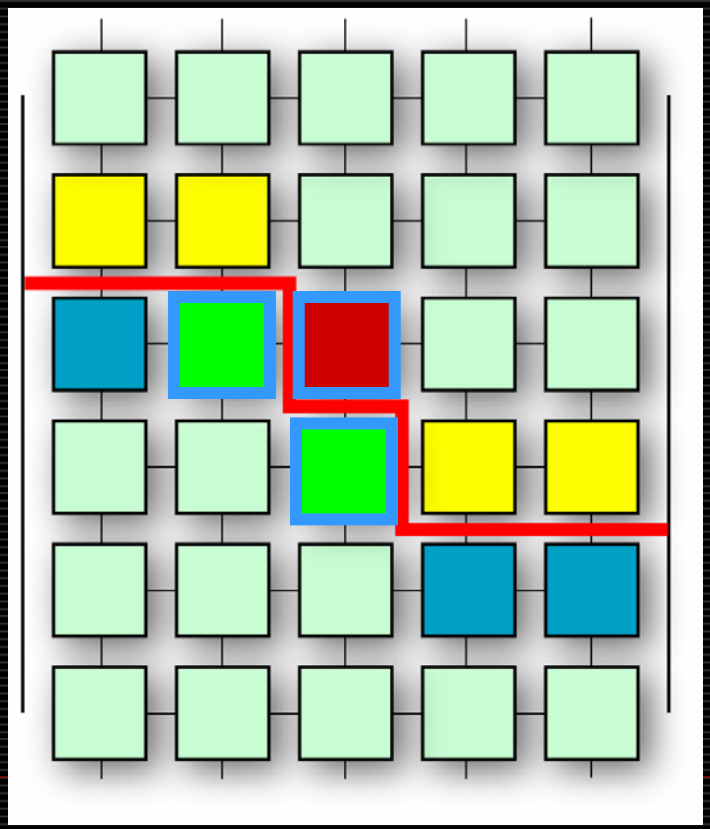




Boundary optimization

A shortest closed-path algorithm

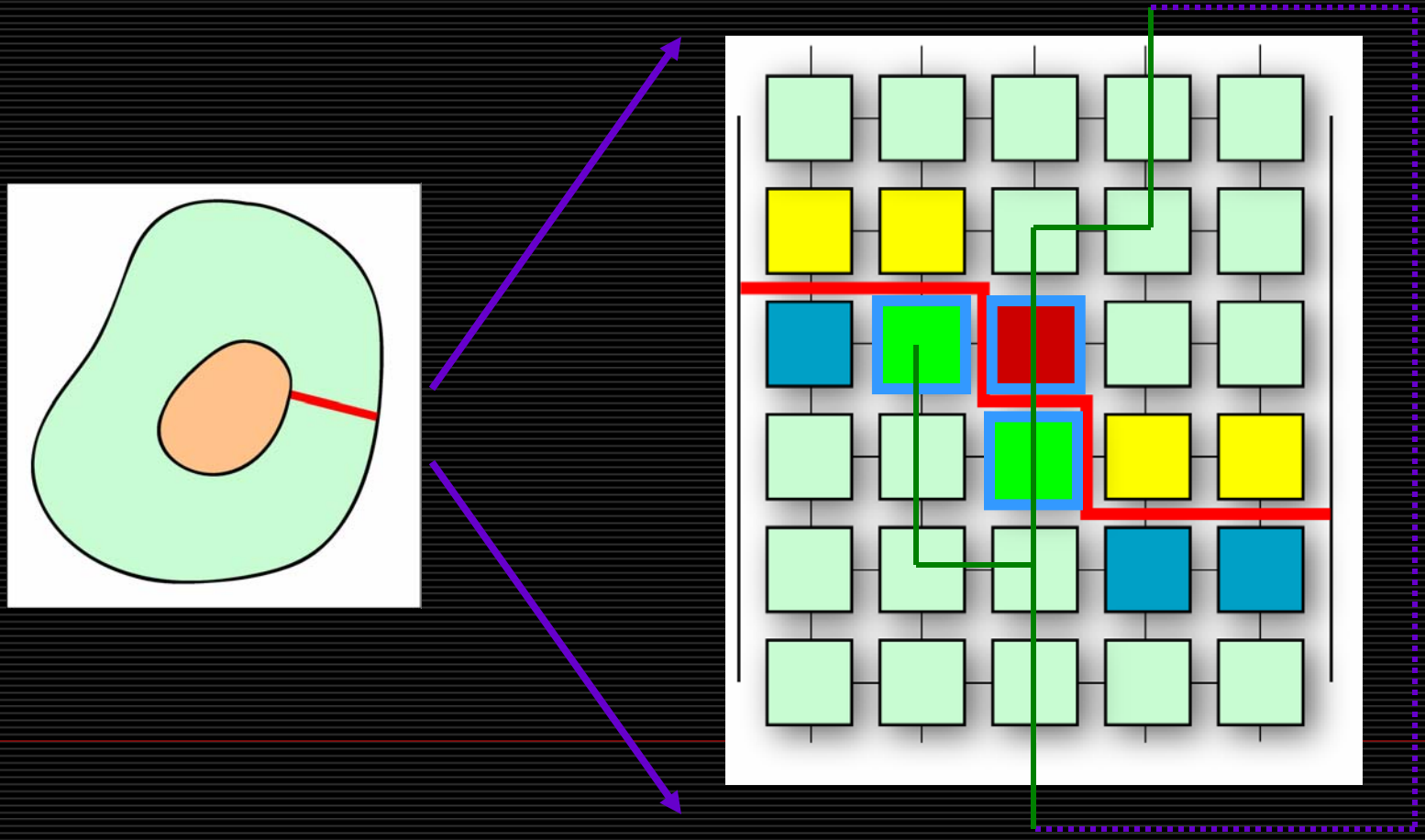




Boundary optimization

A shortest closed-path algorithm

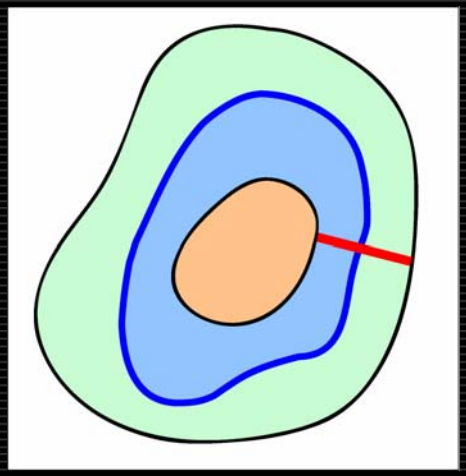
- Computation complexity *O(N)*

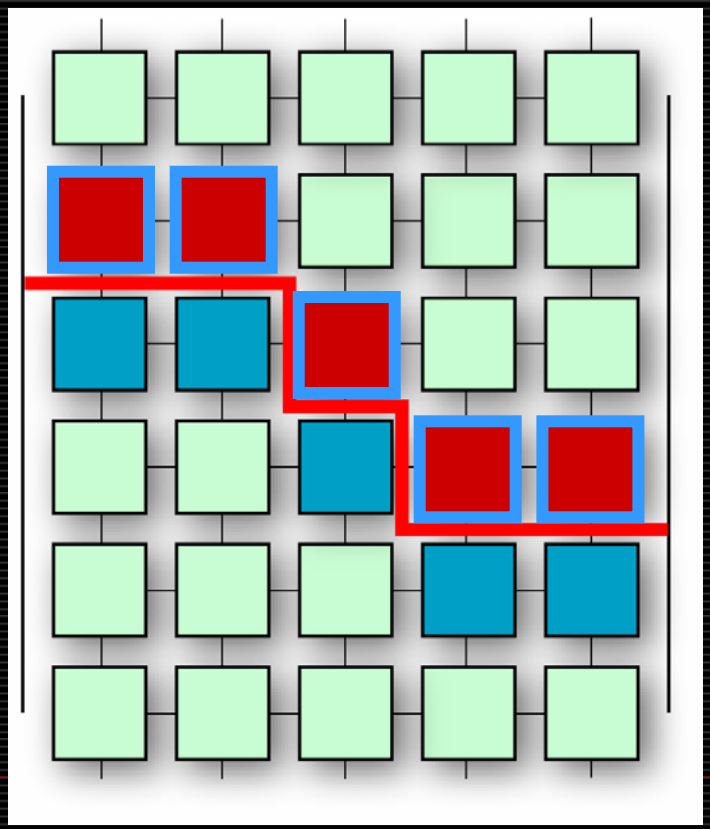


Boundary optimization

A shortest closed-path algorithm

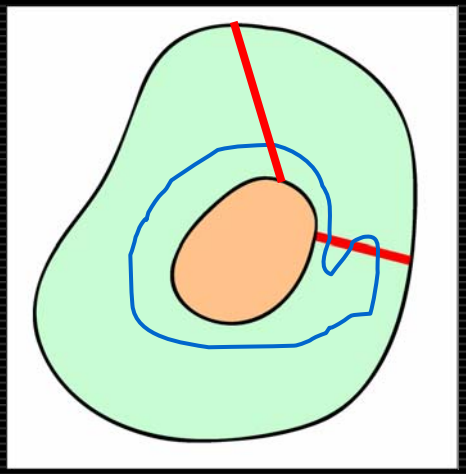
- Total computation complexity *O(NM)*





Boundary optimization discussion

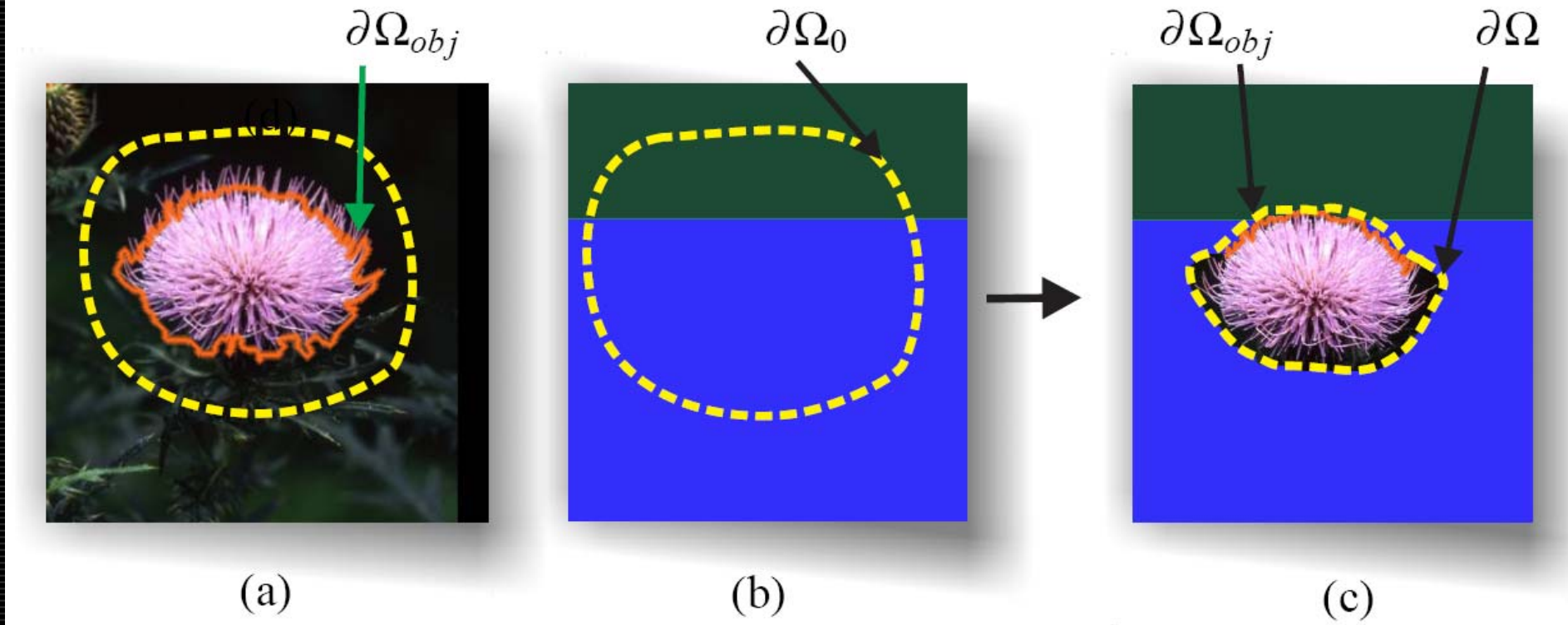
- Optimality
  - Avoiding that the path twists around the cut by selecting the initial cut position.
- How to select the initial cut?
  - Making it short to reduce O(MN)
  - Passing smooth region



One example

Integrating fractional boundary

- Fractional boundary is important in image composting: (transparency)



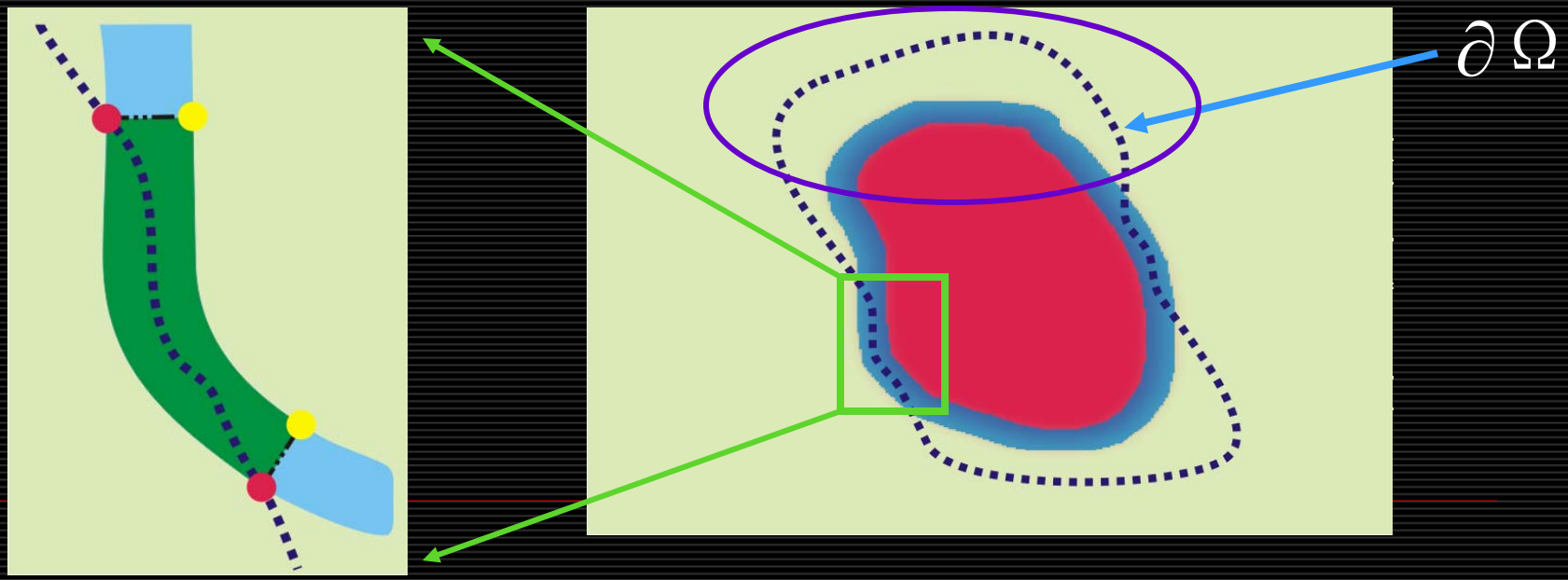
Matting

- Composition Image I is generated by foreground and background with alpha matte
- I = alpha*F + (1-alpha)*B
- Matting is a problem to get alpha,F,B from a given image I.

  User have to devide the image into three region: Foreground, Background and Unknown area. In foreground area, F = I, alpha = 1, B = 0; In background area, F = 0, alpha = 0, B = I. Our task is to get F,B,alpha in unknown area.

Integrating fractional boundary

- Where to use the fractional values?
  - only the pixels where the optimized boundary is near *the blue ribbon*



Integrating fractional boundary

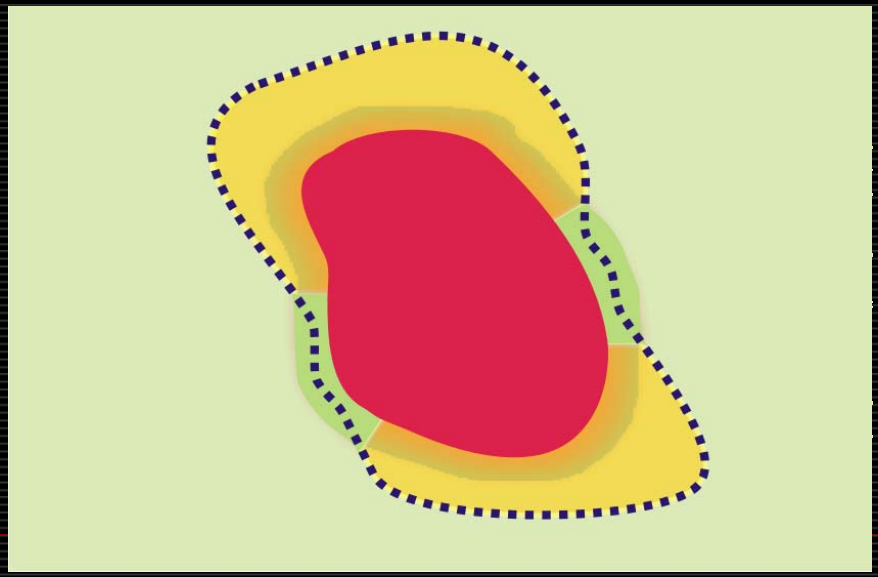
- Where to use the fractional values?
  - only the pixels where the optimized boundary is near *the blue ribbon*

fractional integration:

the green region

otherwise:

the yellow region

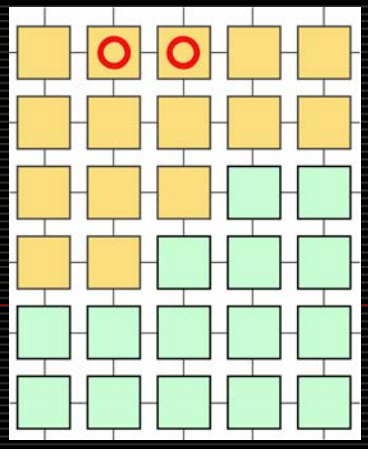


Integrating fractional boundary

- How to integrate the fractional values in Poisson blending?
  - A blended guidance field

$$\nabla_x f(x,y) = f(x+1,y) - f(x,y)$$

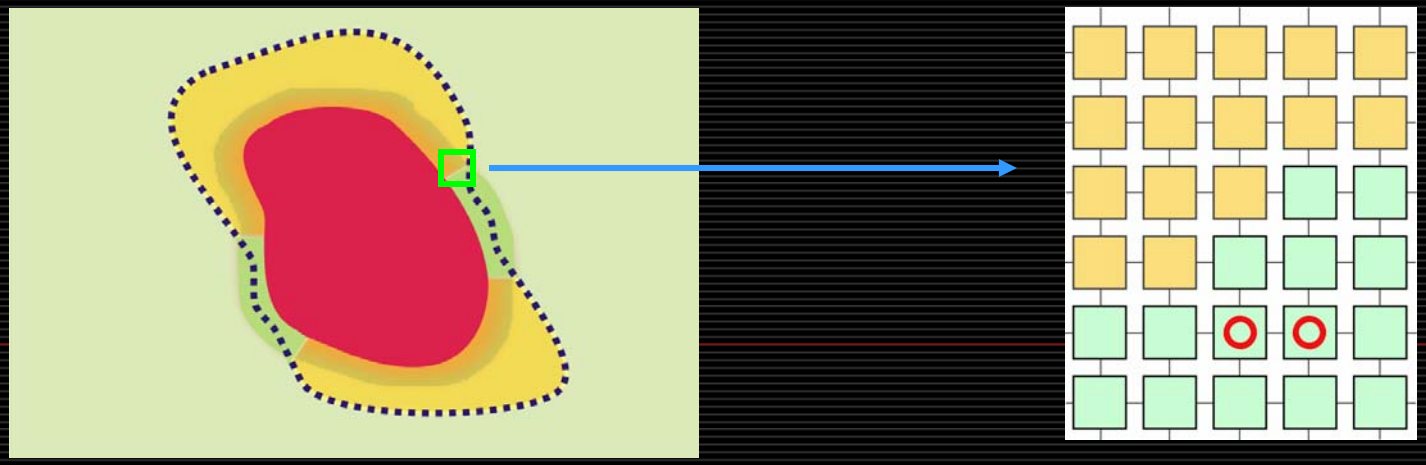
$$v_x'(x,y) = \begin{cases} \nabla_x f_s(x,y) & (x,y),(x+1,y) \in yellow \\ \nabla_x(\alpha f_s + (1-\alpha) f_t) & (x,y),(x+1,y) \in green \\ 0 & \text{otherwise} \end{cases}$$



Integrating fractional boundary

- How to integrate the fractional values in Poisson blending?
  - A blended guidance field

$$v_x'(x,y) = \begin{cases} \nabla_x f_s(x,y) & (x,y),(x+1,y) \in yellow \\ \nabla_x(\alpha f_s + (1-\alpha) f_t) & (x,y),(x+1,y) \in green \\ 0 & \text{otherwise} \end{cases}$$



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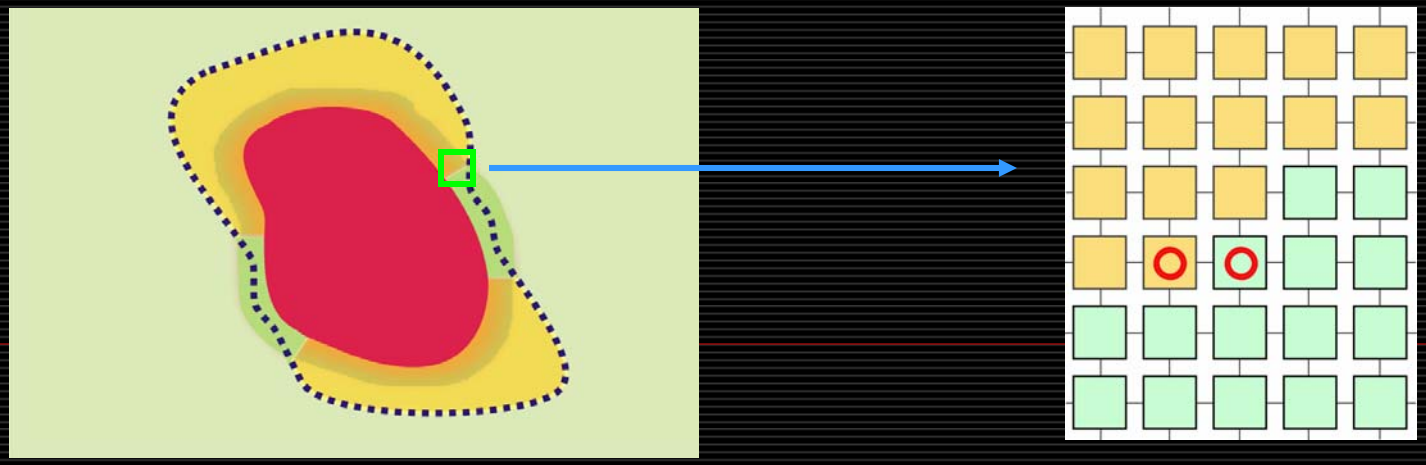


Integrating fractional boundary

- Final minimization:

$$\min_f \int_{p \in \Omega^*} |\nabla f - v'|^2 \, dp \quad \text{with } f|_{\partial\Omega^*} = f_t|_{\partial\Omega^*}$$

- New boundary: $\Omega \cup \Phi$
- Solving the corresponding Poisson equation.

Results and comparison

Alpha blending

Results and comparison

Our method

Poisson blending

Results and comparison

Our method

Alpha blending

Results and comparison

Our method

Poisson blending

Poisson Image Editing Extended

SIGGRAPH 2006 sketch

Daniel Leventhal Brown Univ.
Bernard Gordon Brown Univ.
Peter G. Sibley Brown Univ.

Problem caused by texture

Alpha control

- Alpha in [0, 1], background 0, and foreground 1.
- Obtain unselected area's Alpha by blurring.

$$\forall x \in \Omega, v(x) = \begin{cases} \nabla f^*(x) & \text{if } \|\nabla f^*(x)\| > \alpha \|\nabla g(x)\| \\ \alpha \nabla g(x) & \text{otherwise} \end{cases}$$

- Implemented in YUV rather than RGB

Result




Luminance rescaling

Far from solved…

- Only pure texture, how about mixture of texture and non-texture
- Far from perfect…

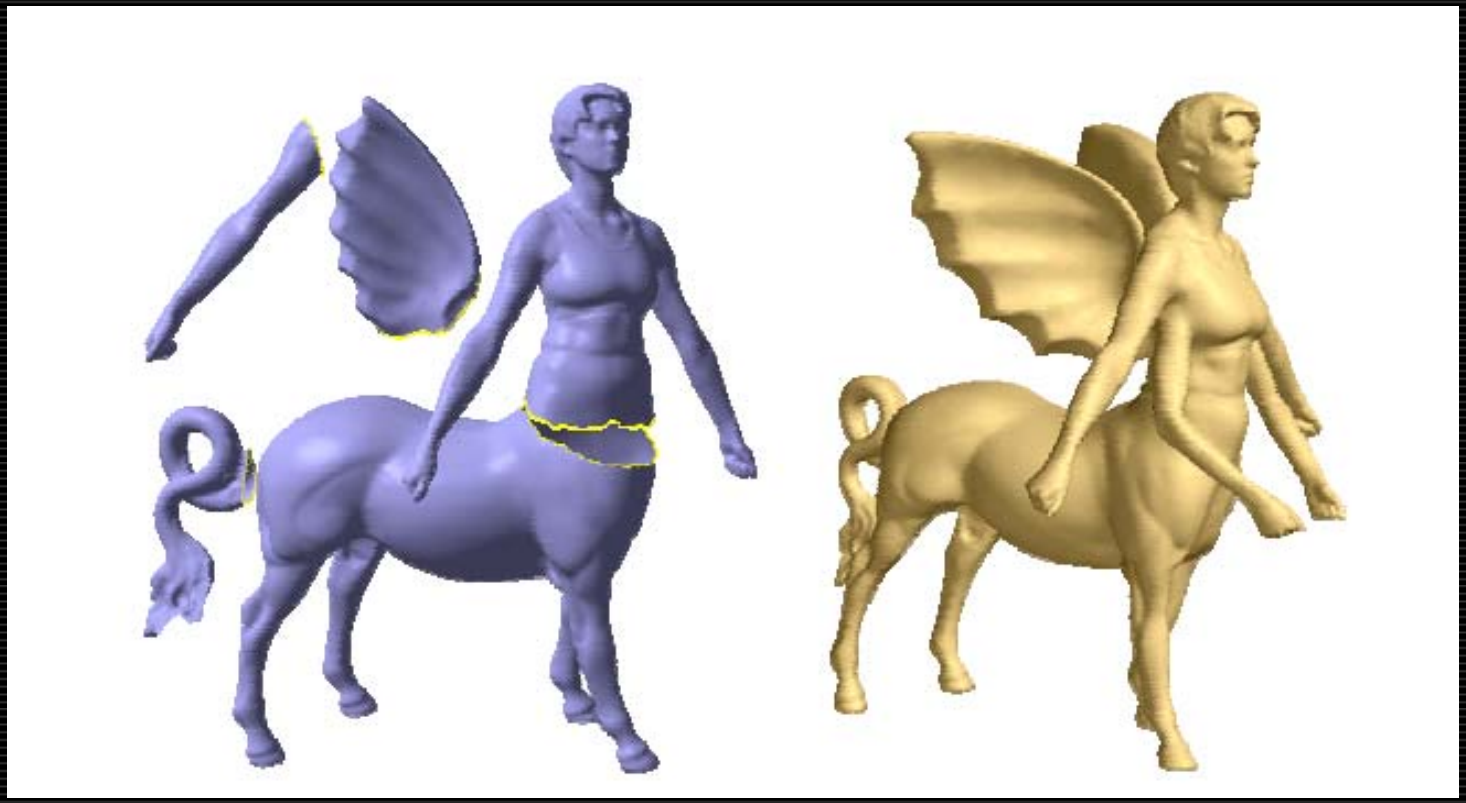
Parallel Method on Mesh

- Poisson based mesh editing
- Optical Boundaries for mesh merging
- Extend from image space to mesh manifold

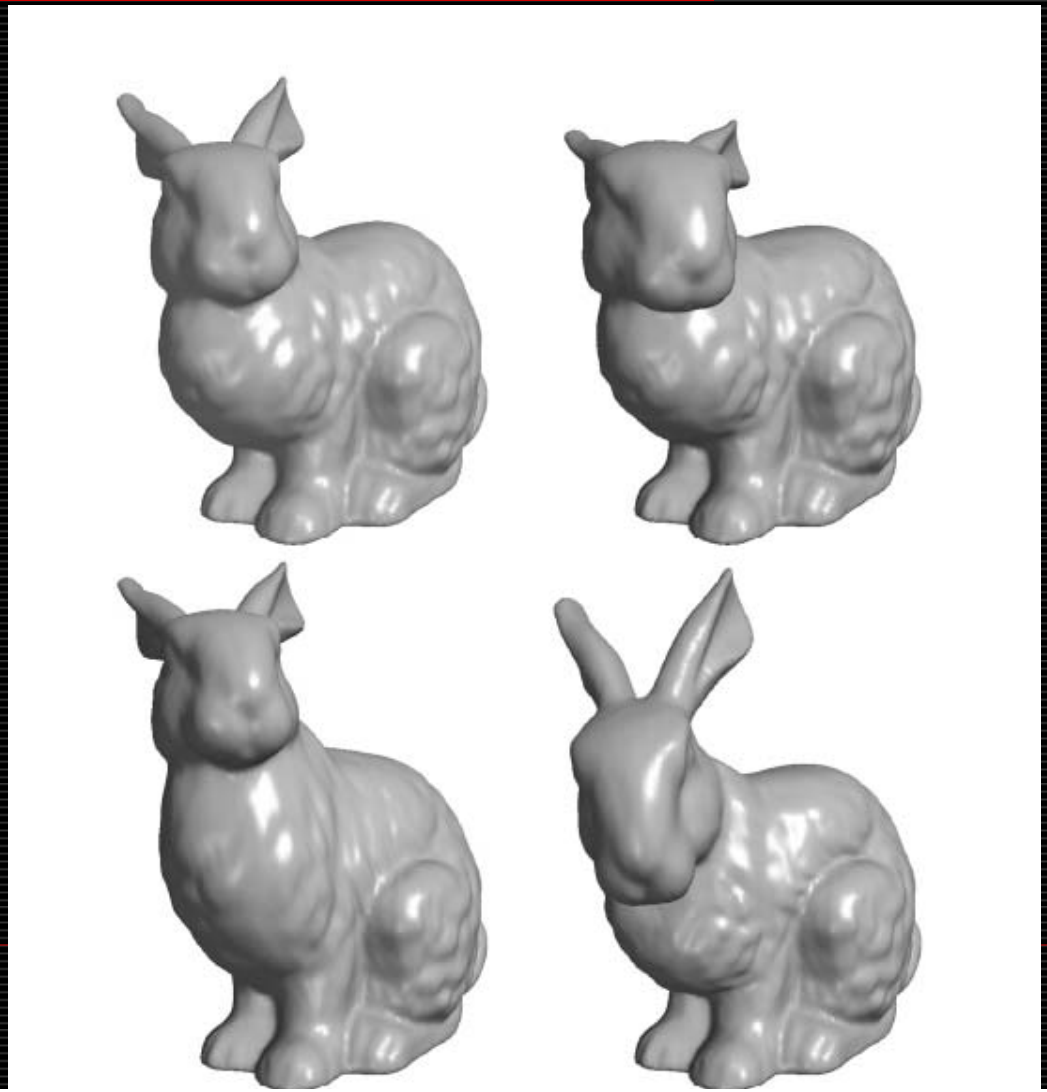
Mesh Editing with Poisson-Based Gradient Field Manipulation

SIGGRAPH 2004
Yizhou Yu UIUC
Kun Zhou MSRA
Dong Xu Zhejiang Univ, MSRA
Xiaohan Shi Zhejiang Univ, MSRA
Hujun Bao Zhejiang Univ.
Baining Guo MSRA
Heung-Yueng Shum MSRA

Mesh merging



Deformation



Basic idea

- Quite similar to the case of image
- Vector field decomposition can be extended to manifold

$$\mathbf{w} = \nabla\phi + \nabla \times \upsilon + \mathbf{h}$$

$$\min_{\phi} \int\int_{T} \|\nabla\phi - \mathbf{w}\|^2 dA,$$

$$\nabla^2\phi = \nabla \cdot \mathbf{w}$$

Discretization

- NOT real gradient and divergence, but it works

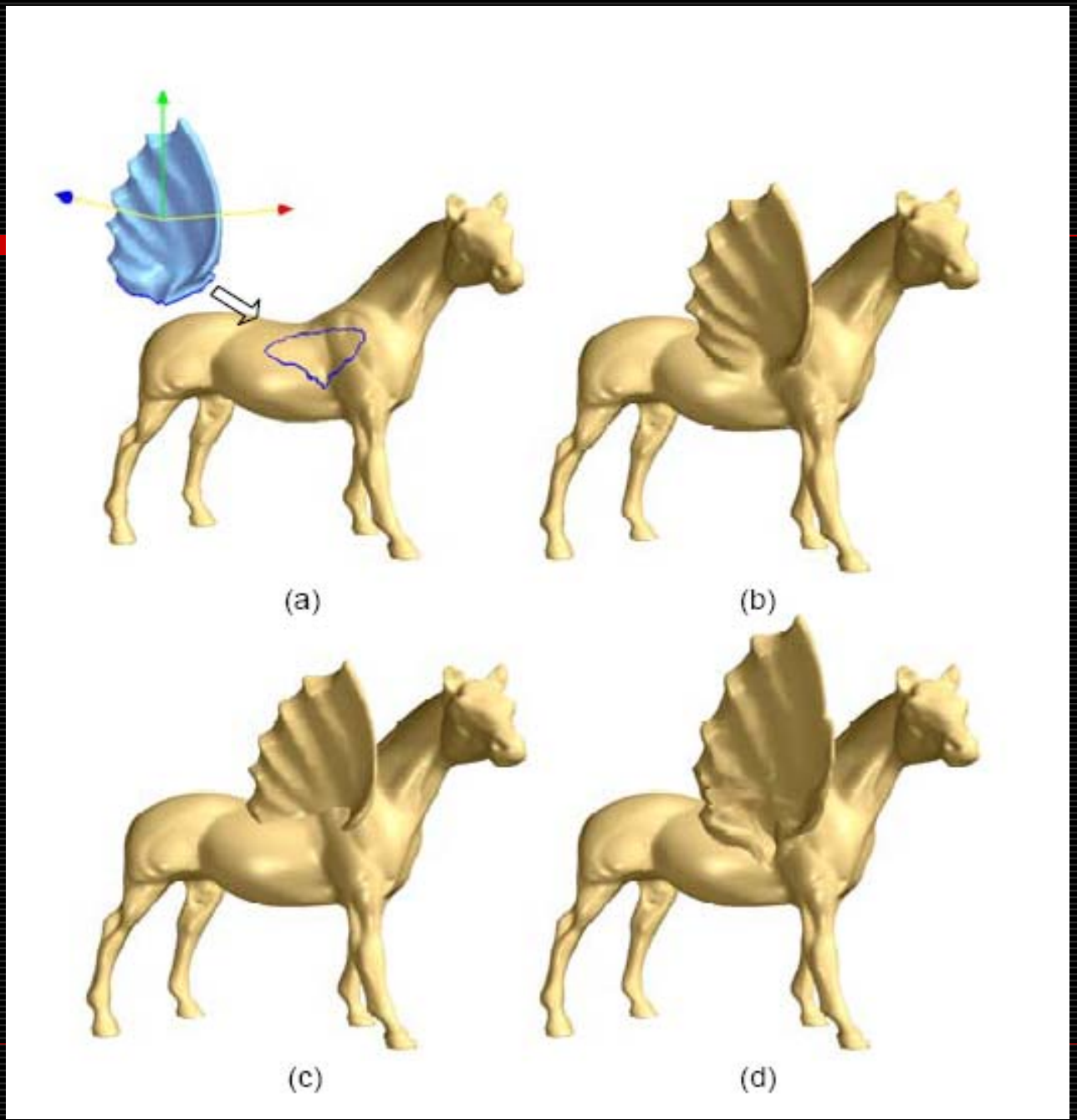
$$(\mathrm{Div}\mathbf{w})(\mathbf{v_i}) = \sum_{T_k \in N(i)} \nabla B_{ik} \cdot \mathbf{w} |T_k|$$

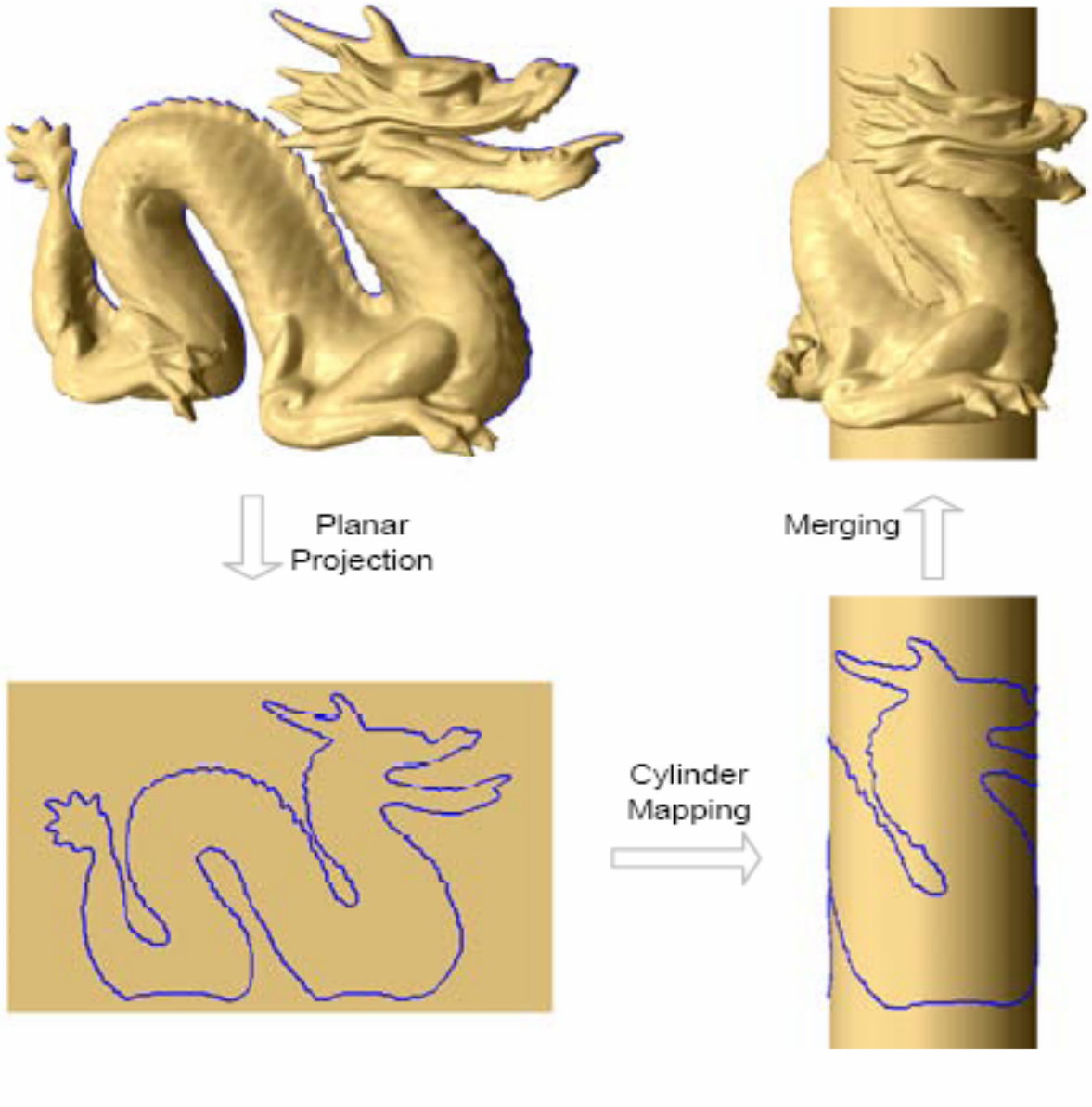
$$\mathrm{Div}(\nabla \phi) = \mathrm{Div}\mathbf{w}$$

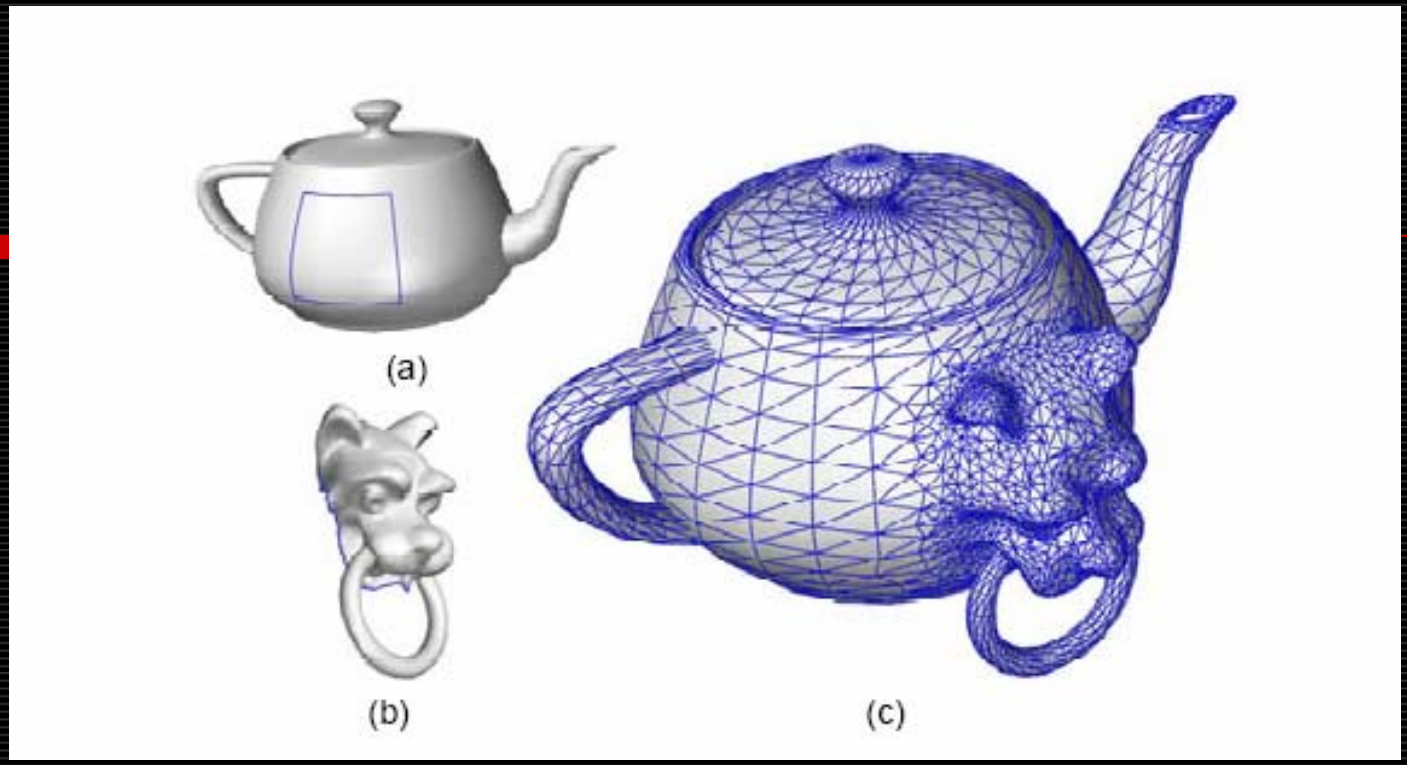
- Sparse linear system

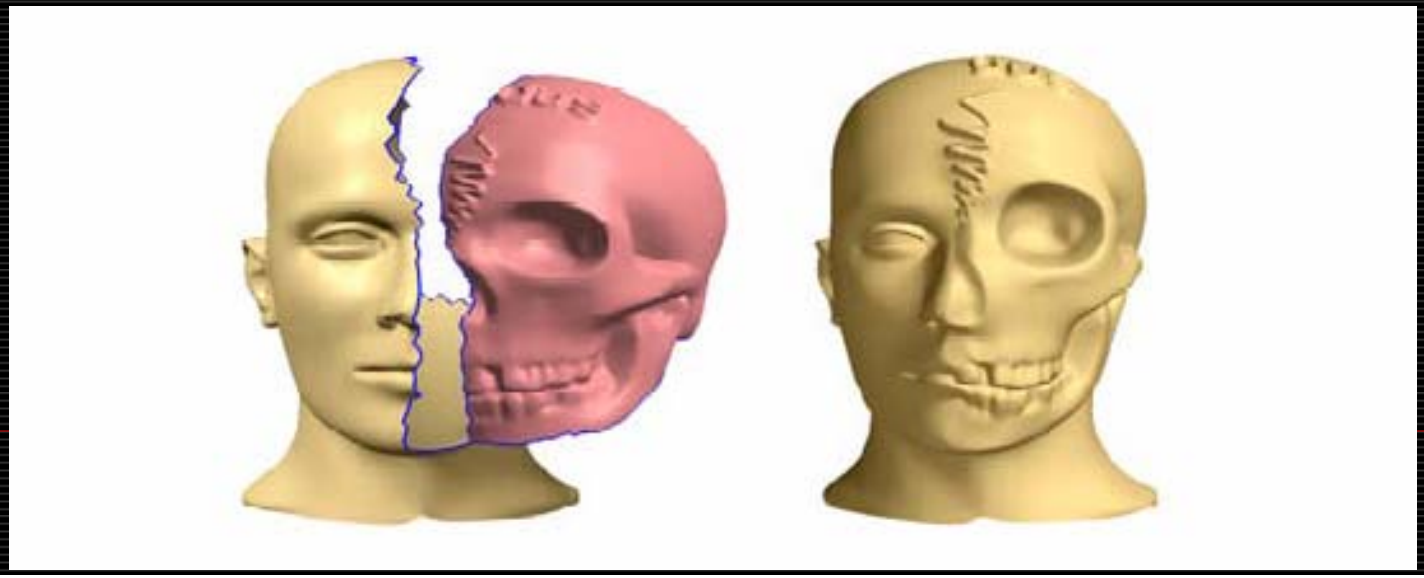
Mesh Merging

- Some ‘ugly’ details
  - Boundary interaction
  - Boundary correspondence
  - Re-parameterization


(a)
(b)
(c)
(d)

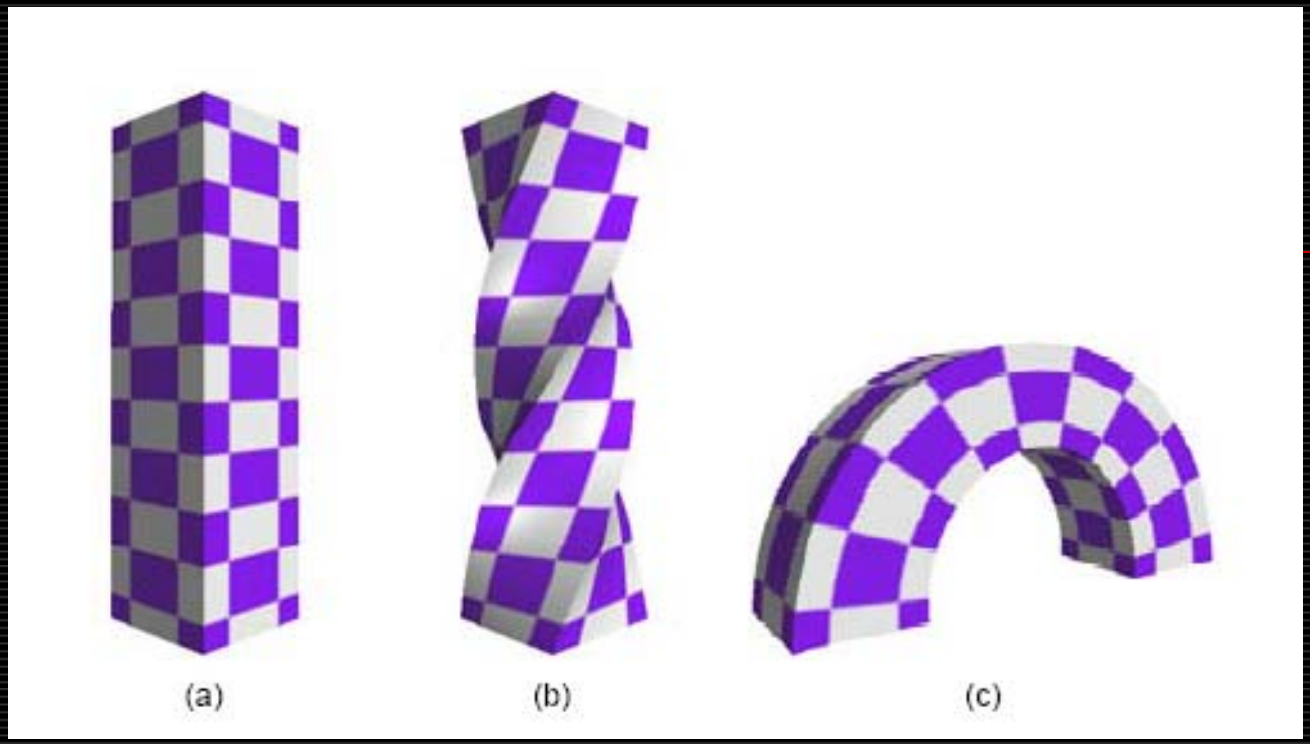

Planar
Projection
Merging
Cylinder
Mapping

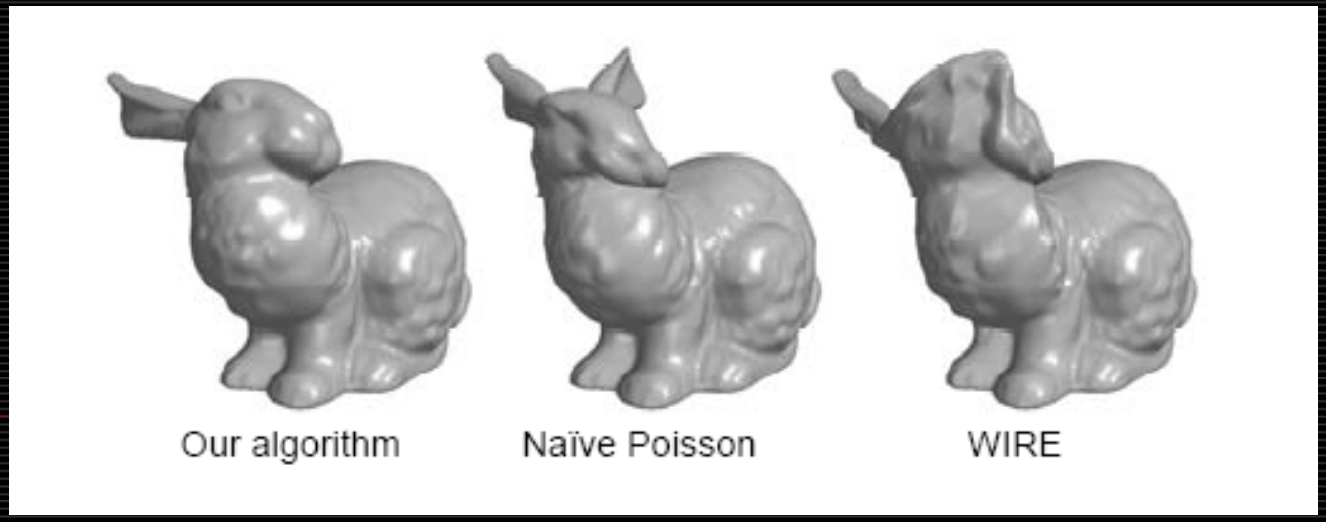

(a)
(b)
(c)



Mesh deformation

- Interactively change guided vector field
  - Change normal on a curve
  - Propagate to other areas
- Smooth normal field – smooth


(a)
(b)
(c)


Our algorithm
Naïve Poisson
WIRE

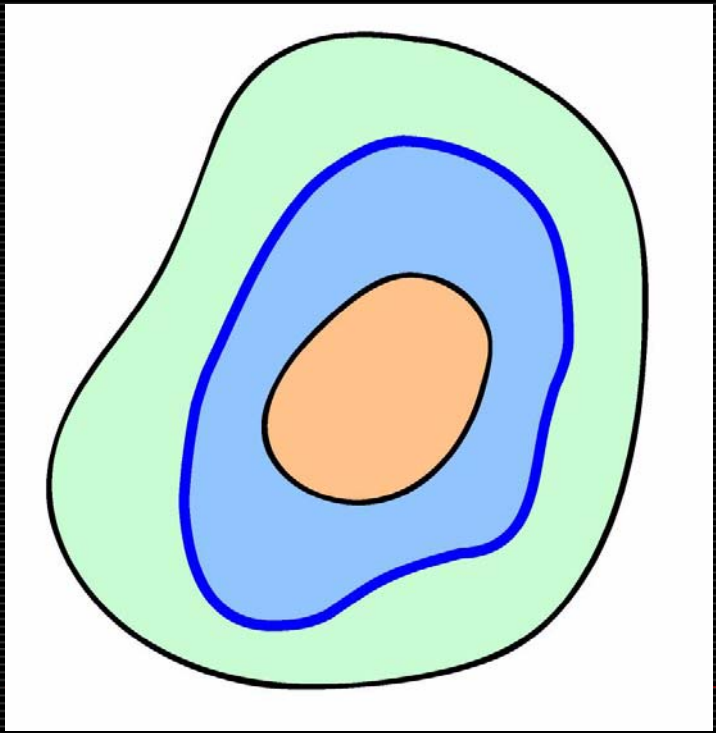
Optimal Boundaries for Poisson Mesh Merging

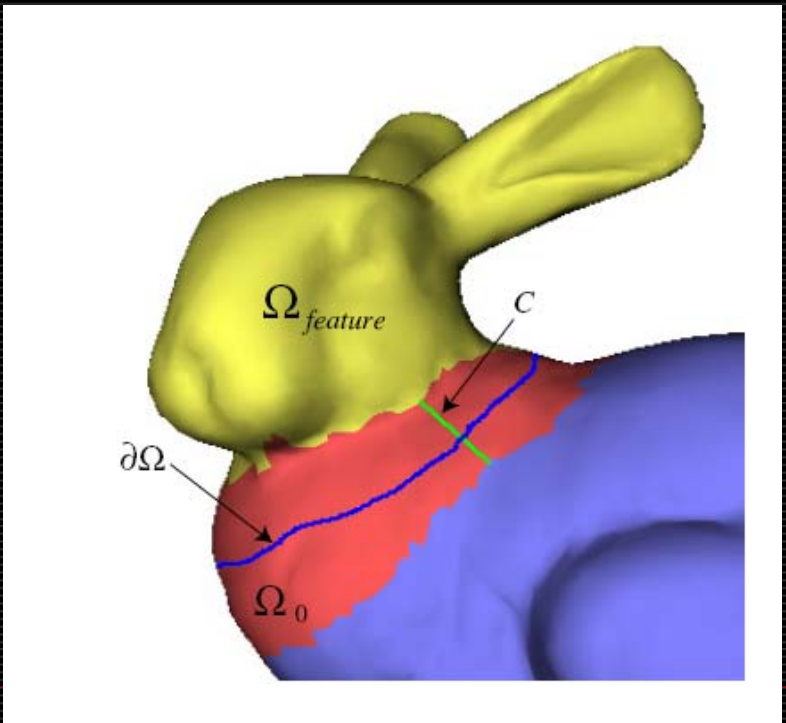
SPM 2007

Xiaohuang Huang Zhejiang Univ. & HUST
Hongbu Fu HUST
Oscar Kin-Chung Au HUST
Chiew-Lan Tai HUST

Basic idea

- Best boundary for Poisson Merging
- Similar to 'Drag-and-Drop Pasting'.





Optimal boundary

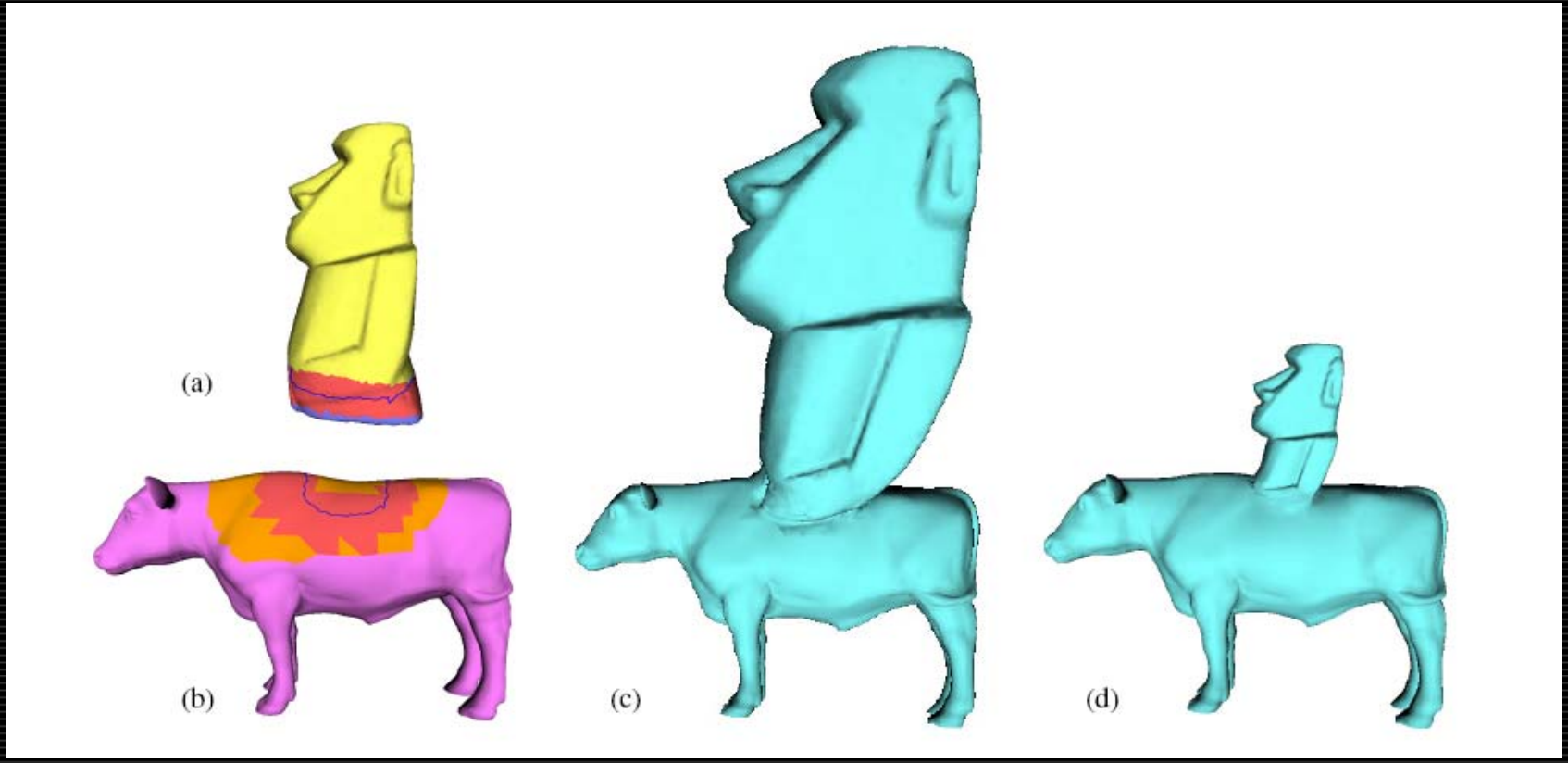
$$E(\partial\Omega, k) = \sum_{p \in \partial\Omega} ((f_t(p) - f_s(p)) - k)^2, \text{ s.t. } \partial\Omega \in \text{blue}$$

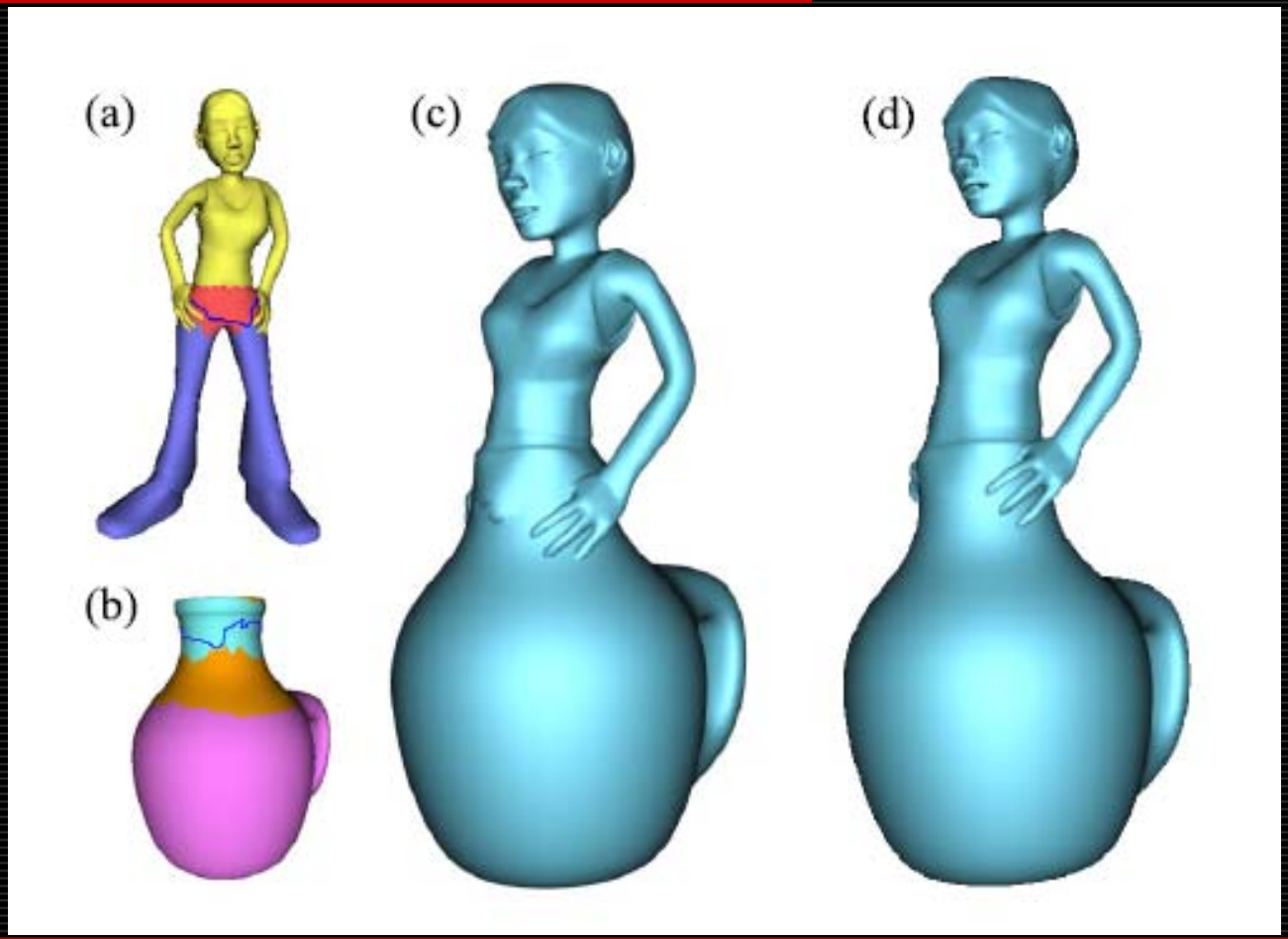
$$E(\partial\Omega, \mathbf{T}) = \sum_{e \in \partial\Omega} \|\mathbf{T}e - e^*\| \cdot length(e), \quad \partial\Omega \subset \Omega_0 \setminus \Omega_{feature}$$

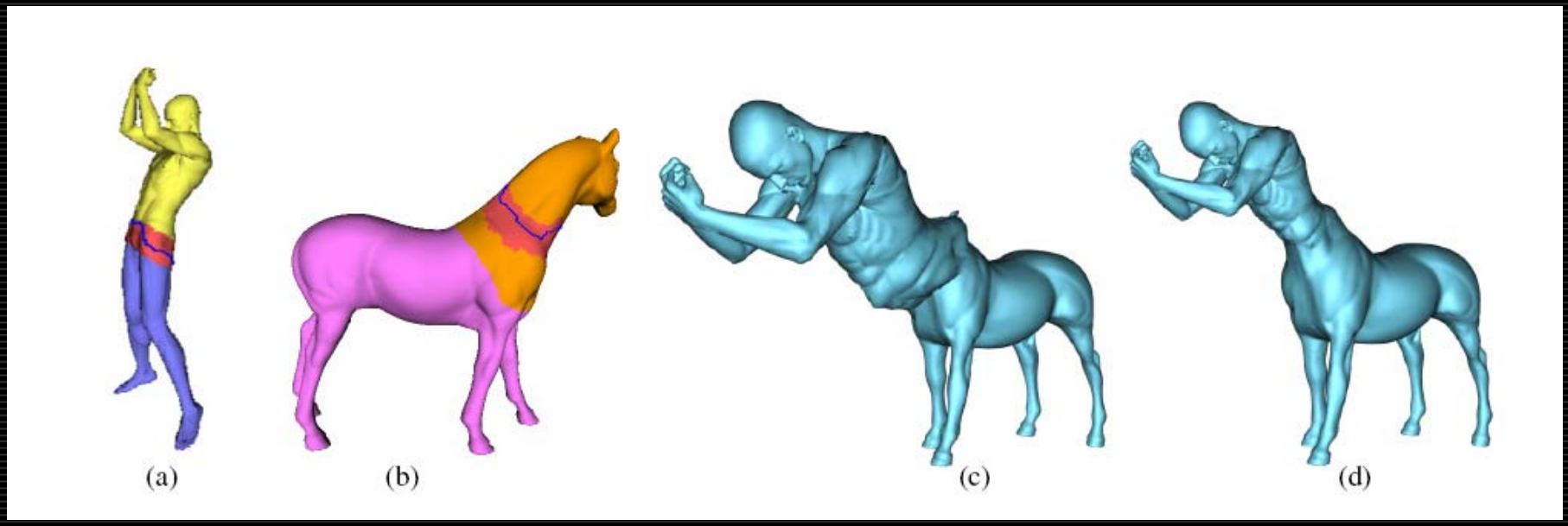
Mesh ‘ugly’ limitation

- Correspondence between source and target images are trivial
- Meaningful correspondence between source and target meshes are VERY difficult.

Results




(a)
(b)
(c)
(d)


(a)
(b)
(c)
(d)

Failure?