



A computational modeling of student cognitive processes in science education



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ABSTRACT

The purpose of this paper is to explain and document the creation of a computational model in the form of an Artificial Neural Network (ANN) capable of simulating student cognition. Specifically, the model simulates students' cognition as they complete activities within a science classroom. This study also seeks to examine the effects, as evidenced in the ANN, of an intervention designed to develop increased levels of critical thinking related to science skills. This model is based on the identification of cognitive attributes and integration of two advanced measurement frameworks: cognitive diagnostics and Item Response Theory. Both frameworks examine student response patterns, providing initial inputs for the ANN portion of the model. Once initial task response patterns are identified, they are parameterized and presented to the ANN. The ANN within this study is the foundational component of a computational model based upon the interaction of multiple, connected, adaptive processing elements known as cognitive attributes. These cognitive attributes process student responses to cognitive tasks within science tasks. Using the Student Task and Cognition Model (STAC-M), the study authors simulated a cognitive training intervention using a randomized control trial design of 100,000 students. Results of the simulation suggest that it is possible to increase levels of student success using a targeted cognitive attribute approach and that computational modeling provides a means to test educational theory for future education research. The paper also discusses limitations of the use of this computational model within education and the possible future directions for educators and researchers.

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1. Introduction

This study examined the creation and characterization of a computational model in the form of an Artificial Neural Network (ANN) capable of simulating student cognition. Specifically, this paper presents the authors' newly created computational model and a subsequent computational experiment (simulation) using this model. The computational experiment relates student learning and cognition in a science classroom. This model arises from the identification of cognitive attributes and integration of two advanced measurement frameworks: cognitive diagnostics and Item Response Theory (IRT). The integration of these two frameworks acts as a means to identify cognitive attributes from data gathered during the play of Serious Educational Games (SEG). The proposed ANN model, called the Student Task and Cognition Model (STAC-M), represents the convergence of two important goals within education. First, to create levels of understanding related to the complex interactions between the student and science-classroom learning tasks. Second, to develop a computationally powerful model of student cognition that can “learn” to perform complex science based tasks for the purposes of developing computational experiments simulating student learning in the science classroom. This form of modeling addresses a key need to assess effectiveness of interventions prior to implementation in the classroom as districts and teachers have limited time and resources. Computational modeling

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of an intervention is used within the natural sciences, engineering, economics, and other fields to provide initial success or failure data related to the intervention. This success and failure data provides a means to preferentially select those interventions more likely to produce success. In addition, using computational modeling reduces the number of subjects used during experimentation, and provides a means to examine outcomes when ethical considerations provide constraints. Educational interventions often suffer from similar constraints as students are a protected class, and schools and teachers are extremely limited in the time they can dedicate to testing educational interventions *in situ*. Perhaps more importantly, there are several ethical considerations when choosing to withhold an intervention thought to have positive effects in the classroom, which clashes with the continuing push to employ randomized control trials in education (Institute of Education Sciences, 2003). Computational modeling allows researchers to sidestep these concerns through the ability to create an experiment from data collected outside the classroom and with no harmful effects on students.

The modeling of science task learning is complex and requires understanding and development of relationships between the cognitive inputs, cognitive attributes (processing streams), and outputs (behaviors) to be measured. Specifically, each individual task requires the assignment of success and failure probabilities related to student cognitive processing tasks. The STAC-M acts to illustrate the role of cognitive attributes as they interact to solve problems by employing artificial intelligence in the form of flexible analysis systems with adaptive gating of data streams related to cognitive attribute activation. Mechanisms of action provide a means to incorporate new information within the system while using pre-existing information as is the case with Bayesian Models (Clark, 2012). Lamb (2013) trained the initial model STAC-M using a version of pedagogy thought to successfully train students to complete tasks within the science classroom. This pedagogy arises from the use of science based SEGs (Annetta, 2008). A second example of the applications of simulations and SEGs in learning is the HAEModynamic SIMulator discussed by Holzinger et al. (Holzinger, Kickmeier-Rust, Wassertheurer, & Hessinger, 2009). The HAEModynamic SIMulator is a SEG used to train and teach medical school students around heart arrhythmias. Lamb in two separate studies in 2011 and 2013 further proposed that the use of experiential learning versus rote learning in the form of SEGs has superior outcomes for student. Specifically because the STAC-M was originally developed from data obtained while students played a science based SEG, the STAC-M more generally exemplifies and relates to reinforcement learning mechanisms due to the constant feedback and data received during play (Lamb, 2013). Within the STAC-M model, cognitive channels decrease cognitive load and increase processing power related to task completion (Kalyuga, 2011). The STAC-M was developed in alignment with the connectivist cognitive framework for emergent properties of the mind (Thomas & Laurillard, 2013).

1.1. Artificial Neural Networks

An Artificial Neural Network (ANN) such as the STAC-M is a computational model based upon the interaction of multiple, connected, processing elements in a non-linear fashion (Gupta, 2010). The connections of the input to the output through propagation or potentiation weight places ANN computational models in a connectionist framework (Neymotin, Jacobs, Fenton, & Lytton, 2011). A key feature of ANN is the connection between input elements and output elements via hidden non-linear Bayesian computational layers (Hanrahan, 2010). However, the elements dealing with the input–output relationships are not fully known or researchers would model these connections directly via traditional modeling techniques such as regression or other more traditional statistical modeling methods (Zangeneh, Omid, & Akram, 2012). Artificial neural network architecture provides for the analysis of complex cognitive constructs, such as critical reasoning, retrieval, and parity judgment (Lamb, Annetta, Vallett, & Sadler, 2014). The ability to work through non-linear complexity and its interconnections makes an ANN the most appropriate base model for development into models of cognition (Chen, Dey, Muller, Sun, & Ye, 2010). The ANN and derived models approximate the architecture of the parallel, non-linear processing found in biologically based cognitive processing systems (Berger et al., 2012).

ANN are represented statistically in the form of a graphic with parallel architecture that provides an understanding of emergent relationships (patterns such as those found in information processing). ANNs are often used in three modes: (1) as a model of biological nervous systems and processing components, (2) real-time, adaptive, signal processors, and (3) as data analytical methods. This study uses ANNs in all three capacities. The generalization of artificial neural network models reduces to underlying functions and algorithms of pattern recognition in dynamic systems. Within this framework, it is important to remember that the author represents the patterns of the STAC-M in terms of numerical values assigned to hidden nodes within the model (Coppola, Rana, Poulton, Szidarovszky, & Uhl, 2005). The numerical values transmitted along the network use these algorithms of pattern recognition to assign propagation or potentiation weights (Ghosh & Adeli, 2009). It is important to differentiate between the ANN models and ANN algorithms. One of the major differences between ANN models and ANN algorithms is the manner in which data are used. ANN models use data to create dynamic systems while ANN algorithms develop the weighing and potentiation related to information transmission. Thus, ANN algorithms are more appropriate to analyze transient data such as computational aspects of cognition thus making them relatively useless as a statistical test procedure and requiring incorporation of psychological measurement under Item Response Theory (Lamb, 2013; Lamb et al., 2014). While there are considerable similarities between the statistical models and ANN models, there are differences within the terminology and language of ANNs.

Developers of Artificial Neural Networks designate nodes using one of three descriptions. The designations are simply a way of designating the manner in which information processes through the model. The designations are input nodes (from the environment), output nodes (outcomes), and hidden nodes (computational nodes) (Yilmaz, Marschalko, Bednarik, Kaynar, & Fojtova, 2012). The nodes link by using the nodes as a multivariate information processing function (Yilmaz & Kaynar, 2011). This multivariate function provides the means for the propagation and transformation of task processing outcomes related to the students via the ANN hidden nodes. In the case of this study, the ANN accomplished the actual transformation of the parameter estimates using learning algorithms that include the use of only forward-feed propagation. This propagation makes the network more flexible using the ratio differences between the expected and actual output to test fit and reduce error (Bilgehan, 2011). In addition to weighting adjustments, it is possible to standardize the output of the maximum propagation weight to 1.00 creating a more probabilistic interpretation based in Bayesian estimates. This adaptive ability allows for flexibility within this model not seen in other modeling techniques such as regression or structural equation modeling.

The movement from a narrow view of ANNs as an information-transmitting program to a cognitive information processor involves the inclusion of probabilistic assumption developed through psychological and educational measurement techniques. The development of

probabilistic assumptions for the ANN in this study derives from the two parameter logistic model (2PLM). IRT parameters derived from student behavioral outcomes reduced to success (1) and failure (0) (Lamb, 2013). The ANN represents the input nodes as patterns, which appear in the input, while the output nodes are recast as resulting samples of a density of higher-dimension, randomized, probability estimations to allow for the inclusion task successes and failures when identifying cognitive attributes as a function of task completion. It is through this bridging to statistics, measurement, and modeling one can link an ANN to descriptions of problems within the actual classroom environment using the cognitive-attribute, task completion probabilities. These linkages also help to visualize the hierarchical relationships between the attributes. It is in this light that the ANN offers answers to a complex array of problems, such as how to characterize the mechanism of action related to learning and cognition. This characterization occurs though its intricate statistical modeling with an emphasis on flexibility. However, this inherent flexibility is sometimes the cause of overfit errors, which increase, as there is an increase in variables. Increasing the number of computational nodes within the training data set creates randomization of components resulting in a decreased performance for future data often seen in poor test data fit (Ajwani et al., 2013). To control for this, it is important that the training and test data have a similar level of, or greater level of, complexity than the existent data used to test the model (Goldstein, 2011).

1.2. Connectivist computational models

Parallels between the STAC-M and actual cognition draw from a biological framework. Within the connectivist framework, the mind is a product of the brain arising from a network of interconnecting neurons that process environmental data streams (Albus, 2010). As we move through and interact with our environment, the mind develops out of the brain as an emergent property of the interconnections and processes of the brain (Galotti, 2013). Within these interactions, the modularity of the mind and brain allows dedicated systems known as cognitive attributes to engage in information processing using selected data channels (Lamb, 2013; Zylberberg, Slezak, Roelfsema, Dehaene, & Sigman, 2010). It is in this processing of information that the innate releasing mechanisms trigger quantifies component parts of fixed behaviors when suitable antecedent stimulations are present (Arciniegas, 2013). Within human cognition, thought, behavior, and affect arise from the complex structured sequences and fragments assembling to achieve goals, such as completing a task in a science classroom (Duncan, 2013). Control of this complex cognition, such as student behaviors related to science task completion, and processing is computationally and symbolically addressed in such systems as an ANN. Extending from this representation of the ANN, the STAC-M is modeled within this connectivist framework.

The interconnection of the brain modeled via ANN at the functional level occurs with the physical action of artificial neurons (Pons, Cantero, Atienza, & Garcia-Ojalvo, 2010). It is the sequential firing and potentiation of artificial neuron activations that allow science information processing functions to occur (Lachaux, Axmacher, Mormann, Halgren, & Crone, 2012). To assist in the characterization of ANN, it may be instructive to draw upon the limited analogy between ANN and biological neural networks within the brain. The human brain is composed of approximately 5×10^9 neurons where the STAC-M only consists of seven artificial neurons (Azevedo et al., 2009). This provides a reference point when considering the complexity of the system the researchers are attempting to model. Within biological systems, neurons serve multiple purposes, however they can generally be classified as either signal propagation or as computational (information processing) components. In this way the ANN, provide similar classification of artificial neurons as signal propagation in the form of input nodes, and processing in the form of hidden nodes. Examination of the ANN in the form of a computational medium is essential to understanding the connectionism framework and the outputs of the STAC-M. It is the computation and signal propagation (connection) aspects of the ANN that provide for the emergent properties associated with the ANN models such as the STAC-M and similarly in biological networks (Hanrahan, 2011). When compared to a biological model at this level, the STAC-M most closely relates to a cognitive production system. A cognitive production system is a set of instructions consisting of conditions and actions held in short-term and long-term memory, accessed when appropriate antecedents (environmental cues and data streams) are applied, creating outcome behaviors in completing science based tasks (Wallach, Franklin, & Allen, 2010).

1.3. Evolutionary algorithms, STAC-M and the future

While the use of a Bayesian approach in combination with ANN provides a beginning to a promising future for educators, the application and development of this model requires further work, refining, and integration. One possible way to improve the model is through the addition of evolutionary algorithms for optimization. Incorporation of evolutionary algorithms (EA) into the STAC-M computational model would allow for future research and development of this model by focusing the processing of task completion away from extant cognitive pathways resulting in little or no success. Holzinger, Palade, Rabadan, and Holzinger (2014) suggest that using evolutionary algorithms will assist in this optimization by increasing the efficiency of interaction between attributes and assisting in the evaluation of individual attribute solutions revising and omitting pathways as needed to solve problems. Evaluation of individual solutions and fit to actual outcomes in classrooms would further drive optimization of STAC-M as a model of student cognitive processes. In this light, the inclusion of evolutionary (selective) algorithms can provide supervised learning for the underlying ANN creating additional research avenues.

1.4. Purpose, research question and hypothesis

The specific purpose of this study was to establish a computational model (STAC-M) allowing for the exploration of the complex cognition and behaviors and seen within a typical high school science classroom. A secondary purpose was to examine STAC-M through the computational model, the effectiveness of a curricular approach designed to train student cognition, specifically critical thinking tasks, framed under four Piagetian conservation tasks. The research questions and associated hypotheses addressed within this computational study are:

Research Question 1 (RQ1): Does the use of the STAC-M allow for the analysis and modeling of complex cognitive and behavioral processes seen within a science classroom?

Research Question 2 (RQ2): How does the use of a curriculum designed to increase levels of critical reasoning impact student science task completion as modeled with the STAC-M?

1.5. Consideration of the research questions results in the following hypotheses

Hypothesis 1 (H1). Using suitable data and theoretical framing it is possible to develop a computational model of student learning with adequate artificial neural network fit, thus allowing for the analysis of the complex system of learning within a science classroom.

Hypothesis 2 (H2). A curriculum designed to increase levels of cognition (critical reasoning) within the science classroom will result in increases in science-task completion as evident in the STAC-M outcomes.

The hypothesis that increases in critical reasoning will result in greater task completion would result in support of the data channel view of cognition and assist in validation of attribution retraining approaches within science education. Within the “data channel view of cognition,” an adjustment to the efficiency in the use of specific cognitive channels through experiential learning will result in greater processing power through increases in bandwidth and neural recruitment. This increase in processing power derives from recruitment of multiple sub-component cognitive processes within critical reasoning possibly acting as a gating attribute.

2. Methods

The STAC-M itself was developed and trained using data derived from student play in a science based Serious Educational Game. Prior to model creation, individual in-game tasks were identified and linked to cognitive attributes based on their characteristics. Learner probability of success for each task was parameterized using a two-parameter Item Response Theory model, with all tasks for each learner developed into an attribute mastery pattern through the use of a Q-Matrix, and the final mastery patterns used to train the ANN. The process for a population level STAC-M mirrored that of the individual models, with population probabilities of success integrated into the IRT stage of the model.

2.1. Data description

The unit of analysis for this study is the subjects' actions and behaviors towards task completion ($n = 158,000$) while playing the SEG. However, for the purposes of clarity, it may be illustrative to identify the human sample characteristics. The target population in the study ($n = 645$) was subjects located in the mid-Atlantic region of the United States. Targeted subjects consist of students enrolled in a full-time traditional high school science program at grade 9–12 levels. Subjects' ages ranged from 14 to 18. Subjects within the study have taken a science class within the last semester. Science classes considered in this study are Earth Science, Biology, Chemistry, or Physics. Criteria for selecting subjects included: (1) taking their current science class for the first time; (2) taking the course as a member of a class and not in an online or virtual capacity; (3) being admitted into the class within the first two weeks of class. This study derived the data set from a preexisting data set using the target population description as means to screen data points. The sample characteristics and by extension IRT-True Score Parameters limit the ANN to modeling student cognition related to these characteristics.

This study used a proportionate stratified sampling approach of science students to generate the computer log data later developed via psychometric analysis for presentation of training data to the ANN. The sample size of each stratum was proportionate to the population size within the school district of interest. This particular sampling technique provides a higher statistical precision compared to simple random sampling and allows for a smaller sample size. In addition, this sampling technique increases the probability of inclusion of specific sub-groups within the sample (Wallander, 2009). Stratum parameters for the stratified sample are grade, gender, and science class level. Due to the sequential analysis of this study, selection of results within each phase results in aggregation of group results to reduce the number of dimensions for analysis.

This population is of interest within this study is due to the increased perception that exposure to Science, Technology, Engineering, and Mathematics (STEM) rich environments increases the likelihood that subjects' select STEM discipline based majors in college and STEM careers after college (Lamb & Annetta, 2009, 2012a, 2012b; Lamb, Annetta, Meldrum, & Vallett, 2011). The National Mathematics Advisory Panel's and the National Science Foundation's assessment of the United States' standing in STEM disciplines is in jeopardy, and will lose its status and place within the early 21st century without increases in outputs from the STEM pipeline (Annetta, 2008).

2.2. Tasks

The novel tasks presented to the STAC-M are modifications of the Piagetian conservation tasks (Piaget, 1970). Within an SEG environment, students were provided with tasks similar to, but not exactly the same, as the conservation tasks presented to the ANN within the context of an investigation of physical properties. The major underlying concepts analyzed are the students' cognition related to physical properties such as how volume is conserved. Each of the conservation tasks are typically mastered between the ages of 5 and 10 with the volume conservation task being the most difficult to master. This task is usually mastered between the ages of 8 and 10.

2.3. Identification of cognitive attributes and training data

Critical reasoning processing is a complex cognitive function and part of a domain with many dynamic components. In light of this, researchers must take care to ensure that modeled tasks are not overly complex and isolate the specific cognitive attributes the researchers are interested in studying. Within a study such as this one, an overly complex task activates other attributes, confounding the identification of studied attributes. While it is computationally possible to model all process attribute types given sufficient computer processing power, it

is not within the purview of this study to do so. The goal within this study is to select tasks that would reflect some of the complexity of the classroom specifically related to student learning and information processing, with the hope of providing generalizable results related to student critical thinking. An artificial intelligence model (STAC-M) was trained with student data obtained from science based Serious Educational Game (SEG) in which students completed conservation type tasks. The use of the SEG provides a means to specifically control environmental stimulus and track all actions the students took while playing. Limiting of task selection occurs intrinsically to the game as tasks already situated within the SEG pertain to science processing critical reasoning. Selected tasks, which meet these criteria, are those that are similar to conservation tasks. In particular, the identified tasks used in this study are similar to the Piagetian Volume Conservation Task, Mass Conservation Task, Liquid Conservation Task, and the Number Conservation Task. Student data were extracted from the server and developed using Item Response Theory (IRT) response pattern analysis. To successfully generate a model and balance the computational concerns with useable tasks, considerable care in task selection is required. During this study, a panel of experts validated core identification of cognitive attributes related to this task. The theoretical attributes were assigned to the tasks and then response patterns related to the students were randomly assigned to the training set to develop the model or test set to test the model. This ensured that there is no crossover between testing sets and training sets. Encoding of the attributes' parameters occurred via the development of the Q-matrix.

2.4. Combining Bayesian models and IRT models

Parameterization of the task completion likelihoods and the use of ANN training models as in this study, assist in the development of more effective targeting of tasks to specific attributes. The probabilities of success developed using the IRT model integrates into the attribute mastery pattern via the Q-matrix. The use of these particular models helps to develop an effective picture (model) of individual subject cognitive processes for simulation and teaching purposes. Effective ANN development provides researchers with an effective means to simulate the mechanisms of learning and test changes within those mechanisms of learning. To create input vectors for the science process tasks (critical reasoning) it is necessary to encode probabilities of successful completion of the task items. Transformation of initial response patterns occurred using an IRT model, specifically the 2PLM model. The study used two-parameter logistic model parameters to compute the population probability using the IRT True-Score method (Dimitrov, 2007). Based upon the results of the IRT True-Score parameterization, task probabilities for the population (high school students) viewed as individual probabilities assigned to cognitive attributes using a Q-matrix. The Q-matrix probabilities are further refined through application of the artificial neural network propagation (potentiation) weightings. Node coding developed using one input node per task actions by flagging the node via a "0" or "1" indicating the presence or absence of the attribute. This type of coding provides a simpler model allowing the ANN to learn the input parameters more efficiently (Bhatt, 2012). Folding all values of the parameters into one node and all constants into another node is a way to represent and account for prior knowledge allowing for the creation of Bayesian estimates (Bauer, Gagneur, & Robinson, 2010). The accounting for prior knowledge within the ANN model STAC-M is a key feature of Bayesian models c.f. (Chib, 2008), which are not present in IRT models (Soares, 2009). Lamb (2013) integrated the IRT model within an ANN model. Since propagation across the network is contingent upon the presence or absence of the cognitive attribute, the attribute values used to determine success are not of consequence to the solution. Thus, the solution to the task does not affect signal propagation across the ANN only the probability of success. Coding input vectors (tasks and attributes) in this manner permits the coding of a large number of examples of science process tasks, which preserve individual identities of the tasks. Due to the potential for a larger number of coded tasks, the STAC-M model becomes more flexible and generalizable as the number of parameters increases. This flexibility allows the use of the STAC-M in novel task presentations using the same cognitive attributes such as the Piagetian Tasks. The use of the same cognitive attributes increases the possibility of measuring transfer through changes in the system dynamics. This also reduces the likelihood that review bias concerning relative importance of one attribute versus the others affected the results in a meaningful way.

A second area of strength, thus indicating the mixing of IRT and Bayesian models as superior, is in the number of attributes required for analysis. Model convergence using an ANN occurs with a smaller number of attribute to item-task ratio as opposed to traditional cognitive diagnostics. Researchers using this modified form of cognitive diagnostics can obtain usable results with fewer suggested attributes resulting in easier interpretation for practitioners (Lamb, 2013; Lamb et al., 2014). Fewer attributes make it more likely that the educator can successfully target those attributes during instruction in a timely manner (Huff & Goodman, 2007). The use of fewer attributes, in-turn, can help to increase interpretability of the data and targeting of content at the classroom level (Roberts & Gierl, 2010). Through a combination of the Bayesian and IRT models researchers are able to capitalize on strengths of each while accounting for weaknesses creating increased reliability for model outcome probabilities.

2.5. Analysis and modeling

Psychometric analysis of subject responses to the task sets within the SEG environment provides the basis of the training data for the STAC-M. This phase involved the use of item response theory to validate the assessment, model fit, item constructs, and item functioning along with population and sample probabilities. In this case, the assessment is the successful completion of tasks associated with tasks related to the conservation tasks within SEGs. Parameterization of the probabilities occurred by calculating the odds of task completion, using a "1" for success and "0" for failure to complete the task. The log-odds of completion provided the information pertaining to the probability of successfully completing conservation tasks. Specifically, the parameterization encompasses a comparison of expected completion, versus actual task completion, measured through χ^2 . Item (task) fit analysis is conducted using a two-parameter model (2PLM) (a and b) with resulting infit and outfit statistics providing model fit information.

The results of phase 1 and 2 inform phase 3, and the development of the Attribute Mastery Pattern (AMP) in the form of a Q-Matrix which is presented to the ANN. Development of the AMP involved identification and validation of the cognitive constructs that underlie the task items via expert review. These phases result in the creation of expected response patterns and attribute probabilities. The study presented the results from this phase to an artificial neural network in the form of test data to establish attribute hierarchy via propagation weightings and model fit, thus validating the STAC-M prior to the simulated curriculum intervention. Post STAC-M development of, the initial weightings associated with the data were recorded and an adjustment of 7% increase to weightings corresponding to the critical reasoning

node were made thus simulating a science based critical reasoning intervention within the classroom. Specifically, the modeled intervention used an attribution retraining curriculum approach. This step essentially allows the authors and other educational researchers to test novel curriculums using ANN simulations to ascertain potential outcomes. This is akin to pre-clinical simulation trials within the medial field. Fig. 1 summarizes the development of the STAC-M and the subsequent simulated intervention.

3. Results

3.1. Attribute identification

The population parameters for the tested conservation tasks are provided in Table 1. Table 1 displays the population proportion of correct response on item i (π), the population estimate of the item error variance $\sigma^2(e_i)$, the population estimate of the item true variance $\sigma^2(\tau_i)$, and the population estimate of item reliability ρ_{ii} . Review of Table 1 provides the overall descriptive statistics for the combined test tasks as $P_1 = 0.366$, $\text{VAR}(e_i) = \sigma^2 e = 4.21$, $\text{VAR}(\tau_i) = \sigma^2 \tau = 51.05$, $RO_{xx} = \rho_{xx} = 0.95$. These results suggest the test population reliability parameter is high ($\rho_{xx} = 0.95$). However, the overall difficulty of the test is moderate with 36.6% of the population correctly completing all tasks. Of the total items included in the final analysis, volume conservation is the most difficult while number conservation is the easiest task to complete. Items, showing difficulty over ± 2 on Table 1, would have resulted in removal due to poor 2PLM model fit. The volume conservation task is the most reliable at $\rho_{ii} = .64$ while the mass conservation task is the least reliable $\rho_{ii} = .50$. It is important to note these data represent the properties of tasks prior to the simulated curricular intervention.

Fit statistics for the 2PLM model suggest adequate model fit for the data ($X^2 = 1.70$, $df = 1$, $p = 0.19$). Estimation of reliability for the measured constructs used the Latent Trait Reliability Method (LTRM) (Dimitrov, 2012; Raykov, 2009; Raykov, Dimitrov, & Asparouhov, 2010). This method (LTRM) provides superior estimation of internal reliability as it does not rely upon the assumptions associated with more common reliability methods such as Cronbach's alpha. More specifically, Cronbach's alpha requires essential tau equivalence and no correlated errors. Within the framework for latent variable modeling, score reliability developed as the ratio of the true-score variance to the observed variance (Dimitrov, 2012). Mplus code for calculation of LTRM is available by contacting the author directly. The reliability of the measured constructs is estimated at $REL = 0.73$, CI 5% [0.71–0.80], SEM 2.10, CI 5% [2.07–2.47]. The computed level of reliability is adequate for this type of measure.

Psychological constructs such as the relationship between attributes and tasks via a Q-matrix, in a computational study such as this one, are reflective of measure performance on tasks during individual learning. Construct validity is the degree to which a scale measures the proposed trait it is thought to measure; when such a test measures a trait, which is difficult to define, such as in a cognitive diagnostic measure, multiple expert reviewers may rate individual pairings of attributes with tasks. Table 2 provides an overview of reviewer agreement related to the relevance of each attribute to each task.

Analysis of reviewer agreement of relevance suggests a task-attribute validity coefficient of .80. This level of task-attribute validity is adequate for an exploratory study such as this one. Equation (1) is a calculation of task-attribute validity using the coefficient of agreement d .

Equation (1) Agreement Coefficient Calculation.

$$d = I_D / \sum_{l=1}^n I_{A-D} \quad (1)$$

A discrete latent attribute model was used to develop an understanding of the place of each cognitive attribute within the current model. This model allows for the modeling of cognitive weighting—via artificial neural network propagation weights—and for inferences about the hierarchical position of the cognitive attributes of the subjects. Within the models, the latent variables conceptualize as a vector of 0s and 1s for each subject. Zero indicates the absence of the trait and 1 indicates the presence of the trait. Table 3 illustrates the hypothetical attributes needed to complete the corresponding tasks. More specifically to describe the model one can draw upon a similar model developed by Tatsuoka (1983), where N examines and J binary task performances variables combine. A fixed set of K cognitive attributes are involved in performing the tasks. Thus, one can understand model parameters in the terms below.

$X_{ij} = 1$ or 0, indicating whether examinee i performed task j correctly;

$Q_{jk} = 1$ or 0 indicating whether attribute k is relevant to task j ; and

$\alpha_{ik} = 1$ or 0, indicating whether examinee i possesses attribute k .

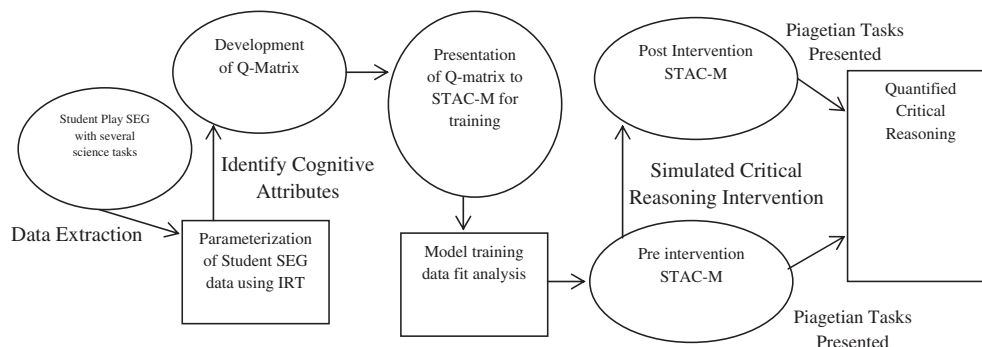


Fig. 1. Study flow chart.

Table 1
Population parameters and psychometric descriptions of each task presented to the ANN.

Task description	Discrimination (a)	Difficulty (b)	π_i	var (e_i)	var (τ_i)	ρ_{ii}
Volume Conservation	0.78	1.98	.17	.12	.03	.64
Liquid Conservation	0.96	1.01	.35	.08	.04	.57
Number Conservation	0.36	0.45	.45	.08	.07	.53
Mass Conservation	0.56	0.63	.33	.08	.06	.50

Note. $\pi_i = 0.366$, $\text{VAR}(e) = 4.212$, $\text{VAR}(\tau) = 51.051$, $P = 0.95$.

Table 2
Review agreement as the relevance of each attribute to each task.

Reviewer 2			
Reviewer 1	Weakly Relevant		Strongly Relevant
	Weakly Relevant	Strongly Relevant	1,2,3,4

Note. Numbers correspond to individual tasks. Letters corrections to relevance grouping; A: WR \times WR, B: WR \times SR, C: SR \times WR and D: SR \times SR.

Analysis of the Q-matrix assists with the standardization of the outputs by fixing term Q_{jk} to 1 prior to insertion into the matrix. The underlying reasoning for fixing term Q_{jk} equal to 1 is similar to the logic associated with the Linear Logistic Test Model (LLTM). From this development, it is important to understand the objective is to infer about the latent cognitive attributes developed via the artificial neural network model weightings. This is not to suggest the examinees do or do not possess traits, but to aggregate the attributes along with suitable tasks to measure them. Note that the matrices are developed out of statistical estimations associated with task response parameters under a 2PLM. Table 3 also displays the odds of successful completion of the task along with the contribution of each attribute to overall task complete. The existence of residual unaccounted for variance is suggestive of the need to include additional attributes. This was not done to concerns related to computational power associated with the computer platform used in this study. Table 3 illustrates the binary Q-matrix.

3.2. Artificial Neural Network

The Artificial Neural Network developed to describe the interconnection between the cognitive attributes and successful task completion arises from a series of interconnected nodes (neurons). The nodes develop the three distinct layers of the ANN- input, hidden, and output. The input layer provides no computational function but distribute stimulus into the neural network. For the purposes of this model, the tasks act as the input. The hidden layer represented by the cognitive attributes assigned to the tasks and the output layer consists of the success and failure probabilities. The hierarchical natures of the weightings found within the ANN are suggestive of a gating attribute.

The ANN used within the portion of the study was designed using SAS as part of the JMP 11 statistical discovery package. Training of the artificial neural network used a random 1/2 n split data approach similar to the validation method for cross validations of data structures with large data sets. Link weights initially consist of randomly weighted values. The weights are limited to random values within the range of $-2/\Omega$, $2/\Omega$ for neurons with Ω inputs (Gallant, 1993). Range limitations ensure the propagations potentials do not become too large and result in selection of one dominating cognitive attribute during model pruning. Post initialization of the network using the random weighting approach, the network was then trained by providing the STAC-M a number of examples from the 1/2 N data set (1/2 N = 77,120) illustrating how the ANN is to behave. This sets the conditions for the computational experiment with the intervention curriculum by establishing the base line behaviors and cognition activation patterns. This also allows the pre-intervention STAC-M to act as a true control group for the purposes of this study.

Review of the results of the trained ANN with the calibrations set suggests an accurate behavioral predictor of subject success outcomes based on the cognitive attributes supplied. Table 4 and Table 5 provide key statistics regarding model fit. The training set shows a .82 and .64 r^2 for the prediction of correctly completing the tasks and incorrectly completing the task. These r^2 values suggest that the ANN model accounts for 82% and 64% of the variance around the sigmoid function used to develop the outputs. The generalized r^2 proves for the aggregation of the predictive ability of the network across the multiple outputs of *correct* and *incorrect*.

Review of Tables 4 and 5 indicates the ANN model used for the test set is less able to predict the output states, *correct* or *incorrect* task completion ($\Delta r^2 = -0.18$). Despite some loss in predictive power associated with the model, there is not a statistically significant difference in the r^2 values ($t(2) = 1.55$, $p = 0.250$, $\alpha = 0.05$). Given the lack of significance for chi-square, the model adequately predicts subject outcomes modeling student cognitive approaches. When tested using the second set of data 1/2n, the model is able to account for 77% of the variance for *correct* outcomes and 69% of the *incorrect* outcomes. Examination of the ΔRMSE (+0.03) term, there is a slight increase in the error term, however this is not considered significant ($t(2) = 1.32$, $p = 0.29$, $\alpha = 0.05$). Review of the correlation coefficient $r = 0.84$ suggests there is a strong linear relationship between the models testing for each task. In an effort to reduce misspecification errors, the author of this paper chose to include a penalty for overfit error at a rate of .05.

Table 3
Probability of task success and the contribution to that success to each cognitive attribute.

Task	Proportion of success	Cognitive Attribute 1	Contribution to p_i	Cognitive Attribute 2	Contribution to p_i	Cognitive Attribute 3	Contribution to p_i
Volume Conservation	.37	Parity Judgment	.03	Critical Reasoning	.48	Retrieval	.13
Liquid Conservation	.49	Parity Judgment	.09	Critical Reasoning	.23	Retrieval	.11
Number Conservation	.53	Parity Judgment	.01	Critical Reasoning	.13	Retrieval	.04
Mass Conservation	.48	Parity Judgment	.07	Critical Reasoning	.21	Retrieval	.09

Table 4
Neural network output (training set, .5 Holdback validations).

Neural network	Correct	Incorrect
R-square	0.84	0.73
RMSE	0.19	0.11
Mean Abs Error	0.12	0.08
Generalized R-Square	0.82	

Table 5
Neural network output (test set, 0.5 Holdback validations).

Neural network	Correct	Incorrect	Average change from training
R-square	0.79	0.63	−0.05
RMSE	0.20	0.23	+0.01
Mean Abs Error	0.13	0.17	+0.01
Generalized R-Square	0.64		
Correlation Coefficient:	0.84		

An Artificial Neural Network derives propagation weights from random assignment to test set data ($1/2n$) for each of the proposed attributes. The weights represent the strength of cognitive activation conceptualized by propagation as the signal moves from node to node within the network. For clarity, the study has standardized STAC-M weights to 1.00, with each subsequent weighting value developed from the standardized value allowing for increased interpretability under Bayesian estimation. Table 6 illustrates the pre- and post-intervention changes in critical reasoning within the simulated classroom trial for a curriculum designed to target increases in this specific cognitive attribute.

Analysis of changes made to the neural network representing student cognition related to each of the tasks. The greatest increase in success is seen in the volume conservation task with a 10% increase. The least increase in success is seen in the number conservation task with only a 3% increase in success. This is mostly due to the contribution of critical reasoning to each task.

4. Discussion

The primary purpose of this study was to develop a computational model of student learning allowing for modeling of complex dynamic systems within the science classroom. A secondary purpose of this study was to computationally examine the effects of cognition-based interventions designed to train critical reasoning in a science classroom.

The results illustrate the authors have developed a suitable ANN model of student cognition relating to science learning which provides usable simulation data for educators. The STAC-M provides a view of the complex processes of learning and sets the conditions to develop further computational models of education. Emergent factors developed from the psychometric analysis of the four tasks using cognitive diagnostics suggests that critical reasoning is a key component in each of the tasks and provides the greatest contribution to task completion. Thus, it naturally follows that interventions designed to retrain attributes, in this case critical reasoning, would provide for the greatest student increases in task completion. From a substantive point of view, this suggests that critical reasoning would play a pivotal role in completing the tasks, as it is foundational to making judgments and drawing conclusions from presented environmental cues. Based upon this foundational role, critical reasoning would seem to be a gating attribute for other attributes further down the hierarchy. Support for this view is established via the weights seen in the STAC-M, meaning, as one is able to successfully train critical reasoning, this attribute may be able to compensate for lesser-accessed attributes thus increasing task success. More to this point, this compensation may occur through increased channel bandwidth for data-streams within the environment and parsing of data streams into recruited attributes further downstream. With the analysis of the STAC-M weightings suggesting this hierarchical relationship, future iterations of the STAC-M model of student cognition should insert this gating mechanism to test this data channel view of cognition. The gating mechanism can provide a potential testable hypothesis for testing temporal differences in activation of critical reasoning regions seen within fMRI studies. Data channels can be modulated via propagate/non-propagate firing of the hidden layers in a multi-layered version of the STAC-M. This will create differentiated patterns of activation with the most successful pathways achieving the highest levels of task success. More precisely, future research can test the role of cognitive attribute gating as a means to mediate the activation of other downstream attributes. Imposition of structures for testing provides a means to establish other relationships between these cognitive components. This approach may provide visualization of mechanisms of action for which seemingly similar educational interventions do not achieve the same results. This means that despite similarities, the interventions may not arouse cognition and, by extension of the Tripartic model of learning, affect and behavior the same way, resulting in differences in data parsing which create differential outcomes related to student success helping to explain individual and contextual differences (Lamb, *in press*). This difference can result due to environmental stimulus cueing in activation of differing cognitive attributes increasing cognitive load and decreasing efficiency in data stream processing.

Table 6
ANN changes in success due to the curricular intervention.

Task	Critical reasoning contribution (pre-intervention)	Critical reasoning contribution (post-intervention)	Change in success
Volume Conservation	.48	.55	+.10
Liquid Conservation	.23	.30	+.06
Number Conservation	.13	.20	+.03
Mass Conservation	.21	.28	+.05

Note: Each task was simulated 100,000 times representing an intervention study of $n = 100,000$.

Modulation of attributes and related affect and behavior may allow one to manipulate variables to understand individual differences in education at a deeper level governed by psychological mechanisms. One important aspect of this research is that individual neurons within the STAC-M are representative of regions of the brain associated with the various cognitive attributes, not individual neurons themselves. Hence, though there is one individual neuron within the hidden layer identified as critical thinking it is actually representative of the regions of the brain associated with the striatum regulated (gated) via the frontal cortex. Recruitment of additional processing centers, via this gated mechanism, allows for an increase in the number of data channels and increasing processing power when presented with difficult problems. This can lead to an assumption within computational modeling that simply adding neurons to the hidden layer will create more outcomes that are positive. However, this assumption becomes problematic because as one increases computational components associated with STAC-M this can lead to overfit errors. The additions of the correct number of computational neurons related to processing also supports the link between affect, cognition, and behavior as portions of the striatum (represented in the STAC-M as a critical thinking neuron) are associated with motivation and behavior in addition to critical reasoning and related attributes.

The computational cognitive model in the form of the STAC-M assists in obtaining information related to science based curriculum and offers additional data related to student learning. The STAC-M exhibits good data fit and approximates human learning related to completion of science related conservation tasks. The STAC-M provides a means to establish the linkage between cognition, affect, and behavior, as a good science curriculum will allow for the interaction of all three of these components. Given the complexity and limited (though important) contribution of critical thinking to science tasks, we should not expect that one method of intervention as suggested here will prove sufficient for developing each component part. While it is possible to teach critical thinking as an isolated skill, it seems best developed when connected to specific domains of knowledge. Limited transfer is seen within the model but is present when examining the success probabilities associated with the different tasks under the same cognitive attributes. Using cognitive attributes as the common currency of learning, one can truly integrate curriculum from multiple related fields of study and in some cases from unrelated curriculum using the same cognitive attributes.

One additional advantage of the STAC-M, and more generally a computational approach, is it is possible to easily establish a true randomized control design for investigation. This design is often seen as a gold standard for experimental research but is quite difficult to obtain in the educational setting. Using the STAC-M, researchers can generate comparisons not otherwise possible due to concerns related to access, funding, and other areas that impede research.

4.1. Limitations

Computational modeling of real world complex systems has increased in recent years with increases in computing power. Computational simulations of the complex factors associated with cognition have become increasingly important in decision-making and pre-analysis of interventions. The successes of these models are often heralded as a means to answer questions using fewer resources; however a model is but a representation of reality. By its very nature it is a simplification and abstraction with nothing more than assumptions and judgments of its creators. More importantly, it is not a physical model but a series of equations consisting of two components: parameters and variables. Within this framework it is important to understand that an initial limitation within a computational model is the judgments necessary to decide what variables should and should not be included within the model. As information about the real-world systems related to cognition becomes less precise, it becomes increasingly difficult to approximate interaction—and, by extension, reduces the precision of outcomes. More explicitly, to state key limitations of computation study models, one has to make two assumptions limiting their use. First, all models are incomplete; and second, models assume that the future outcomes will be similar to past outcomes. This is not to say that the models are useless, as numerous fields have made use of computational models to draw conclusions and direct future research. As with any advancement in a field, modeling is one tool in a series of tools to help us answer questions related to education.

5. Conclusion

The STAC-M demonstrates and models the powerful learning abilities associated with complex and dynamic cognitive systems of students inside of a 9–12 science classroom. The network has been tested on four tasks with simulated curriculum with excellent results. This study provides for future use as a means to analyze curriculum and, more importantly, test the curriculum prior to implementation within the schools. This may be the first time the complex components of attributional retraining, via curricular intervention in science-based learning, have been demonstrated via dynamic computational modeling. In this context, the model acts as a computer representation of cognitive systems giving educational researchers unprecedented access to simulations for experimental purposes. This was demonstrated through a simulation allowing for direct targeting of cognitive attributes within the classroom and testing with novel yet related tasks. This model may also allow for analysis of transfer effects between science-based tasks by converting tasks within science to the common currency of cognitive attributes and giving researchers tools to directly compare those outcomes. The current model (and other related models of student cognition) may allow for increased use of models to test curricular ideas prior to implementation within the classroom. Current models of classroom interaction and student learning focus on relatively simple single-outcome measures of content knowledge and do little to discuss underlying cognition or probing of deeper psychology or learning. Models such as STAC-M allow for testing of tens of thousands of students as well as to probe more deeply into their thinking with little to no cost to time and resources associated with the school systems.

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