

# ML Problem Solving

## Least squares

1. Consider the following dataset, where each row is one input and the measured output is the last column.

$D = [[6, 5, 1], [5, 6, 0], [4, 3, 1], [3, 4, 0], [2, 3, 0], [3, 2, 1]]$

Draw the data points from  $X$  in a coordinate system.

2. Now let's try to manually choose  $\beta \in \mathbb{R}^3$  to minimize the Residual Sum of Squares (RSS). Choose at least 3 different values for  $\beta$  and calculate the associated RSS.

$$\beta_a = [ \text{ \_\_\_\_\_\_ } , \text{ \_\_\_\_\_\_ } , \text{ \_\_\_\_\_\_ } ]$$

$$\beta_b = [ \text{ \_\_\_\_\_\_ } , \text{ \_\_\_\_\_\_ } , \text{ \_\_\_\_\_\_ } ]$$

$$\beta_c = [ \text{ \_\_\_\_\_\_ } , \text{ \_\_\_\_\_\_ } , \text{ \_\_\_\_\_\_ } ]$$

3. Draw the decision boundaries for each model defined by your  $\beta$ . Do it for all three values you chose.

4. Calculate  $\beta = (X^T X)^{-1} X^T y$

5. Considering that  $RSS(\beta) = (y - X\beta)^T (y - X\beta)$ , derive a closed-form  $\beta$  that minimizes  $RSS$ . (optional)