

$$(a, b) = 1$$

$$\varphi \text{ é multiplicativa } (a, b) = 1 \Rightarrow \varphi(ab) = \varphi(a)\varphi(b)$$

$$\text{Suponha } (a, b) = 1$$

$$1 \quad 2 \quad 3 \quad \dots \quad m-1 \quad m$$

$$m+1 \quad m+2 \quad m+3 \dots \quad m+(m-1) \quad 2m$$

$$(m-1)(m+1) (m-1)(m+2) \quad m \cdot n$$

Exigência de Euclides: o que conta de 2  
possíveis

$$\text{Euclid } (a, b)$$



$$(p-1)! \equiv -1 \pmod{p} \stackrel{?}{\iff} p \text{ primo}$$

$$p \text{ primo} \Rightarrow (p-1)! \equiv -1 \pmod{p}$$

Suponha  ~~$(p-1)! \equiv -1 \pmod{p}$~~   $p \text{ primo}$

temos que  $(p-1)! = (p-2)! \cdot (p-1)$

Logo  $(p-1)! \equiv (p-2)! \cdot (p-1) \pmod{p}$

$$\text{Voltando um passo } [(p-2)!]_p \stackrel{?}{=} 1$$

$$\begin{aligned} \text{Logo } (p-1)! &\equiv (p-1) \pmod{p} \\ &\equiv (-1) \pmod{p} \end{aligned}$$

$$(p-1)! \equiv -1 \pmod{p} \stackrel{?}{\iff} p \text{ primo}$$

Suponha  $(n-1)! \equiv -1 \pmod{n}$

~~Logo~~



$$a^p \equiv a \pmod{p} \Rightarrow p \text{ primo}$$

$$2^{341} \equiv 2 \pmod{341}$$

$$341 = 11 \cdot 31$$

Logo 341 composto

$$2^{561} \equiv 2 \pmod{561}$$

$$561 = 11 \cdot 3 \cdot 17$$

$$(a, bc) = 1$$

$$(a, bc) = 1 \xrightarrow{?} (a, b) = 1 \text{ \& } (a, c) = 1$$

$$(a, bc) = 1 \Rightarrow$$

Suponha  $(a, bc) = 1$

$$(a, bc) = (a, b) \cdot (a, c)$$

$$(a, b) = 1 \text{ \& } (a, c) = 1 \Rightarrow (a, bc) = 1$$

$$(a, b) = 1$$

$$(a, c) = 1$$

Suponha  $(a, b) = 1 \text{ \& } (a, c) = 1$

Divisão  
de Euclides

$$(a, m+r)$$

$$(a, m+r, m) \stackrel{!}{=} (r, m)$$

$$a = qm + r \quad (a, m) = (r, m)$$